



## HKCEE Additional Mathematics



| Session 1 | Quadratic Functions and Equations (Part 3/3) |
|-----------|--|
| 1.        | Use of the Absolute Value Sign               |
|           |  |
|           |  |
|           |  |
|           |  |
|           |  |



見 Absolute, 拆 Absolute

⇒ 一定要分 Cases !!!



## **HKCEE Additional Mathematics Syllabus**

### **Section 1: Quadratic Functions and Quadratic Equations**

Section 1: Quadratic Functions and Quadratic Equations

1.3 Use of absolute value sign

Excluding its use in relation to inequalities.

## 1. Definition of Absolute Value Signs

### Definition of Absolute Value Signs

黎 Sir 提提你 :

1.  $|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a & \text{if } a \leq 0 \end{cases}$  e.g.  $|7| = 7$  but  $|-7| = -(-7) = 7$

2. 正負數處理方法不同

⇒ 一定要分 Cases !!!

## Properties of Absolute Value Signs

黎 Sir 提提你 :

### 1. There are 7 properties in Absolute Value Signs

i.  $|x| \geq 0$

ii.  $|x| = |-x|$

iii.  $|xy| = |x||y|$

iv.  $\left| \frac{x}{y} \right| = \frac{|x|}{|y|}$  where  $y \neq 0$

v.  $|x^2| = x^2 = |x|^2$

vi. If  $a \geq 0$ , then  $|x| = a$  means  $x = a$  or  $x = -a$

If  $a < 0$ , then  $|x| = a$  means has no solution

vii.  $|x| = |y| \Rightarrow x = y$  or  $x = -y$

### 2. However, 分 Cases is the most important concept!!!

## 2. Absolute Value Signs in Linear Equations

### “分 Cases” in “Absolute Signs”

黎 Sir 提提你 :

#### 1. 幾時分 Cases?

⇒

e.g.  $|x-1| = 2$ ,  $|x-1| + |2x-1| = 2$ .

#### 2. 分幾多 Cases?

⇒ 一刀兩段! 兩刀三段! 三刀四段!

e.g.  $|x-1| = 2$ ,  $|x-1| + |2x-1| = 2$ ,  $|x+1| + |x-1| + |x+3| = 4$ .

#### 3. 最後, 要 “or” 晒所有 Cases 既 Answers!!!

e.g.  $x=2$  or  $x=1$  or  $x=3$

**Absolute Signs in linear Graphs**

黎 Sir 提提你 :

1.  $y = f(x) \Rightarrow y = |f(x)|$

2.  $y = f(x) \Rightarrow y = f(|x|)$

### 3. Absolute Value Signs in Quadratic Equations

#### Absolute Signs in Quadratic Equations

黎 Sir 提提你 :

##### 1. 分 Cases + Factorization!

e.g.  $|x-3| = |x^2 - 4x + 3|$ ,  $x|x| + 5x + 6 = 0$ ,  $|x^2 - 4| = 4 - x^2$

##### 2. 同 Sum of roots, Product of roots 一齊考

⇒ 有可能 take square! (照樣要分 Cases!)

**Absolute Signs in Quadratic Graphs****黎 Sir 提提你 :**

**1.**  $y = f(x) \Rightarrow y = |f(x)|$

**2.**  $y = f(x) \Rightarrow y = f(|x|)$

**3.**  $y = ax^2 + bx + c \Rightarrow y = ax|x| + bx + c$

**4.**  $y = ax^2 + bx + c \Rightarrow y = ax^2 + b|x| + c$

**5.**  $y = ax^2 + bx + c \Rightarrow y = ax|x| + b|x| + c$

4. Exam Types Questions 考試題種:

黎 Sir 提提你 :

1. Solving linear equations with absolute signs
2. Solving quadratic equations with a absolute signs
3. Transformation of graphs with absolute signs

**1. Solving linear equations with absolute signs**

1.

**Solve**  $|x - 1| = 2$ 

(3 marks)

黎 Sir 提提你 :

**Method 1:**

$$|x - 1| = 2$$

$$x - 1 = +2 \quad \text{or} \quad x - 1 = -2$$

$$x = +2 + 1 \quad \text{or} \quad x = -2 + 1$$

$$x = 3 \quad \text{or} \quad x = -1$$

$$\therefore x = 3 \quad \text{or} \quad x = -1$$

**Method 2:**

$$|x - 1| = 2$$

$$\text{Case 1: } x \geq 1$$

$$x - 1 = 2$$

$$x = 2 + 1$$

$$x = 3$$

$$\therefore x = 3 \quad \text{or} \quad x = -1$$

$$\text{Case 2: } x < 1$$

$$-(x - 1) = 2$$

$$-x + 1 = 2$$

$$x = -1$$

2.

**Solve**  $|x - 1| = 0$ 

(3 marks)

黎 Sir 提提你 :

**Method 1:**

$$|x - 1| = 0$$

$$x - 1 = 0 \quad \text{or} \quad x - 1 = -0$$

$$x = 1 \quad \text{or} \quad x = 1$$

$$\therefore x = 1$$

**Method 2:**

$$|x - 1| = 0$$

$$\text{Case 1: } x \geq 1$$

$$x - 1 = 0$$

$$x = 1$$

$$\therefore x = 1$$

$$\text{Case 2: } x < 1$$

$$-(x - 1) = 0$$

$$-x + 1 = 0$$

$$x = 1$$

3.

Solve  $|x-1| = -1$

(3 marks)

 黎 Sir 提提你 : ∵ Absolute Value ≥ 0!!!

**Method 1:**

$$|x-1| = -1$$

$$x-1 = -1 \text{ or } x-1 = -(-1)$$

$$x = 0 \quad \text{or} \quad x = 2$$

∴  $x = 0$  ? or  $x = 2$  ? (Contradiction!)

**Method 2:**

$$|x-1| = -1$$

$$\text{Case 1: } x \geq 1$$

$$x-1 = -1$$

$$x = 0$$

$$\text{Case 2: } x < 1$$

$$-(x-1) = -1$$

$$-x+1 = -1$$

$$x = 2$$

∴  $x = 0$  ? or  $x = 2$  ? (Contradiction!)

**Method 3:**

∴ Absolute Value ≥ 0

∴ No Solution

4.

a. Solve  $|1-x| = 2$ .b. By considering the cases  $x \leq 1$  and  $x > 1$ , or otherwise, solve  $|1-x| = x-1$ .

(5 marks)

黎 Sir 提提你  : 分 Cases 才是王道!!!

a.

| <b>Method 1:</b>   | <b>Method 2:</b>  |
|--|---|
| $ 1-x  = 2$<br>$1-x = +2 \quad \text{or} \quad 1-x = -2$<br>$x = -1 \quad \text{or} \quad x = +3$<br>$\therefore x = -1 \quad \text{or} \quad x = 3$ | $ 1-x  = 2$<br><b>Case 1:</b> $x \leq 1$ <b>Case 2:</b> $x > 1$<br>$1-x = 2$ $-(1-x) = 2$<br>$x = 1-2$ $-1+x = 2$<br>$x = -1$ $x = +3$<br>$\therefore x = -1 \quad \text{or} \quad x = 3$ |

b.

| <b>Method 1:</b>   | <b>Method 2:</b>   |
|--|--|
| $ 1-x  = x-1$<br>$1-x = +(x-1) \quad \text{or} \quad 1-x = -(x-1)$<br>$1-x = +x-1 \quad \text{or} \quad 1-x = -x+1$<br>$2x = 2 \quad \text{or} \quad 0 = 0 ???$<br>$x = 1 \quad \text{or} \quad x \text{ is all real numbers}$<br>$\therefore x \text{ is all real numbers (Really???)}$ | $ 1-x  = x-1$<br><b>Case 1:</b> $x \leq 1$ <b>Case 2:</b> $x > 1$<br>$1-x = x-1$ $-(1-x) = x-1$<br>$2x = 2$ $-1+x = x-1$<br>$x = 1$ $0 = 0$<br>$\therefore x = 1 \quad \text{or} \quad x > 1$<br>$\therefore x \geq 1$ |

5.

**Solve**  $|x-2| + |2x+1| = 4$

(5 marks)

黎 Sir 提提你 ：兩刀三段!!!

$$\text{Case 1: } x < -\frac{1}{2}$$

$$-(x-2) - (2x+1) = 4$$

$$-x+2-2x-1=4$$

$$-3x=3$$

$$x=-1$$

$$\therefore x = -1 \text{ or } x = 1$$

$$\text{Case 2: } -\frac{1}{2} \leq x \leq 2$$

$$-(x-2) + (2x+1) = 4$$

$$-x+2+2x+1=4$$

$$x=4-1-2$$

$$x=1$$

$$\text{Case 3: } x > 2$$

$$+(x-2) + (2x+1) = 4$$

$$+x-2+2x+1=4$$

$$3x=5$$

$$x=\frac{5}{3} \text{ (Rejected!)}$$

6.

**Solve the following equations:**

a.  $|x - 3| = 1$

b.  $|x - 1| = |x + 1|$

(6 marks)

黎 Sir 提提你 :

a.

**Method 1:**

$|x - 3| = 1$

$x - 3 = +1 \text{ or } x - 3 = -1$

$x = +1 + 3 \text{ or } x = -1 + 3$

$x = +4 \text{ or } x = +2$

$\therefore x = 4 \text{ or } x = 2$

**Method 2:****Case 1:**  $x \geq 3$ 

$x - 3 = 1$

$x = 4$

**Case 2:**  $x < 3$ 

$-(x - 3) = 1$

$-x + 3 = 1$

$x = 2$

$\therefore x = 4 \text{ or } x = 2$

b.

**Method 1:**

$|x - 1| = |x + 1|$

$x - 1 = +(x + 1) \text{ or } x - 1 = -(x + 1)$

$x - 1 = x + 1 \text{ or } x - 1 = -x - 1$

$-1 = 1 ? \text{ or } 2x = 0$

**No Solution** or  $x = 0$ 

$\therefore x = 0$

**Method 2:****Case 1:**  $x < -1$ 

$-(x - 1) = -(x + 1)$

$-x + 1 = -x - 1$

**+1 = -1 (No solution)****Case 2:**  $-1 \leq x \leq +1$ 

$-(x - 1) = x + 1$

$-x + 1 = x + 1$

$2x = 0$

$x = 0$

**Case 3:**  $x \geq +1$ 

$x - 1 = x + 1$

**-1 = 1 (No solution)**

$\therefore x = 0$

7.

Solve the following equations  $|x - 3| = x + 1$ .

(4 marks)

黎 Sir 提提你 :

**Case 1:**  $x < 3$

$$-(x - 3) = x + 1$$

$$-x + 3 = x + 1$$

$$2x = 2$$

$$x = 1$$

**Case 2:**  $x \geq 3$

$$x - 3 = x + 1$$

$$-3 = 1$$

$\therefore$  No solution for  $x \geq 3$

$$\therefore x = 1.$$

8.

Solve the following equations  $|3 - 2x| = 6x - 5$

(4 marks)

黎 Sir 提提你 :

**Case 1:**  $x < \frac{3}{2}$

$$3 - 2x = 6x - 5$$

$$8x = 8$$

$$x = 1$$

**Case 2:**  $x \geq \frac{3}{2}$

$$-3 + 2x = 6x - 5$$

$$2 = 4x$$

$$x = \frac{1}{2} \text{ (Rejected)}$$

9.

Solve the following equations  $|x-2| + |2x+1| = 4$

(6 marks)

黎 Sir 提提你 :

$$\text{Case 1: } x < -\frac{1}{2}$$

$$-(x-2) - (2x+1) = 4$$

$$-x+2-2x-1=4$$

$$-3x=3$$

$$x=-1$$

$$\therefore x = -1 \text{ or } x = 1$$

$$\text{Case 2: } -\frac{1}{2} \leq x \leq 2$$

$$-(x-2) + (2x+1) = 4$$

$$-x+2+2x+1=4$$

$$x=1$$

$$\text{Case 3: } x > 2$$

$$(x-2) + (2x+1) = 4$$

$$3x=5$$

$$x=\frac{5}{3} \text{ (Rejected)}$$

10.

Solve the following equations  $\left|\frac{2x+1}{3-x}\right| = \frac{1}{4}$

(5 marks)

黎 Sir 提提你 :

$$\left|\frac{2x+1}{3-x}\right| = \frac{1}{4}$$

$$\frac{|2x+1|}{|3-x|} = \frac{1}{4}$$

$$4|2x+1|=|3-x|$$

$$\text{Case 1: } x < -\frac{1}{2}$$

$$-4(2x+1)=3-x$$

$$-8x-4-3+x=0$$

$$-7x=7$$

$$x=-1$$

$$\therefore x = -1 \text{ or } x = -\frac{1}{9}$$

$$\text{Case 2: } -\frac{1}{2} \leq x \leq 3$$

$$4(2x+1)=3-x$$

$$9x+1=0$$

$$x=-\frac{1}{9}$$

$$\text{Case 3: } x > 3$$

$$4(2x+1)=-(3-x)$$

$$8x+4+3-x=0$$

$$7x=-7$$

$$x=-1 \text{ (Rejected)}$$

11.

a. Solve  $|x-1| = |x| - 1$ , where  $0 \leq x \leq 1$ .b. Solve  $|x-1| = |x| - 1$ .

(5 marks)

 黎 Sir 提提你 : 幫你分埋 Cases! 重 Easy!

a. For  $0 \leq x \leq 1$ ,

$$-(x-1) = x-1$$

$$-x+1 = x-1$$

$$2x = 2$$

$$x = 1$$

b.

**Case 1:**  $x < 0$ 

$$-(x-1) = -x-1$$

$$-x+1 = -x-1$$

$$1 = -1$$

 $\therefore$  No solution for  $x < 0$ **Case 2:**  $0 \leq x \leq 1$ 

: Part a 做左啦!!!

By part a,  $x = 1$ **Case 3:**  $x > 1$ 

$$x-1 = x-1$$

$$0 = 0$$

 $\therefore$  all real no. for  $x > 1$  $\therefore$  All real numbers for  $x \geq 1$ .

**2. Solving quadratic equations with absolute signs****1.**

Solve the following equality  $|x-3| = |x^2 - 4x + 3|$ .

黎 Sir 提提你 : 分 Cases + Factorization!

$$|x-3| = |x^2 - 4x + 3|$$

$$|x-3| = |(x-3)(x-1)|$$

**Case 1:**  $x < 1$

$$-(x-3) = (x-3)(x-1)$$

$$-x+3 = x^2 - 4x + 3$$

$$x^2 - 3x = 0$$

$$x(x-3) = 0$$

$$x=0 \text{ or } x-3=0$$

$$x=0 \text{ or } x=3 \text{ (Rejected!)}$$

$$x=0$$

$$\therefore x=0 \text{ or } x=2 \text{ or } x=3$$

**Case 2:**  $1 \leq x \leq 3$

$$-(x-3) = -(x-3)(x-1)$$

$$-x+3 = -x^2 + 4x - 3$$

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) = 0$$

$$x-3=0 \text{ or } x-2=0$$

$$x=3 \text{ or } x=2$$

**Case 3:**  $x > 3$

$$(x-3) = (x-3)(x-1)$$

$$x-3 = x^2 - 4x + 3$$

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) = 0$$

$$x-3=0 \text{ or } x-2=0$$

$$x=3 \text{ (Rejected!) or } x=2 \text{ (Rejected!)}$$

No Solution

**2.**

Solve the following equality  $x|x| + 5x + 6 = 0$ .

黎 Sir 提提你 : 分 Cases + Factorization!

**Case 1:**  $x < 0$

$$x(-x) + 5x + 6 = 0$$

$$-x^2 + 5x + 6 = 0$$

$$(-x+6)(x+1) = 0$$

$$-x+6=0 \text{ or } x+1=0$$

$$x=6 \text{ (Rejected!) or } x=-1$$

$$x=-1$$

**Case 2:**  $x \geq 0$

$$x(x) + 5x + 6 = 0$$

$$x^2 + 5x + 6 = 0$$

$$(x+2)(x+3) = 0$$

$$x+2=0 \text{ or } x+3=0$$

$$x=-2 \text{ (Rejected!) or } x=-3 \text{ (Rejected!)}$$

$$\therefore x=-1$$

3.

Solve the following equality  $|x^2 - 4| = 4 - x^2$ .

黎 Sir 提提你 : 分 Cases + Factorization!

$$|x^2 - 4| = 4 - x^2$$

$$|(x-2)(x+2)| = 4 - x^2$$

**Case 1:**  $x < -2$

$$(x-2)(x+2) = 4 - x^2$$

$$x^2 - 4 = 4 - x^2$$

$$2x^2 = 8$$

$$x^2 = 4$$

$$x = +2 \text{ or } x = -2$$

(Rejected) (Rejected)

**Case 2:**  $-2 \leq x \leq 2$

$$-(x-2)(x+2) = 4 - x^2$$

$$-x^2 + 4 = 4 - x^2$$

$$0 = 0$$

$\therefore$  All real no. for  $-2 \leq x \leq 2$

**Case 3:**  $x > 2$

$$(x-2)(x+2) = 4 - x^2$$

$$x^2 - 4 = 4 - x^2$$

$$2x^2 = 8$$

$$x^2 = 4$$

$$x = +2 \text{ or } x = -2$$

(Rejected) (Rejected)

$\therefore$  All real no. for  $-2 \leq x \leq 2$

4.

Solve the following equality  $3(x+1)^2 - 7|x+1| + 2 = 0$ .

黎 Sir 提提你 : 分 Cases + Factorization!

**Case 1:**  $x < -1$

$$3(x+1)^2 + 7(x+1) + 2 = 0$$

$$3x^2 + 6x + 3 + 7x + 7 + 2 = 0$$

$$3x^2 + 13x + 12 = 0$$

$$(3x+4)(x+3) = 0$$

$$3x+4=0 \text{ or } x+3=0$$

$$x = -\frac{4}{3} \text{ or } x = -3$$

**Case 2:**  $x \geq -1$

$$3(x+1)^2 - 7(x+1) + 2 = 0$$

$$3x^2 + 6x + 3 - 7x - 7 + 2 = 0$$

$$3x^2 - x - 2 = 0$$

$$(3x+2)(x-1) = 0$$

$$3x+2=0 \text{ or } x-1=0$$

$$x = -\frac{2}{3} \text{ or } x = 1$$

$$\therefore x = -\frac{4}{3} \text{ or } x = -3 \text{ or } x = -\frac{2}{3} \text{ or } x = 1$$

5.

Solve the following equality  $|x^2 + 3x - 4| = 6$ .

黎 Sir 提提你 :

$$|x^2 + 3x - 4| = 6$$

$$|(x+4)(x-1)| = 6$$

**Case 1:**  $x < -4$

$$x^2 + 3x - 4 = 6$$

$$x^2 + 3x - 10 = 0$$

$$(x+5)(x-2) = 0$$

$$x+5=0 \text{ or } x-2=0$$

$$x=-5 \text{ or } x=2 \text{ (Rejected)}$$

**Case 2:**  $-4 \leq x \leq 1$

$$-(x^2 + 3x - 4) = 6$$

$$-x^2 - 3x + 4 = 6$$

$$-x^2 - 3x - 2 = 0$$

$$(-x-1)(x+2) = 0$$

$$-x-1=0 \text{ or } x+2=0$$

$$x=-1 \text{ or } x=-2$$

**Case 3:**  $x > 1$

$$x^2 + 3x - 4 = 6$$

$$x^2 + 3x - 10 = 0$$

$$(x+5)(x-2) = 0$$

$$x+5=0 \text{ or } x-2=0$$

$$x=-5 \text{ (Rejected)} \text{ or } x=2$$

$$\therefore x = -5 \text{ or } x = -2 \text{ or } x = -1 \text{ or } x = 2$$

6.

$\alpha$  and  $\beta$  are the real roots of the quadratic equation  $x^2 - 5x + k = 0$  such that

$|\alpha - \beta| = 3$ . Find the value(s) of k.

(4 marks)

黎 Sir 提提你 🧐 : 唯一一種要 take square! 小心!

Sum of roots  $\alpha + \beta = -\frac{-5}{1} = 5$ .

Product of roots  $\alpha\beta = \frac{k}{1} = k$ .

$$|\alpha - \beta| = 3$$

$$\left| \sqrt{(\alpha - \beta)^2} \right| = 3$$

$$\left| \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} \right| = 3$$

$$\left| \sqrt{(5)^2 - 4(k)} \right| = 3$$

$$\left| \sqrt{25 - 4k} \right| = 3$$

黎 Sir 提提你 🧐 : 分完 Cases 先至 Take Squares!

Case 1:  $k \leq \frac{25}{4}$

Case 2:  $k > \frac{25}{4}$

No solution! (Do you know why?)

$$\left| \sqrt{25 - 4k} \right| = 3$$

$$25 - 4k = 3^2$$

$$25 - 4k = 9$$

$$4k = 16$$

$$k = 4$$

$$\therefore k = 4$$

**3. Transformation of graphs with absolute signs****1.****a. Stretch  $f(x) = x - 1$ . for  $-5 \leq x \leq 5$** **b. Stretch  $f(x) = |x - 1|$  and  $f(x) = x - 1$  on the same graph for  $-5 \leq x \leq 5$** **c. Stretch  $f(x) = |x| - 1$  and  $f(x) = x - 1$  on the same graph for  $-5 \leq x \leq 5$** **黎 Sir 提提你 :****a.  $f(x) = x - 1$  for  $-5 \leq x \leq 5$** **Slope = 1, y-intercept = -1, x-intercept = 1.** **$x = -5, y = -6, x = 5, y = 4$** **b.  $f(x) = |x - 1|$  for  $-5 \leq x \leq 5$** **: Reflecting about x-axis for “negative y”!****Case 1:  $x < 1, f(x) = -(x - 1)$** 

$$f(x) = -x + 1$$

 **$Slope = -1, y\text{-intercept} = +1, x\text{-intercept} = -1.$**  **$x = -5, y = +6$** **Case 2:  $x \geq 1, f(x) = x - 1$** **c.  $f(x) = |x| - 1$** **: Reflecting about y-axis for “negative x”!****Case 1:  $x < 0, f(x) = -x - 1$**  **$Slope = -1, y\text{-intercept} = -1, x\text{-intercept} = -1.$**  **$x = -5, y = +4,$** **Case 2:  $x \geq 0, f(x) = x - 1$**

2.

a. Stretch the graph  $y = 2|x + 2| + |2x - 5|$  for  $-5 \leq x \leq 5$

b. Use the graph in (a) to solve the following equations.

i.  $2|x + 2| + |2x - 5| = 15$

ii.  $2|x + 2| + |2x - 5| = 9$

iii.  $2|x + 2| + |2x - 5| = 3$

**黎 Sir 提提你 :**

a.  $y = 2|x + 2| + |2x - 5|, -5 \leq x \leq 5$

| Case 1: $x < -2$           | Case 2: $-2 \leq x \leq \frac{5}{2}$ | Case 3: $x > \frac{5}{2}$ |
|----------------------------|--------------------------------------|---------------------------|
| $y = -2(x + 2) - (2x - 5)$ | $y = 2(x + 2) - (2x - 5)$            | $y = 2(x + 2) + (2x - 5)$ |
| $y = -2x - 4 - 2x + 5$     | $y = 2x + 4 - 2x + 5$                | $y = 2x + 4 + 2x - 5$     |
| $y = -4x + 1$              | $y = 9$                              | $y = 4x - 1$              |

b.

i.  $2|x + 2| + |2x - 5| = 15$ , From the graph,  $y = 15$  when  $x < -2$  and  $x > \frac{5}{2}$ .

For  $x < -2$ ,  $y = 15$  when  $x = \frac{14}{-4} = -3.5$ .

For  $x > \frac{5}{2}$ ,  $y = 15$  when  $x = \frac{16}{4} = 4$ .

ii.  $2|x + 2| + |2x - 5| = 9$ , From the graph,  $y = 9$ , when  $-2 \leq x \leq \frac{5}{2}$ .

iii.  $2|x + 2| + |2x - 5| = 3$ , From the graph,  $y = 3$ , there is no solution.

3. Let  $f(x) = x^2$  and  $g(x) = x \cdot |x|$

a. Stretch  $y = f(x)$  for  $-5 \leq x \leq 5$

b. Stretch  $y = f(x)$  and  $y = g(x)$  on the same graph for  $-5 \leq x \leq 5$

(5 marks)

黎 Sir 提提你 :

a.  $f(x) = x^2$ , for  $-5 \leq x \leq 5$ .

$$x = 0, y = 0.$$

$$x = -5, y = 25$$

$$x = 5, y = 25$$

$$\text{Vertex} = (0,0)$$

$a > 0 \Rightarrow \text{Opening Upwards}$

b.  $g(x) = x \cdot |x|$ , for  $-5 \leq x \leq 5$ .

**Case 1:**  $x \leq 0$

$$g(x) = -x^2$$

$a < 0 \Rightarrow \text{Opening Downwards}$

$$x = 0, y = 0.$$

$$x = -5, y = -25$$

$$\text{Vertex} = (0,0)$$

**Case 2:**  $x > 0$

$$g(x) = x^2 = f(x)$$

4. Let  $f(x) = 3x^2 + 4x - 4$
- Stretch  $y = f(x)$  for  $-5 \leq x \leq 5$
  - Stretch  $y = f(x)$  and  $y = |f(x)|$  on the same graph for  $-5 \leq x \leq 5$
  - Stretch  $y = f(x)$  and  $y = f(|x|)$  on the same graph for  $-5 \leq x \leq 5$
  - Find the value of  $k$  such that the graph  $y = f(x) + k$  and  $y = |f(x) + k|$  are the same for  $-5 \leq x \leq 5$

(7 marks)

**黎 Sir 提提你 :**

- a.  $f(x) = 3x^2 + 4x - 4$ , for  $-5 \leq x \leq 5$

When  $3x^2 + 4x - 4 = 0$

$$(3x - 2)(x + 2) = 0$$

$$3x - 2 = 0 \quad \text{or} \quad x + 2 = 0$$

$$x = \frac{2}{3} \quad \text{or} \quad x = -2$$

$$\text{Vertex} = \left( \frac{-b}{2a}, -\frac{b^2 - 4ac}{4a} \right) = \left( -\frac{2}{3}, -\frac{16}{3} \right)$$

$$x = -5, \quad y = 51 \quad \text{and} \quad x = 5, \quad y = 91$$

- b.  $y = |f(x)|$ , for  $-5 \leq x \leq 5$   : Reflecting about x-axis for “negative y”!

**Case 1:**  $-2 \leq x \leq \frac{2}{3}$ ,

$$f(x) = -(3x^2 + 4x - 4)$$

$$f(x) = -3x^2 - 4x + 4$$

**When**  $-3x^2 - 4x + 4 = 0$

$$x = \frac{2}{3} \quad \text{or} \quad x = -2$$

$$\text{Vertex} = \left( \frac{-b}{2a}, -\frac{b^2 - 4ac}{4a} \right) = \left( -\frac{2}{3}, \frac{16}{3} \right)$$

**Case 2:**  $x < -2$  or  $x > \frac{2}{3}$ ,

$$f(x) = 3x^2 + 4x - 4$$

- c.  $y = f(|x|)$ , for  $-5 \leq x \leq 5$   : Reflecting about y-axis for negative x!!!

**Case 1:**  $x \leq 0$ ,  $f(x) = 3x^2 - 4x - 4$

**When**  $3x^2 - 4x + 4 = 0$

$$\Delta = (-4)^2 - 4(3)(4) = -32 < 0$$

$\therefore$  No real root!

$$x = -5, y = +91, \quad x = 0, y = -4$$

**Case 2:**  $x > 0$ ,  $f(x) = 3x^2 + 4x - 4$

- d.

**From the graph,**

**When**  $y = f(x)$  **is shifted upwards by 4 unit vertically,**

**The graph of**  $y = f(x) + k$  **and**

$$y = |f(x) + k| \quad \text{become the same.}$$

$$\therefore k = 4$$

**Do you know how HKEA comment Student's knowledge on “Absolute Value”?**

黎 Sir 提提你 :

1. In 2007 HKCEE Additional Mathematics Subject Report, HKEA states that:

**Q.11**

“Most Candidates didn’t understand the definition of absolute value of a real number x well and wrote mistakenly that  $|x| = \pm x$  instead of  $|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x & \text{if } x \leq 0 \end{cases}$ .”

- a. Many candidates ignored the condition  $0 \leq x \leq 1$  and solved the equation mistakenly by using  $|x| = \pm x$ .
- b. Many candidates solved the equation by simply writing  $|x| = \pm x$  as in (a). Some candidates didn’t know that  $x - 1 = x - 1$  is an identity and concluded that there is no solution while some others rejected  $x - 1 = -x - 1$  instead of concluding that there is no solution.



**The End.**



# 黎 sir 教室 導師簡介:

- 所有數學/物理/綜合科學/經濟科: **黎 sir** BEng (Hons, CUHK)
- 生物/化學科: **高考狀元 Mr. William Cheng** BSc (Hons, HKU)
- 中文科: **工商管理碩士 Miss Fonnie Wong** MBA, BA (Hons, HKU)
- 初中全科導師: **商業管理碩士 Miss Enve Tam** MBA, BBA (Hons)
- **Oral English: Mr. Hugo Ma** Native English Speaker

## 黎 sir 簡介

- 畢業於香港中文大學, 黎 sir 教室創辦人之一.
- 多年教授會考/高考/GCSE/IGCSE/GCE 所有數學/物理/經濟科經驗, 信心保證.
- 現於黎 sir 教室及中學任教補習班, 學生就讀於英文中學, 中文中學, 國際學校及英國留學生.
- 熟悉近年出題趨勢, 教授考試取分技巧; 鼓勵同學獨立思考, 增強同學理解能力
- 善用生活化例子講解, 教法生動, 增加學習趣味; 深入淺出, 明白學生學習上的困難和需要.
- 中英對照筆記, 適合中文和英文中學學生就讀; 精心編制練習和試題, 協助同學盡快掌握答題技巧.
- 黎 sir 在中學和大學時代已是一名傑出學生, 曾獲取多項學業上和運動上的獎學金及獎項; 曾代表香港參加國際性運動比賽, 取得優異成績, 「又讀得又玩得」, 絶不是死讀書的書呆子.
- 黎 sir 在就讀大學時曾於全球最大美資電腦公司任實習生超過一年, 大學畢業後旋即於全港最大英資電腦公司負責主理該公司所代理的全球最大美資電腦公司儲存系統銷售業務(當時黎 sir 只得 24 歲).
- 於短短半年內將該產品線銷售業績提升超過 50%. 同時更被公司評選為"傑出表現員工 Outstanding Performer", 成功將書本上的知識靈活運用於工作上.
- 黎 sir 為了教學理想, 毅然辭去工作, 全身投入教學事業, 希望將自己的一套獨特的學習方法教授學生

## 課程特色

- 小組教學(1 – 6 人), 事半功倍.
- 精心編制筆記, 協助同學盡快掌握答題技巧.
- 課程歡迎自由組合小組上課, 時間及課程內容編排更有彈性.
- 時間及課程請瀏覽以下網址: [www.andylai.hk](http://www.andylai.hk)

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