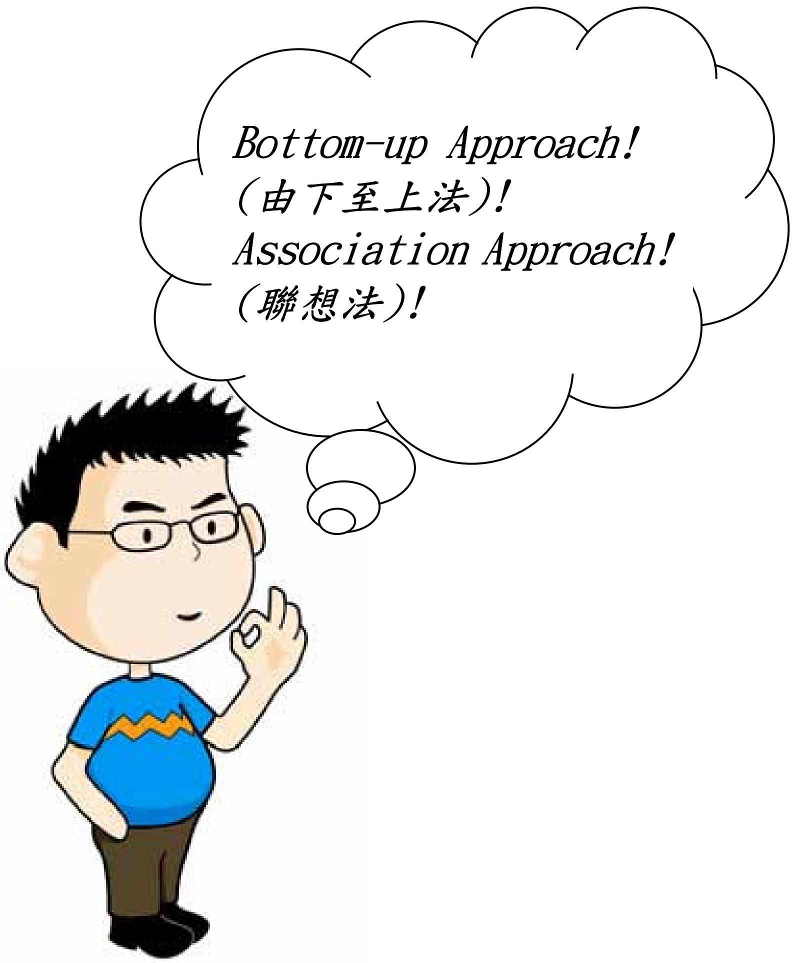
Mathematics

數學科

Deductive Geometry

演繹幾何

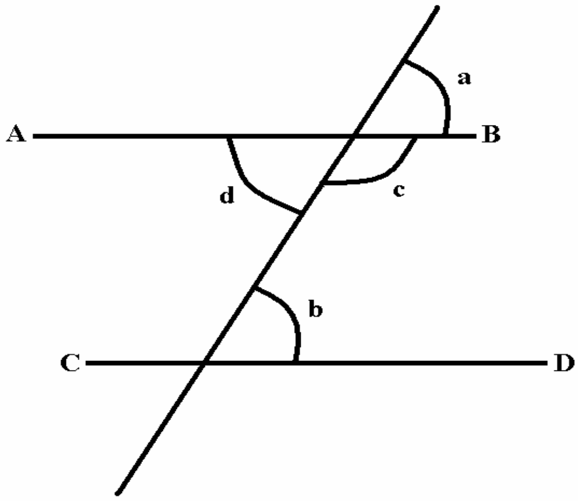


<u>Contents 目錄:</u>

Deductive Geometry 演繹幾何

(A) Summary of Geometry Proving Skills 幾何學證明技巧總結

1. Relationship between parallel lines and its angles.
平行線及其角之關係.

i. If $\angle a=\angle b$ **,then** $AB // CD$ **.** **(Corr.** $\angle s$ **equal.)** 如果 $\angle a=\angle b$,則 $AB // CD$ **.** **(**對應角相等**.)** **ii. If** $\angle b=\angle d$ **,then** $AB // CD$ **.** **(Alt.** $\angle s$ **equal.)** 如果 $\angle b=\angle d$,則 $AB // CD$ **.** **(**錯角相等**.)** **iii. If** $\angle b+\angle c=180^{\circ}$ **,then** $AB // CD$ **.** **(Int.** $\angle s$ **supplementary)** 如果 $\angle b+\angle c=180^{\circ}$,則 $AB // CD$ **.** **(**同旁內角互補**.)**	
iv. If $AB // CD$ **, then** $\angle a=\angle b$ **. (Corr.** $\angle s$ **,** $AB // CD$ **)** 如果 $AB // CD$,則 $\angle a=\angle b$ **. (**對應角**,** $AB // CD$ **)** **v. If** $AB // CD$ **,then** $\angle b=\angle d$ **. (Alt.** $\angle s$ **,** $AB // CD$ **)** 如果 $AB // CD$,則 $\angle b=\angle d$ **. (**錯角**,** $AB // CD$ **)** **vi. If** $AB // CD$ **,then** $\angle b+\angle c=180^{\circ}$ **. (Int.** $\angle s$ **,** $AB // CD$ **)** **If** $AB // CD$ **,then** $\angle b+\angle c=180^{\circ}$ **. (**同旁內角**,** $AB // CD$ **)**	

2. Properties of angles at a point, on a straight line and in a triangle.
角在一點上，一直線上和一三角形內之特性.

<table>
<tr><td>

ii. $\angle a+\angle b+\angle c+\angle d=360^\circ$.

(Angles at a point)

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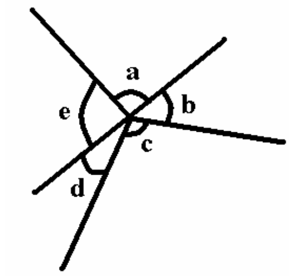

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<tr><td>

ii. $\angle a+\angle b+\angle c+\angle d=180^\circ$.

(Adjacent Angles on a straight line)

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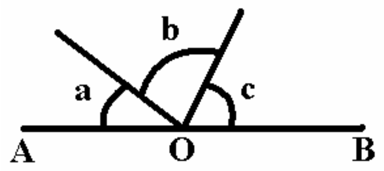

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iv. $\angle a=\angle b$.

(Vert. Oppsite angles equal)

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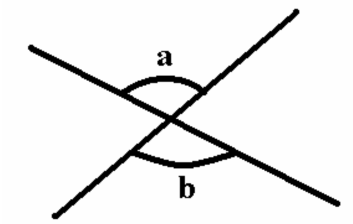

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vi. $\angle a+\angle b+\angle c=180^\circ$.

(Angles sum of triangle.)

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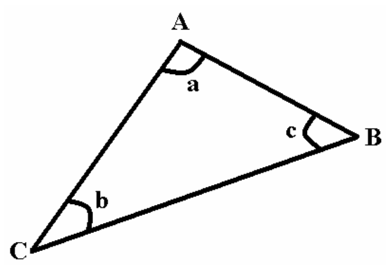

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v. $\angle c=\angle a+\angle b$

(ext. angle of a triangle)

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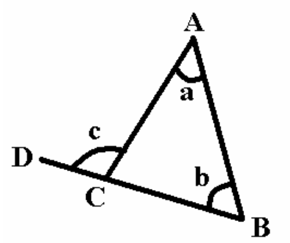

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<tr><td>

vi. ΔABC **is an isosceles** Δ

$\Leftrightarrow$ $AB=AC$ **(Base sides of** △ **).**

$\Leftrightarrow$ $\angle b=\angle c$ **(Bases** $\angle s$ **of** △ **)**

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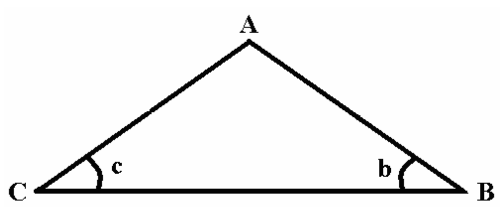

</td></tr>
<tr><td>

vii. ΔABC **is an equilateral** Δ

$\Leftrightarrow$ $AB=BC=AC$ **(sides of** △ **are equal).**

$\Leftrightarrow$ $\angle a=\angle b=\angle c=60^\circ$ **(** $\angle s$ **of** △ **are equal).**

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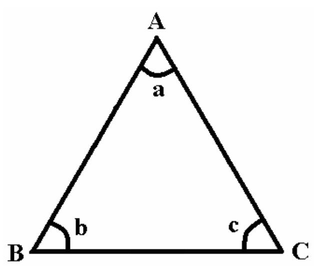

</td></tr>
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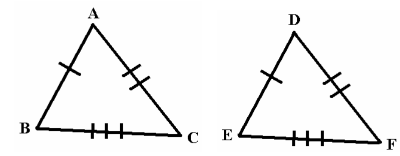
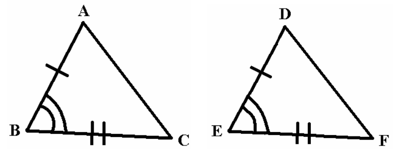
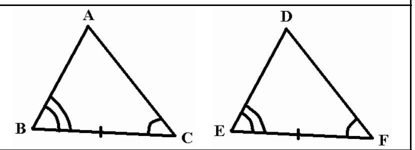
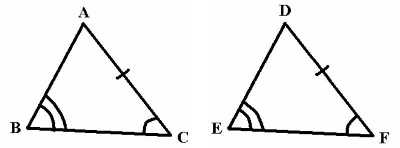
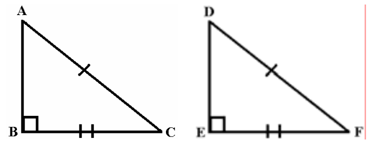
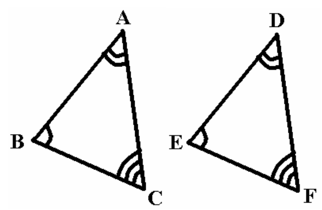
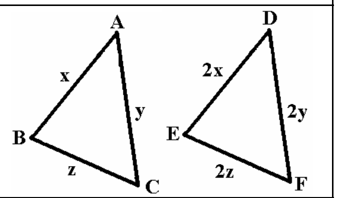
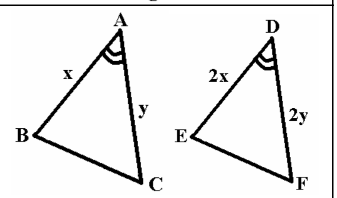
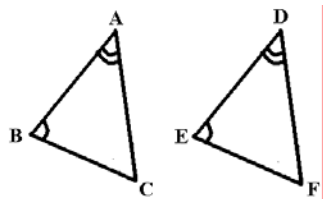
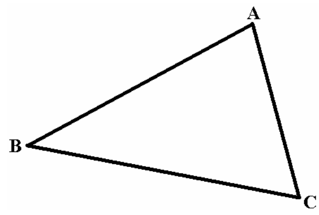
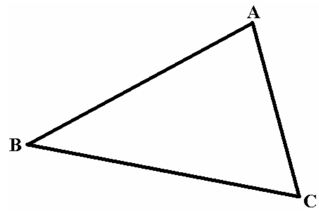
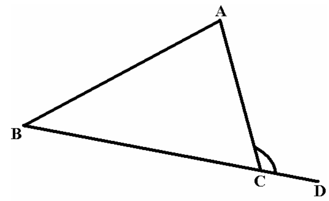
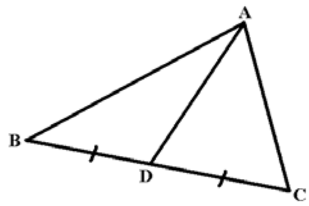
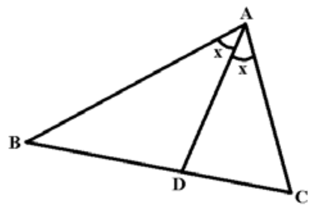
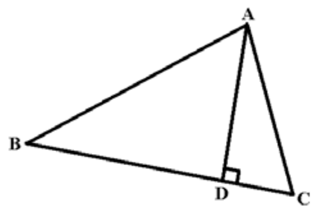
3. Congruent Triangles 全等三角形.

i. $AB = DE$ **and** $BC = EF$ **and** $AC = DF$ $\Leftrightarrow \Delta ABC \cong \Delta DEF \quad (SSS)$	
ii. $AB = DE$ **and** $BC = EF$ **and** $\angle ABC = \angle DEF$ $\Leftrightarrow \Delta ABC \cong \Delta DEF \quad (SAS)$	
iii. $\angle ABC = \angle DEF$ **and** $BC = EF$ **and** $\angle ACB = \angle DFE$ $\Leftrightarrow \Delta ABC \cong \Delta DEF \quad (ASA)$	
iv. $\angle ABC = \angle DEF$ **and** $\angle ACB = \angle DFE$ **and** $AC = DF$ $\Leftrightarrow \Delta ABC \cong \Delta DEF \quad (AAS)$	
v. $\angle ABC = \angle DEF = 90^{\circ}$ **and** $AC = DF$ **and** $BC = EF$ $\Leftrightarrow \Delta ABC \cong \Delta DEF \quad (RHS)$	

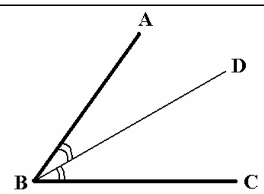
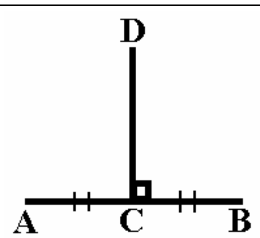
4. Similar Triangles 相似三角形.

i. $\angle ABC = \angle DEF$ **and** $\angle ACB = \angle DFE$ **and** $\angle BAC = \angle EDF$ $\Leftrightarrow \Delta ABC \cong \Delta DEF \quad (AAA)$	
ii. $\dfrac{AB}{DE} = \dfrac{AC}{DF} = \dfrac{BC}{EF} \Leftrightarrow \Delta ABC \cong \Delta DEF$ **(3 sides proportional)**	
iii. $\dfrac{AB}{DE} = \dfrac{AC}{DF}$ **and** $\angle BAC = \angle EDF$ $\Leftrightarrow \Delta ABC \cong \Delta DEF$ **(ratio of 2 sides, included ∠)**	
iv. $\angle ABC = \angle DEF$ **and** $\angle BAC = \angle EDF$ $\Leftrightarrow \Delta ABC \cong \Delta DEF$ **(A.A. Similar)**	

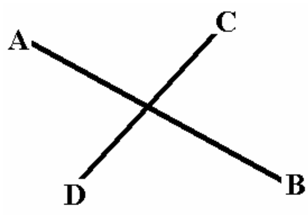
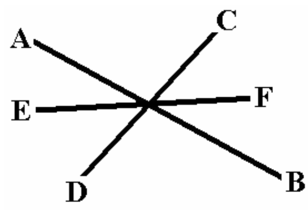
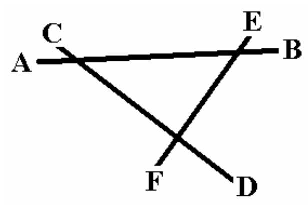
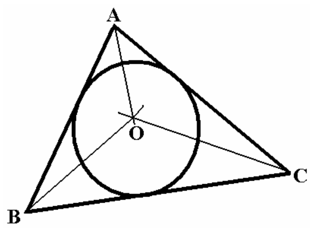
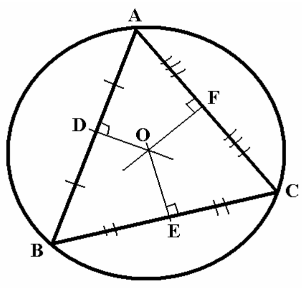
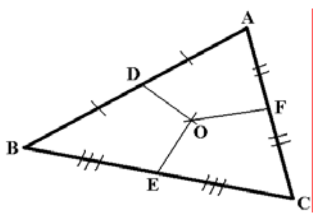
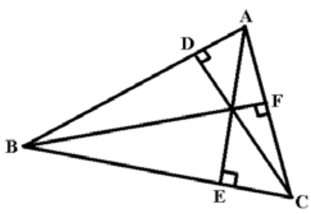
5. Theorem between the sides and angles in a triangle 三角形角度和邊長的定理

i. $AB + BC > AC$ **(Triangle Inequality 三角不等式)**	
ii. $AB > BC$ $\Leftrightarrow \angle ACB > BAC$ **(Bigger interior ∠ , Longer corr. sides)**	
iii. $\angle ACD > \angle ABC$ **and** $\angle ACD > \angle BAC$ **(ext.∠ > corresponding Interior ∠)**	
iv. $BD = DC$ $\Leftrightarrow$ AD **is the median of** Δ**ABC.**	
v. $\angle BAD = \angle CAD$ $\Leftrightarrow$ AD **is the angle bisector of** $\angle A$ **in** Δ**ABC.**	
vi. $AD \perp BC$ $\Leftrightarrow$ AD **is the altitude of** $\angle A$ **in** Δ**ABC.**	

6. Bisectors 平分線

i. BD **is the angle bisector of** $\angle ABC$ $\Leftrightarrow \angle ABD = \angle DBC$	
ii. CD **is the perpendicular bisector of** AB**.** $\Leftrightarrow$ $AC = BC$ **and** $\angle ACD = 90^\circ$**.**	

7. Point of Intersection of straight lines 直線相交點之特性

i. I is the point of intersection of AB and CD.	
ii. AB, CD and EF are concurrent at point I.	
iii. AB, CD and EF are not concurrent.	
iv. O is the In centre of $\triangle$ABC **⇔ AO, BO and CO are the angle bisector of** $\angle A$, $\angle B$ **and** $\angle C$ **respectively.**	
v. O is the Circumcenter of $\triangle$ABC **⇔ DO, EO and FO are the perpendicular bisector of AB, BC and AC respectively.**	
vi. O is the Centroid of $\triangle$ABC **⇔ DO, EO and FO are the perpendicular bisector of AB, BC and AC respectively.**	
vii. O is the Orthocenter of $\triangle$ABC **⇔ DO, EO and FO are altitude of $\triangle$ABC.**	

(B) <u>Method for solving Geometry questions 解幾何問題方法</u>

1. There are mainly two methods for solving Geometry questions.
解幾何問題主要有兩個方法**.**

i. Bottom-up Approach (由下至上法**).**

ii. Association Approach (聯想法**).**

iii. Applying the answers from the previous parts (應用之前部份的答案**).**

黎 Sir 提提你  **:**

1. **Bottom-up Approach (**由下至上法**):**

   ____

   ____

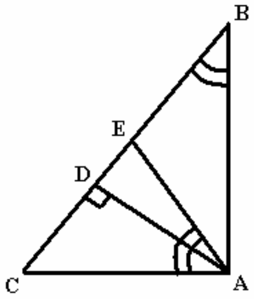
   ____

2. **Association Approach (**聯想法**):**______________________________
3. **Applying the answers from the previous parts (**應用之前部份的答案**):**

   ____

4. 條條大道通羅馬，殊途同歸**:**______________________________
5. **Deductive Geometry(**演繹幾何**)** ⇔ ______________________________

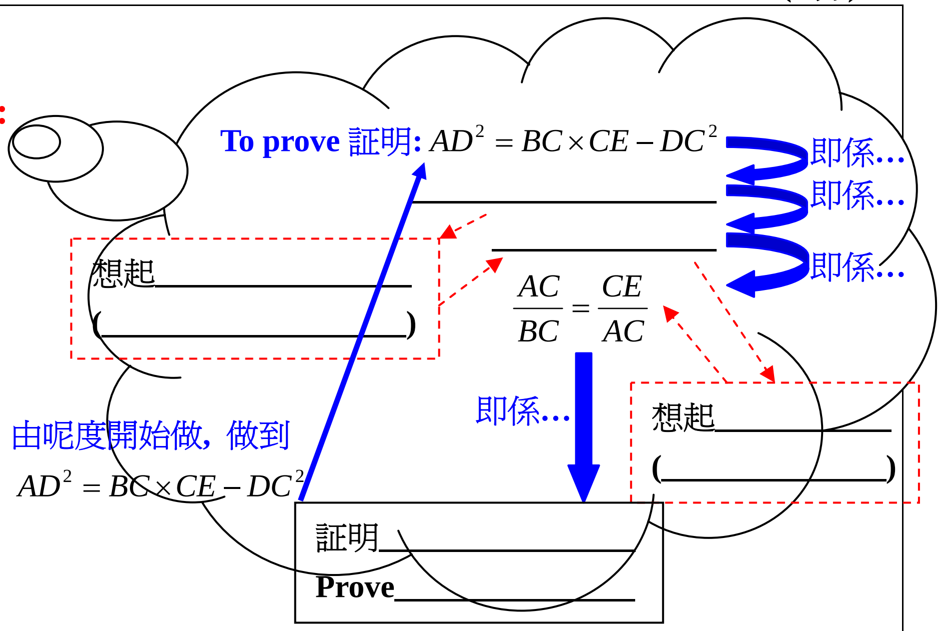
e.g. 1 例子一



In the figure above, prove that $AD^2 = BC \times CE - DC^2$ **(5 marks)**

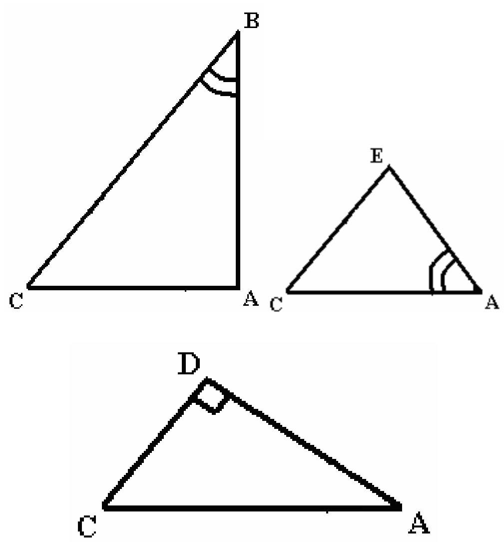
在上圖，証明 $AD^2 = BC \times CE - DC^2$ **(5 分)**

黎 sir 教你諗:



藍箭嘴: Bottom-up Approach 由下至上法 **紅箭嘴: Association Approach 聯想法.**

Solutions 題解:

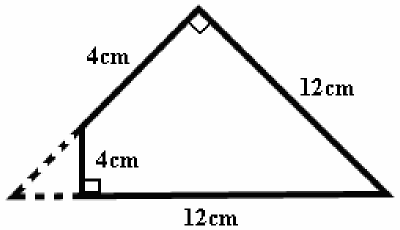


$AD^2 = BC \times CE - DC^2$ ------------------ **(1A)**

黎 sir 提提你: **1. (side 邊長)^2 + ⊿ ⇔ ____**

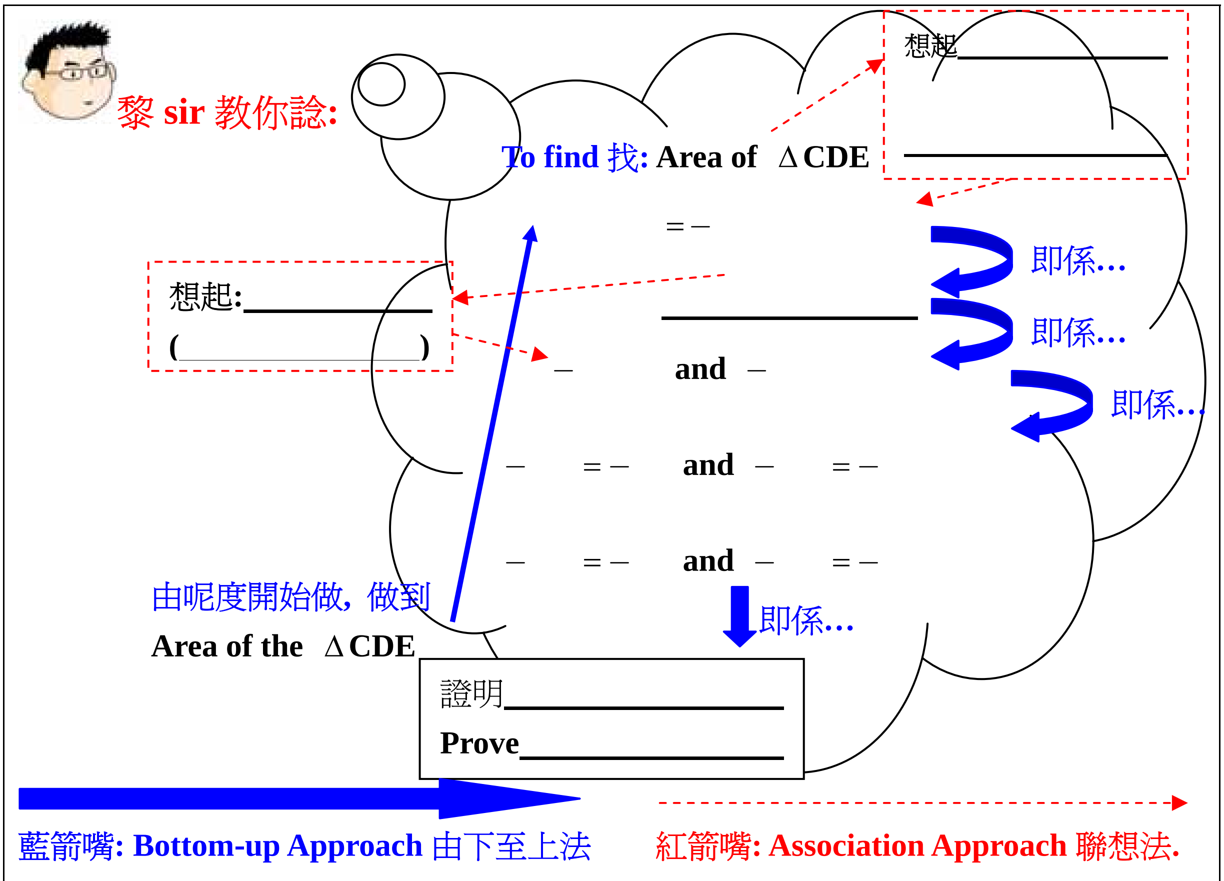
2. Ratio of sides 邊長比例 ⇔ ____

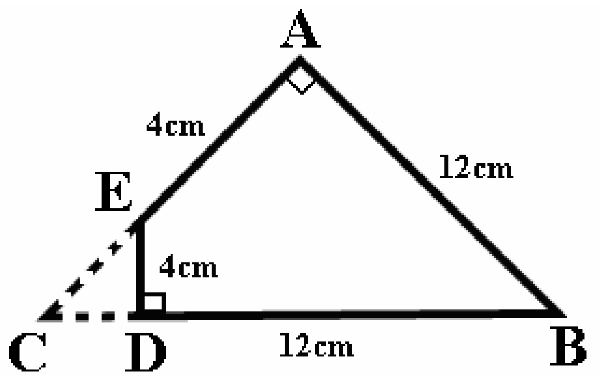
e.g. 2 例子二



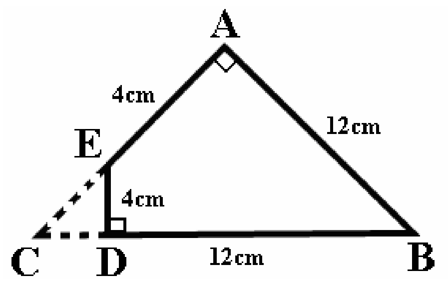
After teaching 10 lessons, Andy is very hungry and so he eats a sandwich. When he eat a part of the sandwich (as shown in the figure), he find that the sandwich goes bad. So he throws it away. Find out the area of the part of sandwich where Andy ate. (8 marks)

上完 **10** 堂後, **Andy** 好肚餓所以他吃了一件三文治. 當他吃了一部份後, 他發現這三文治原來已經變壞了. 所以他便棄掉它. 試找出給 **Andy** 吃了的那一部份三文治的面積. **(8** 分**)**





Solutions 題解:

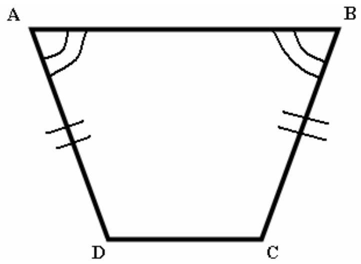


黎 sir 提提你:

1. 找Δ面積, 一定想起Δ面積公式: ______________________

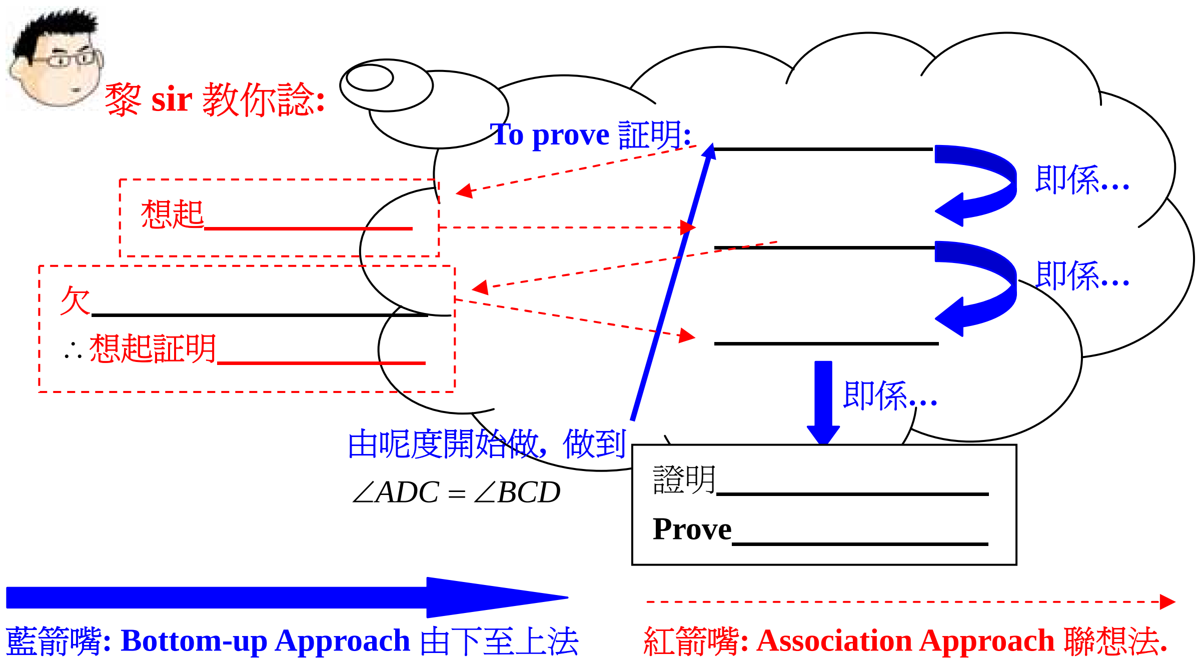
2. 找Δ邊長, 有兩個Δ+冇邊相等 ⇔ ______________________

e.g. 3 例子三

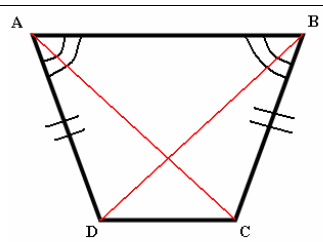


In the figure above, prove that $\angle ADC = \angle BCD$. **(6 marks)**

在上圖，証明 $\angle ADC = \angle BCD$. **(6 分)**

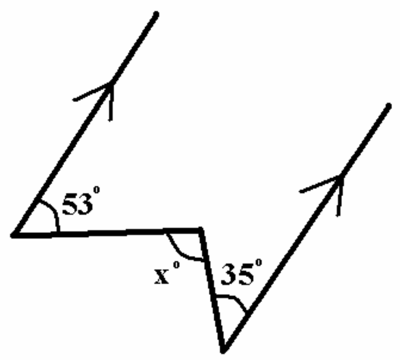


Solutions 題解:



黎 sir 提提你:

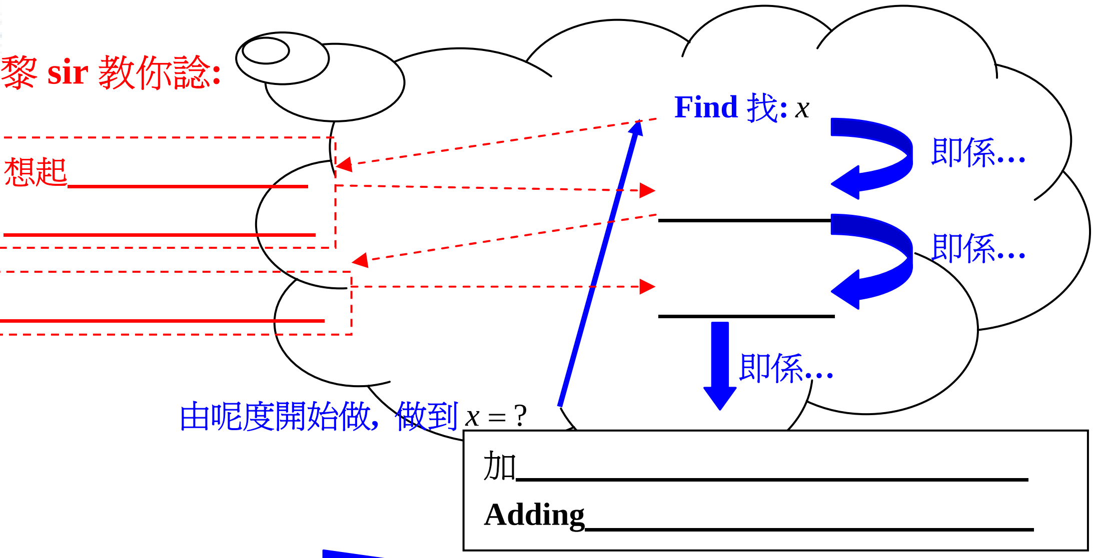
1. 找Δ邊長，有兩個Δ+有邊相等 ⇔ ____________________



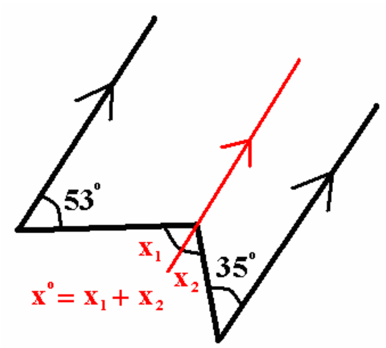
e.g. 4 例子四

Find x. 找 x. **(3 marks / 3 分)**

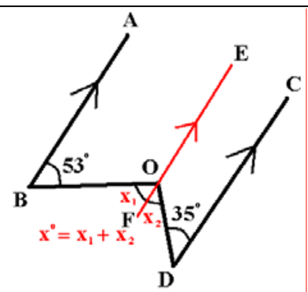
黎 sir 教你諗:



藍箭嘴: Bottom-up Approach 由下至上法 **紅箭嘴: Association Approach 聯想法.**

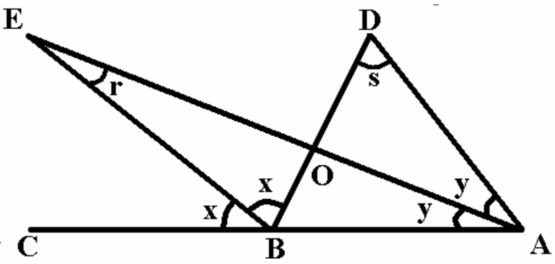


Solutions 題解:



黎 sir 提提你:

1. 有平衡線, 找∠ ⇔ ________



e.g. 6 例子六

In the figure, ABC, AOE and BOD are straight lines and $x - y = 50^{\circ}$. Find $r + s$. **(3 marks)**

在上圖, **ABC, AOE** 和 **BOD** 是直線和 $x - y = 50^{\circ}$. 求 $r + s$. **(3 分)**

黎 sir 教你諗:

Find 找: $r + s$ 即係… 即係… 即係…

想起________________

____

想起________________

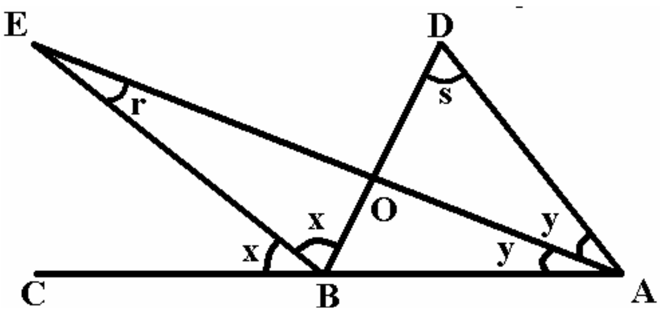
即係…

由呢度開始做, 做到 $r + s = ?$

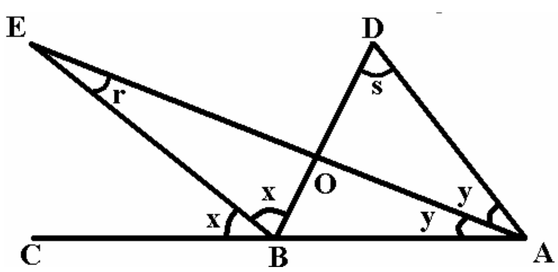
r, s ________________

r, s ________________

藍箭嘴: **Bottom-up Approach** 由下至上法　　紅箭嘴: **Association Approach** 聯想法.



Solutions 題解:

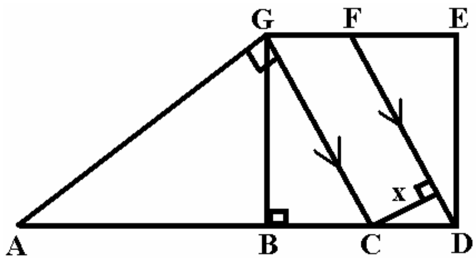


黎 sir 提提你:

1. 有外角(**ext.** ∠), ________________

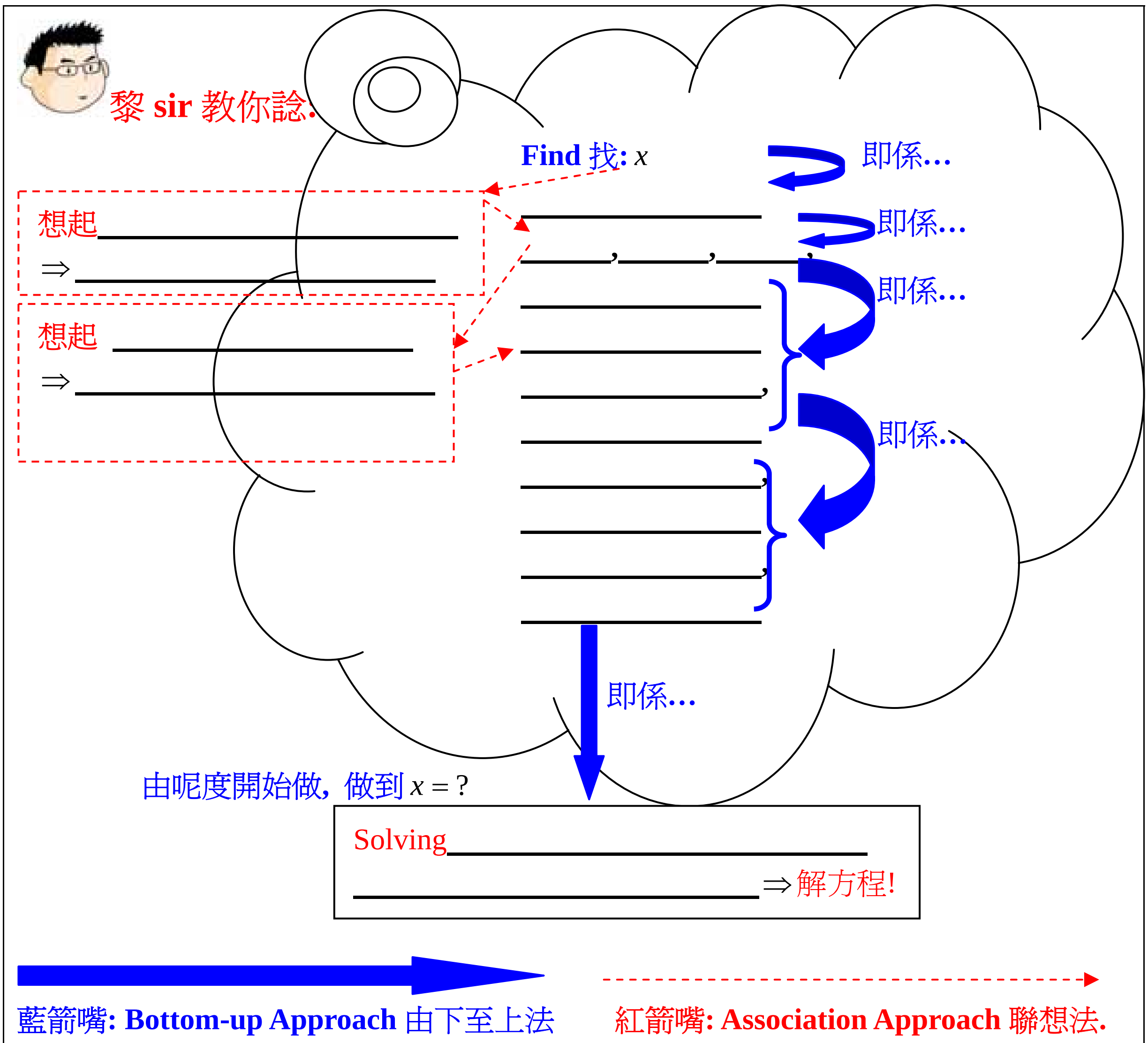
2. 題目已知既, ________________

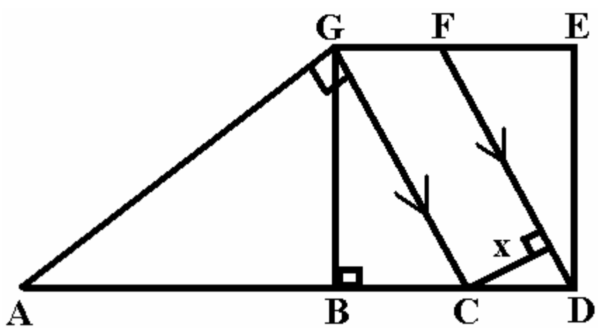
e.g.7 例子七



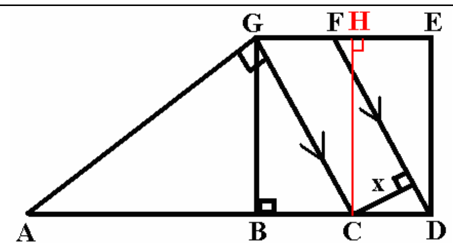
在上圖中, AD⊥BC, AG⊥GC, x⊥DF, CG//DF, BDEG 是正方形. AB=8, BC=2. 求 x. (8 分)

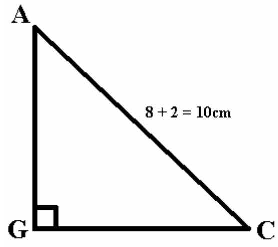
In the figure, AD⊥BC, AG⊥GC, x⊥DF, CG//DF, BDEG is a square. AB=8m BC=2. Find x. (8 marks)

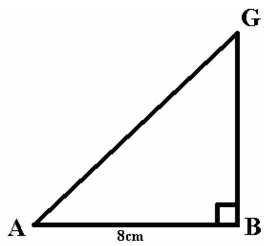


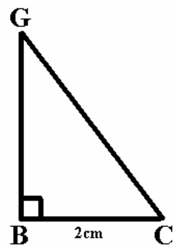


Solutions 題解:










黎 sir 提提你:

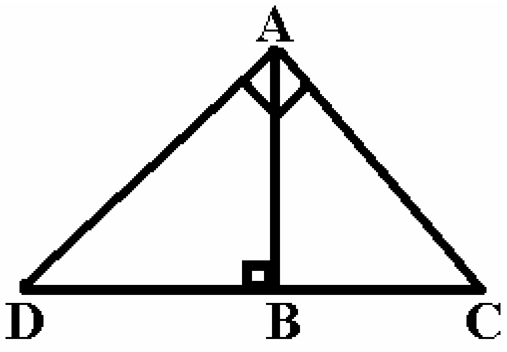
1. 平行四邊形的高 ⇒ ________________________________

2. 平行四邊形 ⇒ ________________________________

3.  ⇒ ________________________________

4. 3 equations 方程, 3 unknowns 變數 ⇒ ________________________________

5. 最後答案 ⇒ ________________________________



e.g.8 例子八

In the figure, CD⊥AB, $\angle ACB = 90^{\circ}$ **, AD = 5c**

在上圖中, **CD⊥AB,** $\angle A$

____

(______________)

黎 sir 教你諗:

CD, AD ⇒ ______________

DB ⇒ ______________

Find 找:_____ 即係…

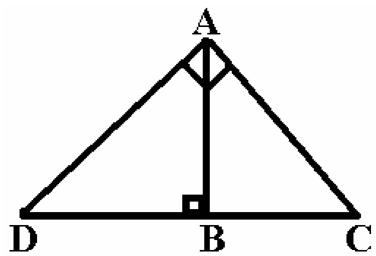
______________,

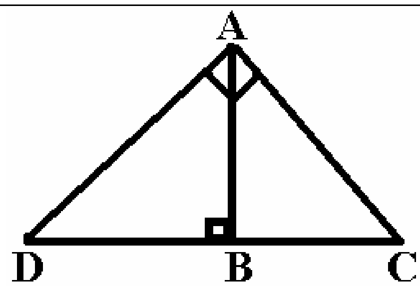
即係…

由呢度開始做，做到 $CD = ?$

Prove 証明______________

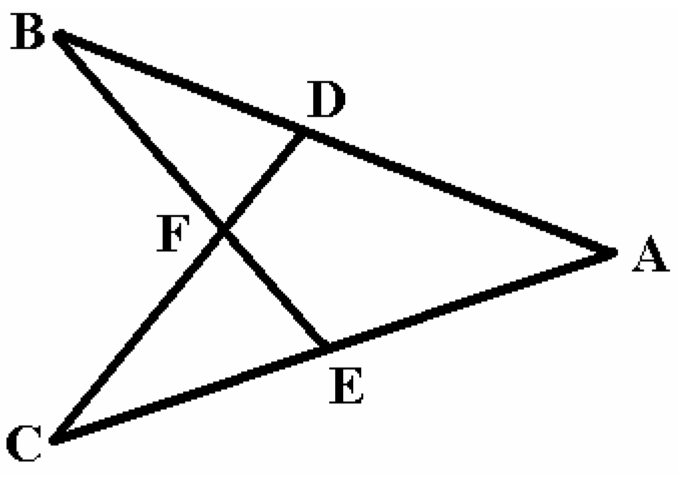
藍箭嘴: **Bottom-up Approach** 由下至上法 紅箭嘴: **Association Approach** 聯想法.



Solutions 題解:	
Consider Δ**ACD and** Δ**BAD**	

黎 **sir** 提提你:

1. 找Δ邊長，有兩個Δ+冇邊相等 ⇔ ______________



e.g. 9 例子九

In the figure, AB=AC, AD=AE, Prove BF=CF. **(3 marks)**

在上圖中, **AB=AC, AD=AE,** 證明 **BF=CF.** **(3 分)**

黎 sir 教你諗:

Prove 証明: ____________

____________,　即係…

____________　即係…

____________　即係…

想起: ____________

(____________)

BF ⇒ ____________

CF ⇒ ____________

想起: ____________

(____________)

$\angle DBF$, AB ⇒ ________

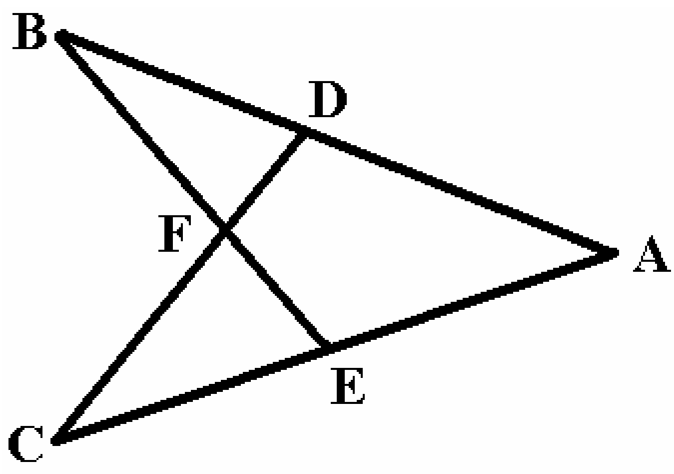
$\angle ECF$, AC ⇒ ________

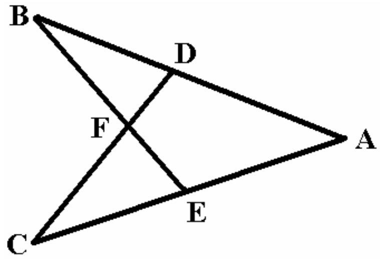
由呢度開始做, 做到 $BF = CF$

Prove 証明____________

藍箭嘴: **Bottom-up Approach** 由下至上法

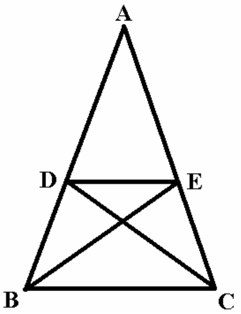
紅箭嘴: **Association Approach** 聯想法.



Solutions 題解:	
	

黎 sir 提提你:

1. 找Δ邊長, 有兩個Δ+冇邊相等 ⇔ ____________________



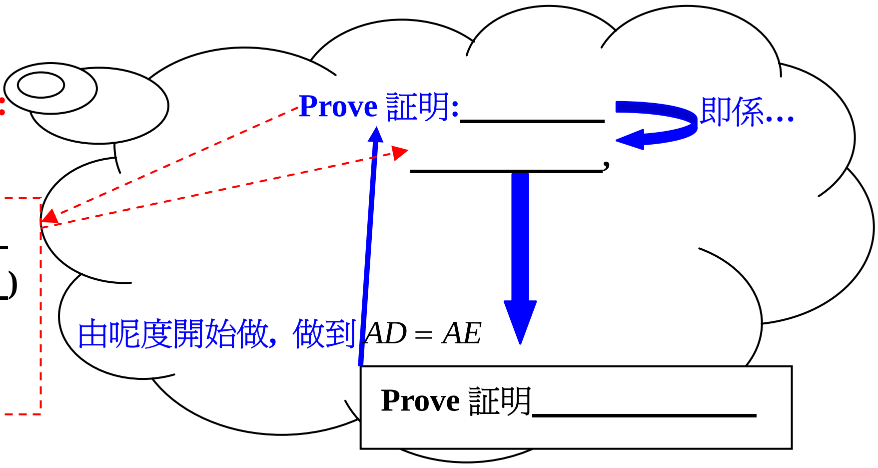
e.g. 10 例子十

In the figure, AB=AC, $\angle ABE = \angle ACD$ **. Prove that AD = AE.** **(2 marks)**

在上圖中, **AB=AC,** $\angle ABE = \angle ACD$ **.** 証明 **AD = AE.** **(2** 分**)**

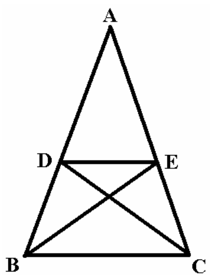
黎 sir 教你諗:

想起:________________
(____________________)
AD ⇒ ______________
AE ⇒ ______________

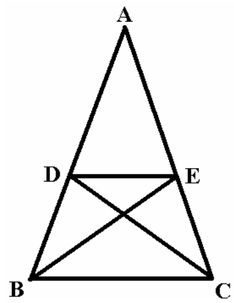


藍箭嘴: **Bottom-up Approach** 由下至上法

紅箭嘴: **Association Approach** 聯想法.

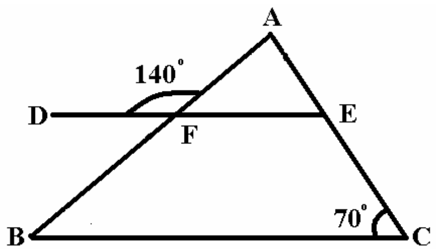


Solutions 題解:



黎 sir 提提你:

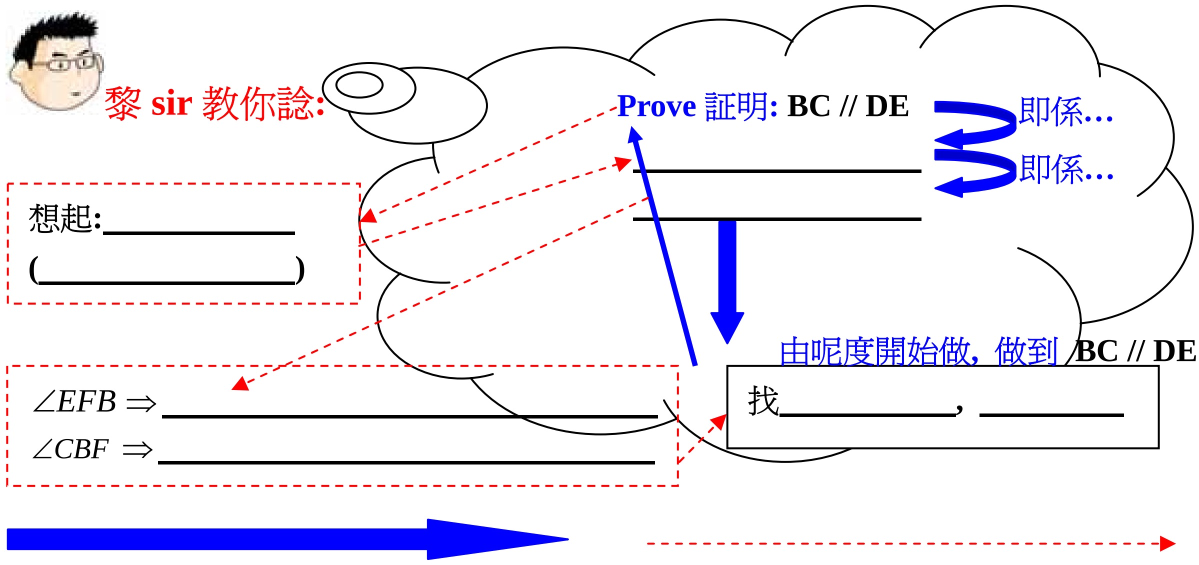
1. 找Δ邊長, 有兩個Δ+有邊相等 ⇔ ____________________



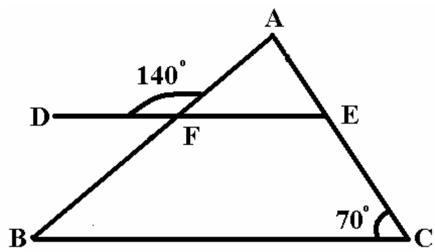
e.g. 11 例子十一

In the figure, AB=BC. Prove BC // DE. **(4 marks)**

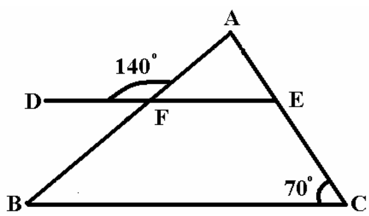
在上圖中, **AB=BC.** 証明 **BC // DE.** **(4 分)**



藍箭嘴: **Bottom-up Approach** 由下至上法　　紅箭嘴: **Association Approach** 聯想法.

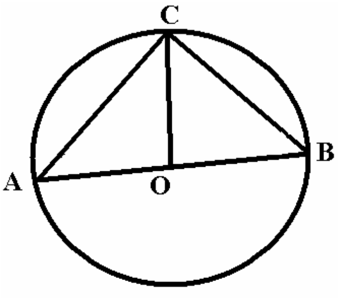


Solutions 題解:



黎 **sir** 提提你:

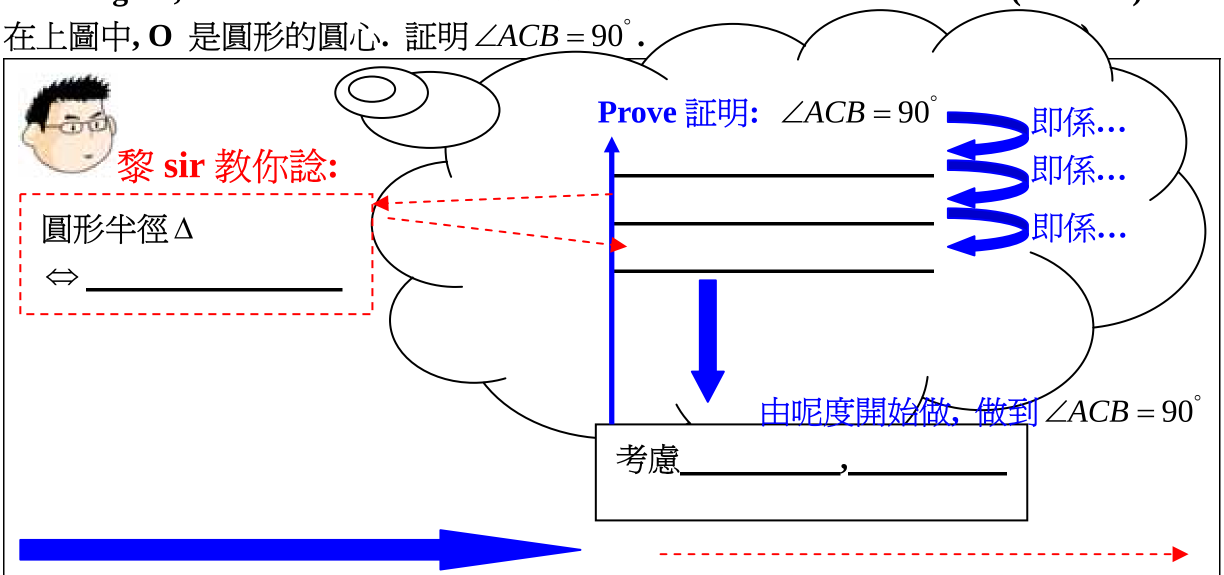
1. 找平行邊 ⇔ ____________



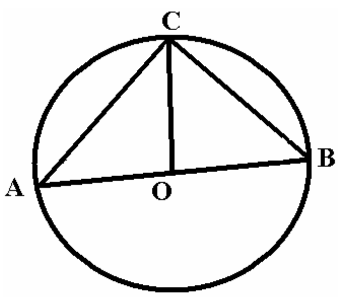
e.g. 12 例子十二

In the figure, O is the center of the circle. Prove $\angle ACB = 90^{\circ}$ **(4 marks)**

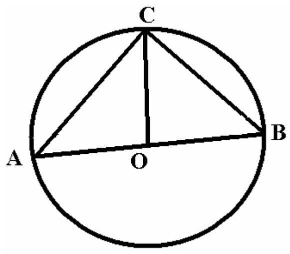
在上圖中, **O** 是圓形的圓心. 証明 $\angle ACB = 90^{\circ}$.



藍箭嘴: **Bottom-up Approach** 由下至上法 紅箭嘴: **Association Approach** 聯想法.



Solutions 題解:



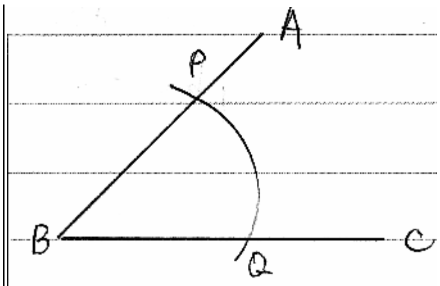
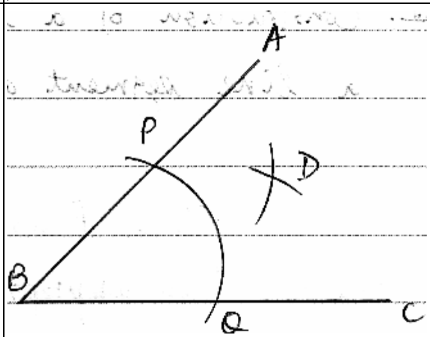
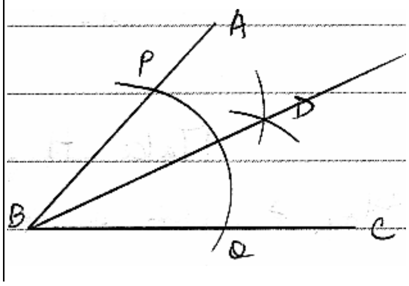
黎 sir 提提你:

1. 圓形半徑Δ ⇔ ______________________________

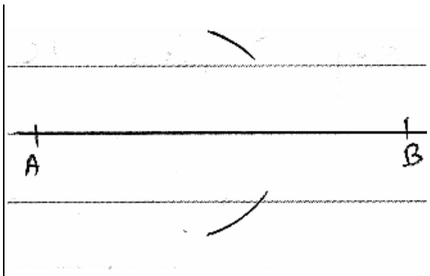
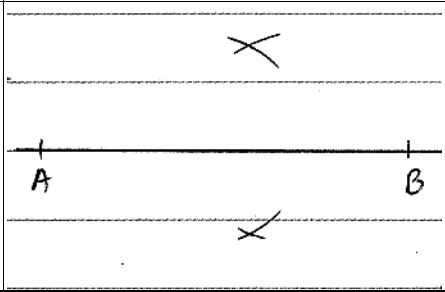
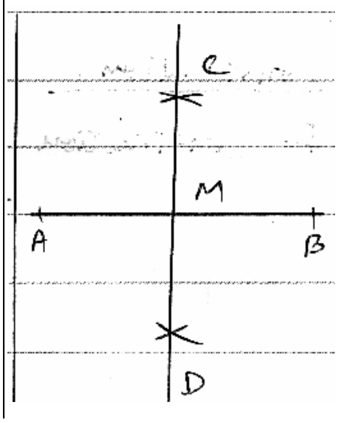
2. 找Δ內角 ⇔ ______________________________

(C) Construction of Geometry 幾何繪圖

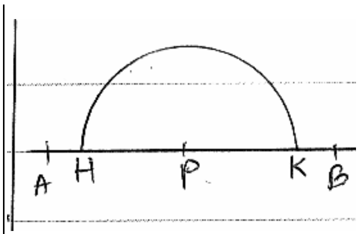
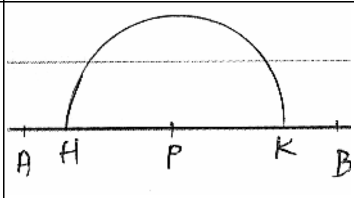
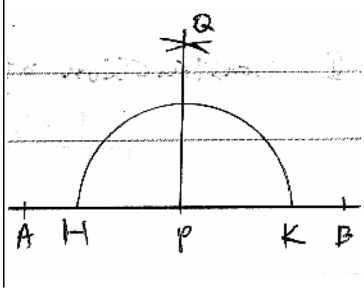
1. Construction of Angle bisector of an angle.

Step 1	Take B as the center. Choose an appropriate radius, and draw an arc to intersect BA and BC at P and Q respectively.	
Step 2	Take P and Q as centers, select a radius longer than $\frac{1}{2}$ PQ and draw two arcs such that these two arcs intersect at D.	
Step 3	Join BD. BD is the angle bisector of $\angle ABC$	

2. Construction of perpendicular bisector of a line segment.

Step 1	Take A as the center. Choose a radius longer than $\frac{1}{2}AB$ and draw an arc above and below the line segment AB.	
Step 2	Take B as the center. Use the same radius and draw another two ones to intersect the arcs at C and D.	
Step 3	Join CD. CD intersects AB at M, M is the mid-point of AB. CD is the perpendicular bisector of AB	

3. Construction of a line passing through a given point on a line segment and perpendicular to that line segment.

Step 1	Take P as the center. Choose an appropriate radius and draw an arc to intersect AB at H and K.	
Step 2	Take H and K as centers. Use a radius longer than $\frac{1}{2}HK$ and draw two arcs such that they meet at Q.	
Step 3	Join PQ. Then $PQ \perp AB$.	

4. Construction of a line passing through a point lying outside a line segment and perpendicular to that line segment.

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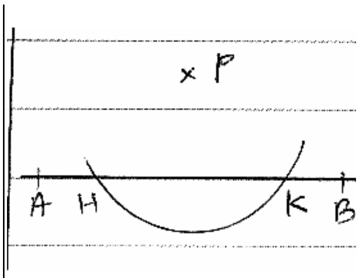
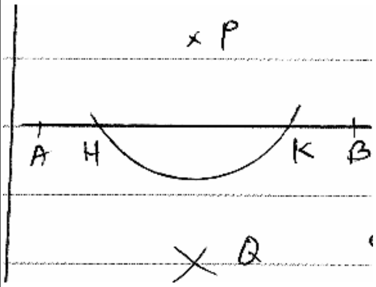
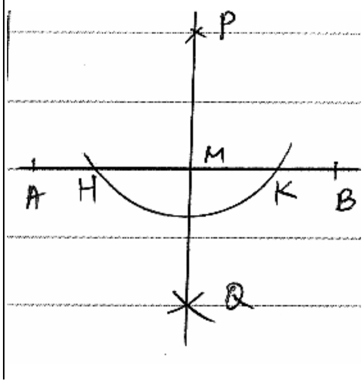
<td>Step 1</td>
<td>Take P as the center. Choose an appropriate radius and draw an arc to intersect AB at H and K.</td>
<td>
</td>
</tr>
<tr>
<td>Step 2</td>
<td>Take H and K as centers. Choose a radius longer than $\frac{1}{2}HK$ and draw two arcs such that the two arcs meet at Q.</td>
<td>
</td>
</tr>
<tr>
<td>Step 3</td>
<td>Join PQ. The line PQ intersects AB at M, and $PQ \perp AB$</td>
<td>
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