香 港 考 試 及 評 核 局
HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

2 0 0 9 年 香 港 中 學 會 考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2009

附加數學

ADDITIONAL MATHEMATICS

評 卷 參 考

MARKING SCHEME

本評卷參考乃香港考試及評核局專爲今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視爲標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations and Assessment Authority for markers' reference. The Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Authority is counting on the co-operation of markers/teachers in this regard.

General Instructions To Markers

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept / technique had been used.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. Unless the form of the answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they are correct.

5. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

6. In the marking scheme, marks are classified into the following three categories:

'M' marks	–	awarded for applying correct methods
'A' marks	–	awarded for the accuracy of the answers
Marks without 'M' or 'A'	–	awarded for correctly completing a proof or arriving at an answer given in the question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. (I.e. Markers should follow through candidates' work in awarding 'M' marks.) However, 'A' marks for the corresponding answers should NOT be awarded, unless otherwise specified.

7. In the marking scheme, steps which can be skipped are enclosed by dotted rectangles, whereas alternative answers are enclosed by solid rectangles.

8. (a) Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.
   (b) In case a certain degree of accuracy had been specified in the question, answers not accurate up to that degree should not be accepted. For answers with an excess degree of accuracy, deduct 1 mark (*pp*) for the first time it happened. In any case, do not deduct any marks for excess degree of accuracy in those steps where candidates failed to score any marks.

9. Marks may be deducted for poor presentation (*pp*). The symbol (*pp*−1) should be used to denote 1 mark deducted for *pp*.
   (a) In section A, at most deduct 1 mark for *pp* in each question, up to a maximum of 2 marks. For similar *pp*, deduct 1 mark for the first time that it occurs. Do not penalize candidates twice in section A for the same *pp*.
   (b) In section B, at most deduct 1 mark for *pp* in the whole section.
   (c) In any case, do not deduct any marks for *pp* in those steps where candidates could not score any marks.

10. In section B, marks may be deducted for wrong / no unit (*u*). The symbol (*u*−1) should be used to denote 1 mark deducted for *u*.
   (a) At most deduct 1 mark for *u* in the whole section B.
   (b) In any case, do not deduct any marks for *u* in those steps where candidates could not score any marks.

11. Marks entered in the Page Total Box should be the NET total scored on that page.

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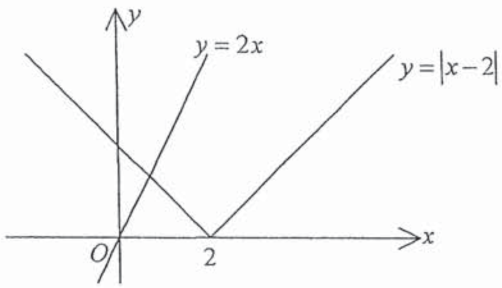
Solution	Marks	Remarks
1. (a) $\int(4x+1)^2\,dx=\frac{(4x+1)^3}{3\cdot 4}+C$	1M	For $\int x^n\,dx=\frac{x^{n+1}}{n+1}$
$=\frac{(4x+1)^3}{12}+C$	1A	Withhold 1A if either one of these C's was omitted
Alternative Solution $\int(4x+1)^2\,dx=\int(16x^2+8x+1)\,dx$	1M	
$=\frac{16}{3}x^3+4x^2+x+C$	1A	Withhold 1A if either one of these C's was omitted
(b) $\int\sin 3\theta\cos\theta\,d\theta=\int\frac{\sin 4\theta+\sin 2\theta}{2}\,d\theta$	1M+1A	
$=\frac{-\cos 4\theta}{8}-\frac{\cos 2\theta}{4}+C$	1A	Withhold 1A if either one of these C's was omitted
	(5)	
2. (a) $y^2+5y-6\geq 0$		
$(y+6)(y-1)\geq 0$	1A	
$y\leq -6$ or $y\geq 1$	1A	
(b) From (a), put $y=x^2$.	1M	
$\therefore$ $x^2\leq -6$ or $x^2\geq 1$		
no solution or $x\leq -1$ or $x\geq 1$	1M	For no solution
$x\leq -1$ or $x\geq 1$	1A	
	(5)	
3. (a) The required family is $x-3y+7+k(3x-y-11)=0$, where k is a real number.	1A	OR $3x-y-11+k(x-3y+7)=0$
(b) $\therefore$ $2-3(1)+7+k[3(2)-1-11]=0$	1M	
which gives $k=1$	1A	
Hence the required line is $x-3y+7+3x-y-11=0$		
i.e. $x-y-1=0$	1A	
	(4)	
4. $2x=\lvert x-2\rvert$		
Case 1: $x\geq 2$ $2x=x-2$ $x=-2$ (rejected) i.e. no solution	1A	For considering 2 cases
Case 2: $x<2$	1M	For considering 2 cases
$2x=2-x$ $x=\frac{2}{3}$	1A	
Conclusively, $x=\frac{2}{3}$.	1A	

Solution	Marks	Remarks
Alternative Solution (1)		
$2x = \|x-2\|$		
$2x = \pm(x-2)$	1M	
$x=-2$ or $\frac{2}{3}$	1A	
By checking, $x=-2$ should be rejected.	1A	
Conclusively, $x=\frac{2}{3}$.	1A	
Alternative Solution (2)		
	1A 1A	For $y=2x$ For $y=\|x-2\|$
From the graph, the solution can be obtained by solving $y=2x$ and $y=2-x$.	1M	
i.e. $x=\frac{2}{3}$	1A	
	(4)	
5. For $n=1$,		
L.H.S. $=1\times 4=4$		
R.H.S. $=\frac{1}{3}(1)(1+1)(1+5)=4$		
$\therefore$ L.H.S. = R.H.S. and so the statement is true for $n=1$.	1	
Assume $1\times 4+2\times 5+3\times 6+\cdots+k(k+3)=\frac{1}{3}k(k+1)(k+5)$, where k is a positive integer.	1	
$1\times 4+2\times 5+3\times 6+\cdots+k(k+3)+(k+1)(k+1+3)$		
$=\frac{1}{3}k(k+1)(k+5)+(k+1)(k+4)$ (by the assumption)	1	
$=\frac{1}{3}(k+1)(k^2+5k+3k+12)$		
$=\frac{1}{3}(k+1)(k+2)(k+6)$	1	
$=\frac{1}{3}(k+1)(k+1+1)(k+1+5)$		
Hence the statement is true for $n=k+1$.		
By the principle of mathematical induction, the statement is true for all positive integers n .	1	Follow through
	(5)	
6. (a) $y^3+x^3y=10$		
$3y^2\frac{dy}{dx}+3x^2y+x^3\frac{dy}{dx}=0$	1A+1A	1A for either of the last 2 terr 1A for all correct
$\frac{dy}{dx}=\frac{-3x^2y}{x^3+3y^2}$	1A	

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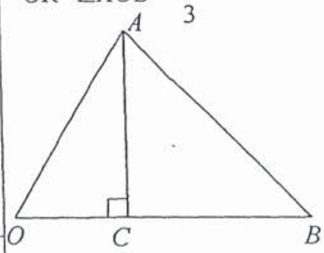
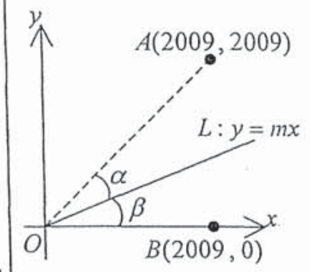
Solution	Marks	Remarks
Alternative Solution		
$x^3y+y^3=10$		
$x=\left(\frac{10}{y}-y^2\right)^{\frac{1}{3}}$		
$\frac{dx}{dy}=\frac{1}{3}\left(\frac{10}{y}-y^2\right)^{\frac{-2}{3}}\left(\frac{-10}{y^2}-2y\right)$	1A+1A	
$=\frac{-2}{3}\left(\frac{y}{10-y^3}\right)^{\frac{2}{3}}\left(\frac{y^3+5}{y^2}\right)$		
$\frac{dy}{dx}=\frac{-3y^{\frac{4}{3}}(10-y^3)^{\frac{2}{3}}}{2(y^3+5)}$	1A	
(b) $\left.\frac{dy}{dx}\right\|_{(1,2)}=\frac{-3(1)^2(2)}{(1)^3+3(2)^2}=\frac{-6}{13}$	1M	
Hence the equation of the tangent to C at $(1, 2)$ is		
$y-2=\frac{-6}{13}(x-1)$		
i.e. $6x+13y-32=0$	1A	
	(5)	
7. (a) $\mathbf{a}\cdot\mathbf{b}=\|\mathbf{a}\|\|\mathbf{b}\|\cos\angle AOB$		
$24=6\cdot 8\cdot\cos\angle AOB$	1M	
$\cos\angle AOB=\frac{1}{2}$		
$\angle AOB=60°$	1A	OR $\angle AOB=\frac{\pi}{3}$
(b) $\|\mathbf{c}\|=\|\mathbf{a}\|\cos\angle AOC$		
$=6\cdot\frac{1}{2}$	1M	
$=3$	1A	
	(4)	
8. $\frac{d}{dx}(\sqrt{x+1})=\lim_{h\to 0}\frac{\sqrt{x+h+1}-\sqrt{x+1}}{h}$	1A	Withhold 1A if $\lim_{h\to 0}$ was omitted
$=\lim_{h\to 0}\frac{(x+h+1)-(x+1)}{h(\sqrt{x+h+1}+\sqrt{x+1})}$	1M	For rationalisation
$=\lim_{h\to 0}\frac{1}{\sqrt{x+h+1}+\sqrt{x+1}}$	1A	For $\frac{1}{\sqrt{x+h+1}+\sqrt{x+1}}$
$=\frac{1}{2\sqrt{x+1}}$	1A	
	(4)	

Solution	Marks	Remarks
9. The slope of OA is 1 .	1A	
Let the angle between of OA and L be α and the inclination of L be β .		
Since L bisects $\angle AOB$, $\alpha = \beta$.		
$\tan\alpha = \tan\beta$	1A	
$\left\|\frac{1-m}{1+m\cdot 1}\right\| = m$	1M	Absolute sign can be omitted
$1-m = m+m^2$ or $-1+m = m+m^2$	1A	
$m^2+2m-1=0$		
$m=\frac{-2+\sqrt{8}}{2}$ or $m=\frac{-2-\sqrt{8}}{2}$ (rejected)		
i.e. $m=\sqrt{2}-1$	1A	
Alternative Solution (1)		
The equations of OA and OB are $x-y=0$ and $y=0$ respectively.	1A	For BOTH
Hence the equation of the angle bisectors are		
$\left\|\frac{x-y}{\sqrt{1^2+(-1)^2}}\right\| = \left\|\frac{y}{\sqrt{0^2+1^2}}\right\|$	1M+1A	
$x-y=\pm\sqrt{2}y$	1A	
$y=\frac{1}{1+\sqrt{2}}x$ or $y=\frac{1}{1-\sqrt{2}}x$ (rejected)		
i.e. $m=\frac{1}{1+\sqrt{2}}$ $=\sqrt{2}-1$	1A	
Alternative Solution (2)		
The slope of OA is 1 .	1A	
$\therefore$ $\angle AOB = 45°$		
$\therefore$ $m=\tan 22.5°$	1A	
Consider $\tan 45° = \tan(2\times 22.5°)$	1M	
i.e. $1=\frac{2m}{1-m^2}$	1A	
$m^2+2m-1=0$		
$m=\frac{-2+\sqrt{8}}{2}$ or $\frac{-2-\sqrt{8}}{2}$ (rejected)		
i.e. $m=\sqrt{2}-1$	1A	
	(5)	
10. (a) $8\cos x = \sec^2 x$		
$\cos^3 x = \frac{1}{8}$	1M	
$\cos x = \frac{1}{2}$		
$x=\frac{\pi}{3}$ for $0<x<\frac{\pi}{2}$	1A	

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Solution	Marks	Remarks
(b) The area of the shaded region		
$= \int_0^{\frac{\pi}{3}} (8\cos x - \sec^2 x)\,dx$	1M	For $\int (y_1 - y_2)\,dx$
$= [8\sin x - \tan x]_0^{\frac{\pi}{3}}$	1A	For primitive function
$= 8 \cdot \frac{\sqrt{3}}{2} - \sqrt{3}$		
$= 3\sqrt{3}$	1A	
	(5)	
11. The general term in the expansion of $\left(x^2 + \frac{1}{x}\right)^{20}$ is ${}_{20}C_r (x^2)^{20-r} \left(\frac{1}{x}\right)^r$	1M	OR ${}_{20}C_r (x^2)^r \left(\frac{1}{x}\right)^{20-r}$
$= {}_{20}C_r x^{40-3r}$	1A	OR $= {}_{20}C_r x^{3r-20}$
(a) In the term x^{16}, $40 - 3r = 16$ $\therefore\ r = 8$	1M	OR $3r - 20 = 16$ OR $r = 12$
Hence the coefficient is ${}_{20}C_8 = 125970$	1A	
(b) In the constant term, $40 - 3r = 0$		OR $3r - 20 = 0$
$r = \frac{40}{3}$ which is not an integer	1A	OR $r = \frac{20}{3}$ which is not …
Hence the constant term = 0 .	1A	OR there is no constant term
<u>Alternative Solution</u>		
$\left(x^2+\frac{1}{x}\right)^{20} = (x^2)^{20} + {}_{20}C_1 (x^2)^{19}\left(\frac{1}{x}\right) + {}_{20}C_2 (x^2)^{18}\left(\frac{1}{x}\right)^2 + {}_{20}C_3 (x^2)^{17}\left(\frac{1}{x}\right)^3$ $+ {}_{20}C_4 (x^2)^{16}\left(\frac{1}{x}\right)^4 + {}_{20}C_5 (x^2)^{15}\left(\frac{1}{x}\right)^5 + {}_{20}C_6 (x^2)^{14}\left(\frac{1}{x}\right)^6$ $+ {}_{20}C_7 (x^2)^{13}\left(\frac{1}{x}\right)^7 + {}_{20}C_8 (x^2)^{12}\left(\frac{1}{x}\right)^8 + {}_{20}C_9 (x^2)^{11}\left(\frac{1}{x}\right)^9$ $+ {}_{20}C_{10} (x^2)^{10}\left(\frac{1}{x}\right)^{10} + {}_{20}C_{11} (x^2)^9\left(\frac{1}{x}\right)^{11} + {}_{20}C_{12} (x^2)^8\left(\frac{1}{x}\right)^{12}$ $+ {}_{20}C_{13} (x^2)^7\left(\frac{1}{x}\right)^{13} + {}_{20}C_{14} (x^2)^6\left(\frac{1}{x}\right)^{14} + \cdots$	2M	
$= \cdots + {}_{20}C_8 \cdot x^{16} + \cdots + {}_{20}C_{13} \cdot x + {}_{20}C_{14} \cdot \frac{1}{x^2} + \cdots$	2A	
(a) In the expansion, the coefficient of x^{16} is ${}_{20}C_8 = 125970$	1A	
(b) In the expansion, the constant term = 0 .	1A	OR there is no constant term
	(6)	

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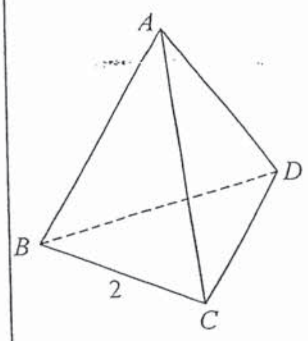
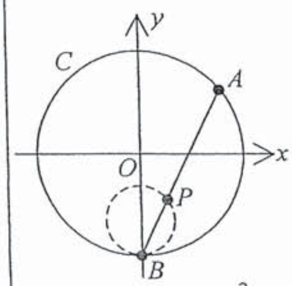
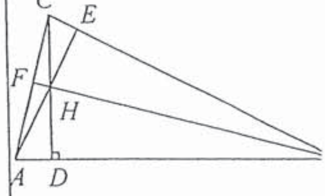
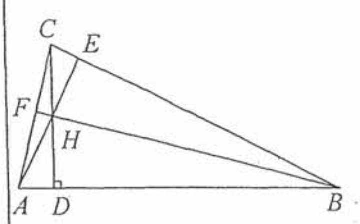
Solution	Marks	Remarks
12. Let M be the mid-point of BC.		
$AM \perp BC$ and $DM \perp BC$	} 1M	
Hence the required angle is $\angle AMD$.		
$AM = DM = \sqrt{2^2 - 1^2}$		
$= \sqrt{3}$	1A	
In ΔAMD, $2^2 = \sqrt{3}^2 + \sqrt{3}^2 - 2(\sqrt{3})(\sqrt{3})\cos\angle AMD$ (cosine formula)	1M	
$\cos\angle AMD = \frac{1}{3}$	1A	
$\angle AMD \approx 71°$ (correct to the nearest degree)	1A	
Alternative Solution Let N be the mid-point of AD.		
In ΔAMN, $\sin\angle AMN = \frac{1}{\sqrt{3}}$	1M+1A	
$\angle AMN = 35.26438968°$		
$\therefore \angle AMD = 2\angle AMN \approx 71°$ (correct to the nearest degree)	1A	
	(5)	
13. (a) $x = \frac{1(h)+2(0)}{1+2}$ and $y = \frac{1(k)+2(-1)}{1+2}$	1M	
$\therefore$ $h = 3x$ and $k = 3y + 2$	1A+1A	
(b) Since $A(h, k)$ lies on C, $h^2 + k^2 = 1$.		
Hence, $(3x)^2 + (3y+2)^2 = 1$.	1M	
i.e. $3x^2 + 3y^2 + 4y + 1 = 0$ which is the equation of the locus of P.	1A	OR $x^2 + \left(y + \frac{2}{3}\right)^2 = \frac{1}{9}$
	(5)	
14. (a) $\overrightarrow{AH} = \text{p} + \text{q}$	1A	
	(1)	
(b) $\overrightarrow{AC} = \text{p} + 2\text{q}$	1A	
$\therefore$ $\overrightarrow{AE} = \frac{r(\text{p}+2\text{q}) + 1(\lambda\text{p})}{r+1}$		
$= \frac{(r+\lambda)\text{p} + 2r\text{q}}{r+1}$	1A	
Since A, H and E are collinear,		
$\frac{\left(\frac{r+\lambda}{r+1}\right)}{1} = \frac{\left(\frac{2r}{r+1}\right)}{1}$	1M	
$r = \lambda$	1	
	(4)	

Solution	Marks	Remarks
(c) (i) Since H is the orthocentre of ΔABC, $\overrightarrow{AH}\cdot\overrightarrow{BC}=0$.		OR $\overrightarrow{BH}\cdot\overrightarrow{AC}=0$
$(\mathbf{p}+\mathbf{q})\cdot(\mathbf{p}+2\mathbf{q}-\lambda\mathbf{p})=0$	1M	OR $(\mathbf{p}+\mathbf{q}-\lambda\mathbf{p})\cdot(\mathbf{p}+2\mathbf{q})=0$
$(1-\lambda)\lvert\mathbf{p}\rvert^2+2\lvert\mathbf{q}\rvert^2=0$ ($\mathbf{p}\cdot\mathbf{q}=0$ since $CD\perp AB$)	1A	
$(1-\lambda)(1)^2+2(2)^2=0$		
$\lambda=9$	1A	
By (b), $r=9$.		
$\therefore\ \overrightarrow{AE}=\dfrac{(9+9)\mathbf{p}+2\cdot 9\mathbf{q}}{9+1}$		
$=\dfrac{9}{5}(\mathbf{p}+\mathbf{q})$	1A	
(ii) $\overrightarrow{BH}=(\mathbf{p}+\mathbf{q})-9\mathbf{p}$		
$=\mathbf{q}-8\mathbf{p}$	1A	
Let $\dfrac{AF}{FC}=s$.		
$\therefore\ \overrightarrow{BF}=\dfrac{s\overrightarrow{BC}+\overrightarrow{BA}}{s+1}$		
$=\dfrac{s(\mathbf{p}+2\mathbf{q}-9\mathbf{p})-9\mathbf{p}}{s+1}$	1M	
$=\dfrac{2s\mathbf{q}-(8s+9)\mathbf{p}}{s+1}$		
Since B, H and F are collinear,		
$\dfrac{\left(\dfrac{2s}{s+1}\right)}{1}=\dfrac{-\left(\dfrac{8s+9}{s+1}\right)}{-8}$		
$16s=8s+9$		
$s=\dfrac{9}{8}$	1A	
i.e. $\dfrac{AF}{FC}=\dfrac{9}{8}$		
<u>Alternative Solution</u>		
Let $\overrightarrow{BF}=\alpha\overrightarrow{BH}$		
$=\alpha(\mathbf{q}-8\mathbf{p})$		
$\overrightarrow{AF}=\overrightarrow{AB}+\overrightarrow{BF}$		
$=9\mathbf{p}+\alpha(\mathbf{q}-8\mathbf{p})$	1M	
$=(9-8\alpha)\mathbf{p}+\alpha\mathbf{q}$		
Since A, F and C are collinear,		
$\dfrac{9-8\alpha}{1}=\dfrac{\alpha}{2}$		
$\alpha=\dfrac{18}{17}$		
$\therefore\ \overrightarrow{AF}=\dfrac{9}{17}(\mathbf{p}+2\mathbf{q})$		
$\therefore\ \dfrac{AF}{FC}=\dfrac{\frac{9}{17}}{1-\frac{9}{17}}=\dfrac{9}{8}$	1A	
	(7)	

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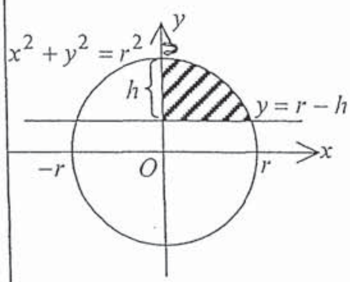
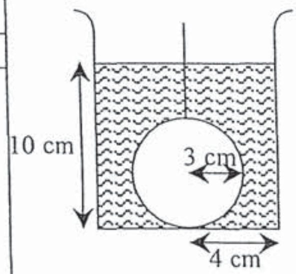
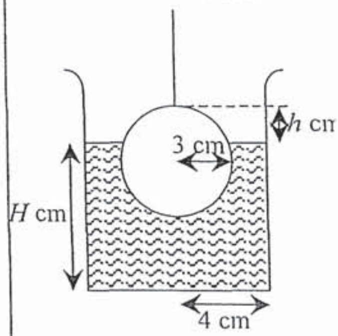
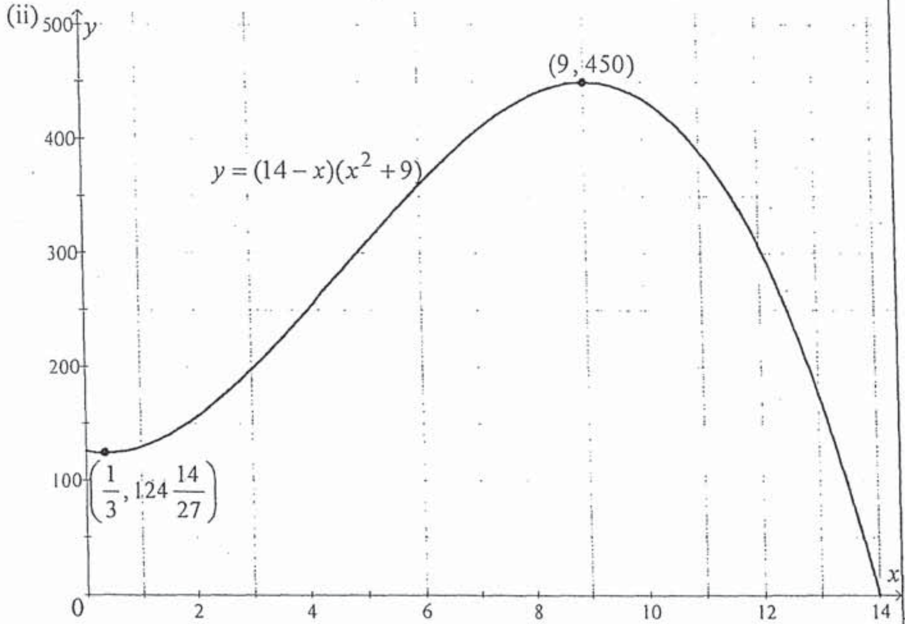
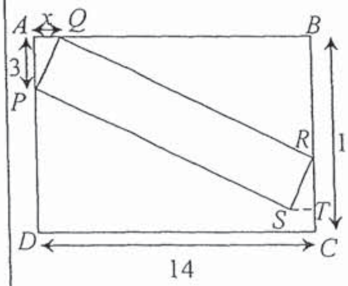
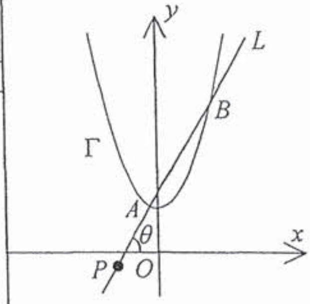
Solution	Marks	Remarks
15. (a) Volume $=\pi\int_{r-h}^{r}(r^2-y^2)\mathrm{d}y$	1M	For $V=\pi\int_a^b x^2\mathrm{d}y$
$=\pi\left[r^2y-\frac{y^3}{3}\right]_{r-h}^{r}$	1A	
$=\pi\left[r^3-\frac{r^3}{3}-r^2(r-h)+\frac{(r-h)^3}{3}\right]$	1A	
$=\pi\left(r^3-\frac{r^3}{3}-r^3+r^2h+\frac{r^3}{3}-r^2h+rh^2-\frac{h^3}{3}\right)$		
$=\pi rh^2-\frac{\pi h^3}{3}$	1	
	(4)	
(b) (i) Sum of the volumes of the water and the sphere $=\pi(4)^2(10)$	1A	
$\therefore\ \pi(4)^2H+\pi(3)h^2-\frac{\pi h^3}{3}=\pi(4)^2(10)$	1M	
$16H=\frac{h^3}{3}-3h^2+160$		
$H=\frac{1}{48}(h^3-9h^2+480)$ ---------- (1)	1	
(ii) Differentiate (1) with respect to t:		
$\frac{\mathrm{d}H}{\mathrm{d}t}=\frac{1}{48}(3h^2-18h)\frac{\mathrm{d}h}{\mathrm{d}t}$	1M	
When $h=3$, $\frac{\mathrm{d}H}{\mathrm{d}t}=\frac{-9}{16}\cdot\frac{\mathrm{d}h}{\mathrm{d}t}$ ---------- (2)	1A	
Since the sphere is being pulled out at the rate of $\frac{1}{4}$ cm s^{-1},		
$\frac{\mathrm{d}}{\mathrm{d}t}(H+h)=\frac{1}{4}$		
$\frac{\mathrm{d}H}{\mathrm{d}t}+\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{1}{4}$ ---------- (3)	1A	
Solving (2) and (3), we have $\frac{\mathrm{d}H}{\mathrm{d}t}=\frac{-9}{28}$ and $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{4}{7}$.		
(1) The rate of change of the water depth $=\frac{-9}{28}$ cm s^{-1}.	1A	OR <u>decreasing</u> at $\frac{9}{28}$ cm s^{-1}
(2) The rate of change of the distance between the top of the sphere and the water surface $=\frac{4}{7}$ cm s^{-1}.	1A	
	(8)	
16. (a) (i) $y=(14-x)(x^2+9)$		
$\frac{\mathrm{d}y}{\mathrm{d}x}=-(x^2+9)+(14-x)(2x)$		
$=-3x^2+28x-9$		
$\therefore\ \frac{\mathrm{d}y}{\mathrm{d}x}=0$ when $x=\frac{1}{3}$ or 9	1M	
$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-6x+28$		
$\left.\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right\|_{x=\frac{1}{3}}=26>0$ and $\left.\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right\|_{x=9}=-26<0$	1M	OR by using sign test

Solution	Marks	Remarks
$\therefore$ the minimum point is $\left(\frac{1}{3}, 124\frac{14}{27}\right)$ and the maximum point is $(9, 450)$.	1A	
(ii) 	1M+1A	1M for shape 1A for all correct
	(5)	
(b) (i) $\because \Delta APQ \sim \Delta BQR$, $\therefore \frac{AQ}{AP} = \frac{BR}{BQ}$		
i.e. $\frac{x}{3} = \frac{BR}{14-x}$		
$BR = \frac{x(14-x)}{3}$	1A	
$g(x) = PQ \times QR$		
$= \sqrt{x^2+3^2} \cdot \sqrt{(14-x)^2 + \left[\frac{x(14-x)}{3}\right]^2}$	1M	
$= \sqrt{x^2+9} \cdot (14-x)\sqrt{1+\frac{x^2}{9}}$ (since $x \le 14$)		
$= \frac{(14-x)(x^2+9)}{3}$	1	
(ii) Since S lies inside the cardboard, $BR + RT \le BC$.		
$\frac{x(14-x)}{3} + 3 \le 11$	1M	
$x^2 - 14x + 24 \ge 0$		
$x \le 2$ or $x \ge 12$		
$\because$ $0 \le x \le 14$		
$\therefore$ $0 \le x \le 2$ or $12 \le x \le 14$	1	
(iii) From(a)(ii), the curve $y = f(x)$ has no maximum points in the above range. Therefore, the greatest value of $f(x)$ is attained at one of the boundaries.	1M	
$f(0) = 126$, $f(2) = 156$, $f(12) = 306$, $f(14) = 0$		
Hence the greatest value of $g(x)$ is $\frac{f(12)}{3} = 102$.	1A	
	(7)	

Solution	Marks	Remarks
17. (a) $PQ=\sqrt{(-1+r\cos\theta+1)^2+\left(\frac{-1}{3}+r\sin\theta+\frac{1}{3}\right)^2}$	1M	
$=\sqrt{r^2(\cos^2\theta+\sin^2\theta)}$		
$=\lvert r\rvert$	1A	
	(2)	
(b) (i) Since A and B lies on L, they are in the form $\left(-1+r\cos\theta,\frac{-1}{3}+r\sin\theta\right)$, where $r=r_1$ or r_2.		
Since A lies on Γ, $\frac{-1}{3}+r\sin\theta=3(-1+r\cos\theta)^2+2$	1M	
$-1+3r\sin\theta=9r^2\cos^2\theta-18r\cos\theta+9+6$		
$9r^2\cos^2\theta-3(\sin\theta+6\cos\theta)r+16=0$ which has roots r_1 and r_2	1	
(ii) $AB^2=(r_2-r_1)^2$		
$=(r_1+r_2)^2-4r_1r_2$	1M	
$=\left[\frac{3(\sin\theta+6\cos\theta)}{9\cos^2\theta}\right]^2-4\left(\frac{16}{9\cos^2\theta}\right)$	1A	
$=\frac{\sin^2\theta+12\sin\theta\cos\theta-28\cos^2\theta}{9\cos^4\theta}$		
$=\frac{(\sin\theta-2\cos\theta)(\sin\theta+14\cos\theta)}{9\cos^4\theta}$	1	
(iii) Since L_1 is a tangent to Γ from P, $AB=0$.		
By (ii), $\sin\theta-2\cos\theta=0$ or $\sin\theta+14\cos\theta=0$.		
$\therefore\ \tan\theta=2$ or -14		
Hence the possible slopes of L_1 are 2 and -14.	1A	
When $\tan\theta=2$, $\sin\theta=\frac{2}{\sqrt5}$ and $\cos\theta=\frac{1}{\sqrt5}$.	1M	Either one
$\therefore$ the equation in (i) becomes $9r^2\left(\frac{1}{5}\right)-3\left(\frac{2}{\sqrt5}+\frac{6}{\sqrt5}\right)r+16=0$	1M	Either one
$9r^2-24\sqrt5r+80=0$		
$(3r-4\sqrt5)^2=0$		
$\therefore\ PR=r=\frac{4\sqrt5}{3}$		
When $\tan\theta=-14$, $\sin\theta=\frac{14}{\sqrt{197}}$ and $\cos\theta=\frac{-1}{\sqrt{197}}$.		
$\therefore$ the equation in (i) becomes $9r^2\left(\frac{1}{197}\right)-3\left(\frac{14}{\sqrt{197}}-\frac{6}{\sqrt{197}}\right)r+16=0$		Both
$9r^2-24\sqrt{197}r+3152=0$		
$(3r-4\sqrt{197})^2=0$		
$\therefore\ PR=r=\frac{4\sqrt{197}}{3}$	1A	

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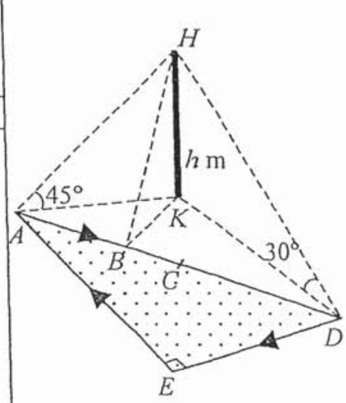
Solution	Marks	Remarks
Alternative Solution		
By (a), $PR = r$, where $r\ (>0)$ is the double root of the equation in (i).		
$\therefore\ r^2 = \dfrac{16}{9\cos^2\theta}$	1M	OR $2r = \dfrac{3(\sin\theta + 6\cos\theta)}{9\cos^2\theta}$
$= \dfrac{16(1+\tan^2\theta)}{9}$	1M	OR by considering a right-angled triangle
$= \dfrac{16(1+2^2)}{9}$ or $\dfrac{16[1+(-14)^2]}{9}$		
$\therefore\ PR = r = \dfrac{4\sqrt{5}}{3}$ or $\dfrac{4\sqrt{197}}{3}$	1A	
(iv) The slopes of tangents are -2 and 14 .	1A	
	(10)	
18. (a) $\dfrac{h}{DK} = \tan 30°$		
$\therefore\ DK = \sqrt{3}h$ m	1A	
	(1)	
(b) Let $AB = x$ m.		
Hence $BD = \dfrac{20}{10}x = 2x$	1M	
$AK = \dfrac{h}{\tan 45°} = h$		
Since B is closest to K, $KB \perp AD$.		
In ΔABK , $BK^2 = AK^2 - AB^2$.		
In ΔDBK , $BK^2 = DK^2 - DB^2$.		
$\therefore\ h^2 - x^2 = (\sqrt{3}h)^2 - (2x)^2$	1M	
$x = \sqrt{\dfrac{2}{3}}h$		
i.e. $AB = \sqrt{\dfrac{2}{3}}h$ m	1A	
	(3)	
(c) In ΔKAB, $BK^2 = h^2 - \left(\sqrt{\dfrac{2}{3}}h\right)^2 = \dfrac{h^2}{3}$	1M	
$BC = \dfrac{5}{10}AB = \dfrac{h}{\sqrt{6}}$		
Hence in ΔKBC, $KC = \sqrt{\dfrac{h^2}{3} + \dfrac{h^2}{6}} = \dfrac{h}{\sqrt{2}}$	1A	
Alternative Solution		
In ΔKAB, $\cos\angle KAB = \sqrt{\dfrac{2}{3}}$	1M	
$AC = \dfrac{15}{10}AB = \sqrt{\dfrac{3}{2}}h$		
Hence in ΔKAC, $KC = \sqrt{h^2 + \left(\sqrt{\dfrac{3}{2}}h\right)^2 - 2h\left(\sqrt{\dfrac{3}{2}}h\right)\sqrt{\dfrac{2}{3}}} = \dfrac{h}{\sqrt{2}}$	1A	



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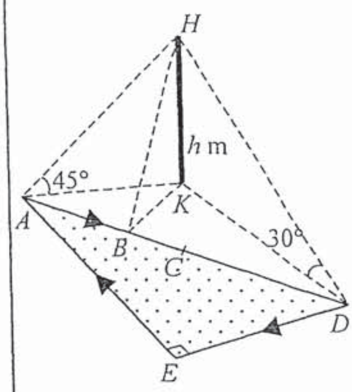
Solution	Marks	Remarks
$\therefore \tan \angle HCK = \dfrac{h}{\frac{h}{\sqrt{2}}} = \sqrt{2}$		
Alternative Solution		
In ΔHAB, $HB^2 = 2h^2 - \left(\sqrt{\frac{2}{3}}h\right)^2 = \frac{4}{3}h^2$	1M	
$BC^2 = \left(\frac{5}{10}AB\right)^2 = \frac{h^2}{6}$		
Hence in ΔHBC, $HC = \sqrt{\frac{4}{3}h^2 + \frac{h^2}{6}} = \sqrt{\frac{3}{2}}h$	1A	
$\therefore \sin \angle HCK = \dfrac{h}{\sqrt{\frac{3}{2}}h} = \sqrt{\frac{2}{3}}$		
$\therefore \angle HCK = 54.73561032°$		
i.e. the angle of elevation of H from C is $55°$ (correct to the nearest degree)	1A	
	(3)	
(d) (i) Let $DE = y$ m and $EA = z$ m.		
$y + z = \frac{40}{10}AB = 4\sqrt{\frac{2}{3}}h$	1A	For both
$y^2 + z^2 = \left(\frac{30}{10}AB\right)^2 = 6h^2$ (Pythagoras theorem)		
Hence y and z are the roots of the equation		
$r^2 + \left(4\sqrt{\frac{2}{3}}h - r\right)^2 = 6h^2$		
$2r^2 - 8\sqrt{\frac{2}{3}}hr + \frac{14}{3}h^2 = 0$		
$\therefore yz = \frac{7}{3}h^2$	1A	
Alternative Solution		
$2yz = (y+z)^2 - (y^2 + z^2)$		
$= \left(4\sqrt{\frac{2}{3}}h\right)^2 - 6h^2$		
$\therefore yz = \frac{7}{3}h^2$	1A	
On the other hand, the area of the park $= \frac{1}{2}yz = 9450$		
$\therefore \frac{1}{2} \cdot \frac{7}{3}h^2 = 9450$		
$h = 90$	1A	
(ii) ΔAED can be inscribed in a semi-circle with diameter AD (converse of $\angle$ in semi-circle)		
Hence the vertical pole should be located at C, the centre of the semi-circle.	1M	
Let θ be the required angle.		
$\tan \theta = \dfrac{h-3}{\frac{h}{\sqrt{2}}} = \frac{29}{30}\sqrt{2}$		
i.e. $\theta = 54°$ (correct to the nearest degree)	1A	
	(5)	