2009-CE
A MATH

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2009

ADDITIONAL MATHEMATICS

Question-Answer Book

8.30 am – 11.00 am (2½ hours)

This paper must be answered in English

INSTRUCTIONS

1. Write your Candidate Number in the space provided on Page 1.
2. Stick barcode labels in the spaces provided on Pages 1, 3, 5, 7 and 9.
3. This paper consists of two sections, Section A and Section B. Section A carries 62 marks and Section B carries 48 marks.
4. Answer **ALL** questions in Section A. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
5. Answer any **FOUR** questions in Section B. Write your answers in the CE(A) answer book.
6. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string **INSIDE** the book.
7. The Question-Answer book and the CE(A) answer book must be handed in separately at the end of the examination.
8. All working must be clearly shown.
9. Unless otherwise specified, numerical answers must be **exact**.
10. In this paper, vectors may be represented by bold-type letters such as **u**, but candidates are expected to use appropriate symbols such as $\vec{u}$ in their working.
11. The diagrams in this paper are not necessarily drawn to scale.

Please stick the barcode label here.

Candidate Number							

	Marker's Use Only	Examiner's Use Only
	Marker No.	Examiner No.
Question No.	Marks	Marks
1		
2		
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4		
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6		
7		
8		
9		
10		
11		
12		
13		
Total (A)		

Checker's Use Only	Checker No.	Total (A)

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A\cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A\cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A\sin B = \cos(A-B) - \cos(A+B)$	

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Section A (62 marks)
Answer **ALL** questions in this section and write your answers in the spaces provided in this Question-Answer Book.

1. Find
   (a) $\int (4x+1)^2 \, dx$,
   (b) $\int \sin 3\theta \cos\theta \, d\theta$.

(5 marks)

2. Solve
   (a) $y^2 + 5y - 6 \geq 0$,
   (b) $x^4 + 5x^2 - 6 \geq 0$.

(5 marks)

3. Consider the straight lines $L_1 : x - 3y + 7 = 0$ and $L_2 : 3x - y - 11 = 0$.
   (a) Write down the equation of the family of straight lines passing through the point of intersection of L_1 and L_2 .
   (b) Find the equation of the straight line in the family in (a) which passes through the point $(2, 1)$.

(4 marks)

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4. Solve $2x = |x-2|$.

(4 marks)

5. Prove by mathematical induction that for all positive integers n ,

$$1 \times 4 + 2 \times 5 + 3 \times 6 + \cdots + n(n+3) = \frac{1}{3}n(n+1)(n+5) .$$

(5 marks)

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6. C is a curve with equation $y^3 + x^3 y = 10$.

(a) Find $\frac{dy}{dx}$.

(b) Find the equation of the tangent to the curve C at the point $(1, 2)$.

(5 marks)

7.

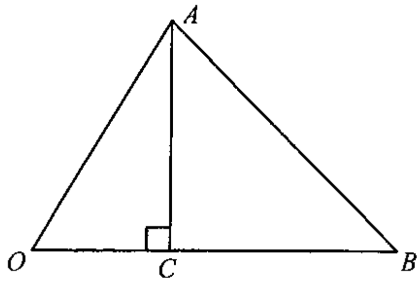

Figure 1

In Figure 1, AC is an altitude of ΔOAB . Let $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ be $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ respectively. It is given that $|\mathbf{a}| = 6$, $|\mathbf{b}| = 8$ and $\mathbf{a} \cdot \mathbf{b} = 24$. Find

(a) $\angle AOB$,

(b) $|\mathbf{c}|$.

(4 marks)

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8. Find $\frac{\mathrm{d}}{\mathrm{d}x}(\sqrt{x+1})$ from first principles.

(4 marks)

9. $A(2009, 2009)$, $B(2009, 0)$ are two points and $L: y = mx$ is a straight line passing through the origin O . If L bisects $\angle AOB$, find the value of m in surd form.

(5 marks)

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10. (a) Solve $8\cos x = \sec^2 x$ for $0 < x < \frac{\pi}{2}$.

(b)

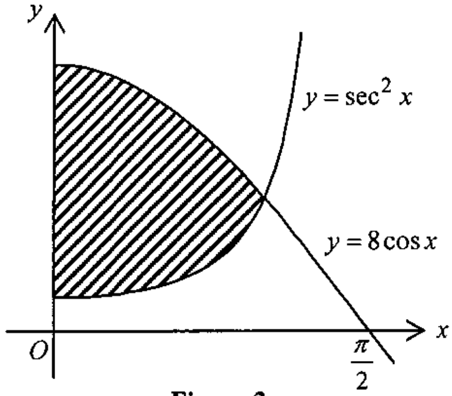

Figure 2

Figure 2 shows the graphs of $y = 8\cos x$ and $y = \sec^2 x$.
Find the area of the shaded region.

(5 marks)

11. In the binomial expansion of $\left(x^2 + \frac{1}{x}\right)^{20}$, find

(a) the coefficient of x^{16} ,
(b) the constant term.

(6 marks)

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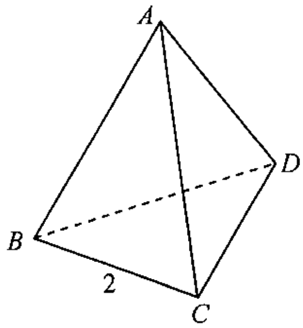

Figure 3

In Figure 3, $ABCD$ is a regular tetrahedron with length of each side 2 .
Find the angle between the planes ABC and BCD correct to the nearest degree.

(5 marks)

13. C is a circle with equation $x^2 + y^2 = 1$. $A(h, k)$ and $B(0, -1)$ are two points on C . Let $P(x, y)$ be the point dividing AB in ratio $2:1$.

(a) Express h in terms of x and k in terms of y .

(b) Find the equation of the locus of P as A moves on C .

(5 marks)

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Section B (48 marks)
Answer any **FOUR** questions in this section and write your answers in the CE(A) answer book.
Each question carries 12 marks.

14.

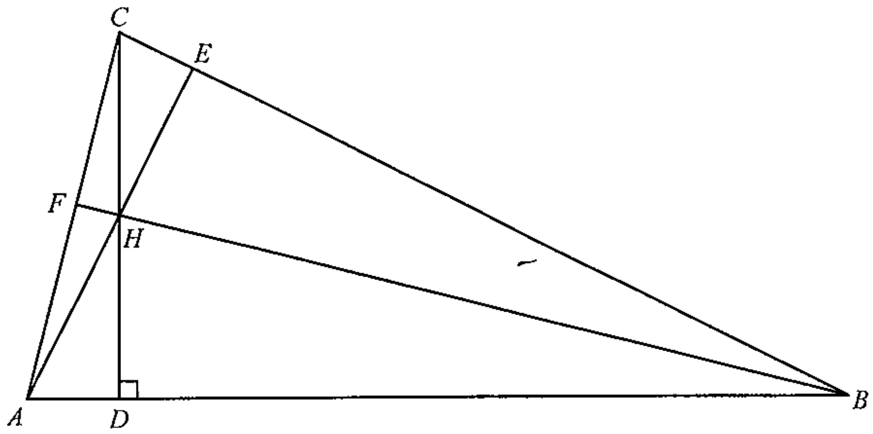

Figure 4

In Figure 4, CD is an altitude of ΔABC and H is the mid-point of CD. AH and BH are produced to meet BC and AC at E and F respectively.

Let $\mathbf{p}$, $\lambda\mathbf{p}$ $(\lambda > 1)$ and $\mathbf{q}$ be $\overrightarrow{AD}$, $\overrightarrow{AB}$ and $\overrightarrow{DH}$ respectively. Let $\dfrac{BE}{EC} = r$.

(a) Find $\overrightarrow{AH}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

(1 mark)

(b) Express $\overrightarrow{AE}$ in terms of λ, r, $\mathbf{p}$ and $\mathbf{q}$.

Hence show that $r = \lambda$.

(4 marks)

(c) It is given that $|\mathbf{p}| = 1$, $|\mathbf{q}| = 2$ and H is the orthocentre of ΔABC.

(i) Find $\overrightarrow{AE}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

(ii) Find $\dfrac{AF}{FC}$.

(7 marks)

15. (a)

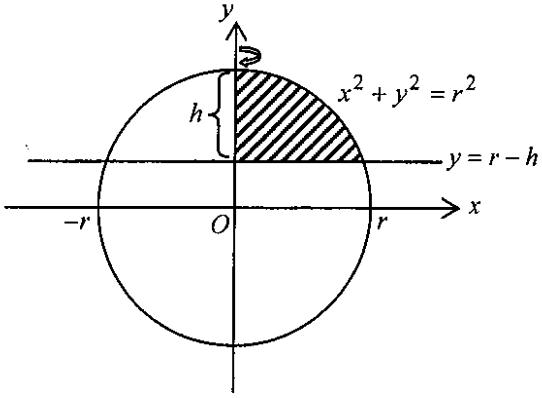

Figure 5

In Figure 5, the shaded region is bounded by the circle $x^2 + y^2 = r^2$, the y-axis and the straight line $y = r - h$, where $0 \le h \le 2r$. Show that the volume of the solid generated by revolving the shaded region about the y-axis is $\pi r h^2 - \dfrac{\pi h^3}{3}$.

(4 marks)

(b)

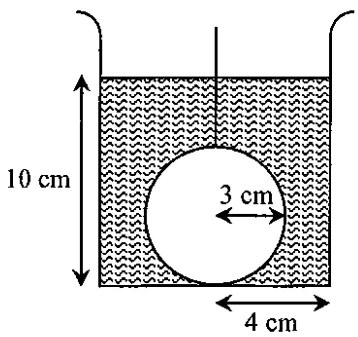

Figure 6

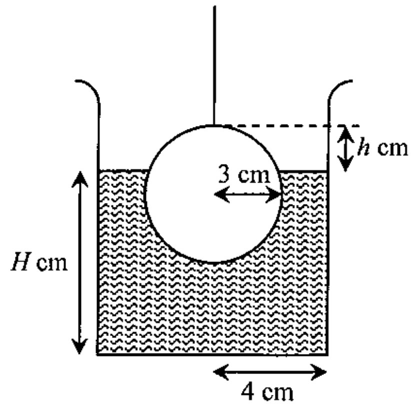

Figure 7

A metal sphere of radius 3 cm, with a thin string attached, is placed inside a circular cylindrical container of base radius 4 cm. Water is poured into the container until the depth of the water is 10 cm (see Figure 6). The sphere is then being pulled vertically out of the water. Let H cm and h cm be the depth of the water and the distance between the top of the sphere and the water surface respectively (see Figure 7).

(i) Prove that $H = \dfrac{1}{48}(h^3 - 9h^2 + 480)$.

(ii) The sphere is being pulled at a constant speed of $\dfrac{1}{4}$ cm s^{-1} . At the instant when $h = 3$, find the rate of change of

(1) the depth of the water,

(2) the distance between the top of the sphere and the water surface.

(8 marks)

16. (a) Let $f(x)=(14-x)(x^2+9)$.

(i) Find the coordinates of all the maximum and minimum points of the curve $y=f(x)$.

(ii) Sketch the graph of $y=f(x)$ for $0\le x\le 14$ in the answer book.
(Note: the suggested range of values of y is $0\le y\le 500$.)

(5 marks)

(b)

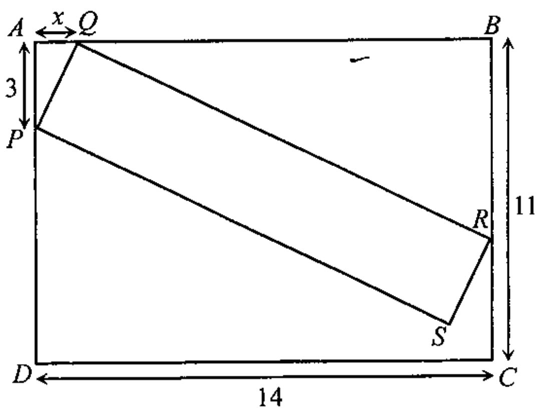

Figure 8

Figure 8 shows a rectangular cardboard $ABCD$ with $BC=11$ and $DC=14$.
A variable rectangle $PQRS$ is cut from the cardboard according to the following rules:

[1] P is a fixed point on AD such that $AP=3$,
[2] Q and R are points on AB and BC respectively.

Let x be the length of AQ and $g(x)$ be the area of the rectangle $PQRS$.

(i) By considering ΔAPQ and ΔBQR , express BR in terms of x .
Hence show that $g(x)=\dfrac{(14-x)(9+x^2)}{3}$.

(ii) By considering the fact that point S lies inside the cardboard $ABCD$, show that the range of values of x is given by

$$0\le x\le 2 \quad \text{or} \quad 12\le x\le 14 \ .$$

(iii) Using (a)(ii), find the greatest value of $g(x)$ in the range shown in (b)(ii).

(7 marks)

17. Let L be the straight line passing through $P\left(-1, \frac{-1}{3}\right)$ with angle of inclination θ. It is known that the coordinates of any point Q on L are in the form $\left(-1 + r\cos\theta, \frac{-1}{3} + r\sin\theta\right)$, where r is a real number.

(a) Find the length of PQ in terms of r.

(2 marks)

(b)

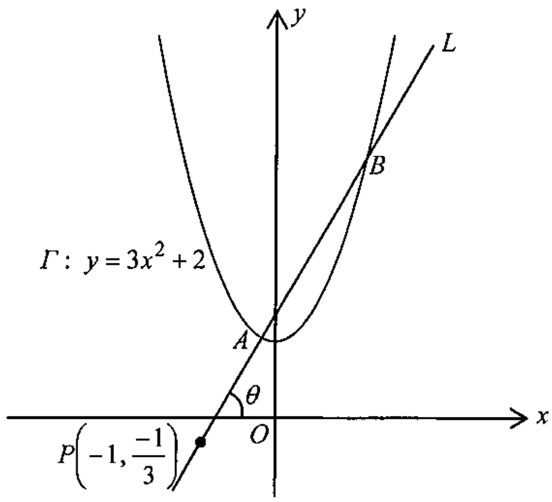

Figure 9

In Figure 9, L cuts the parabola $\Gamma: y = 3x^2 + 2$ at points A and B. Let $PA = r_1$ and $PB = r_2$.

(i) Show that r_1 and r_2 are the roots of the equation

$$9r^2\cos^2\theta - 3(\sin\theta + 6\cos\theta)r + 16 = 0 \ .$$

(ii) Using (b)(i), show that $AB^2 = \dfrac{(\sin\theta - 2\cos\theta)(\sin\theta + 14\cos\theta)}{9\cos^4\theta}$.

(iii) Let L_1 be a tangent to Γ from P, with point of contact R.
Using the above results, find the two possible slopes of L_1 and the corresponding lengths of PR.

(iv) Let L_2 be a tangent to Γ passing through the point $\left(1, \frac{-1}{3}\right)$. Write down the two possible slopes of L_2.

(10 marks)

18.

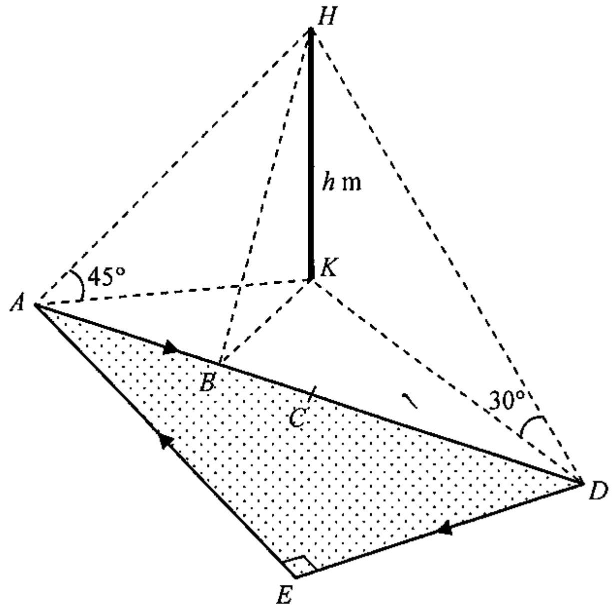

Figure 10

Figure 10 shows a park AED on a horizontal ground. The park is in the form of a right-angled triangle surrounded by a walking path with negligible width. Henry walks along the path at a constant speed. He starts from point A at 7:00 am. He reaches points B, C and D at 7:10 am, 7:15 am and 7:30 am respectively and returns to A via point E. The angles of elevation of H, the top of a tower outside the park, from A and D are $45°$ and $30°$ respectively. At point B, Henry is closest to the point K which is the projection of H on the ground. Let $HK = h$ m.

(a) Express DK in terms of h.

(1 mark)

(b) Show that $AB = \sqrt{\frac{2}{3}}h$ m.

(3 marks)

(c) Find the angle of elevation of H from C correct to the nearest degree.

(3 marks)

(d) Henry returns to A at 8:10 am. It is known that the area of the park is 9450 m^2.

(i) Find h.

(ii) A vertical pole of length 3 m is located such that it is equidistant from A, D and E. Find the angle of elevation of H from the top of the pole correct to the nearest degree.

(5 marks)

END OF PAPER