2008-CE
A MATH

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2008

ADDITIONAL MATHEMATICS

Question-Answer Book

8.30 am – 11.00 am (2½ hours)
This paper must be answered in English

1. Write your Candidate Number in the space provided on Page 1.

2. Stick barcode labels in the spaces provided on Pages 1, 3, 5, 7 and 9.

3. This paper consists of **TWO** sections, Section A and Section B. Section A carries 62 marks and Section B carries 48 marks.

4. Answer **ALL** questions in Section A. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

5. Answer any **FOUR** questions in Section B. Write your answers in the CE(B) answer book.

6. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, fill in the question number and stick a barcode label on each sheet, and fasten them with string **INSIDE** the book.

7. The Question-Answer Book and the CE(B) answer book must be handed in separately at the end of the examination.

8. All working must be clearly shown.

9. Unless otherwise specified, numerical answers must be **exact**.

10. In this paper, vectors may be represented by bold-type letters such as **u**, but candidates are expected to use appropriate symbols such as $\vec{u}$ in their working.

11. The diagrams in the paper are not necessarily drawn to scale.

Please stick the barcode label here.

Candidate Number							

	Marker's Use Only	Examiner's Use Only
	Marker No.	Examiner No.
Section A Question No.	Marks	Marks
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
Section A Total		

Checker's Use Only	Section A Total	

Checker No.	

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

****

Section A (62 marks)
Answer **ALL** questions in this section and write your answers in the spaces provided in this Question-Answer Book.

1. Find $\int (8x+5)^{250}\,dx$.

(2 marks)

2. (a) Expand $\left(2x+\frac{1}{x}\right)^3$.

   (b) Find the coefficient of x in the expansion of $(3x^2-x-5)\left(2x+\frac{1}{x}\right)^3$.

(4 marks)

3. Find the value of $\tan 22.5°$ in surd form.

(4 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Page Total

Please stick the barcode label here.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Go on to the next page

Page Total

4.

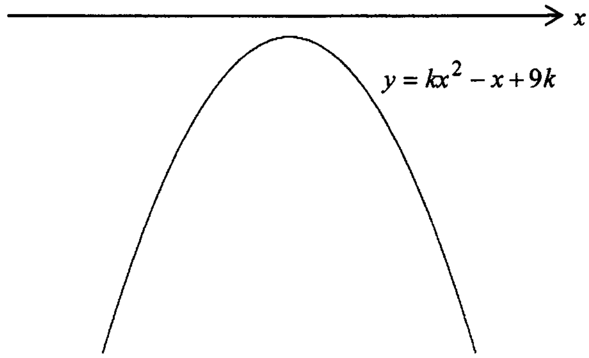

Figure 1

The graph of $y = kx^2 - x + 9k$ lies below the x-axis, where $k \neq 0$ (see Figure 1). Find the range of possible values of k.

(4 marks)

5. Prove, by mathematical induction, that

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{1}{4}n^2(n+1)^2$$

for all positive integers n.

(5 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Page Total

Please stick the barcode label here.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

6. Find the equation of the tangent to the curve $y=\frac{3x}{x^2+2}$ at the point $(2, 1)$.

(5 marks)

7. It is given that $\overrightarrow{OA}=2\mathbf{i}+3\mathbf{j}$ and $\overrightarrow{OB}=5\mathbf{i}+6\mathbf{j}$. If P is a point on AB such that $\overrightarrow{PB}=2\overrightarrow{AP}$, find the unit vector in the direction of $\overrightarrow{OP}$.

(5 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Page Total

Please stick the barcode label here.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

8. $A(1, 3)$ and $B(2, 2)$ are two points on a rectangular coordinate plane. P is a point on the straight line $2x + y - 3 = 0$ such that the area of ΔAPB is 1. If P lies on the second quadrant, find the coordinates of P.

(5 marks)

9. (a) Express $\sin x + \sqrt{3}\cos x$ in the form $r\sin(x + \alpha)$, where $r > 0$ and $0° < \alpha < 90°$.

(b) Using (a), find the least and the greatest values of $\sin x + \sqrt{3}\cos x$ for $0° \le x \le 90°$.

(5 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Page Total

Please stick the barcode label here.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Page Total

10.

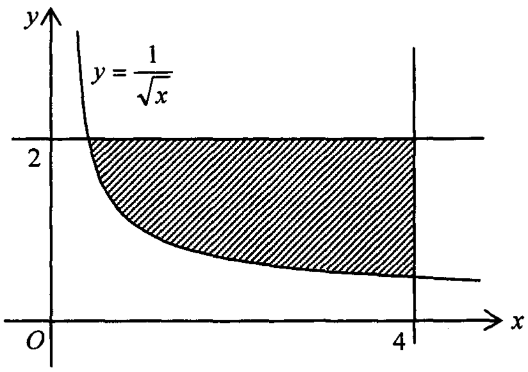

Figure 2

Find the area of the shaded region bounded by the curve $y = \frac{1}{\sqrt{x}}$ and the straight lines $y = 2$, $x = 4$ (see Figure 2).

(5 marks)

11. (a) Solve $|x-3|+3 = x$, where $x \le 3$.

(b) Solve $|x-3|+3 = x$.

(5 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Page Total

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Go on to the next page

Page Total

12. There is a variable straight line $L_1 : y = kx + 1$, where k is real. Let L_2 be the straight line passing through the origin and perpendicular to L_1 . L_1 and L_2 intersect at the point P .

(a) Find the coordinates of P in terms of k .

(b) Find the equation of the locus of P as k varies.

(6 marks)

13. Let $f(x) = x(x-6)^2$.

(a) Find the maximum and minimum points of the graph of $y = f(x)$.

(b) Sketch the graph of $y = f(x)$.

(7 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Page Total

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

SECTION B (48 marks)
Answer any **FOUR** questions in this section. Each question carries 12 marks.
Write your answers in the CE(B) answer book.

14. There is a family of circles $F: x^2 + y^2 - 10x + k(2x + y) = 0$, where k is real.

(a) It is known that all circles in F pass through the origin O and a fixed point P. Find the coordinates of P.

(2 marks)

(b) Find the equation of the locus of the centres of all circles in F.

(4 marks)

(c) Two circles in F have two common tangents L_1 and L_2. It is given that the equation of L_1 is $y + 5 = 0$. Find

(i) the coordinates of the intersecting point of L_1 and L_2,

(ii) the equation of L_2.

(6 marks)

15.

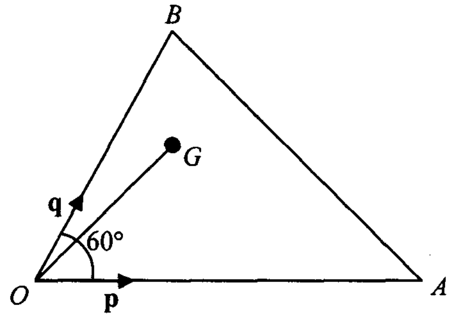

Figure 3

In Figure 3, **p** and **q** are unit vectors with angle between them 60° . Let $\overrightarrow{OA} = 4\mathbf{p}$, $\overrightarrow{OB} = 3\mathbf{q}$ and $\overrightarrow{OG} = \frac{2}{3}\mathbf{p} + \frac{5}{3}\mathbf{q}$.

(a) Find $\mathbf{p} \cdot \mathbf{q}$.

(1 mark)

(b) Show that $OG \perp AB$.

Hence show that G is the orthocentre of ΔOAB .

(5 marks)

(c)

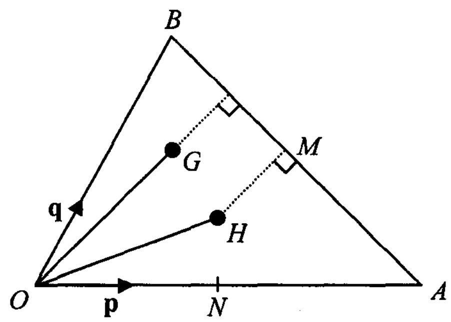

Figure 4

In Figure 4, H is the circumcentre of ΔOAB , M and N are the mid-points of AB and OA respectively. Let $HM : OG = t : 1$.

By expressing $\overrightarrow{HM}$ and $\overrightarrow{HN}$ in terms of t, **p** and **q**, find $\overrightarrow{OH}$ in terms of **p** and **q**.

(6 marks)

16.

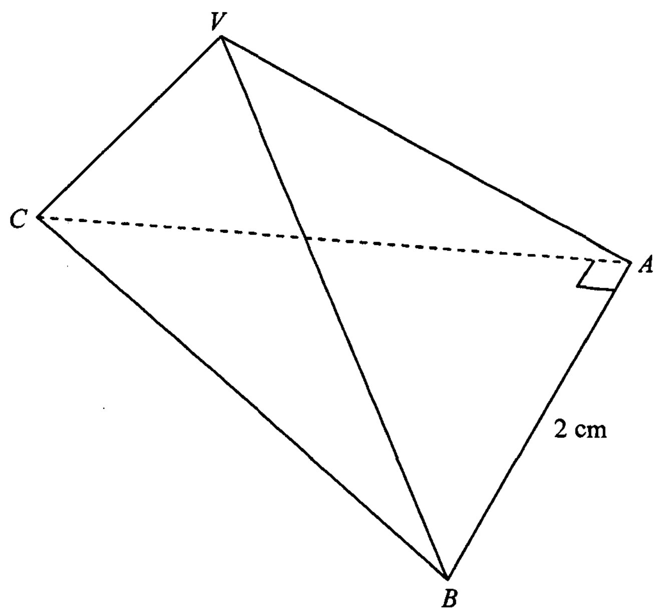

Figure 5

Figure 5 shows a triangular pyramid $VABC$. The base of the pyramid is a right-angled triangle with $AB = 2$ cm and $\angle BAC = 90°$. ΔVAB and ΔVAC are equilateral triangles.

(a) Explain why the angle between the planes VAB and ABC **cannot** be represented by $\angle VAC$.

(1 mark)

(b) Let D and E be the mid-points of AB and BC respectively.

(i) Show that the angle between the planes VAB and ABC can be represented by $\angle VDE$.

(ii) Show that $\angle VED = 90°$.

(7 marks)

(c) Find the distance between the point C and the plane VAB.

(4 marks)

17. (a) Let $f(x) = x\sin x$.

(i) Show that $\int_0^{\pi}[f(x) + f(\pi - x)]dx = 2\pi$.

(ii)

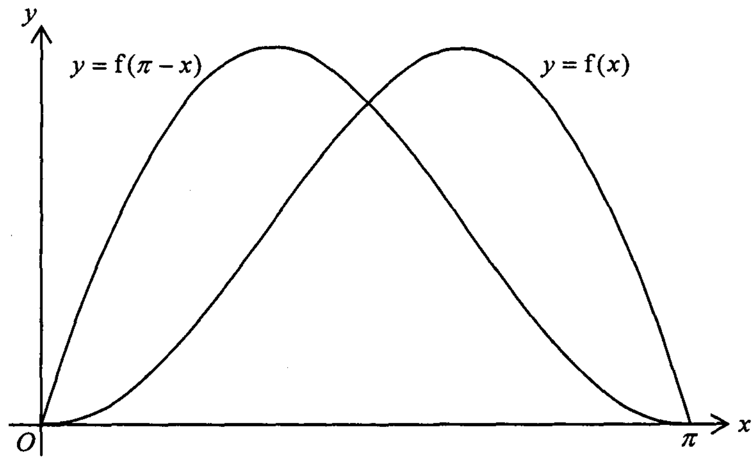

Figure 6

Figure 6 shows the graphs of $y = f(x)$ and $y = f(\pi - x)$ for $0 \le x \le \pi$.

Using (a)(i), and by considering the symmetry of the graphs of $y = f(x)$ and $y = f(\pi - x)$, write down the value of $\int_0^{\pi} x\sin x\,dx$.

(5 marks)

(b) (i) Find $\frac{d}{dx}(x^2 \sin x)$, and hence evaluate $\int_0^{\pi} x^2 \cos x\,dx$.

(ii) R denotes the region bounded between $y = x\sin\frac{x}{2}$ and the x-axis for $0 \le x \le \pi$. Find the volume of the solid formed by revolving R about the x-axis.

(7 marks)

18.

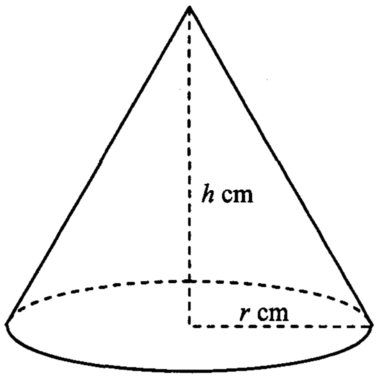

Figure 7

In a Winter Carnival, a display item is in the shape of a right circular cone. It is made of ice and a stabilizer so that the display remains in the shape of a right circular cone with the volume remaining constant. Within the duration of the Carnival, the height of the cone decreases at a constant rate of 2 cm per day. At time t days after the beginning of the Carnival, the base radius and height of the cone are r cm and h cm respectively (see Figure 7).

(a) Show that $\frac{dr}{dt} = \frac{r}{h}$.

(4 marks)

(b) Let S cm^2 be the curved surface area of the cone.

(i) Show that $\frac{d}{dt}(S^2) = \frac{2\pi^2 r^2}{h}(2r^2 - h^2)$.

(ii) At the beginning of the Carnival, the height of the cone is 1.2 times the base radius. The gatekeeper of the Carnival claims that the curved surface area of the display increases during the whole period of the Carnival.

Do you agree with the gatekeeper? Explain your answer.

(8 marks)

END OF PAPER