# 2007 附加數學 (只設英文版本)

## 評卷參考 *

## Marking Scheme

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| This document was prepared for markers' reference. It should not be regarded as a set of model answers. Candidates and teachers who were not involved in the marking process are advised to interpret its contents with care. |

### General Instructions To Markers

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates would use alternative methods not specified in the marking scheme. Markers should be patient in marking these alternative answers. In general, a correct alternative answer merits all the marks allocated to that part, unless a particular method was specified in the question.

2. In the marking scheme, marks are classified as follows :

   'M' marks – awarded for knowing a correct method of solution and attempting to apply it

   'A' marks – awarded for the accuracy of the answer

   Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. The symbol $(pp-1)$ should be used to denote marks deducted for poor presentation (*pp*). Note the following points:

   (a) In Section A, at most deduct 1 mark for *pp* in each question, up to a maximum of 2 marks.
   In Section B, at most deduct 1 mark for *pp* in the whole section.

   (b) In Section A, deduct only 1 mark for similar *pp*s for the first time that it occurs, i.e. do not penalise candidates twice in Section A for the same *pp*.

   (c) In any case, do not deduct any marks for *pp* in those steps where candidates failed to score any marks.

   (d) Some cases in which marks should be deducted for pp are specified in the marking scheme. However, the lists are by no means exhaustive. Markers should exercise their professional judgment to give *pp*s whenever applicable.

5. In Section A, The symbol $(u-1)$ should be used to denote marks deducted for wrong/no units in the final answers (if applicable). Note the following points:

   (a) In Section A, at most deduct 1 mark for wrong/no units.

   (b) Do not deduct any marks for wrong/no units in case candidate's answer was already wrong.

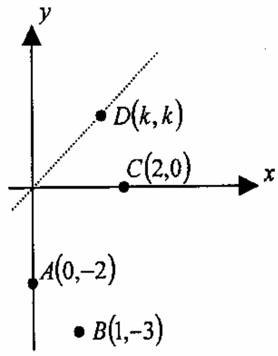
6. (a) Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.

   (b) In case a certain degree of accuracy had been specified in the question, answers not accurate up to that degree should not be accepted. For answers with an excess degree of accuracy, deduct 1 mark for the first time if happened. In any case, do not deduct any marks for excess degree of accuracy in those steps where candidates failed to score any marks.

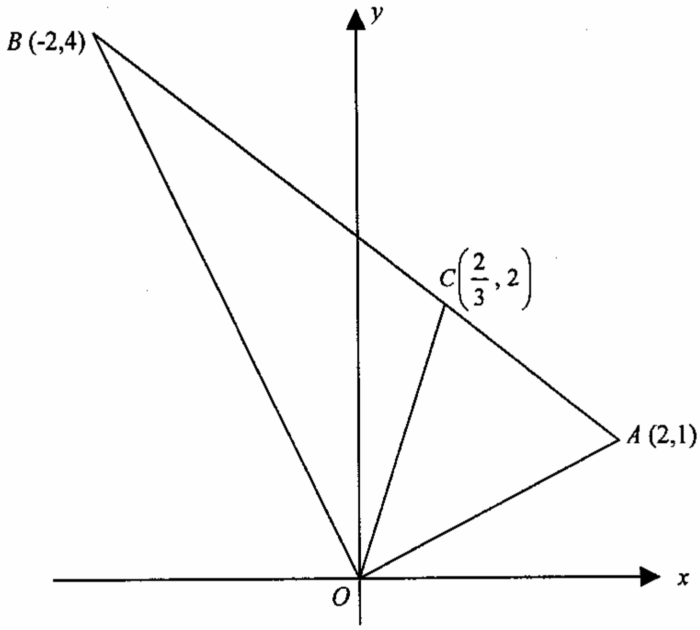
7. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

8. Unless the form of answer was specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they were correct.

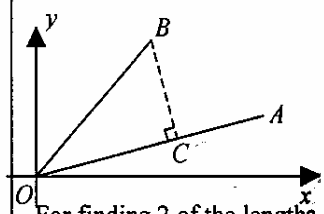
*此部分只設英文版本。

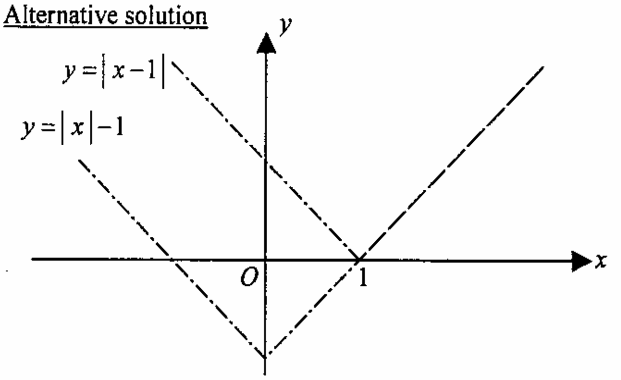
| Solution | Marks | Remarks |
|---|---|---|
| 1. $\int \frac{x^4+1}{x^2}\,\mathrm{d}x = \int\left(x^2+\frac{1}{x^2}\right)\mathrm{d}x$ | 1A | |
| $=\frac{x^3}{3}-\frac{1}{x}+c$ | 1M+1A | 1M for $\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1}$<br>(pp-1) if $c$ was omitted |
| | (3) | |
| 2. The area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 0 & -2 \\ 1 & -3 \\ 2 & 0 \\ 0 & -2 \end{vmatrix}$ | 1M | Either one |
| $=\frac{1}{2}(-4+2+6)$<br>$=2$ | 1A | |
| The area of $\Delta ACD = \frac{1}{2}\begin{vmatrix} 0 & -2 \\ 2 & 0 \\ k & k \\ 0 & -2 \end{vmatrix}$ | | |
| $=\frac{1}{2}(2k-2k+4)$<br>$=2$ | | |
| Hence the area of the quadrilateral $ABCD = 2+2=4$ | 1A | |
| Alternative Solution<br>The area of the quadrilateral $ABCD = \frac{1}{2}\begin{vmatrix} 0 & -2 \\ 1 & -3 \\ 2 & 0 \\ k & k \\ 0 & -2 \end{vmatrix}$ | 1M |  |
| $=\frac{1}{2}(2k-2k+2+6)$ | 1A | |
| $=4$ | 1A | |
| | (3) | |
| 3. $\cos x-\sqrt{2}\cos 2x+\cos 3x=0$ | | |
| $2\cos 2x\cos x-\sqrt{2}\cos 2x=0$ | 1M | For sum to product formula |
| Alternative solution<br>$\cos x-\sqrt{2}\cos 2x+\cos 3x=0$<br>$\cos x-\sqrt{2}\cos 2x+\cos 2x\cos x-\sin 2x\sin x=0$<br>$\cos x-\cos 2x\left(\sqrt{2}-\cos x\right)-2\sin^2 x\cos x=0$<br>$\cos x\left(1-2\sin^2 x\right)+\cos 2x\left(\cos x-\sqrt{2}\right)=0$ | 1M | For compound angle formula |
| $\cos 2x\left(2\cos x-\sqrt{2}\right)=0$ | | |
| $\cos 2x=0$ or $\cos x=\frac{\sqrt{2}}{2}$ | 1A | |
| $2x=360^\circ n\pm 90^\circ$ (or $180^\circ n+90^\circ$) or $x=360^\circ n\pm 45^\circ$ | 1M | For any general solution |
| $x=180^\circ n\pm 45^\circ$ (or $90^\circ n+45^\circ$) or $360^\circ n\pm 45^\circ$ | 1A | Accept radian measure: |
| i.e. $x=180^\circ n\pm 45^\circ$ (or $90^\circ n+45^\circ$) | | $n\pi\pm\frac{\pi}{4}$ or $2n\pi\pm\frac{\pi}{4}$ |
| | (4) | |

| Solution | Marks | Remarks |
|---|---|---|
| 4. $\frac{d}{dx}(x^2+1)=\lim_{\Delta x\to 0}\frac{[(x+\Delta x)^2+1]-[x^2+1]}{\Delta x}$ | 1M | For $\frac{f(x+\Delta x)-f(x)}{\Delta x}$ |
| $=\lim_{\Delta x\to 0}\frac{x^2+2x\Delta x+(\Delta x)^2-x^2}{\Delta x}$ | 1M | For expanding $(x+\Delta x)^2$ |
| OR $=\lim_{\Delta x\to 0}\frac{(x+\Delta x-x)(x+\Delta x+x)}{\Delta x}$ | 1M | For factorizing $(x+\Delta x)^2-x^2$ |
| $=\lim_{\Delta x\to 0}(2x+\Delta x)$ | 1A | (pp-1) if $\lim_{\Delta x\to 0}$ was omitted |
| $=2x$ | 1A | or written improperly |
| | (4) | |
| 5. For $n=1$, | | |
| L.H.S. $=\frac{1}{a-1}-\frac{1}{a}=\frac{a-(a-1)}{a(a-1)}=\frac{1}{a(a-1)}$ | | |
| R.H.S. $=\frac{1}{a(a-1)}$ | | |
| $\therefore$ L.H.S. = R.H.S. and so the statement is true for $n=1$. | 1 | |
| Assume $\frac{1}{a-1}-\frac{1}{a}-\frac{1}{a^2}-\cdots-\frac{1}{a^k}=\frac{1}{a^k(a-1)}$, where $k$ is a positive integer. | 1 | |
| $\therefore \frac{1}{a-1}-\frac{1}{a}-\frac{1}{a^2}-\cdots-\frac{1}{a^{k+1}}=\frac{1}{a^k(a-1)}-\frac{1}{a^{k+1}}$ by the assumption | 1 | |
| $=\frac{a-(a-1)}{a^{k+1}(a-1)}$ | | |
| $=\frac{1}{a^{k+1}(a-1)}$ | 1 | Follow through |
| Hence the statement is also true for $n=k+1$. | | |
| By the principle of mathematical induction, the statement is true for all positive integers $n$. | 1 | |
| | (5) | |
| 6. $\sin 2x=\cos x$ | 1M | For finding intersection |
| $2\sin x\cos x=\cos x$ | | |
| $(2\sin x-1)\cos x=0$ | | |
| $\sin x=\frac{1}{2}$ or $\cos x=0$ | | |
| Alternative solution | | |
| $\sin 2x=\cos x$ | | |
| $\cos\left(\frac{\pi}{2}-2x\right)=\cos x$ (or $\sin 2x=\cos\left(\frac{\pi}{2}-x\right)$) | 1M | |
| $\frac{\pi}{2}-2x=x$ (or $2x=\frac{\pi}{2}-x$) (for $0<x<\frac{\pi}{2}$) | | |
| $x=\frac{\pi}{6}$ (for $0<x<\frac{\pi}{2}$) or $x=\frac{\pi}{2}$ | 1A | Accept $x=30°$ |
| Hence the shaded area $=\int_0^{\frac{\pi}{6}}(\cos x-\sin 2x)dx$ | 1M | For $A=\int_a^b(y_2-y_1)dx$ |
| $=\left[\sin x+\frac{\cos 2x}{2}\right]_0^{\frac{\pi}{6}}$ | 1A | For $\pm\left(\sin x+\frac{\cos 2x}{2}\right)$ |
| $=\frac{1}{4}$ | 1A | |
| | (5) | |

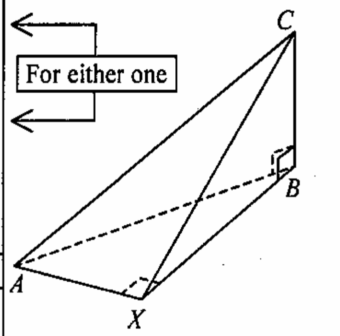
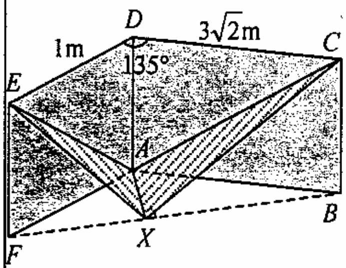
| Solution | Marks | Remarks |
|---|---|---|
| 7.  | | |
| (a) $C$ is $\left(\frac{1(-2)+2(2)}{1+2},\frac{1(4)+2(1)}{1+2}\right)=\left(\frac{2}{3},2\right)$ | 1A | |
| (b) The slopes of $OA$, $OC$ and $OB$ are $\frac{1}{2}$, 3 and $-2$ respectively. | 1M | For finding 2 of the slopes |
| $\tan\angle COA=\left\lvert\frac{(3)-\left(\frac{1}{2}\right)}{1+(3)\left(\frac{1}{2}\right)}\right\rvert=1$<br>$\tan\angle BOC=\left\lvert\frac{(-2)-(3)}{1+(-2)(3)}\right\rvert=1$ | 1M+<br>1A | 1M for $\tan\theta=\left\lvert\frac{m_1-m_2}{1+m_1m_2}\right\rvert$<br>(accept without absolute sign)<br>1A for either value correct |
| $\therefore \tan\angle COA=\tan\angle BOC$<br>Hence $\angle COA=\angle BOC$ and so $OC$ is the angle bisector of $\angle AOB$. | 1 | Follow through |
| Alternative solution (1)<br>$OA=\sqrt{5}, OC=\frac{2\sqrt{10}}{3}, OB=2\sqrt{5}, AC=\frac{5}{3}, CB=\frac{10}{3}$ | 1M | For finding 2 of the lengths |
| $\cos\angle COA=\frac{(\sqrt{5})^2+\left(\frac{2\sqrt{10}}{3}\right)^2-\left(\frac{5}{3}\right)^2}{2(\sqrt{5})\left(\frac{2\sqrt{10}}{3}\right)}=\frac{1}{\sqrt{2}}$<br>$\cos\angle BOC=\frac{(2\sqrt{5})^2+\left(\frac{2\sqrt{10}}{3}\right)^2-\left(\frac{10}{3}\right)^2}{2(2\sqrt{5})\left(\frac{2\sqrt{10}}{3}\right)}=\frac{1}{\sqrt{2}}$ | 1M+<br>1A | 1M for cosine formula<br>1A for either value correct |
| $\therefore \cos\angle COA=\cos\angle BOC$<br>Hence $\angle COA=\angle BOC$ and so $OC$ is the angle bisector of $\angle AOB$. | 1 | Follow through |

| Solution | Marks | Remarks |
| --- | --- | --- |
| Alternative solution (2) | | |
| The equations of $OA$ and $OB$ are $y=\frac{1}{2}x$ and $y=-2x$ respectively. | 1M | For finding both equations |
| The distance from $C$ to $OA$ is $\left\|\frac{\left(\frac{2}{3}\right)-2(2)}{\sqrt{1^2+2^2}}\right\|=\frac{2\sqrt{5}}{3}$ | 1M+<br>1A | 1M for distance formula<br>1A for either value correct |
| The distance from $C$ to $OB$ is $\left\|\frac{2\left(\frac{2}{3}\right)+(2)}{\sqrt{2^2+1^2}}\right\|=\frac{2\sqrt{5}}{3}$ | | |
| Since $C$ is equidistant from $OA$ and $OB$, so $\angle COA=\angle BOC$. | | |
| Hence $OC$ is the angle bisector of $\angle AOB$. | 1 | Follow through |
| Alternative solution (3) | | |
| The equations of $OA$, $OB$ and $OC$ are $y=\frac{1}{2}x$, $y=-2x$ and $y=3x$ respectively. | 1M | For finding 2 of the equations |
| The equation(s) of the angle bisector(s) of lines $OA$ and $OB$ is / are $\frac{2x+y}{\sqrt{2^2+1^2}}=-\frac{x-2y}{\sqrt{2^2+1^2}}$ or $\frac{2x+y}{\sqrt{2^2+1^2}}=\frac{x-2y}{\sqrt{2^2+1^2}}$ | 1M | |
| i.e. $y=3x$ or $x=-3y$ | 1A | |
| Hence $OC$ is the angle bisector of $\angle AOB$. | 1 | Follow through |
| Alternative solution (4) | | |
| $\overrightarrow{OA}=2\mathbf{i}+\mathbf{j}$, $\overrightarrow{OC}=\frac{2}{3}\mathbf{i}+2\mathbf{j}$, $\overrightarrow{OB}=-2\mathbf{i}+4\mathbf{j}$ | 1M | For finding 2 of the position vectors |
| $\overrightarrow{OA}\cdot\overrightarrow{OC}=\|\overrightarrow{OA}\|\cdot\|\overrightarrow{OC}\|\cos\angle COA$ | | |
| $\frac{4}{3}+2=\sqrt{5}\sqrt{\frac{40}{9}}\cos\angle COA$ | 1M | |
| $\cos\angle COA=\frac{1}{\sqrt{2}}$ | 1A | For either one |
| $\overrightarrow{OB}\cdot\overrightarrow{OC}=\|\overrightarrow{OB}\|\cdot\|\overrightarrow{OC}\|\cos\angle BOC$ | | |
| $\frac{-4}{3}+8=\sqrt{20}\sqrt{\frac{40}{9}}\cos\angle BOC$ | | |
| $\cos\angle BOC=\frac{1}{\sqrt{2}}$ | | |
| $\therefore\cos\angle COA=\cos\angle BOC$ | | |
| Hence $\angle COA=\angle BOC$ and so $OC$ is the angle bisector of $\angle AOB$. | 1 | Follow through |
| Alternative solution (5) | | |
| The areas of $\Delta COA:\Delta BOC=AC:CB$ | 1M | |
| $\therefore\frac{1}{2}(OC)(OA)\sin\angle COA:\frac{1}{2}(OC)(OB)\sin\angle BOC=AC:CB$ | | |
| $\frac{1}{2}(OC)\sqrt{2^2+1^2}\sin\angle COA:\frac{1}{2}(OC)\sqrt{(-2)^2+4^2}\sin\angle BOC=AC:CB$ | 1M | For area formula |
| $\sqrt{5}\sin\angle COA:2\sqrt{5}\sin\angle BOC=1:2$ | | |
| $\sin\angle COA=\sin\angle BOC$ | 1A | |
| Hence $\angle COA=\angle BOC$ and so $OC$ is the angle bisector of $\angle AOB$. | 1 | Follow through |
| | (5) | |

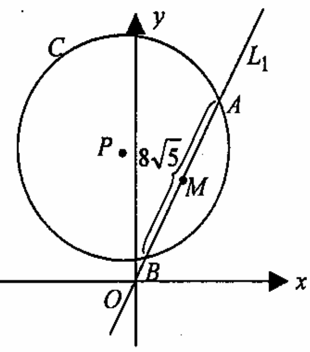
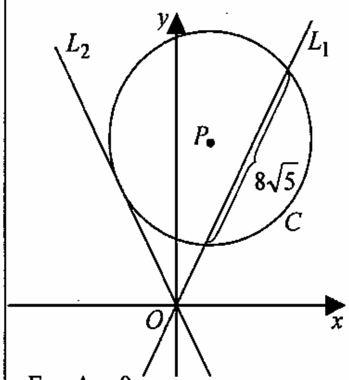
| Solution | Marks | Remarks |
|---|---|---|
| 8. (a) $\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}$ | 1A | OR $\overrightarrow{BC}=\overrightarrow{BO}+\overrightarrow{OC}$ |
| $=k(6\mathbf{i}+3\mathbf{j})-(2\mathbf{i}+6\mathbf{j})$ | | |
| $=(6k-2)\mathbf{i}+(3k-6)\mathbf{j}$ | 1A | |
| (b) $\because BC\perp OA,\ \therefore \overrightarrow{BC}\cdot\overrightarrow{OA}=0$ | | Accept $\overrightarrow{BC}\cdot\overrightarrow{OC}=0$ |
| $[(6k-2)\mathbf{i}+(3k-6)\mathbf{j}]\cdot(6\mathbf{i}+3\mathbf{j})=0$ | 1M |  |
| $(6k-2)(6)+(3k-6)(3)=0$ | 1M | |
| $k=\frac{2}{3}$ | 1A | |
| Alternative solution | | |
| $OA^2=45$, $OB^2=40$, $OC^2=45k^2$, $BC^2=(6k-2)^2+(3k-6)^2$ | 1M | For finding 2 of the lengths |
| Since $\Delta OBC$ is a right-angled triangle, | | |
| so $45k^2+(36k^2-24k+4)+(9k^2-36k+36)=40$ (Pythagoras theorem) | 1M | |
| $90k^2-60k=0$ | | |
| $k=\frac{2}{3}$ or 0 (rej.) | 1A | (pp-1) if arrow sign was omitted in most cases |
| | (5) | |
| 9. (a) By Pythagoras Theorem, $\frac{x^2}{4}+y^2=25$ | 1A | |
| (b) $\therefore \frac{x}{2}\cdot\frac{dx}{dt}+2y\cdot\frac{dy}{dt}=0$ | 1M+1A | 1M for differentiation |
| $\frac{dx}{dt}=\frac{-4y}{x}\cdot\frac{dy}{dt}$ | | |
| Alternative solution | | |
| $\frac{x}{2}+2y\cdot\frac{dy}{dx}=0$ | 1M | 1M for differentiation |
| $\frac{dy}{dx}=\frac{-x}{4y}$ | | |
| $\frac{dx}{dt}=\frac{\frac{dy}{dt}}{\frac{dy}{dx}}=\frac{-4y}{x}\cdot\frac{dy}{dt}$ | 1A | |
| By (a), when $y=3$, $x=8$ | 1M | For finding $x$ |
| $\therefore \left.\frac{dx}{dt}\right\|_{y=3}=\frac{-4\cdot 3}{8}(-2)=3$ | | |
| Alternative solution | | |
| $x=2\sqrt{25-y^2}$ | | |
| $\frac{dx}{dt}=\frac{2(-2y)}{2\sqrt{25-y^2}}\cdot\frac{dy}{dt}$ | 1M+1A | 1M for differentiation |
| $\left.\frac{dx}{dt}\right\|_{y=3}=\frac{-4(3)}{2\sqrt{25-3^2}}\cdot(-2)=3$ | 1M | 1M for $\sqrt{25-3^2}$ |
| Hence the rate of change of the distance between $A$ and $B$ is $3\ \mathrm{m\,s^{-1}}$ | 1A | Accept $\left.\frac{dx}{dt}\right\|_{y=3}=3\ \mathrm{m\,s^{-1}}$<br>(u-1) if $\mathrm{m\,s^{-1}}$ was omitted |
| | (5) | |

| Solution | Marks | Remarks |
|---|---|---|
| 10. (a) The equation of the given line is $\frac{x}{4}+\frac{y}{2}=1$. | 1M | For any straight line form |
| $\therefore f'(x)=y=2-\frac{x}{2}$ | | |
| Hence the slope of the tangent at $x=1$ is $f'(1)=\frac{3}{2}$. | 1M+1A | 1M for substituting $x=1$ |
| (b) There is only one turning point, with $x$-coordinate 4. | 1A | (pp-1) for $(4,0)$ |
| It is a maximum point since the slope changes from positive to negative. | 1A | OR since $f''(x)=\frac{-1}{2}<0$ |
| | (5) | |
| 11. (a) For $0\le x\le 1$, $\lvert x-1\rvert=\lvert x\rvert-1$ becomes | | |
| $1-x=x-1$ | 1A | |
| $x=1$ | 1A | |
| (b) For $x<0$, $\lvert x-1\rvert=\lvert x\rvert-1$ becomes | | |
| $1-x=-x-1$ | | |
| Hence there is no solution in this case. [For either one] | 1A | For considering BOTH $x<0$ and $x>1$ |
| For $x>1$, $\lvert x-1\rvert=\lvert x\rvert-1$ becomes | 1M | |
| $x-1=x-1$ | | |
| which is true for all real $x$ | | |
| Hence the solution in this case is $x>1$. [For either one] | | |
| Combining all three cases, the overall solution is $x\ge 1$. | 1A | |
| Alternative solution | | |
|  | 1M | For attempting to use graphical method |
| | 1A | For either graph |
| From the graph, we see that the condition for $\lvert x-1\rvert=\lvert x\rvert-1$ is $x\ge 1$. | 1A | |
| | (5) | |
| 12. $(1-2x+x^2)^n=(1-x)^{2n}$ | 1M | For $1-2x+x^2=(1-x)^2$ |
| $=1-{}_{2n}C_1\,x+{}_{2n}C_2\,x^2-{}_{2n}C_3\,x^3+\cdots$ | 1M | For bin. expansion up to $x^2$ |
| OR General term $={}_{2n}C_r(-1)^r x^r$ | 1M | |
| $\therefore \frac{2n(2n-1)}{2}=66$ | 1M | For coefficient of $x^2=66$ |
| $2n^2-n-66=0$ | | |
| $n=6$ or $\frac{-11}{2}$ (rej.) | 1A | |
| Hence the coefficient of $x^3=-{}_{12}C_3$ | 1M | |
| $=-220$ | 1A | |

| Solution | Marks | Remarks |
|---|---|---|
| Alternative solution | | |
| $\left(1-2x+x^2\right)^n=\left[1+\left(-2x+x^2\right)\right]^n$ | 1M | For grouping the trinomial expression into binomial |
| $=1+{}_nC_1\left(-2x+x^2\right)+{}_nC_2\left(-2x+x^2\right)^2+{}_nC_3\left(-2x+x^2\right)^3+\cdots$ | 1M | For binomial expansion up to $\left(-2x+x^2\right)^2$ |
| $=1+{}_nC_1\left(-2x+x^2\right)+{}_nC_2\left(4x^2-4x^3+\cdots\right)+{}_nC_3\left(-8x^3+\cdots\right)+\cdots$ | | |
| $=1-2{}_nC_1x+\left({}_nC_1+4{}_nC_2\right)x^2+\left(-4{}_nC_2-8{}_nC_3\right)x^3+\cdots$ | | |
| $\therefore n+4\dfrac{n(n-1)}{2}=66$ | 1M | For coefficient of $x^2=66$ |
| $2n^2-n-66=0$ | | |
| $(n-6)(2n+11)=0$ | | |
| $n=6$ or $\dfrac{-11}{2}$ (rej.) | 1A | |
| Hence the coefficient of $x^3=-4{}_6C_2-8{}_6C_3$ | 1M | |
| $=-220$ | 1A | (pp-1) if dots were omitted in most cases |
| | (6) | |
| 13. (a) $\begin{cases} y=x^2 \\ y=mx-2m \end{cases}$ | | |
| $\therefore x^2-mx+2m=0$ ------------------------------------------------ (*) | 1M | For eliminating $y$ |
| Since $C$ intersects $L$ at 2 distinct points, so $\Delta>0$ | | |
| i.e. $(-m)^2-4(1)(2m)>0$ | 1M | Accept $\Delta\geq 0$ |
| Alternative solution | | |
| $\therefore y^2+m(4-m)y+4m^2=0$ | 1M | For eliminating $x$ |
| Since $C$ intersects $L$ at 2 distinct points, so $\Delta>0$ | | |
| i.e. $m^2(4-m)^2-4(1)(4m^2)>0$ | 1M | Accept $\Delta\geq 0$ |
| $m(m-8)>0$ | | |
| $m<0$ or $m>8$ | 1A | |
| (b) (i) Let $A$, $B$ and $M$ be $(x_1,y_1)$, $(x_2,y_2)$ and $(p,q)$ respectively. | | |
| Since $x_1$ and $x_2$ are roots of (*), so $x_1+x_2=m$. | 1M | |
| Alternative solution | | |
| $x_1=\dfrac{m+\sqrt{m^2-8m}}{2}$ and $x_2=\dfrac{m-\sqrt{m^2-8m}}{2}$ (or vice versa) | 1M | |
| $\therefore x_1+x_2=m$ | | |
| $\therefore p=\dfrac{x_1+x_2}{2}=\dfrac{m}{2}$ | | |
| $\therefore q=m\left(\dfrac{m}{2}\right)-2m=\dfrac{m^2-4m}{2}$ | | |
| i.e. $M$ is $\left(\dfrac{m}{2},\dfrac{m^2-4m}{2}\right)$ | 1 | |
| (ii) If $AB$ is bisected by the straight line $x+y=5$, then | | |
| $\dfrac{m}{2}+\dfrac{m^2-4m}{2}=5$ | 1A | |
| $m^2-3m-10=0$ | | |
| $m=-2$ or 5 (rej. by (a)) | | |
| i.e. $m=-2$ | 1A | |
| | (7) | |

| Solution | Marks | Remarks |
|---|---|---|
| 4. (a) $\because AX \perp XB$, $\therefore AX^2 + XB^2 = AB^2$ (Pythagoras Theorem) | 1M | For either one |
| $AX^2 + XB^2 + BC^2 = AB^2 + BC^2$ | 1M | |
| $\because XB^2 + BC^2 = XC^2$ and $AB^2 + BC^2 = AC^2$ (Pythagoras Theorem) | | |
| $\therefore AX^2 + XC^2 = AC^2$ | | <br> |
| Hence $AX \perp XC$ (Converse of Pythagoras Theorem) | 1 | |
| | (3) | |
| (b) (i) $FB = \sqrt{(1)^2 + (3\sqrt{2})^2 - 2(1)(3\sqrt{2})\cos 135°} = 5$ | 1M | For cosine formula |
| $\dfrac{AF}{\sin \angle ABF} = \dfrac{FB}{\sin \angle FAB}$ | | |
| $\dfrac{1}{\sin \angle ABF} = \dfrac{5}{\sin 135°}$ | 1M | <br> |
| $\sin \angle ABF = \dfrac{1}{5\sqrt{2}}$ | | |
| $\therefore \cos \angle ABF = \dfrac{\sqrt{(5\sqrt{2})^2 - 1^2}}{5\sqrt{2}} = \dfrac{7}{5\sqrt{2}}$ | 1A | |
| Hence $XB = AB \cos \angle ABF$ | | |
| $= (3\sqrt{2})\left(\dfrac{7}{5\sqrt{2}}\right) = \dfrac{21}{5}$ m | 1 | |
| <u>Alternative solution (1)</u> | | |
| $FB = \sqrt{(1)^2 + (3\sqrt{2})^2 - 2(1)(3\sqrt{2})\cos 135°} = 5$ | 1M | For cosine formula |
| The area of $\Delta AFB = \dfrac{1}{2}(1)(3\sqrt{2})\sin 135°$ | 1A | |
| $= \dfrac{3}{2}$ | | |
| Hence $\dfrac{1}{2}(5)(AX) = \dfrac{3}{2}$ | 1M | |
| $AX = \dfrac{3}{5}$ | | |
| Therefore $XB = \sqrt{(3\sqrt{2})^2 - \left(\dfrac{3}{5}\right)^2} = \dfrac{21}{5}$ m | 1 | |
| <u>Alternative solution (2)</u> | | |
| $FB = \sqrt{(1)^2 + (3\sqrt{2})^2 - 2(1)(3\sqrt{2})\cos 135°} = 5$ | 1M | For cosine formula |
| Let $XB = x$ and therefore $FX = 5 - x$ | | |
| In $\Delta AXB$, $AX^2 = (3\sqrt{2})^2 - x^2$ | } 1A | |
| In $\Delta AXF$, $AX^2 = (1)^2 - (5-x)^2$ | | |
| $\therefore 18 - x^2 = 1 - 25 + 10x - x^2$ | 1M | |
| $x = \dfrac{21}{5}$ m | 1 | |

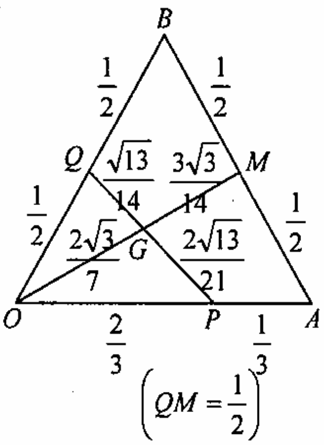
| Solution | Marks | Remarks |
|---|---|---|
| (ii) By (a), since $AX \perp XB$, so $AX \perp XC$; and since $AX \perp XF$, so $AX \perp XE$. Hence the required angle $(\theta)$ is $\angle CXE$. | 1A | |
| $\tan \angle CXB = \dfrac{\left(\frac{7}{5}\right)}{\left(\frac{21}{5}\right)}$ $= \dfrac{1}{3}$ | 1A | Either one |
| $FX = 5 - \dfrac{21}{5} = \dfrac{4}{5}$ $\tan \angle EXF = \dfrac{\left(\frac{7}{5}\right)}{\left(\frac{4}{5}\right)}$ $= \dfrac{7}{4}$ | | |
| $\therefore \tan\theta = \tan(180° - \angle CXB - \angle EXF)$ | 1M | For $\theta = 180° - \angle CXB - \angle EXF$ |
| $= -\tan(\angle CXB + \angle EXF)$ $= -\dfrac{\frac{1}{3}+\frac{7}{4}}{1-\frac{1}{3}\cdot\frac{7}{4}}$ | 1M | |
| $= -5$ | 1A | |
| <u>Alternative solution</u> By (a), since $AX \perp XB$, so $AX \perp XC$; and since $AX \perp XF$, so $AX \perp XE$. Hence the required angle $(\theta)$ is $\angle CXE$. | 1A | |
| $CX = \sqrt{\left(\frac{7}{5}\right)^2 + \left(\frac{21}{5}\right)^2}$ $= \dfrac{7\sqrt{10}}{5}$ | 1A | Either one |
| $FX = 5 - \dfrac{21}{5} = \dfrac{4}{5}$ $EX = \sqrt{\left(\frac{7}{5}\right)^2 + \left(\frac{4}{5}\right)^2}$ $= \dfrac{\sqrt{65}}{5}$ | | |
| $\therefore EC^2 = CX^2 + EX^2 - 2(CX)(EX)\cos\theta$ | | |
| $5^2 = \left(\frac{7\sqrt{10}}{5}\right)^2 + \left(\frac{\sqrt{65}}{5}\right)^2 - 2\left(\frac{7\sqrt{10}}{5}\right)\left(\frac{\sqrt{65}}{5}\right)\cos\theta$ | 1M | |
| $\cos\theta = \dfrac{-1}{\sqrt{26}}$ | | |
| $\therefore \tan\theta = \dfrac{\sqrt{26-1^2}}{-1}$ | 1M | |
| $= -5$ | 1A | |
| | (9) | |

| Solution | Marks | Remarks |
|---|---|---|
| 15. (a) The distance from $P$ to $L_1$ is $\left\|\dfrac{2a-b}{\sqrt{2^2+(-1)^2}}\right\|$ ---------------------------- (*). | 1A |   |
| Let $A$ and $B$ be the intersections of $C$ and $L_1$, and $M$ be the foot of perpendicular of $P$ to $AB$. Then $AM=\dfrac{1}{2}AB$ (line from centre ⊥ chord bisects chord). | | |
| Hence $\left(\dfrac{2a-b}{\sqrt{5}}\right)^2+\left(4\sqrt{5}\right)^2=r^2$ (Pythagoras theorem) | 1M | |
| i.e. $r^2=\dfrac{4a^2-4ab+b^2+400}{5}$ | 1 | Withhold if the absolute sign of (*) is missed |
| | (3) | |
| (b) (i) The distance from $P$ to $L_2$ is $\left\|\dfrac{2a+b}{\sqrt{2^2+1^2}}\right\|$. | 1A | Withhold 1A if the absolute sign was omitted OR $(x, y)$ was used instead of $(a, b)$ |
| Since $C$ touches $L_2$, so $r=\left\|\dfrac{2a+b}{\sqrt{5}}\right\|$. | 1M |   |
| $\therefore r^2=\dfrac{4a^2+4ab+b^2}{5}$ | 1A | |
| <u>Alternative Solution</u> | | |
| Solving $C:(x-a)^2+(y-b)^2=r^2$ and $L_2: y=-2x$, we have | | |
| $x^2-2ax+a^2+4x^2+4bx+b^2=r^2$. | | |
| $5x^2-2(a-2b)x+a^2+b^2-r^2=0$ | 1A | |
| $\Delta=4(a-2b)^2-4\cdot 5\left(a^2+b^2-r^2\right)=0$ | 1M | For $\Delta=0$ |
| $a^2-4ab+4b^2=5\left(a^2+b^2-r^2\right)$ | | |
| $\therefore r^2=\dfrac{4a^2+4ab+b^2}{5}$ | 1A | |
| By (a), $\dfrac{4a^2-4ab+b^2+400}{5}=\dfrac{4a^2+4ab+b^2}{5}$ | 1M | |
| i.e. $ab=50$ | 1A | |
| Hence the equation of the locus of $P$ is $xy=50$ . | | |
| (ii) $C$ is smallest when the chord is in fact the diameter of $C$, i.e. when $P$ lies on $L_1$. | 1M | OR when $r=\sqrt{80}$ |
| Hence $P$ satisfies $xy=50$ (from (b)(i)) and $L_1: y=2x$ | 1M | |
| Solving, $(x,y)=(-5,-10)$ or $(5,10)$ | 1A | |

| Solution | Marks | Remarks |
| --- | --- | --- |
| **Alternative Solution** | | |
| From (a) and (b)(i), $r^2 = \frac{1}{5}\left[4a^2 - 4(50) + \left(\frac{50}{a}\right)^2 + 400\right]$ | 1M | |
| $= \frac{4}{5}a^2 + 40 + \frac{500}{a^2}$ | | |
| $\therefore 2r\frac{\mathrm{d}r}{\mathrm{d}a} = \frac{8a}{5} - \frac{1000}{a^3}$ ---------------------------------------------- (*) | 1M | |
| When $\frac{\mathrm{d}r}{\mathrm{d}a} = 0$ , $a^4 = 625$ which gives $a = \pm 5$ . | | |
| From (*), $r\frac{\mathrm{d}^2r}{\mathrm{d}a^2} + \left(\frac{\mathrm{d}r}{\mathrm{d}a}\right)^2 = \frac{4}{5} + \frac{1500}{a^4}$ | | |
| $\left.\frac{\mathrm{d}^2r}{\mathrm{d}a^2}\right\|_{a=\pm 5} = \frac{1}{r}\left[\frac{4}{5} + \frac{1500}{(\pm 5)^4}\right] > 0$ | | Accept using sign test |
| Hence when $C$ is smallest, $r$ is minimum and that occurs when $a = \pm 5$ | | |
| Therefore the centre is $(a, b) = (-5, -10)$ or $(5, 10)$ | 1A | Withhold 1A if the condition of minimum was unchecked |
| $\therefore$ by (a), $r^2 = 80$ | | |
| Hence $C$ is $(x+5)^2 + (y+10)^2 = 80$ or $(x-5)^2 + (y-10)^2 = 80$ | 1A | |
| i.e. $x^2 + y^2 \pm 10x \pm 20y + 45 = 0$ | | |
| | (9) | |

| Solution | Marks | Remarks |
|---|---|---|
| 16. (a) (i) The area of the minor segment enclosed by $\overset{\frown}{PR}$ and $PR$ | | |
| $=\frac{1}{2}(1)^2(2\theta)-\frac{1}{2}(1)(1)(\sin 2\theta)$ | 1A | |
| $=\theta-\frac{1}{2}\sin 2\theta$ | | |
| $\therefore A=\pi(1)^2-\pi(\cos\theta)^2-\left(\theta-\frac{1}{2}\sin 2\theta\right)$ | 1M | For $A=C_1-C_2-$ segment |
| $=\pi\sin^2\theta-\theta+\frac{1}{2}\sin 2\theta$ | 1 | |
| (ii) $\frac{dA}{d\theta}=\pi(2\sin\theta\cos\theta)-1+\frac{1}{2}(\cos 2\theta)(2)$ | 1A | |
| $=\pi\sin 2\theta-1+\left(1-2\sin^2\theta\right)$ | | |
| $=\pi\sin 2\theta-2\sin\theta\cos\theta\tan\theta$ | | |
| $=(\pi-\tan\theta)\sin 2\theta$ | 1 | Follow through |
| | (5) | |
| (b) Since $0<2\theta<\pi$, so $\sin 2\theta>0$. | 1M | |
| $\therefore \frac{dA}{d\theta}=0$ when $\tan\theta=\pi$ | 1M | For $\frac{dA}{d\theta}=0$ |
| $\frac{d^2A}{d\theta^2}=(\pi-\tan\theta)(2\cos 2\theta)+\left(-\sec^2\theta\right)\sin 2\theta$ | | |
| $=2(\pi-\tan\theta)\cos 2\theta-2\tan\theta$ | | |
| $\left.\frac{d^2A}{d\theta^2}\right\|_{\tan\theta=\pi}=0-2\pi<0$ | | |
| Alternative Solution: $\because \frac{dA}{d\theta}>0$ when $0<\tan\theta<\pi$, and $\frac{dA}{d\theta}<0$ when $\tan\theta>\pi$, | | |
| Therefore $A$ attains its greatest value when $\tan\theta=\pi$. | 1A | |
| | (3) | |
| (c) The perimeter is $s=\overset{\frown}{PQR}+PR+$ circumference of $C_2$ | | |
| $=2\pi-2\theta+2\sin\theta+2\pi\cos\theta$ | 1A | |
| When $A$ is max., $\tan\theta=\pi$ which gives $\cos\theta=\frac{1}{\sqrt{1+\pi^2}}$ and $\sin\theta=\frac{\pi}{\sqrt{1+\pi^2}}$ | | |
| $\frac{ds}{d\theta}=-2+2\cos\theta-2\pi\sin\theta$ | 1M | OR $\frac{ds}{d\theta}=-2(1-\cos\theta+\pi\sin\theta)$ |
| $\therefore \left.\frac{ds}{d\theta}\right\|_{\tan\theta=\pi}=-2+\frac{2}{\sqrt{1+\pi^2}}-\frac{2\pi^2}{\sqrt{1+\pi^2}}$ | | |
| $\approx -7.38$ $\neq 0$ | 1M | $<0$ for any $0<\theta<\frac{\pi}{2}$ |
| Alternative solution: At $\theta=\tan^{-1}\pi\approx 1.26$, $s\approx 2\pi-2(1.26)+2\sin(1.26)+2\pi\cos(1.26)$ $\approx 7.57$ | 1M | |
| At $\theta=1.2$, $s=2\pi-2(1.2)+2\sin(1.2)+2\pi\cos(1.2)$ $\approx 8.02>7.57$ | 1M | Can use any $\theta$ for $0<\theta<1.26$ |
| Hence $s$ will not attain its greatest value when $A$ attains its greatest value. The student is incorrect. | 1A | Follow through |
| | (4) | |

| Solution | Marks | Remarks |
|---|---|---|
| 17. (a) $\overrightarrow{OM}=\dfrac{\mathbf{a}+\mathbf{b}}{2}$ | 1A | |
| | (1) | |
| (b) (i) $\overrightarrow{OP}=\dfrac{2}{3}\mathbf{a}$ and $\overrightarrow{OQ}=k\mathbf{b}$ | | |
| $\therefore \overrightarrow{OG}=\dfrac{3\left(\dfrac{2}{3}\mathbf{a}\right)+4(k\mathbf{b})}{3+4}$ | 1M | |
| $=\dfrac{2\mathbf{a}+4k\mathbf{b}}{7}$ | 1A | |
| (ii) Since $O$, $G$ and $M$ are collinear, so | | |
| $\dfrac{\left(\dfrac{2}{7}\right)}{\left(\dfrac{1}{2}\right)}=\dfrac{\left(\dfrac{4k}{7}\right)}{\left(\dfrac{1}{2}\right)}$ | 1M | |
| Alternative Solution<br>Since $O$, $G$ and $M$ are collinear, so<br>$\overrightarrow{OG}\cdot\overrightarrow{AB}=0$<br>$\dfrac{2\mathbf{a}+4k\mathbf{b}}{7}\cdot(\mathbf{b}-\mathbf{a})=0$<br>$4k\lvert\mathbf{b}\rvert^2+(2-4k)\mathbf{a}\cdot\mathbf{b}-2\lvert\mathbf{a}\rvert^2=0$<br>$\therefore \mathbf{a}\cdot\mathbf{b}=(1)(1)\cos 60^\circ=\dfrac{1}{2}$<br>$\therefore 4k(1)^2+(2-4k)\left(\dfrac{1}{2}\right)-2(1)^2=0$ | 1M | |
| $k=\dfrac{1}{2}$ | | |
| $\therefore \overrightarrow{OQ}=\dfrac{1}{2}\mathbf{b}$ | | |
| $\therefore \overrightarrow{PQ}=\dfrac{1}{2}\mathbf{b}-\dfrac{2}{3}\mathbf{a}$ | 1 | |
| | (4) | |
| (c) (i) $\mathbf{a}\cdot\mathbf{b}=(1)(1)\cos 60^\circ=\dfrac{1}{2}$ | 1A | |
| $\lvert\overrightarrow{PQ}\rvert^2=\left(\dfrac{1}{2}\mathbf{b}-\dfrac{2}{3}\mathbf{a}\right)\cdot\left(\dfrac{1}{2}\mathbf{b}-\dfrac{2}{3}\mathbf{a}\right)$ | 1A | |
| $=\dfrac{1}{4}\lvert\mathbf{b}\rvert^2-\dfrac{2}{3}\mathbf{a}\cdot\mathbf{b}+\dfrac{4}{9}\lvert\mathbf{a}\rvert^2$ | | |
| $=\dfrac{1}{4}(1)^2-\dfrac{2}{3}\left(\dfrac{1}{2}\right)+\dfrac{4}{9}(1)^2$ | | |
| $=\dfrac{13}{36}$ | | |
| $\therefore \lvert\overrightarrow{PQ}\rvert=\dfrac{\sqrt{13}}{6}$ | 1A | |

| Solution | Marks | Remarks |
|---|---|---|
| (ii) $\left\lvert\overrightarrow{OM}\right\rvert = \sqrt{1^2 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$ | 1A | |
| $\overrightarrow{PQ}\cdot\overrightarrow{OM} = \left\lvert\overrightarrow{PQ}\right\rvert\left\lvert\overrightarrow{OM}\right\rvert\cos\angle QGM$ | | |
| $\left(\frac{1}{2}\mathbf{b} - \frac{2}{3}\mathbf{a}\right)\cdot\left(\frac{\mathbf{a}+\mathbf{b}}{2}\right) = \left(\frac{\sqrt{13}}{6}\right)\left(\frac{\sqrt{3}}{2}\right)\cos\angle QGM$ | 1M | |
| $\frac{\sqrt{39}}{12}\cos\angle QGM = \frac{1}{4}\lvert\mathbf{b}\rvert^2 - \frac{1}{12}\mathbf{a}\cdot\mathbf{b} - \frac{1}{3}\lvert\mathbf{a}\rvert^2$ | | |
| $= \frac{1}{4}(1)^2 - \frac{1}{12}\left(\frac{1}{2}\right) - \frac{1}{3}(1)^2$ | | |
| $= \frac{-1}{8}$ | 1A | |
| $\therefore \cos\angle QGM = \frac{-3}{2\sqrt{39}}$ | | |
| $\therefore \angle QGM = 104°$ (correct to the nearest degree) | 1A | |
| **Alternative Solution** | |  |
| $OQ = \left\lvert\frac{1}{2}\mathbf{b}\right\rvert = \frac{1}{2}$ | | |
| $\frac{OQ}{\sin\angle OPQ} = \frac{QP}{\sin\angle QOP}$ | | |
| $\frac{\frac{1}{2}}{\sin\angle OPQ} = \frac{\frac{\sqrt{13}}{6}}{\sin 60°}$ | 1M | |
| $\sin\angle OPQ = \frac{3\sqrt{3}}{2\sqrt{13}}$ | 1A | |
| **Alternative Solution** | | |
| $OP = 1\cdot\left(\frac{2}{2+1}\right) = \frac{2}{3}$ | | |
| $\because OQ^2 = OP^2 + PQ^2 - 2(OP)(PQ)\cos\angle OPQ$ | | **Marking Criteria** |
| $\therefore \left(\frac{1}{2}\right)^2 = \left(\frac{2}{3}\right)^2 + \left(\frac{\sqrt{13}}{6}\right)^2 - 2\left(\frac{2}{3}\right)\left(\frac{\sqrt{13}}{6}\right)\cos\angle OPQ$ | 1M | 1A for any ONE relevant side / angle correctly found |
| $\cos\angle OPQ = \frac{5}{2\sqrt{13}}$ | 1A | 1A for ALL relevant sides / angles correctly found |
| $\angle OPQ \approx 46.1°$ (since $\angle OPQ$ is acute from the figure) | | 1M for any trigo. method |
| Since $M$ is the mid-point of $AB$, so $\angle AOM = \angle BOM = 30°$ | | 1A for $\angle QGM$ |
| $\therefore \angle OGP \approx 180° - 30° - 46.1° = 103.9°$ | 1A | |
| Hence $\angle QGM = \angle OGP = 104°$ (correct to the nearest degree) | 1A | |
| | (7) | |

| Solution | Marks | Remarks |
| --- | --- | --- |
| 18. (a) $y = 2\sqrt{x} - x$ | | |
| $\frac{dy}{dx} = \frac{1}{\sqrt{x}} - 1$ | 1A | |
| For horizontal tangent, $\left.\frac{dy}{dx}\right\vert_{x=r} = 0$. | | |
| $\therefore \frac{1}{\sqrt{r}} - 1 = 0$ which gives $r = 1$ | 1 | |
| | (2) | |
| (b) (i) Area under $C_1$ is $\int_0^1 \left(2\sqrt{x} - x\right) dx$ | 1A | |
| $= \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{1}{2}x^2\right]_0^1$ | | |
| $= \frac{5}{6}$ | 1A | |
| Area under $C_2$ is $\int_2^3 \left[2\sqrt{3-x} - (3-x)\right] dx$ | 1A | |
| $= \left[2 \cdot \frac{2}{-3}(3-x)^{\frac{3}{2}} - 3x + \frac{1}{2}x^2\right]_2^3$ | | |
| $= \frac{5}{6}$ | | |
| Alternative Solution<br>Since $C_2$ is the reflection of $C_1$ about the line $x = \frac{3}{2}$, so the area under $C_2$ equals the area under $C_1$. | 1M | |
| So the area of $S = 4\left(\frac{5}{6}\right) + 2(1 \cdot 1)$ | | |
| $= \frac{16}{3}$ | 1A | |
| (ii) Volume $= \frac{1}{2}\left\{\pi \int_0^1 \left(2\sqrt{x} - x\right)^2 dx + \pi(1)^2(1) + \pi \int_2^3 \left[2\sqrt{3-x} - (3-x)\right]^2 dx\right\}$ | 1M+1M | 1M for $V_1 = \pi \int_a^b y^2 dx$<br>1M for $V = \frac{1}{2}\left[V_1 + \pi(1)^2(1) + V_2\right]$ |
| $= \frac{\pi}{2}\int_0^1 \left(4x - 4x^{\frac{3}{2}} + x^2\right) dx + \frac{\pi}{2} + \frac{\pi}{2}\int_2^3 \left[4(3-x) - 4(3-x)^{\frac{3}{2}} + (3-x)^2\right] dx$ | | |
| $= \frac{\pi}{2}\left[2x^2 - \frac{8}{5}x^{\frac{5}{2}} + \frac{1}{3}x^3\right]_0^1 + \frac{\pi}{2} + \frac{\pi}{2}\left[-2(3-x)^2 + \frac{8}{5}(3-x)^{\frac{5}{2}} - \frac{1}{3}(3-x)^3\right]_2^3$ | 1A | |
| $= \frac{37}{30}\pi$ | 1 | |

| Solution | Marks | Remarks |
|---|---|---|
| **Alternative Solution** | | |
| Volume $= \frac{1}{2}\left[2\cdot\pi\int_0^1 \left(2\sqrt{x}-x\right)^2 dx + \pi(1)^2(1)\right]$ | 1M+1M | 1M for $V_1 = \pi\int_a^b y^2 dx$; 1M for $V = \frac{1}{2}\left[2V_1 + \pi(1)^2(1)\right]$ |
| $= \pi\int_0^1\left(4x - 4x^{\frac{3}{2}} + x^2\right)dx + \frac{\pi}{2}$ | | |
| $= \pi\left[2x^2 - \frac{8}{5}x^{\frac{5}{2}} + \frac{1}{3}x^3\right]_0^1 + \frac{\pi}{2}$ | 1A | |
| $= \frac{37}{30}\pi$ | 1 | |
| (iii) The volume of the middle part is $\frac{37}{90}\pi$ (by (ii)). | | |
| Area of $S_1$ is $\frac{1}{2}\pi(1)^2 = \frac{\pi}{2}$ | | |
| Hence the length of the middle part is $\left(\frac{37\pi}{90}\right)\Big/\left(\frac{\pi}{2}\right) = \frac{37}{45}$ | 1M | |
| $\therefore OQ = \frac{1}{2}\left(3 - \frac{37}{45}\right) = \frac{49}{45}$ | | |
| $\therefore OQ : OP = \frac{49}{45} : 3 = 49 : 135$ | 1A | |
| **Alternative Solution** | | |
| The volume of the first part is $\frac{37}{90}\pi$ (by (ii)). | | |
| By (ii), the volume formed by $C_1$ is $\pi\int_0^1\left(4x - 4x^{\frac{3}{2}} + x^2\right)dx = \frac{11\pi}{30}$. | | |
| Area of $S_1$ is $\frac{1}{2}\pi(1)^2 = \frac{\pi}{2}$ | | |
| Hence $OQ - 1 = \left(\frac{37\pi}{90} - \frac{11\pi}{30}\right)\Big/\frac{\pi}{2}$ | 1M | For RHS |
| $\therefore OQ = \frac{49}{45}$ | | |
| $\therefore OQ : OP = \frac{49}{45} : 3 = 49 : 135$ | 1A | |
| | (10) | |

## 考生表現

**甲部**（必答題）

| 題號 | | | 一般表現 |
|---|---|---|---|
| 1 | | | 良好 |
| 2 | | | 一般 |
| 3 | | | 一般 |
| 4 | | | 良好 |
| 5 | | | 一般 |
| 6 | | | 良好 |
| 7 | (a) | | 甚佳 |
| | (b) | | 尙可 |
| 8 | (a) | | 良好 |
| | (b) | | 良好 |
| 9 | (a) | | 良好 |
| | (b) | | 尙可 |
| 10 | (a) | | 尙可 |
| | (b) | | 良好 |
| 11 | (a) | | 差劣 |
| | (b) | | 差劣 |
| 12 | | | 一般 |
| 13 | (a) | | 良好 |
| | (b) | (i) | 良好 |
| | | (ii) | 一般 |

**乙部**（5 題選答 4 題）

| 題號 | | | 選題百分率 | 一般表現 |
|---|---|---|---|---|
| 14 | (a) | | 80 | 良好 |
| | (b) | (i) | | 良好 |
| | | (ii) | | 差劣 |
| 15 | (a) | | 78 | 甚佳 |
| | (b) | (i) | | 良好 |
| | | (ii) | | 差劣 |

| 題號 | | | 選題百分率 | 一般表現 |
|---|---|---|---|---|
| 16 | (a) | (i) | 74 | 尙可 |
| | | (ii) | | 良好 |
| | (b) | | | 良好 |
| | (c) | | | 差劣 |
| 17 | (a) | | 98 | 甚佳 |
| | (b) | (i) | | 甚佳 |
| | | (ii) | | 尙可 |
| | (c) | (i) | | 良好 |
| | | (ii) | | 尙可 |
| 18 | (a) | | 70 | 甚佳 |
| | (b) | (i) | | 尙可 |
| | | (ii) | | 尙可 |
| | | (iii) | | 甚劣 |

考生在每題的表現

Q.1 大部分考生能夠運用法則 $\int x^n dx = \frac{x^{n+1}}{n+1} + c$ 求出答案。然而，當 $n$ 是負數時，部分考生未能應用法則，並且把法則誤解爲 $\int x^{-2} dx = \frac{x^{-3}}{-3} + c$。部分考生在答案中沒有分裂被積函數而分別積其分子及分母。部分考生仍然遺漏了積分常數。

Q.2 很多考生把四邊形誤以爲平行四邊形，然後在求它的面積前先求出 $k$ 的值。部分考生因爲忽略了已知點的方向，或計算失誤，以致失分。

Q.3 大部分考生嘗試把 $\cos x + \cos 3x$ 的和轉爲積。部分考生弄錯了 $\cos(-x) = -\cos x$。部分考生消去了因子 cos2x 因而遺漏了其中一個通解。少數考生採用以 $\cos x$ 表 $\cos 3x$ 的冗長方法，而他們大部分都未能完成答題。部分考生以 $x = 360°n \pm 45°$ 作爲 $\cos 2x = 0$ 的通解。少數考生在答案內同時使用度和弧度。

Q.4 考生的整體表現良好，惟部分考生在簡化 $\frac{(x + \Delta x)^2 + 1 - x^2 - 1}{\Delta x}$ 時出錯。少數考生拙劣地把導數表達爲 $\lim_{\Delta x \to 0} = \frac{(x+\Delta x)^2 + 1 - x^2 - 1}{\Delta x}$，而部分考生甚至遺漏了極根記號。

Q.5　近年來，考生在以數學歸納法證明命題方面的表現普遍有進步，但考生的答題中仍有出現下列錯誤或表達拙劣的命題：

(1) 設 $n = 1$ 的命題爲真（沒有加以證明）。

(2) 假設 $n = k$ 爲真。

(3) 假設 $n = k$ 的命題爲真，其中 $k$ 是任何一個整數。

(4) 假設命題對所有正整數/所有整數均爲真。

(5) 按照數學歸納法的原則，這命題對所有正整數/所有實數均爲真。

在所提出的證明中，部分考生混淆了未知數 $a$ 和 $k$，設了 $a = 2$ 或誤以爲對 $n = 1$，L.S. = $\frac{1}{a-1}$。這些錯誤顯示考生在開始寫出證明之前，未明白命題的含義。

Q.6　考生常犯的錯誤包括 $\int \sin 2x\,dx = \frac{\cos 2x}{2} + c$ 、 $\int \sin 2x\,dx = 2\cos 2x + c$ 和 $\int \sin 2x\,dx = \cos 2x + c$ 。部分考生甚至錯誤地把須求出的面積表達爲 $\int_{0^\circ}^{30^\circ} (\cos x - \sin 2x)\,dx$ ，因爲他們不明白定積分的含義。

Q.7　(b) 很多考生未能利用計算機準確求出有關的角度，因而失分。部分考生利用錯誤的公式 $\tan\theta = \frac{m_1 + m_2}{1 - m_1 m_2}$ 求兩條線之間的角度。

Q.8　(a) 部分考生未能以事實 $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$ 求出 $\overrightarrow{BC}$，而部分考生錯誤地寫出 $\overrightarrow{BC} = \overrightarrow{OB} - \overrightarrow{OC}$ 。

(b) 大部分考生成功利用 $BC \perp OA$ 求出 $k$ 的值，惟部分考生誤解了 $\overrightarrow{BC} \cdot \overrightarrow{OA} = -1$ 或 $\overrightarrow{BC} \times \overrightarrow{OA} = 0$ 。

Q.9　(b) 很多考生未能應用微分法的鏈式法則求出複合函數的導數，及以 $y$ 的變率來表 $x$ 的變率。部分考生未能說明「下墜速度」與 $y$ 的變率之間的關係，或寫出 $\frac{dy}{dt} = 2\,\text{ms}^{-1}$ 而不是 $\frac{dy}{dt} = -2\,\text{ms}^{-1}$ 。

Q.10 (a) 部分考生嘗試以積分法求 $f(x)$，而沒有注意 $y=f'(x)$ 的圖像是一條直線，其方程可利用該直線的截距求出。

(b) 大部分考生寫出轉向點，而不是它的 $x$ 坐標。部分考生只寫下 $x$ 坐標而沒有說明理由。

Q.11 大部分考生不清楚明白實數 $x$ 絕對值的定義，並且錯誤地寫出 $|x|=\pm x$，而不是 $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$。

(a) 很多考生忽略了條件 $0 \leq x \leq 1$，並且錯誤利用 $|x|=\pm x$ 來求解方程。

(b) 很多考生如(a)般簡單寫下 $|x|=\pm x$ 來求解方程。部分考生不知道 $x-1=x-1$ 是恆等式，然後作出沒有解的結論，而部分其他考生排除了 $1-x=-x-1$，而不是作出沒有解的結論。

Q.12 大部分考生成功利用二項式定理展開三項式，雖然差不多所有考生都沒有察覺 $(1-2x+x^2)^n$ 可表為 $(1-x)^{2n}$。然而，部分考生在處理併項及展開時，在符號方面出錯。部分考生因在展式保留的項不足夠，或因運算的錯誤而失分。很多考生誤解了 ${}_nC_3=\frac{n(n-1)(n-2)}{3}$。

Q.13 (a) 很多考生忽略了「相異」一詞而設 $\Delta \geq 0$。部分考生錯誤把值域表達為 $m<0, m>8$ 而沒有寫上連接詞「或」。

(b) 部分考生忽略了在(a)得出的條件而忘記了排除 $m=5$。

Q.14 (b)(i) 部分考生嘗試從計算機得出的近似角求出已知的真確答案。

(ii) 很多考生未能利用(a)得出的結果，並且未能辨識所須求出的角 $\angle EXC$。

Q.15 (a) 部分考生忘記加上絕對值符號，而部分考生寫下 $d=\left|\frac{2a-b}{\sqrt{5}}\right|=\frac{2a-b}{\sqrt{5}}$。

(b)(ii) 部分考生錯誤排除了 $a$ 和 $b$ 的負值。

Q.16 (a)(i) 部分考生在求扇形面積時混淆了度和弧度。

(b) 部分考生沒有排除 $\sin 2\theta = 0$，這顯示他們對函數極值的理解很弱。

Q.17 (b)(ii) 很多考生未能應用基本性質 $OM \parallel OG$ 來求 $k$。

(c)(i) 部分考生利用餘弦公式來求 $PQ$。

(ii) 很多考生沒有察覺 $\angle QGM$ 是鈍角。其他常犯的錯誤包括

- 錯誤假設 $\mathbf{QP} \cdot \mathbf{OM} = |\mathbf{QP}||\mathbf{OM}| \cos \angle QGM$
- 錯誤假設 $\mathbf{a} = \mathbf{i}$ 和 $\mathbf{b} = \mathbf{j}$
- $|\mathbf{PQ}|^2 = (\frac{1}{2}\mathbf{b} - \frac{2}{3}\mathbf{a})^2$

Q.18 (b)(i) 部分考生沒有察覺須求出的面積是曲線下方面積的兩倍。

一般評論及建議

1. 考生應小心閱讀題目，以免錯解題目，並應依從題目所說明的指示作答。

2. 考生應注意以下各點，以改善答題技巧：加上適當的括號；運用適當的單位；寫出真確值作為答案；顯示必須的步驟；以及利用箭號來表示向量。

3. 考生應加強處理涉及變率問題的能力，尤其是涉及實際處境的情況。

4. 除非題目已說明某些準確程度，否則考生應以真確值寫出數值的答案。對一些要求考生提供真確答案的題目，考生應確保寫出真確值，即使是中途的步驟，亦須注意這點。

5. 考生應清楚顯示所有運算步驟，並在表達解法時加以必須的解釋，特別是要求考生作出證明的部分。

6. 考生應完全明白向量的表達以及點積的基本運算。

7. 考生應注意不要在同一數式內同時使用弧度和度。