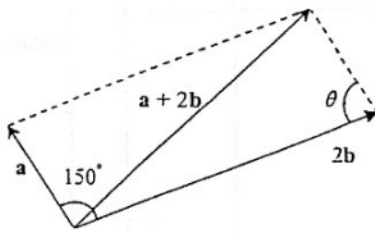


Solution	Marks	Remarks
$1. \frac{d}{dx} \left[\frac{\sin(2x+1)}{x} \right] = \frac{x \frac{d}{dx} \sin(2x+1) - \sin(2x+1) \frac{d}{dx} (x)}{x^2}$ $= \frac{x[2 \cos(2x+1)] - \sin(2x+1)}{x^2}$ <div style="border: 1px dashed black; padding: 5px; width: fit-content; margin: 10px auto;"> $= \frac{2x \cos(2x+1) - \sin(2x+1)}{x^2}$ </div>	1M 1M 1A (3)	For quotient / product rule $\frac{d}{dx} \sin u = \cos u \frac{du}{dx}$ (accept without $\frac{du}{dx}$)
$2. \cos^2 x - \cos^2 y$ $= \frac{1}{2}(1 + \cos 2x) - \frac{1}{2}(1 + \cos 2y)$ $= \frac{1}{2}(\cos 2x - \cos 2y)$ $= \frac{1}{2} \left(-2 \sin \frac{2x+2y}{2} \sin \frac{2x-2y}{2} \right)$ $= -\sin(x+y) \sin(x-y)$	1A 1M 1	Marking criteria: First trig formulas 1A Second trig formula 1M Completed proof 1
<u>Alternative Solution (1)</u> $\cos^2 x - \cos^2 y$ $= (\cos x + \cos y)(\cos x - \cos y)$ $= \left(2 \cos \frac{x+y}{2} \cos \frac{x-y}{2} \right) \left(-2 \sin \frac{x+y}{2} \sin \frac{x-y}{2} \right)$ $= - \left(2 \sin \frac{x+y}{2} \cos \frac{x+y}{2} \right) \left(2 \sin \frac{x-y}{2} \cos \frac{x-y}{2} \right)$ $= -\sin(x+y) \sin(x-y)$	1A 1M + 1	
<u>Alternative Solution (2)</u> $-\sin(x+y) \sin(x-y)$ $= -\frac{1}{2} \{ \cos[x+y-(x-y)] - \cos[x+y+(x-y)] \}$ $= -\frac{1}{2} (\cos 2y - \cos 2x)$ $= -\frac{1}{2} [(2 \cos^2 y - 1) - (2 \cos^2 x - 1)]$ $= \cos^2 x - \cos^2 y$	1A 1M 1	

Solution	Marks	Remarks
<p><u>Alternative Solution (3)</u></p> $-\sin(x+y)\sin(x-y)$ $= -(\sin x \cos y + \cos x \sin y)(\sin x \cos y - \cos x \sin y)$ $= -\sin^2 x \cos^2 y + \cos^2 x \sin^2 y$ $= -(1 - \cos^2 x)\cos^2 y + \cos^2 x(1 - \cos^2 y)$ $= -\cos^2 y + \cos^2 x + \cos^2 x - \cos^2 x \cos^2 y$ $= \cos^2 x - \cos^2 y$	<p>1A</p> <p>1M</p> <p>1</p>	
<p>$\therefore \cos^2 x - \cos^2 y = -\sin(x+y)\sin(x-y)$</p> <p>(3)</p>		
<p>3. $(1-2x+3x^2)^n$</p> $= [1+(-2x+3x^2)]^n$ $= 1 + {}_n C_1(-2x+3x^2) + {}_n C_2(-2x+3x^2)^2 + \dots$ $= 1 + n(-2x+3x^2) + \frac{n(n-1)}{2}(4x^2 + \dots) + \dots$ $= 1 - 2nx + (2n^2 + n)x^2 + \dots \text{-----} (*)$ $-2n = -10$ $n = 5$ $k = 2n^2 + n$ $= 2(5)^2 + 5$ $= 55$	<p>1M</p> <p>1M</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>(5)</p>	<p>Accept $[(1-2x)+3x^2]^n$ etc.</p> <p>For binomial expansion</p> <p>(pp-1) if dots were omitted in all cases</p> <p>either one</p> <p>Award ONLY when (*) is correct</p>
<p>4. $kx^2 + x + k > 0$</p> $\Delta = 1 - 4k^2$ <p>For $kx^2 + x + k > 0$ for all real values of x,</p> <p>$k > 0$ and $1 - 4k^2 < 0$</p> <p>$k > 0$ and $k^2 > \frac{1}{4}$</p> <p>$k > 0$ and $(k > \frac{1}{2} \text{ or } k < -\frac{1}{2})$</p> <p>$\therefore k > \frac{1}{2}$</p>	<p>1A+1M</p> <p>1M</p> <p>1A</p> <p>(4)</p>	<p>1A for $k > 0$, 1M for $\Delta < 0$</p> <p>For solving a quadratic inequality correctly</p>

Solution	Marks	Remarks
5. $\begin{cases} y = x^2 \\ y = x+k \end{cases}$ $x^2 = x+k$ $x^2 - x - k = 0$ Let x_1 and x_2 be the x -coordinates of P and Q . $x_1 + x_2 = 1$ Let (a, b) be a point on the locus. $a = \frac{x_1 + x_2}{2}$ $= \frac{1}{2}$	1M 1M 1M	
OR $x = \frac{1 \pm \sqrt{1+4k}}{2}$ $a = \frac{1}{2}(x_1 + x_2)$ $= \frac{1}{2} \left(\frac{1 + \sqrt{1+4k}}{2} + \frac{1 - \sqrt{1+4k}}{2} \right)$ $= \frac{1}{2}$	1M 1M	
Alternative Solution $\begin{cases} y = x^2 \\ y = x+k \end{cases}$ $y = (y-k)^2$ $y^2 - (2k+1)y + k^2 = 0$ Let y_1 and y_2 be the y -coordinates of P and Q . $y_1 + y_2 = 2k+1$ Let (a, b) be a point on the locus. $b = \frac{2k+1}{2}$ $a = b - k = \frac{1}{2}$	1M 1M 1M	
\therefore the equation of L is $x = \frac{1}{2}$.	1A (4)	OR $\begin{cases} x = \frac{1}{2} \\ y = \frac{1}{2} + k \end{cases}$

Solution	Marks	Remarks
<p>6. $x x +5x+6=0$</p> <p>Consider the following cases : (1) $x \geq 0$ (2) $x < 0$</p> <p>Case (1) : The equation becomes $x(x)+5x+6=0$ $x^2+5x+6=0$ $x=-3$ or $x=-2$ Since $x \geq 0$, the two values are rejected.</p>	1A	
<p><u>OR</u></p> <p>$x^2+5x+6=0$ Since $x \geq 0$, so this equation has no solution.</p>	1M	For checking
<p>Case (2) : The equation becomes $x(-x)+5x+6=0$ $x^2-5x-6=0$ $x=-1$ or $x=6$ Since $x < 0$, $x=-1$</p> <p>Combining the two cases, $x=-1$.</p>	1A	either one
<p><u>Alternative solution</u></p> <p>$x x +5x+6=0$ $x(x)+5x+6=0$ or $x(-x)+5x+6=0$ $x^2+5x+6=0$ $x^2-5x-6=0$ $x=-3$ or $x=-2$ $x=-1$ or $x=6$</p>	2A	For BOTH correct
<p><u>OR</u></p> <p>$x x = -(5x+6)$ $x^4 = 25x^2 + 60x + 36$ $x^4 - 25x^2 - 60x - 36 = 0$ $(x+1)(x^3 - x^2 - 24x - 36) = 0$ $(x+1)(x-6)(x^2 + 5x + 6) = 0$ $(x+1)(x-6)(x+2)(x+3) = 0$</p>	1A	For any two correct linear factors
<p>$x = -1$ or $x = 6$ or $x = -2$ or $x = -3$</p> <p>Put $x = -3$, LHS = $-18 \neq 0$ (rejected) Put $x = -2$, LHS = $-8 \neq 0$ (rejected) Put $x = -1$, LHS = $0 = \text{RHS}$ Put $x = 6$, LHS = $72 \neq 0$ (rejected)</p> <p>$\therefore x = -1$</p>	1M	For checking
	(4)	

Solution	Marks	Remarks
<p>7. (a) $\mathbf{a} \cdot \mathbf{b}$</p> $= \sqrt{3} (2) \cos 150^\circ$ $= -3$ <p>(b) $\mathbf{a} + 2\mathbf{b} ^2 = (\mathbf{a} + 2\mathbf{b}) \cdot (\mathbf{a} + 2\mathbf{b})$</p> $= \mathbf{a} \cdot \mathbf{a} + 4 \mathbf{a} \cdot \mathbf{b} + 4 \mathbf{b} \cdot \mathbf{b}$ $= (\sqrt{3})^2 + 4(-3) + 4(2)^2$ $= 7$ $ \mathbf{a} + 2\mathbf{b} = \sqrt{7}$	<p>1A</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1A</p>	<p>For $\mathbf{a} \cdot \mathbf{a} = \sqrt{3}^2$ and $\mathbf{b} \cdot \mathbf{b} = 2^2$</p>
<p><u>Alternative solution</u></p>  <p>$\mathbf{a} + 2\mathbf{b} ^2 = \mathbf{a} ^2 + 2\mathbf{b} ^2 - 2 \mathbf{a} 2\mathbf{b} \cos\theta$</p> $= (\sqrt{3})^2 + 4^2 - 2(\sqrt{3})(4)\cos 30^\circ$ $= 7$ <p>$\mathbf{a} + 2\mathbf{b} = \sqrt{7}$</p>	<p>1M+1A</p> <p>1A</p>	<p>1 M for cosine formula</p> <p>1A for $\mathbf{a} = \sqrt{3}$ and $2\mathbf{b} = 4$</p> <p>Omitting vector sign / dot sign in most cases, or using the expression \mathbf{v}^2 or \mathbf{v}^{-2}, (pp-1)</p>
(5)		

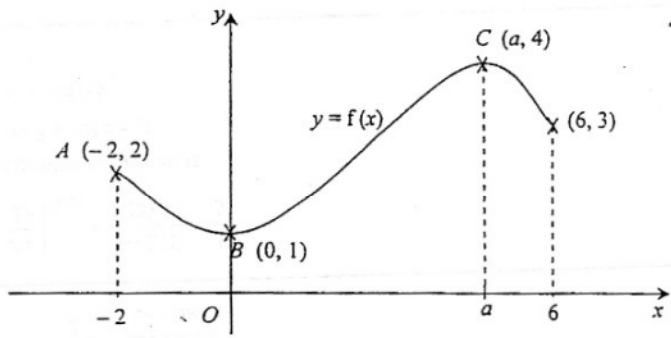
Solution	Marks	Remarks
8. For $n=1$, $n^3 - n + 3 = 1^3 - 1 + 3 = 3$, which is divisible by 3. \therefore the statement is true for $n=1$.	1	
Assume $k^3 - k + 3$ is divisible by 3, where k is a positive integer. (OR Assume $k^3 - k + 3 = 3m$, where k and m are positive integers.)	1	
$(k+1)^3 - (k+1) + 3$ $= k^3 + 3k^2 + 3k + 1 - (k+1) + 3$ $= (k^3 - k + 3) + 3k^2 + 3k$ $= 3(m + k^2 + k)$	1M 1	For using the assumption
$\therefore (k+1)^3 - (k+1) + 3$ is also divisible by 3. The statement is also true for $n = k+1$ if it is true for $n = k$. By the principle of mathematical induction, the statement is true for all positive integers n.	1	Not awarded if anyone of the above marks was withheld.
<div style="border: 1px solid black; padding: 5px;"> <p><u>Alternative solution</u></p> $n^3 - n + 3$ $= n(n^2 - 1) + 3$ $= (n-1)(n)(n+1) + 3$ $(n-1)(n)(n+1)$ is the product of 3 consecutive integers, and hence is divisible by 3. As 3 is also divisible by 3, $\therefore n^3 - n + 3$ is divisible by 3 for all positive integers n. </div>	2A 2A 1	
(5)		

Solution	Marks	Remarks
9. (a) $\cos \theta - \sqrt{3} \sin \theta = r \cos(\theta + \alpha)$ $= r(\cos \theta \cos \alpha - \sin \theta \sin \alpha)$ $\begin{cases} r \cos \alpha = 1 & \dots (1) \\ r \sin \alpha = \sqrt{3} & \dots (2) \end{cases}$ $r = 2$ $\alpha = 60^\circ$ $\therefore \cos \theta - \sqrt{3} \sin \theta = 2 \cos(\theta + 60^\circ)$	1A 1M 1A	Accept $2 \cos(\theta + \frac{\pi}{3})$
OR $\cos \theta - \sqrt{3} \sin \theta = 2(\frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta)$ $= 2(\cos 60^\circ \cos \theta - \sin 60^\circ \sin \theta)$ $= 2 \cos(\theta + 60^\circ)$	1M 1A 1A	For the form of $r(\frac{a}{r} \cos \theta - \frac{b}{r} \sin \theta)$ Accept $2 \cos(\theta + \frac{\pi}{3})$
(b) $\cos 2x - \sqrt{3} \sin 2x = 1$ $2 \cos(2x + 60^\circ) = 1$ $\cos(2x + 60^\circ) = \frac{1}{2}$ $2x + 60^\circ = 360n^\circ \pm 60^\circ$, where n is an integer. $x = 180n^\circ$ or $x = 180n^\circ - 60^\circ$	1M 1M 1A	using (a) General solution of $\cos \phi = k$ Accept radian (pp-1) for mixing degree and radian measures in answer
Alternative Solution (1) $\cos 2x - \sqrt{3} \sin 2x = 1$ $2 \sin(2x + 150^\circ) = 1$ $\sin(2x + 150^\circ) = \frac{1}{2}$ $2x + 150^\circ = 180n^\circ + (-1)^n(30^\circ)$, where n is an integer. $x = 90n^\circ + (-1)^n(15^\circ) - 75^\circ$	1A 1M 1A	
Alternative Solution (2) $\cos 2x - \sqrt{3} \sin 2x = 1$ $1 - 2 \sin^2 x - \sqrt{3}(2 \sin x \cos x) = 1$ $2 \sin x(\sin x + \sqrt{3} \cos x) = 0$ $\sin x = 0$ or $\tan x = -\sqrt{3}$ $x = 180n^\circ$ or $x = 180n^\circ - 60^\circ$	1A 1M+1A	1M for either one
	(6)	

Solution	Marks	Remarks
<p>10. $y = \int (3 + 2 \cos 2x) dx$</p> <p>$= 3x + \sin 2x + c$, where c is a constant.</p> <p>Put $x = \frac{\pi}{4}$, $y = \frac{3\pi}{4}$:</p> <p>$\frac{3\pi}{4} = 3\left(\frac{\pi}{4}\right) + \sin 2\left(\frac{\pi}{4}\right) + c$</p> <p>$c = -1$</p> <p>$\therefore$ the equation of the curve is $y = 3x + \sin 2x - 1$.</p>	<p>1M</p> <p>1M+1A</p> <p>1M</p> <p>1A</p> <p>(5)</p>	<p>(pp-1) for dx omitted 1M awarded even if y was omitted</p> <p>1M for $\int \cos u \, du = \sin u$ 1A awarded even if c was omitted</p>
<p>11. (a) Slope of $L_1 = 2$</p> <p>Let m be the slopes of L_2 and L_3.</p> <p>$\left \frac{m-2}{1+2m} \right = \tan 45^\circ$</p> <p>$\frac{m-2}{1+2m} = \pm 1$</p> <p>$m = -3$ or $m = \frac{1}{3}$</p> <p>\therefore the equation of L_2 and L_3 are $y = \frac{1}{3}x$ and $y = -3x$.</p>	<p>1M+1A</p> <p>1A</p>	<p>1M for LHS Accept omitting absolute sign</p>
<p><u>Alternative solution</u></p> <p>Slope of $L_1 = 2$</p> <p>Let L_1 and L_2 make an angle θ_1 and θ_2 with the x-axis respectively.</p> <p>$\theta_2 = \theta_1 - 45^\circ$</p> <p>$\tan \theta_2 = \tan(\theta_1 - 45^\circ)$</p> <p>$= \frac{\tan \theta_1 - \tan 45^\circ}{1 + \tan \theta_1 \tan 45^\circ}$</p> <p>$= \frac{2-1}{1+2(1)} = \frac{1}{3}$</p> <p>$\therefore$ the equation of L_2 is $y = \frac{1}{3}x$.</p> <p>Since $L_3 \perp L_2$, slope of $L_3 = -3$,</p> <p>\therefore the equation of L_3 is $y = -3x$.</p>	<p>1A</p> <p>1M</p> <p>1A</p>	

Solution	Marks	Remarks
<p>(b) $\begin{cases} y = 2x - 5 \\ y = \frac{1}{3}x \end{cases}$</p> <p>$x = 3, y = 1 \therefore L_1$ and L_2 intersect at $P(3, 1)$.</p> <p>$\begin{cases} y = 2x - 5 \\ y = -3x \end{cases}$</p> <p>$x = 1, y = -3 \therefore L_1$ and L_3 intersect at $Q(1, -3)$.</p> <p>Area = $\frac{1}{2} \begin{vmatrix} 0 & 0 \\ 1 & -3 \\ 3 & 1 \\ 0 & 0 \end{vmatrix}$</p> <p>$= \frac{1}{2} [1 - (-9)]$</p> <p>$= 5$</p>	<p>1M</p> <p>1M</p> <p>1A</p>	<p>Marking Criteria:</p> <p>1M for finding a point of intersection</p> <p>1M for the correct method of finding the area</p> <p>1A for the answer</p>
<p><u>Alternative solution (1)</u></p> <p>L_1 and L_2 intersect at $P(3, 1)$.</p> <p>$OP = \sqrt{3^2 + 1^2} = \sqrt{10}$ and $OQ = \sqrt{10}$</p> <p>Area = $\frac{1}{2} (OP)(OQ)$</p> <p>$= \frac{1}{2} (\sqrt{10})(\sqrt{10})$</p> <p>$= 5$</p>	<p>1M</p> <p>1M</p> <p>1A</p>	<p>Same as above</p>
<p><u>Alternative solution (2)</u></p> <p>L_1 and L_2 intersect at $P(3, 1)$.</p> <p>L_1 and L_3 intersect at $Q(1, -3)$.</p> <p>$OP = \sqrt{3^2 + 1^2} = \sqrt{10}$, $PQ = \sqrt{(3-1)^2 + (1+3)^2} = \sqrt{20}$</p> <p>Area = $\frac{1}{2} (\sqrt{10})(\sqrt{20}) \sin 45^\circ$</p> <p>$= 5$</p>	<p>1M</p> <p>1M</p> <p>1A</p>	<p>Same as above</p>
<p><u>Alternative solution (3)</u></p> <p>The distance from O to L_1 is</p> <p>$d = \left -\frac{5}{\sqrt{5}} \right = \sqrt{5}$</p> <p>(The angles are 45°), the length of (the hypotenuse) PQ is</p> <p>$2d = 2\sqrt{5}$</p> <p>The area of $\triangle OPQ$ is</p> <p>$\frac{1}{2} (\sqrt{5})(2\sqrt{5}) = 5$</p>	<p>1M</p> <p>1M</p> <p>1A</p>	
(6)		

Solution	Marks	Remarks
12. (a) $x^2 - xy + y^2 = 7$ $2x - (x \frac{dy}{dx} + y) + 2y \frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{y-2x}{2y-x}$	1A+1A 1A	1A for $\frac{d}{dx}(xy)$, 1A for the other terms
<div style="border: 1px solid black; padding: 5px;"> <p>Alternative solution</p> $x^2 - xy + y^2 = 7$ $y^2 - xy + x^2 - 7 = 0$ $y = \frac{x \pm \sqrt{x^2 - 4(x^2 - 7)}}{2}$ $= \frac{x \pm \sqrt{28 - 3x^2}}{2}$ $\frac{dy}{dx} = \frac{1}{2} \pm \frac{1}{2} \left(\frac{-6x}{2\sqrt{28 - 3x^2}} \right)$ $= \frac{1}{2} \pm \frac{3x}{2\sqrt{28 - 3x^2}}$ </div>	1A 1A 1A	
(b) $\frac{dy}{dx} \Big _{(1,3)} = \frac{3-2(1)}{2(3)-1} = \frac{1}{5}$ Equation of normal is $y-3 = -5(x-1)$ $y = -5x+8$	} 1M } 1A (5)	

Solution	Marks	Remarks
13. (a) The curve attains a maximum at $x = a$. The curve attains a minimum at $x = 0$.	1A 1A	Withhold 1A if answered in coordinates form
(b) (i) $\int_0^a f'(x) dx$ $= [f(x)]_0^a$ $= f(a) - f(0)$ $= f(a) - 1$ (Since $f(0) = 1$) Since the area of $R_1 = 3$, $f(a) = 4$.	1A 1A	
(ii) $[f(x)]_0^6 = \text{Area of } R_1 - \text{Area of } R_2$ $f(6) - f(0) = 3 - 1$ $f(6) = 3$	1A	
	1A 1M (7)	Shape For points A, B and C

Solution	Marks	Remarks
14. (a) (i) The centre of J is $(0, 0)$ and the radius is r . Distance from $(0, 0)$ to $L = r$ $\frac{ m(0) - 0 + c }{\sqrt{m^2 + 1}} = r$ $\frac{c^2}{m^2 + 1} = r^2$ $c^2 = r^2(m^2 + 1)$	1M+1M 1	1M for distance formula 1M for $d = r$ Accept omitting absolute sign
<div style="border: 1px solid black; padding: 5px;"> <p><u>Alternative solution</u></p> $\begin{cases} x^2 + y^2 = r^2 \\ y = mx + c \end{cases}$ $x^2 + (mx + c)^2 = r^2$ $(m^2 + 1)x^2 + 2mcx + c^2 - r^2 = 0$ $\Delta = (2mc)^2 - 4(m^2 + 1)(c^2 - r^2) = 0$ $m^2c^2 - (m^2c^2 - m^2r^2 + c^2 - r^2) = 0$ $c^2 = r^2(m^2 + 1)$ </div>	1M 1M 1	For elimination of x (or y)
(ii) Since L passes through (h, k) , $k = mh + c$ $c = k - mh$ Substitute into $c^2 = r^2(m^2 + 1)$: $(k - mh)^2 = r^2(m^2 + 1) \dots (*)$	1	
(b) (i) Equation of PR : $y + 5 = \frac{4 + 5}{7 + 5}(x + 5)$ $3x - 4y - 5 = 0$ $\text{radius of } J = \frac{ 3(0) - 4(0) - 5 }{\sqrt{3^2 + 4^2}}$ $= 1$	(4) 1M 1A	
<div style="border: 1px solid black; padding: 5px;"> <p><u>Alternative solution</u></p> <p>Equation of PR:</p> $y + 5 = \frac{4 + 5}{7 + 5}(x + 5)$ $y = \frac{3}{4}x - \frac{5}{4}$ <p>Using (a) (i),</p> $\left(-\frac{5}{4}\right)^2 = r^2\left[\left(\frac{3}{4}\right)^2 + 1\right]$ $r^2 = 1$ <p>\therefore the radius of J is 1.</p> </div>	1M 1A	

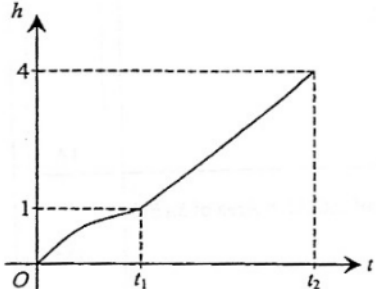
Solution	Marks	Remarks
<p>(ii) Let m be the slope of PQ. Put $r = 1$, $h = 7$, $k = 4$ into the result of (a)(ii): $(4 - 7m)^2 = m^2 + 1$</p> <p>$48m^2 - 56m + 15 = 0$ $m = \frac{3}{4}$ (rejected) or $m = \frac{5}{12}$ \therefore the slope of PQ is $\frac{5}{12}$.</p>	1M 1A	Withhold 1M if $(h, k) \neq (7, 4)$
<p><u>Alternative solution</u> Slope of $PR = \frac{4+5}{7+5} = \frac{3}{4}$ Slope of $OP = \frac{4}{7}$ Let the slope of PQ be m. OP bisects $\angle QPR$.</p> $\frac{\frac{3}{4} - \frac{4}{7}}{1 + (\frac{3}{4})(\frac{4}{7})} = \frac{\frac{4}{7} - m}{1 + (\frac{4}{7})m}$ $\frac{1}{8} = \frac{4 - 7m}{7 + 4m}$ $m = \frac{5}{12}$	1M 1A	
<p>(iii) Put $r = 1$, $h = -5$, $k = -5$ into (*): $(-5 + 5m)^2 = m^2 + 1$</p> <p>$24m^2 - 50m + 24 = 0$ $m = \frac{3}{4}$ (rejected) or $m = \frac{4}{3}$ \therefore the slope of QR is $\frac{4}{3}$.</p>	1M	Withhold 1M if $(h, k) \neq (-5, -5)$
<p><u>Alternative solution</u> Slope of $PR = \frac{3}{4}$ Slope of $OR = 1$ Let the slope of QR be n.</p> $\frac{1 - \frac{3}{4}}{1 + \frac{3}{4}} = \frac{n - 1}{1 + n}$ $n = \frac{4}{3}$	1M	

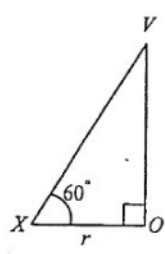
Solution	Marks	Remarks
<p>The equation of PQ is</p> $y - 4 = \frac{5}{12}(x - 7)$ $5x - 12y + 13 = 0 \quad (1)$ <p>The equation of QR is</p> $y + 5 = \frac{4}{3}(x + 5)$ $4x - 3y + 5 = 0 \quad (2)$	1M	For finding equations of PQ and QR
$(1) \times 4 - (2) \times 5 :$ $-33y + 27 = 0$		
$y = \frac{9}{11}$		
<p>Substitute $y = \frac{9}{11}$ into (1), $x = \frac{-7}{11}$.</p>		
<p>\therefore the coordinates of Q are $(\frac{-7}{11}, \frac{9}{11})$.</p>	1A	
	(3)	

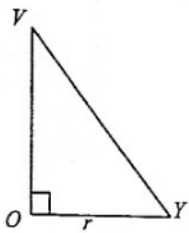
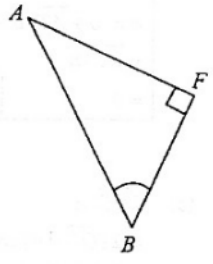
Solution	Marks	Remarks						
15. (a) $l = PS + QS + NS$ $= 2(\sqrt{1^2 + x^2} + (\sqrt{2})^2) + 2 - x$ $= 2\sqrt{x^2 + 3} + 2 - x$ $\frac{dl}{dx} = 2\left(\frac{2x}{2\sqrt{x^2 + 3}}\right) - 1$ $= \frac{2x}{\sqrt{x^2 + 3}} - 1$	1A 1							
	(2)							
(b) (i) $\frac{dl}{dx} = 0$ $\frac{2x}{\sqrt{x^2 + 3}} - 1 = 0$ $4x^2 = x^2 + 3$ $x = -1$ (rejected) or $x = 1$	1M 1A							
<table border="1" style="width: 100%; text-align: center;"> <tr> <td style="width: 33%;">$0 \leq x < 1$</td> <td style="width: 33%;">$x = 1$</td> <td style="width: 33%;">$1 < x \leq \frac{3}{2}$</td> </tr> <tr> <td>$\frac{dl}{dx} < 0$</td> <td>$\frac{dl}{dx} = 0$</td> <td>$\frac{dl}{dx} > 0$</td> </tr> </table>	$0 \leq x < 1$	$x = 1$	$1 < x \leq \frac{3}{2}$	$\frac{dl}{dx} < 0$	$\frac{dl}{dx} = 0$	$\frac{dl}{dx} > 0$	1M	$\frac{d^2l}{dx^2} = \frac{6}{(x^2 + 3)^{\frac{3}{2}}}$ $\frac{d^2l}{dx^2} \Big _{x=1} = \frac{3}{4} > 0$
$0 \leq x < 1$	$x = 1$	$1 < x \leq \frac{3}{2}$						
$\frac{dl}{dx} < 0$	$\frac{dl}{dx} = 0$	$\frac{dl}{dx} > 0$						
\therefore As l has only one turning point, it attains the least value at $x = 1$.	1A							
(ii) The greatest value of l occurs at one of the end-points. At $x = 0$, $l = 2\sqrt{3} + 2$ (≈ 5.46) At $x = \frac{3}{2}$, $l = \sqrt{21} + \frac{1}{2}$ (≈ 5.08) $< 2\sqrt{3} + 2$ $\therefore l$ attains the greatest value at $x = 0$.	1M 1A							
	(6)							
(c) (i) $E = \frac{k}{(2-x)^2} + \frac{2k}{x^2 + 3}$	1A	<u>OR</u> $E = \frac{k(3x^2 - 8x + 11)}{(2-x)^2(x+3)}$ Accept $E = \frac{k_1}{(2-x)^2} + \frac{2k_2}{x^2 + 3}$						

Solution	Marks	Remarks
<p>(ii) From (b), l attains the least value at $x=1$.</p> <p>At $x=1, E = \frac{k}{1} + \frac{2k}{4}$ $= \frac{3k}{2}$</p> <p>At $x = \frac{3}{2}, E = \frac{8k}{21} + 4k > \frac{3k}{2}$</p> <p>$\therefore E$ will not attain its greatest value at $x=1$. The student is incorrect.</p>	<p>1M</p> <p>1M</p> <p>1A</p>	<p>Accept other counter examples: e.g. $E(1.1) \approx 1.71k > \frac{3k}{2}$.</p>
<p><u>Alternative solution</u></p> $\frac{dE}{dx} = \frac{-2k(-1)}{(2-x)^3} + \frac{-2k(2x)}{(x^2+3)^2}$ $= \frac{2k}{(2-x)^3} - \frac{4kx}{(x^2+3)^2}$ <p>From (b), l attains the least value at $x=1$.</p> <p>At $x=1, \frac{dE}{dx} = 2k - \frac{4k}{16}$ $= \frac{7k}{4} > 0$</p> <p>E is increasing at $x=1$.</p> <p>$\therefore E$ will not attain its greatest value at $x=1$. The student is incorrect.</p>	<p>1M</p> <p>1M</p> <p>1A</p>	<p>OR solving $\frac{dE}{dx} = 0$ successfully Accept $\frac{dE}{dx} = \frac{7k}{4} \neq 0$</p>
(4)		

Solution	Marks	Remarks
16. (a) (i) $y = \frac{1}{3}x^2 - \frac{4}{3}x + 1$ $x^2 - 4x + 3 - 3y = 0$ $x = \frac{4 \pm \sqrt{16 - 4(3-3y)}}{2}$ $= 2 \pm \sqrt{3y+1}$ As $0 \leq x \leq 1$, the equation of C_1 is $x = 2 - \sqrt{3y+1}$.	1	
(ii) The equation of C_2 is $x = 2 + \sqrt{3y+1}$.	1A	
	(2)	
(b) (i) (1) $V = \pi \int_0^h [(2 + \sqrt{3y+1})^2 - (2 - \sqrt{3y+1})^2] dy$ $= \pi \int_0^h 8\sqrt{3y+1} dy$ $= 8\pi \left[\frac{2}{3} \left(\frac{1}{3} \right) (3y+1)^{\frac{3}{2}} \right]_0^h$ $= \frac{16\pi}{9} [(3h+1)^{\frac{3}{2}} - 1]$	1M+1A	1M for $V = \pi \int_a^b x^2 dy$
	1A	
	1	
(2) $\frac{dV}{dt} = \frac{16\pi}{9} \left(\frac{3}{2} \right) (3h+1)^{\frac{1}{2}} \left(3 \frac{dh}{dt} \right)$ $= 8\pi (3h+1)^{\frac{1}{2}} \frac{dh}{dt}$ Put $\frac{dV}{dt} = 8\pi$: $8\pi = 8\pi (3h+1)^{\frac{1}{2}} \left(\frac{dh}{dt} \right)$ $\frac{dh}{dt} = \frac{1}{(3h+1)^{\frac{1}{2}}}$	1M	for $\frac{dV}{dt} = \frac{dV}{dh} \frac{dh}{dt}$
	1A	
(ii) When $h=1$, $V = \frac{112\pi}{9}$ When $1 < h < 4$, $V = \frac{112\pi}{9} + \pi(3)^2(h-1)$ $= 9\pi h + \frac{31\pi}{9}$ $\frac{dV}{dt} = 9\pi \frac{dh}{dt}$ $\frac{dh}{dt} = \frac{8}{9}$	1M	1M for $V =$ volume of lower part $+ \pi(3)^2(h-1)$
	1A	

Solution	Marks	Remarks
<p><u>Alternative Solution</u></p> $\frac{dh}{dt} = \frac{\frac{dV}{dt}}{\left(\text{cross-sectional area of the cylindrical part}\right)}$ $= \frac{8\pi}{\pi(3)^2}$ $= \frac{8}{9}$	<p>1M</p> <p>1A</p>	
<p>(iii)</p> 	<p>1A+1A</p>	<p>1A for the curved shape for $0 \leq t \leq t_1$</p> <p>1A for the straight line for $t_1 \leq t \leq t_2$</p>
	<p>(10)</p>	<p>pp-1 if labeling incomplete (Must show $h, t, t_1, t_2, 1, 4$)</p>

Solution	Marks	Remarks
17. (a) (i) Using Heron's formula, $\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$ $= \sqrt{9(9-5)(9-6)(9-7)}$ $= 6\sqrt{6}$	1M 1A	$(s = \frac{5+6+7}{2} = 9)$
<div style="border: 1px solid black; padding: 5px;"> <p><u>Alternative solution</u></p> $\cos \angle BAC = \frac{5^2 + 6^2 - 7^2}{2(5)(6)}$ $= \frac{1}{5}$ <p>Area of $\Delta ABC = \frac{1}{2}(AB)(AC)\sin \angle BAC$</p> $= \frac{1}{2}(6)(5)\left(\frac{\sqrt{5^2-1}}{5}\right)$ $= 6\sqrt{6}$ </div>	1M 1A	$\cos \angle ABC = \frac{5}{7}$ $\cos \angle ACB = \frac{19}{35}$
(ii) Area of ΔAOB + Area of ΔBOC + Area of ΔCOA = Area of ΔABC $\frac{1}{2}(6)(r) + \frac{1}{2}(7)(r) + \frac{1}{2}(5)(r) = 6\sqrt{6}$ $9r = 6\sqrt{6}$ $r = \frac{2\sqrt{6}}{3}$	1M 1 (4)	
(b) (i) Let X be the point of contact of AB and the inscribed circle. $\angle VXO = 60^\circ$ Consider ΔVXO : $\tan 60^\circ = \frac{VO}{r}$ $VO = \frac{2\sqrt{6}}{3}(\sqrt{3})$ $= 2\sqrt{2}$ <p>Volume of $VABC = \frac{1}{3}(\text{area of } \Delta ABC)(VO)$</p> $= \frac{1}{3}(6\sqrt{6})(2\sqrt{2})$ $= 8\sqrt{3}$	1A 1M 1A	 <p>Must have found VO</p>

Solution	Marks	Remarks
<p>(ii) Let Y be the point of contact of BC and the inscribed circle. Consider $\triangle VOY$:</p> $VY^2 = VO^2 + OY^2 = (2\sqrt{2})^2 + \left(\frac{2\sqrt{6}}{3}\right)^2$ $VY = \frac{4\sqrt{6}}{3}$		
<p><u>Alternative Solution</u> $\triangle VOX \cong \triangle VOY$ (SAS) $\angle VYO = \angle VXO = 60^\circ$ Consider $\triangle VOY$:</p> $\cos 60^\circ = \frac{r}{VY}$ $VY = \frac{r}{\cos 60^\circ} = \left(\frac{2\sqrt{6}}{3}\right)(2) = \frac{4\sqrt{6}}{3}$		
<p>Area of $\triangle VBC = \frac{1}{2}(BC)(VY)$</p> $= \frac{1}{2}(7)\left(\frac{4\sqrt{6}}{3}\right)$ $= \frac{14\sqrt{6}}{3}$	<p>1M 1A</p>	<p>Must have found VY</p>
<p>(iii) Let F be the foot of perpendicular from A to the plane VBC. $\angle ABF$ represents the angle between AB and the plane VBC.</p> $\frac{1}{3}(\text{area of } \triangle VBC)(AF) = \text{volume of tetrahedron}$ $\frac{1}{3}\left(\frac{14\sqrt{6}}{3}\right)(AF) = 8\sqrt{3}$ $AF = \frac{18\sqrt{2}}{7} (\approx 3.637)$ $\sin \angle ABF = \frac{AF}{AB}$ $= \frac{18\sqrt{2}}{7}$ $= \frac{3\sqrt{2}}{7} (\approx 0.606)$	<p>1A 1M</p>	
<p>$\angle ABF = 37^\circ$ (correct to the nearest degree)</p>	<p>1A (8)</p>	

Solution	Marks	Remarks
18. (a) $\overline{OG} = \frac{2\overline{OM} + \overline{OB}}{2+1}$ $= \frac{2\left(\frac{a}{2}\right) + b}{3}$ $= \frac{a+b}{3}$	1A	
(1)		
(b) $\overline{OT} \cdot a - b \cdot a$ $= (\overline{OB} + \overline{BT}) \cdot a - b \cdot a$ $= b \cdot a + \overline{BT} \cdot a - b \cdot a$ $= \overline{BT} \cdot a$ $= 0 \quad \text{(Since } \overline{BT} \perp a \text{)}$	1A	
<u>Alternative solution</u> $\overline{OT} \cdot a - b \cdot a$ $= (\overline{OT} - b) \cdot a$ $= \overline{BT} \cdot a$ $= 0 \quad \text{(Since } \overline{BT} \perp a \text{)}$	1A	$\therefore \overline{BT} \perp a$ $\therefore \overline{BT} \cdot a = 0$ $\therefore (\overline{OT} - b) \cdot a = 0$ $\text{i.e. } \overline{OT} \cdot a - b \cdot a = 0$
Similarly, $\overline{OT} \cdot b - a \cdot b = 0$.	1A	
<u>Alternative solution</u> $\overline{OT} \cdot b - a \cdot b$ $= (\overline{OA} + \overline{AT}) \cdot b - a \cdot b$ $= a \cdot b + \overline{AT} \cdot b - a \cdot b$ $= \overline{AT} \cdot b$ $= 0 \quad \text{(Since } \overline{AT} \perp b \text{)}$	1A	
(3)		
(c) $2\overline{OC} \cdot a$ $= 2 \overline{OC} a \cos\angle COM$ $= 2(OM) a $ $= a ^2$	1A	
<u>Alternative solution</u> $2\overline{OC} \cdot a = 2(\overline{OM} + \overline{MC}) \cdot a$ $= 2\overline{OM} \cdot a + 2\overline{MC} \cdot a$ $= a \cdot a + 0 \quad \text{(Since } \overline{MC} \perp a \text{)}$ $= a ^2$	1A	
Similarly, $\overline{OC} \cdot b = \frac{ b ^2}{2}$.	1A	
(3)		

Solution	Marks	Remarks
<p>(d) (i) $(\overrightarrow{GT} - 2\overrightarrow{CG}) \cdot \mathbf{a}$ $= [\overrightarrow{GO} + \overrightarrow{OT} - 2(\overrightarrow{CO} + \overrightarrow{OG})] \cdot \mathbf{a}$ $= (\overrightarrow{OT} + 2\overrightarrow{OC} - 3\overrightarrow{OG}) \cdot \mathbf{a}$ $= \overrightarrow{OT} \cdot \mathbf{a} + 2\overrightarrow{OC} \cdot \mathbf{a} - 3\overrightarrow{OG} \cdot \mathbf{a}$ Since $\overrightarrow{OT} \cdot \mathbf{a} = \mathbf{b} \cdot \mathbf{a}$ (result of (b)) $2\overrightarrow{OC} \cdot \mathbf{a} = \mathbf{a} ^2$ (result of (c)) and $\overrightarrow{OG} = \frac{\mathbf{a} + \mathbf{b}}{3}$, (result of (a)) $(\overrightarrow{GT} - 2\overrightarrow{CG}) \cdot \mathbf{a} = \mathbf{b} \cdot \mathbf{a} + \mathbf{a} ^2 - (\mathbf{a} ^2 + \mathbf{b} \cdot \mathbf{a})$ $= 0$ Similarly, $(\overrightarrow{GT} - 2\overrightarrow{CG}) \cdot \mathbf{b} = 0$.</p>	<p>1A 1A 1M 1A 1A</p>	<p>1A for $\overrightarrow{GT} = \overrightarrow{GO} + \overrightarrow{OT}$ and $\overrightarrow{CG} = \overrightarrow{CO} + \overrightarrow{OG}$ For using either one Need $(\overrightarrow{GT} - 2\overrightarrow{CG}) \cdot \mathbf{a} = 0$ manipulated correctly</p>
<p>(ii) Since $(\overrightarrow{GT} - 2\overrightarrow{CG}) \cdot \mathbf{a} = (\overrightarrow{GT} - 2\overrightarrow{CG}) \cdot \mathbf{b} = 0$ and \mathbf{a}, \mathbf{b} are non-parallel, $\overrightarrow{GT} - 2\overrightarrow{CG} = \mathbf{0}$, i.e. $\overrightarrow{GT} = 2\overrightarrow{CG}$ As $\overrightarrow{GT} = \lambda\overrightarrow{CG}$ for a scalar λ, $\overrightarrow{GT} \parallel \overrightarrow{CG}$. G, T and C are collinear.</p>	<p>1</p>	<p>Omitting vector sign / dot sign in most cases, or using the expression v^2 or v^{-2}, (pp-1)</p>
(5)		