

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$
$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$
$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$	

Section A (62 marks)

Answer ALL questions in this section and write your answers in the spaces provided in this Question-Answer Book.

1. Find $\frac{d}{dx} \left[\frac{\sin(2x+1)}{x} \right]$. (3 marks)

2. Prove the identity $\cos^2 x - \cos^2 y = -\sin(x+y)\sin(x-y)$. (3 marks)

3. It is given that $(1-2x+3x^2)^n = 1-10x+kx^2 + \text{terms involving higher powers of } x$,
 where n is a positive integer and k is a constant. Find the values of n and k . (5 marks)

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10. The slope at any point (x, y) of a curve is given by $\frac{dy}{dx} = 3 + 2 \cos 2x$. If the curve passes through the point $(\frac{\pi}{4}, \frac{3\pi}{4})$, find its equation. (5 marks)

11. Let L_1 be the straight line $y = 2x - 5$. L_2 and L_3 are two straight lines passing through the origin and each makes an angle of 45° with L_1 (see Figure 1).
(a) Find the equations of L_2 and L_3 .
(b) Find the area of the triangle bounded by L_1 , L_2 and L_3 . (6 marks)

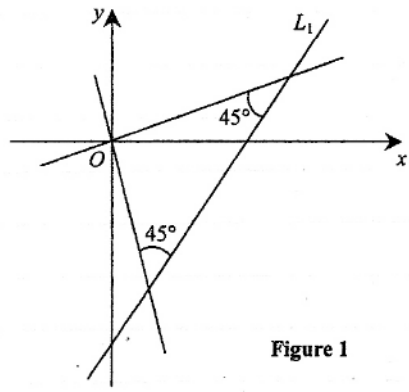


Figure 1

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12. (a) Let $x^2 - xy + y^2 = 7$. Find $\frac{dy}{dx}$.

(b) Find the equation of the normal to the curve $x^2 - xy + y^2 = 7$ at the point $(1, 3)$.

(5 marks)

13. Let $f(x)$ be a polynomial. Figure 2 shows a sketch of the curve $y = f'(x)$, where $-2 \leq x \leq 6$. The curve cuts the x -axis at the origin and $(a, 0)$, where $0 < a < 6$. It is known that the areas of the shaded regions R_1 and R_2 as shown in Figure 2 are 3 and 1 respectively.

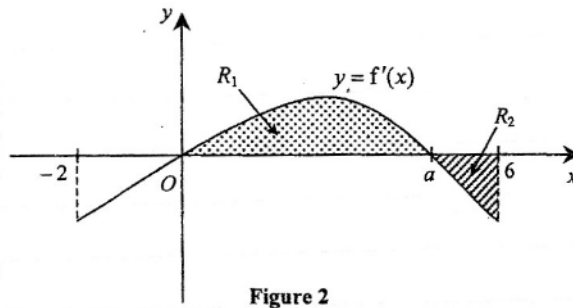


Figure 2

(a) Write down the x -coordinates of the maximum and minimum points of the curve $y = f(x)$ for $-2 < x < 6$.

(b) It is known that $f(-2) = 2$ and $f(0) = 1$.

(i) By considering $\int_0^a f'(x) dx$, find the value of $f(a)$.

(ii) In Figure 3, sketch the curve $y = f(x)$ for $-2 \leq x \leq 6$.

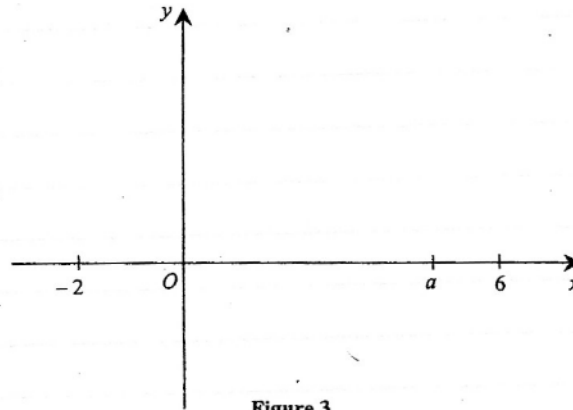


Figure 3

(7 marks)

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SECTION B (48 marks)

Answer any **FOUR** questions in this section. Each question carries 12 marks.
Write your answers in the CE(B) answer book.

14. Let J be the circle $x^2 + y^2 = r^2$, where $r > 0$.

(a) Suppose that the straight line $L: y = mx + c$ is a tangent to J .

(i) Show that $c^2 = r^2(m^2 + 1)$.

(ii) If L passes through a point (h, k) , show that $(k - mh)^2 = r^2(m^2 + 1)$.

(4 marks)

(b)

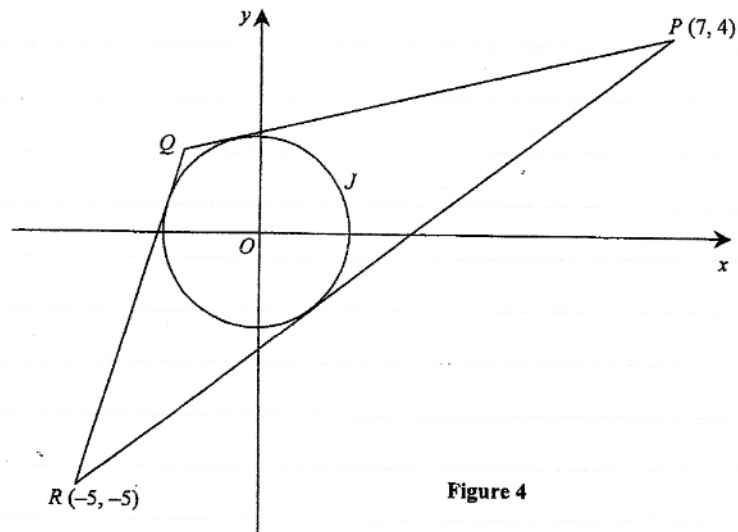


Figure 4

J is inscribed in a triangle PQR (see Figure 4). The coordinates of P and R are $(7, 4)$ and $(-5, -5)$ respectively.

(i) Find the radius of J .

(ii) Using (a) (ii), or otherwise, find the slope of PQ .

(iii) Find the coordinates of Q .

(8 marks)

15.

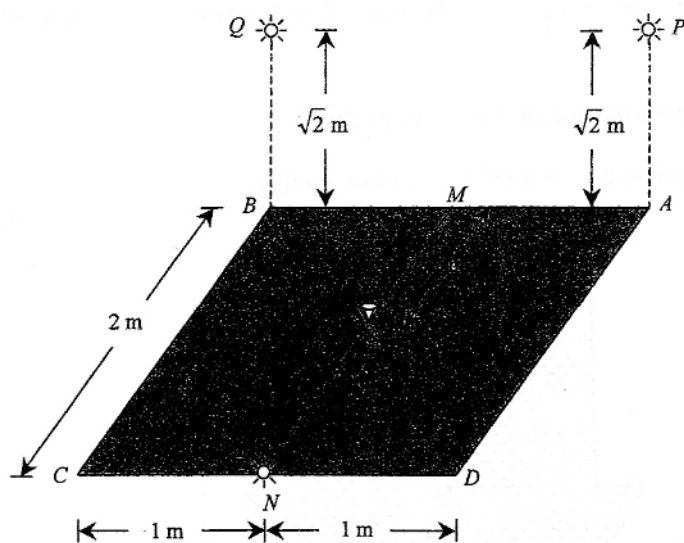


Figure 5

In Figure 5, $ABCD$ is a horizontal square board of side 2 m for displaying diamonds. Let M, N be the mid-points of BA and CD respectively. Three identical small bulbs are located at points N, P and Q respectively for illumination purpose, where P and Q are at a height $\sqrt{2}$ m vertically above A and B respectively. A diamond is placed at a point S along MN and $MS = x$ m, where $0 \leq x \leq \frac{3}{2}$. Let $PS + QS + NS = \ell$ m.

- (a) Express ℓ in terms of x .

Hence show that
$$\frac{d\ell}{dx} = \frac{2x}{\sqrt{x^2 + 3}} - 1.$$

(2 marks)

- (b) Find the values of x at which ℓ attains

- (i) the least value, and
(ii) the greatest value.

(6 marks)

- (c) Suppose that the intensity of light energy received by the diamond from each bulb varies inversely as the square of the distance of the bulb from the diamond, with $k (> 0, \text{ in suitable unit})$ being the variation constant. Let E (in suitable unit) be the total intensity of light energy received by the diamond from the three bulbs.

- (i) Express E in terms of k and x .
(ii) A student guesses that when ℓ attains its least value, E will attain its greatest value. Explain whether the student's guess is correct or not.

(4 marks)

16. Let C be the curve $y = \frac{1}{3}x^2 - \frac{4}{3}x + 1$. C_1 is a part of C with $0 \leq x \leq 1$ and C_2 is a part of C with $3 \leq x \leq 4$.

- (a) (i) Show that the equation of C_1 is $x = 2 - \sqrt{3y+1}$.
(ii) Write down the equation of C_2 in the form $x = f(y)$.

(2 marks)

(b)

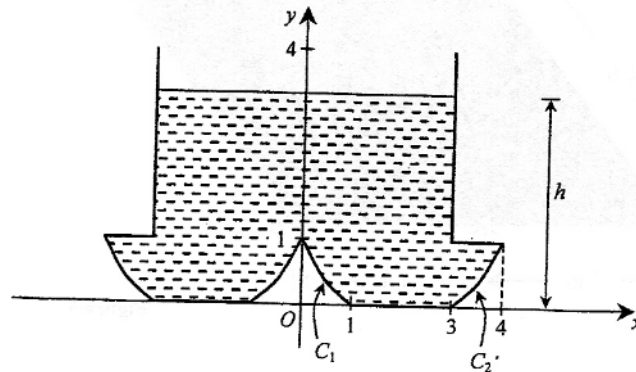


Figure 6

A container is formed by revolving C_1 , the line segment $y=0$ (for $1 \leq x \leq 3$), C_2 , the line segment $y=1$ (for $3 \leq x \leq 4$) and the line segment $x=3$ (for $1 \leq y \leq 4$) about the y -axis (see Figure 6). Starting from time $t=0$, water is poured into the container at a constant rate of 8π cubic units per minute. Let the volume and depth of water in the container at time t minutes be V cubic units and h units respectively.

- (i) Consider $0 < h < 1$.

(1) Show that $V = \frac{16\pi}{9} [(3h+1)^{\frac{3}{2}} - 1]$.

(2) Find $\frac{dh}{dt}$ in terms of h .

- (ii) Consider $1 < h < 4$. Find $\frac{dh}{dt}$.

- (iii) It is known that $h=1$ at $t=t_1$ and $h=4$ at $t=t_2$. Sketch a graph to show how h varies with t for $0 \leq t \leq t_2$. (You are not required to find the values of t_1 and t_2 .)

(10 marks)

17. (a)

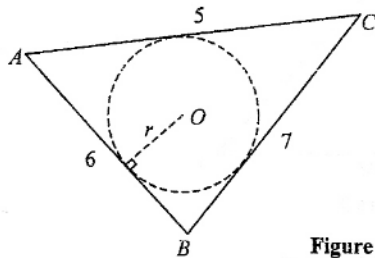


Figure 7

ABC is a triangle with $AB=6$, $BC=7$ and $CA=5$. A circle is inscribed in the triangle (see Figure 7). Let O be the centre of the circle and r be its radius.

(i) Find the area of $\triangle ABC$.

(ii) By considering the areas of $\triangle AOB$, $\triangle BOC$ and $\triangle COA$, show that $r = \frac{2\sqrt{6}}{3}$.

(4 marks)

(b)

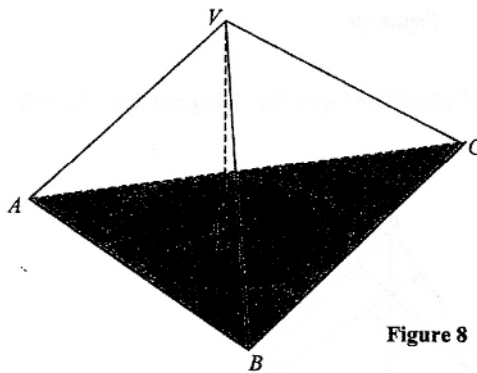


Figure 8

$VABC$ is a tetrahedron with the $\triangle ABC$ described in (a) as the base (see Figure 8). Furthermore, point O is the foot of perpendicular from V to the plane ABC . It is given that the angle between the planes VAB and ABC is 60° .

(i) Find the volume of the tetrahedron $VABC$.

(ii) Find the area of $\triangle VBC$.

(iii) Find the angle between the side AB and the plane VBC , giving your answer correct to the nearest degree.

(8 marks)

18.

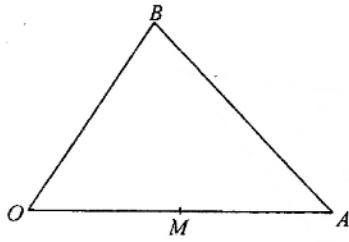


Figure 9

Figure 9 shows a triangle OAB . Let $\vec{OA} = \mathbf{a}$, $\vec{OB} = \mathbf{b}$ and M be the mid-point of OA .

(a)

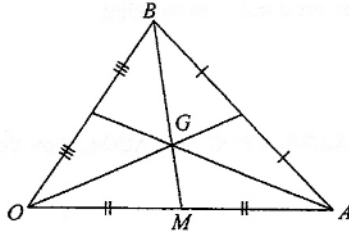


Figure 10

Let G be the centroid of $\triangle OAB$ (see Figure 10). It is given that $BG : GM = 2 : 1$. Express \vec{OG} in terms of \mathbf{a} and \mathbf{b} .

(1 mark)

(b)

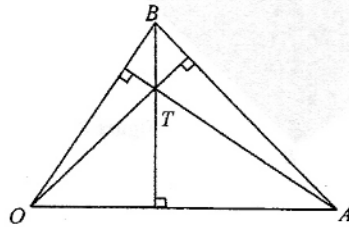


Figure 11

Let T be the orthocentre of $\triangle OAB$ (see Figure 11). Show that $\vec{OT} \cdot \mathbf{a} - \mathbf{b} \cdot \mathbf{a} = 0$ and write down the value of $\vec{OT} \cdot \mathbf{b} - \mathbf{a} \cdot \mathbf{b}$.

(3 marks)

(c)

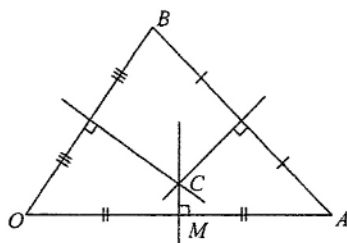


Figure 12

Let C be the circumcentre of $\triangle OAB$ (see Figure 12). Show that $2\overrightarrow{OC} \cdot \mathbf{a} = |\mathbf{a}|^2$ and find $\overrightarrow{OC} \cdot \mathbf{b}$ in terms of $|\mathbf{b}|$.

(3 marks)

(d) Consider the points G , T and C described in (a), (b) and (c) respectively.

(i) Using the above results, find the values of $(\overrightarrow{GT} - 2\overrightarrow{CG}) \cdot \mathbf{a}$ and $(\overrightarrow{GT} - 2\overrightarrow{CG}) \cdot \mathbf{b}$.

(ii) Show that G , T and C are collinear.

Note : You may use the following property for vectors in the two-dimensional space :

If $\mathbf{w} \cdot \mathbf{u} = \mathbf{w} \cdot \mathbf{v} = 0$, where \mathbf{u} and \mathbf{v} are non-parallel, then $\mathbf{w} = \mathbf{0}$.

(5 marks)

END OF PAPER