香港考試及評核局
HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

2005年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2005

附加數學
ADDITIONAL MATHEMATICS

本評卷參考乃香港考試及評核局專爲今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視爲標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations and Assessment Authority for markers' reference. The Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Authority is counting on the co-operation of markers/teachers in this regard.

考試結束後，各科評卷參考將存放於教師中心，供教師參閱。
After the examinations, marking schemes will be available for reference at the teachers' centre.

GENERAL INSTRUCTIONS TO MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates would use alternative methods not specified in the marking scheme. Markers should be patient in marking these alternative solutions. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method is specified in the question.

2. In the marking scheme, marks are classified as follows :

   'M' marks – awarded for knowing a correct method of solution and attempting to apply it;

   'A' marks – awarded for the accuracy of the answer;

   Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. The symbol (pp–1) should be used to denote marks deducted for poor presentation (p.p.). Note the following points:

   (a) At most deduct 1 mark for p.p. in each question, up to a maximum of 2 marks for each section.

   (b) For similar p.p., deduct only 1 mark for the first time that it occurs, i.e. do not penalise candidates twice in the whole paper for the same p.p.

   (c) In any case, do not deduct any marks for p.p. in those steps where candidates could not score any marks.

   (d) Some cases in which marks should be deducted for p.p. are specified in the marking scheme. However, the lists are by no means exhaustive. Markers should exercise their professional judgement to give p.p.s in other situations.

5. The symbol (u–1) should be used to denote marks deducted for wrong/no units in the final answers (if applicable).

   Note the following points:

   (a) At most deduct 1 mark for wrong/no units for each section.

   (b) Do not deduct any marks for wrong/no units in case candidate's answer was already wrong.

6. Marks entered in the Page Total Box should be the net total score on that page.

7. In the Marking Scheme, steps which can be omitted are enclosed by dotted rectangles [dotted rectangle],

   whereas alternative answers are enclosed by solid rectangles [solid rectangle].

8. (a) Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.

   (b) In case a certain degree of accuracy had been specified in the question, answers not accurate up to that degree should not be accepted. For answers with an excess degree of accuracy, deduct 1 mark for the first time if happened. In any case, do not deduct any marks for excess degree of accuracy in those steps where candidates could not score any marks.

9. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

10. Unless the form of answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they were correct.

Solution	Marks	Remarks
1. $\int (2x-3)^7 \mathrm{d}x$		
$= \frac{(2x-3)^8}{8}(\frac{1}{2}) + c$, where c is a constant	1M	For $\int u^n \mathrm{d}u = \frac{u^{n+1}}{n+1}$
$= \frac{1}{16}(2x-3)^8 + c$	1A	Withhold 1A if c was omitted in Q.1 or/and Q.13 (a)
	2	
2. (a) $(1+y)^5$		
$= 1 + {}_5C_1 y + {}_5C_2 y^2 + {}_5C_3 y^3 + {}_5C_4 y^4 + y^5$		
$= 1 + 5y + 10y^2 + 10y^3 + 5y^4 + y^5$	1A	
(b) $(1+x+2x^2)^5$		
$= [1+(x+2x^2)]^5$	1M	
$= 1 + 5(x+2x^2) + 10(x+2x^2)^2 + \cdots$	1M	For expanding up to sufficient terms
$= 1 + 5(x+2x^2) + 10(x^2 + \cdots) + \cdots$		
$= 1 + 5x + 10x^2 + 10x^2 + \cdots$		
$= 1 + 5x + 20x^2 + \cdots$	1A	(pp–1) if dots were omitted in all cases
Alternative solution		
$(1+x+2x^2)^5$		
$= [(1+x)+2x^2]^5$	1M	Accept $= [(1+2x^2)+x]^5$
$= (1+x)^5 + 5(1+x)^4(2x^2) + \cdots$	1M	
$= (1+5x+10x^2+\cdots) + 5(1+\cdots)(2x^2) + \cdots$		
$= 1 + 5x + 20x^2 + \cdots$	1A	
	4	
3. $\frac{\mathrm{d}}{\mathrm{d}x}(\frac{1}{x}) = \lim_{\Delta x \to 0} \frac{1}{\Delta x}(\frac{1}{x+\Delta x} - \frac{1}{x})$	1A	Withhold this mark if $\lim_{\Delta x \to 0}$ was omitted
$= \lim_{\Delta x \to 0} \frac{1}{\Delta x}[\frac{x-(x+\Delta x)}{x(x+\Delta x)}]$	1A	For simplification only
$= \lim_{\Delta x \to 0} \frac{-1}{x(x+\Delta x)}$	1A	For simplification only
$= \frac{-1}{x^2}$	1A	
	4	

	Solution	Marks	Remarks
4.	$\sin(\theta+30°)=\cos\theta$		
	$\sin\theta\cos30°+\sin30°\cos\theta=\cos\theta$	1A	
	$\frac{\sqrt{3}}{2}\sin\theta+\frac{1}{2}\cos\theta=\cos\theta$		
	$\frac{\sqrt{3}}{2}\sin\theta=\frac{1}{2}\cos\theta$		
	$\tan\theta=\frac{1}{\sqrt{3}}$	1A	
	$\theta=180n°+30°$, where n is an integer.	1M+1A	1M for the correct form of a general solution Withhold 1A for "n is a constant" etc.
	OR		
	:	1A	(Same as above)
	$\frac{\sqrt{3}}{2}\sin\theta=\frac{1}{2}\cos\theta$		
	$\cos\theta-\sqrt{3}\sin\theta=0$		
	$2\cos(\theta+60°)=0$	1A	$2\sin(\theta-30°)=0$
	$\theta+60°=360n°\pm90°$, where n is an integer.	1M	$\theta-30°=180n°+(-1)^n(0)$
	$\theta=360n°\pm90°-60°$ OR $\theta+60°=180n°+90°$		
	$\theta=360n°+30°$ or $\theta=-360n°-150°$ OR $\theta=180n°+30°$	1A	$\theta=180n°+30°$
	Alternative solution (1)		
	$\sin(\theta+30°)=\cos\theta$		
	$\cos[90°-(\theta+30°)]=\cos\theta$	1A	
	$60°-\theta=360n°\pm\theta$, where n is an integer.	1M+1A	
	$60°-\theta=360n°+\theta$ or $60°-\theta=360n°-\theta$ (rejected)		
	$2\theta=-360n°+60°$		
	$\theta=-180n°+30°$	1A	
	Alternative solution (2)		
	$\sin(\theta+30°)=\cos\theta$		
	$\sin(\theta+30°)=\sin(90°-\theta)$	1A	
	$\theta+30°=180n°+(-1)^n(90°-\theta)$, where n is an integer.	1M+1A	
	When n is odd, $\theta+30°=180n°-90°+\theta$ (rejected)		
	When n is even, $\theta+30°=180n°+90°-\theta$		
	$\theta=90n°+30°$	1A	Accept radian measures. (pp−1) for mixing degree and radian
		4	
5.	$x^2-x-1>k(x-2)$		
	$x^2-(k+1)x+2k-1>0$	1M	For expressing in $ax^2+bx+c>0$
	$\Delta=(k+1)^2-4(2k-1)$	1M	
	$k^2-6k+5<0$	1A	
	$(k-1)(k-5)<0$		
	$1<k<5$	1A	
		4	

Solution	Marks	Remarks
6. (a) Slope of $L_1 = -2$		
$\tan(\pi - \theta) = -2$		
$\tan\theta = 2$	1A	
Alternative solution The coordinates of P are (3, 0). The coordinates of Q are (0, 6). $\tan\theta = \frac{6}{3}$ $= 2$	1A	
(b) Let the equation of L_2 be $y = mx$.		
$\angle OQP = \angle OPQ = \theta$		
$\angle POQ = \pi - 2\theta$	1M	
$m = \tan(\pi - 2\theta)$ $= -\tan 2\theta$ $= -(\frac{2\tan\theta}{1-\tan^2\theta})$		
$= -\frac{2(2)}{1-(2)^2}$	1M	
$= \frac{4}{3}$		
$\therefore$ the equation of L_2 is $y = \frac{4}{3}x$.	1A	
Alternative solution (1) Let the equation of L_2 be $y = mx$. $\angle OQP = \theta$		
$\left\lvert \frac{m-(-2)}{1+m(-2)} \right\rvert = 2$	1M+1A	1M for $\tan\phi = \left\lvert \frac{m_1 - m_2}{1 + m_1 - m_2} \right\rvert$ 1A : Accept omitting absolute sign
$\frac{m+2}{1-2m} = 2$ or $\frac{m+2}{1-2m} = -2$		
$m = 0$ (rejected) or $m = \frac{4}{3}$		
$\therefore$ the equation of L_2 is $y = \frac{4}{3}x$.	1A	

Solution	Marks	Remarks
Alternative solution (2)		
Let the equation of L_2 be $y = mx$, where $m > 0$.		
$\begin{cases} 2x + y - 6 = 0 \\ y = mx \end{cases}$		
$x = \dfrac{6}{2+m}$, $y = \dfrac{6m}{2+m}$		
$OQ = \sqrt{(\dfrac{6}{2+m})^2 + (\dfrac{6m}{2+m})^2}$	1M	
$= \dfrac{6\sqrt{1+m^2}}{2+m}$		
$OQ = OP = 3$		
$\dfrac{6\sqrt{1+m^2}}{2+m} = 3$	1M	
$4(1+m)^2 = (2+m)^2$		
$3m^2 = 4m$		
$m = 0$ (rejected) or $\dfrac{4}{3}$		
$\therefore$ the equation of L_2 is $y = \dfrac{4}{3}x$.	1A	
Alternative solution (3)		
Let the coordinates of Q be (h, k).		
Since $OQ = 3$, $h^2 + k^2 = 9$	1M	
$\begin{cases} h^2 + k^2 = 9 \\ 2h + k - 6 = 0 \end{cases}$		
$h^2 + (6 - 2h)^2 = 9$	1M	
$5h^2 - 24h + 27 = 0$		
$h = 3$ (rejected) or $h = \dfrac{9}{5}$		
When $h = \dfrac{9}{5}$, $k = \dfrac{12}{5}$		
$\therefore$ the equation of L_2 is		
$\dfrac{y-0}{x-0} = \dfrac{\frac{12}{5} - 0}{\frac{9}{5} - 0}$		
$y = \dfrac{4}{3}x$	1A	
	4	

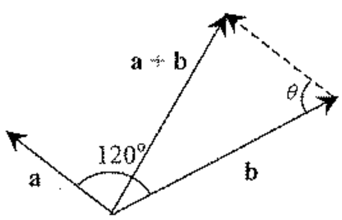
Solution	Marks	Remarks
7. $\lvert x-x^2 \rvert = -4x$		
$x-x^2 = -4x$ or $x-x^2 = 4x$	1M	
$x^2-5x = 0$ $\quad$ $x^2+3x = 0$		
$x=0$ or $x=5$ $\quad$ $x=-3$ or $x=0$	1A+1A	Awarded even if 1M was not awarded.
Since $-4x \geq 0$, $x \leq 0$. $\therefore$ $x=5$ is rejected.	1M	For checking
OR Put $x=-3$, LHS = RHS = 12 Put $x=0$, LHS = RHS = 0 Put $x=5$, LHS = 20 RHS = −20 (rejected)	1M	
$\therefore$ $x=-3$ or $x=0$	1A	
Alternative solution (1)		
$\lvert x-x^2 \rvert = -4x$		
Since $-4x \geq 0$, $x \leq 0$	2A	
$x-x^2 \leq 0$	1A	
The equation becomes		
$-(x-x^2) = -4x$ $\quad$ OR $x-x^2 = 4x$	1A	
$x(x+3) = 0$		
$x=-3$ or $x=0$	1A	
Alternative solution (2)		
$\lvert x-x^2 \rvert = -4x$		
$\lvert x(1-x) \rvert = -4x$		
Considering the following cases : (1) $x<0$ (2) $0 \leq x \leq 1$ (3) $x>1$	1M	Accept omitting = sign
Case (1) : The equation becomes		
$-x(1-x) = -4x$	1A	
$x(x+3) = 0$		
$x=-3$ or $x=0$		
Since $x<0$, $x=-3$		
Case (2) : The equation becomes		
$x(1-x) = -4x$	1A	
$x(x-5) = 0$		
$x=0$ or $x=5$		
Since $0 \leq x \leq 1$, $x=0$		
Case (3) : The equation becomes		
$-x(1-x) = -4x$	1A	
$x(x+3) = 0$		
$x=-3$ or $x=0$		
Since $x>1$, the two values are rejected.		
OR Since $-4x \geq 0$, $x \leq 0$, there is no solution for $x>1$.	1A	
Combining the 3 cases, $x=-3$ or $x=0$	1A	

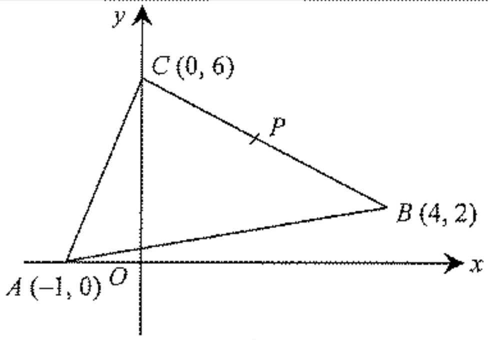
Solution	Marks	Remarks
Alternative solution (3)		
$\lvert x-x^2 \rvert = -4x$		
Consider the following cases : (1) $x \le 0$ (2) $x > 0$	1M	(OR $x < 0$)
Case (1) : The equation becomes		
$-x(1-x) = -4x$	1A	
$x(x+3) = 0$		
$x = -3$ or $x = 0$		
Since $x \le 0$, $x = -3$ or $x = 0$		
Case (2) : Since $x > 0$, $-4x < 0$		
$\lvert x-x^2 \rvert = -4x < 0$ (impossible)		
$\therefore$ there is no solution for $x > 0$.	2A	
Combining the two cases, $x = -3$ or $x = 0$.	1A	
Alternative solution (4)		
$\lvert x-x^2 \rvert = -4x$		
$(x-x^2)^2 = (-4x)^2$	1M	
$x^2 - 2x^3 + x^4 = 16x^2$		
$x^4 - 2x^3 - 15x^2 = 0$	1A	
$x^2(x^2 - 2x - 15) = 0$		
$x^2(x+3)(x-5) = 0$		
$x = 0$ or $x = -3$ or $x = 5$	1A	
Since $-4x \ge 0$, $x \le 0$	1M	
$\therefore$ $x = 5$ is rejected.	1A	
$\therefore$ $x = -3$ or $x = 0$		
	5	

Solution	Marks	Remarks
8. For $n=1$, LHS $=\dfrac{1\times 2}{2\times 3}=\dfrac{1}{3}$ RHS $=\dfrac{2^2}{3}-1=\dfrac{1}{3}$ $\therefore$ the statement is true for $n=1$.	1	
Assume $\dfrac{1\times 2}{2\times 3}+\dfrac{2\times 2^2}{3\times 4}+\dfrac{3\times 2^3}{4\times 5}+\cdots+\dfrac{k\times 2^k}{(k+1)(k+2)}$ $=\dfrac{2^{k+1}}{k+2}-1$, where k is a positive integer.	1	
$\dfrac{1\times 2}{2\times 3}+\dfrac{2\times 2^2}{3\times 4}+\dfrac{3\times 2^3}{4\times 5}+\cdots+\dfrac{k\times 2^k}{(k+1)(k+2)}+\dfrac{(k+1)\times 2^{k+1}}{(k+2)(k+3)}$ $=\dfrac{2^{k+1}}{k+2}-1+\dfrac{(k+1)\times 2^{k+1}}{(k+2)(k+3)}$	1	
$=\dfrac{2^{k+1}}{k+2}(1+\dfrac{k+1}{k+3})-1$ $=\dfrac{2^{k+1}}{k+2}(\dfrac{2k+4}{k+3})-1$ $=\dfrac{2^{k+2}}{k+3}-1$	1	
The statement is also true for $n=k+1$ if it is true for $n=k$. By the principle of mathematical induction, the statement is true for all positive integers n.	1 5	Not awarded if any one of the above marks was withheld.

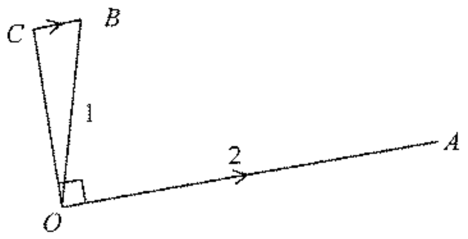
Solution	Marks	Remarks
9. (a) $\frac{d}{dx}\sin^3(x^2+1)$		
$=\frac{d}{d\sin(x^2+1)}\sin^3(x^2+1)\frac{d}{d(x^2+1)}\sin(x^2+1)\frac{d}{dx}(x^2+1)$	1M	For chain rule
$=3\sin^2(x^2+1)\cos(x^2+1)(2x)$	1M	For $\frac{d}{du}u^3=3u^2$ OR $\frac{d}{du}\sin u=\cos u$
$=6x\sin^2(x^2+1)\cos(x^2+1)$	1A	
(b) $xy+y^2=2005$		
$x\frac{dy}{dx}+y+2y\frac{dy}{dx}=0$	1A + 1A	1A for $\frac{d}{dx}(xy)$, 1A for $\frac{d}{dx}(y^2)$ and $\frac{d}{dx}(2005)$
$\frac{dy}{dx}=\frac{-y}{x+2y}$	1A	
Alternative solution (1)		
$y^2+xy-2005=0$		
$y=\frac{-x\pm\sqrt{x^2+8020}}{2}$		
$\frac{dy}{dx}=\frac{1}{2}[-1\pm\frac{1}{2\sqrt{x^2+8020}}(2x)]$	1M+1A	
$=\frac{1}{2}(-1\pm\frac{x}{\sqrt{x^2+8020}})$	1A	
Alternative solution (2)		
$x=\frac{2005-y^2}{y}$		
$\frac{dx}{dy}=\frac{y(-2y)-(2005-y^2)}{y^2}$	1M+1A	
$=\frac{-(2005+y^2)}{y^2}$		
$\frac{dy}{dx}=\frac{-y^2}{2005+y^2}$	1A	
	6	

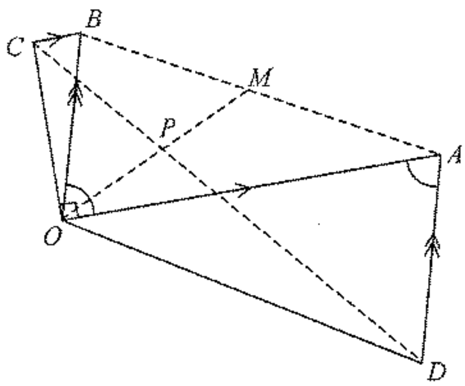
Solution	Marks	Remarks
10. (a) $\frac{d}{dx}[x(x+1)^n]$		
$= x\frac{d}{dx}(x+1)^n + (x+1)^n \frac{d}{dx}x$		
$= xn(x+1)^{n-1} + (x+1)^n$	1A	
$= (x+1)^{n-1}(nx+x+1)$		
$= (x+1)^{n-1}[(n+1)x+1]$	1	
(b) $y = \int (x+1)^{2004}(2006x+1)\,dx$	1M	
Put $n = 2005$ in (a) :		
$\frac{d}{dx}[x(x+1)^{2005}] = (x+1)^{2004}(2006x+1)$		
$\therefore\ y = x(x+1)^{2005} + k$, where k is a constant.	1A	For primitive function, awarded even if k was omitted
Put $x = -1$, $y = 1$:		
$1 = 0 + k \qquad \therefore\ k = 1$	1M	
$\therefore$ the equation of C is $y = x(x+1)^{2005} + 1$.	1A	Accept $y = (x+1)^{2006} - (x+1)^{2005} + 1$
	6	

Solution	Marks	Remarks
11. (a) $\mathbf{a}\cdot\mathbf{b}=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos 120^\circ$		
$=3(5)\cos 120^\circ$	1A	
$=-\frac{15}{2}$	1A	
(b) $\mathbf{a}+\mathbf{b}+\mathbf{c}=\mathbf{0}$		
$\mathbf{c}=-(\mathbf{a}+\mathbf{b})$	1A	
$\lvert\mathbf{c}\rvert=\lvert\mathbf{a}+\mathbf{b}\rvert$		
$\lvert\mathbf{c}\rvert^2=(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}+\mathbf{b})$		
$=\mathbf{a}\cdot\mathbf{a}+2\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{b}$	1M	
$=3^2+2(-\frac{15}{2})+5^2$	1A	For $\mathbf{a}\cdot\mathbf{a}=3^2$ and $\mathbf{b}\cdot\mathbf{b}=5^2$
$=19$		
$\therefore\ \lvert\mathbf{c}\rvert=\sqrt{19}$	1A	
Alternative Solution		
$\mathbf{a}+\mathbf{b}+\mathbf{c}=\mathbf{0}$		
$\mathbf{c}=-(\mathbf{a}+\mathbf{b})$	1A	
$\lvert\mathbf{c}\rvert=\lvert\mathbf{a}+\mathbf{b}\rvert$		
		
$\lvert\mathbf{a}+\mathbf{b}\rvert^2=\lvert\mathbf{a}\rvert^2+\lvert\mathbf{b}\rvert^2-2\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta$		
$=3^2+5^2-2(3)(5)\cos 60^\circ$	1M+1A	
$=19$		
$\therefore\ \lvert\mathbf{c}\rvert=\sqrt{19}$	1A	
		Omit vector sign/dot sign in most cases (pp−1)
	6	

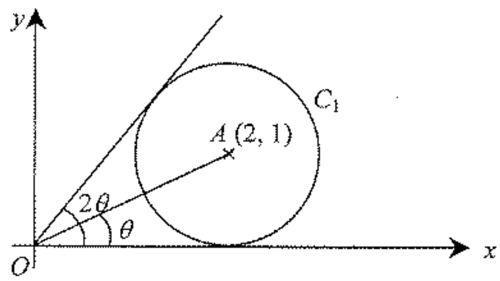
Solution	Marks	Remarks
12. (a) 		
Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} -1 & 0 \\ 4 & 2 \\ 0 & 6 \\ -1 & 0 \end{vmatrix}$		
$= \frac{1}{2}(-2+24+6)$	1M	
$= 14$	1A	
(b) Since area of ΔAPC : area of $\Delta ABC = 1:4$,		
$CP : CB = 1 : 4$	1A	
$\therefore\ CP : PB = 1 : 3$	1A	
The coordinates of P are		
$\left(\frac{3(0)+1(4)}{3+1}, \frac{3(6)+1(2)}{3+1}\right)$	1M	
i.e. (1, 5).	1A	
Alternative Solution		
Area of $\Delta APC = \frac{1}{4}$ (area of ΔABC)		
$= \frac{1}{4}(14)$	1M	
$= \frac{7}{2}$		
Let the coordinates of P be (x, y).		
$\frac{1}{2}\begin{vmatrix} -1 & 0 \\ x & y \\ 0 & 6 \\ -1 & 0 \end{vmatrix} = \frac{7}{2}$		
$\frac{1}{2}(-y+6x+6) = \frac{7}{2}$	1M	$\frac{1}{2}(-y+6x+6) = \pm\frac{7}{2}$ $6x-y=1$ OR $6x-y=-13$ (rejected)
$6x - y = 1$		
Equation of BC is		
$\frac{y-6}{x-0} = \frac{6-2}{0-4}$		
$x + y = 6$		
$\begin{cases} 6x - y = 1 \\ x + y = 6 \end{cases}$		
$6x + x = 6 + 1$	1M	
$x = 1$		
$y = 6 - x$		
$= 5$		
$\therefore$ the coordinates of P are (1, 5).	1A	
	6	

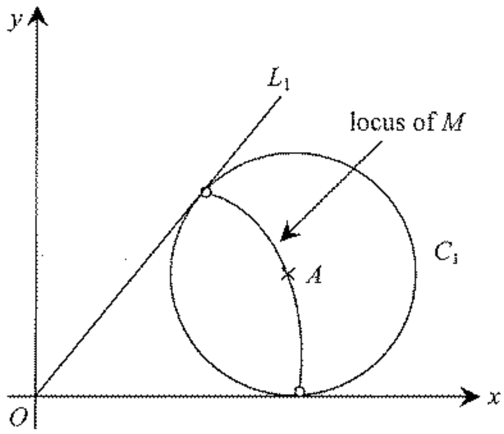
Solution	Marks	Remarks
13. (a) $\int \sin \pi x \, dx = -\frac{1}{\pi}\cos \pi x + c$, where c is a constant.	1M + 1A	1M for $\int \sin u \, du = -\cos u$ withhold 1A if c was omitted in Q.1 or/and Q.13 (a)
(b) Area $= \int_0^1 [f(x) - g(x)]dx + \int_1^2 [g(x) - f(x)]dx$	1M	1M for area between two curves $= \int_a^b [h_1(x) - h_2(x)]dx$
$= \int_0^1 2\sin \pi x \, dx + \int_1^2 -2\sin \pi x \, dx$	1A	
$= [-\frac{2}{\pi}\cos \pi x]_0^1 + [\frac{2}{\pi}\cos \pi x]_1^2$	1M	For the two integrands
$= (\frac{2}{\pi} + \frac{2}{\pi}) + (\frac{2}{\pi} + \frac{2}{\pi})$		
$= \frac{8}{\pi}$	1A	
	6	

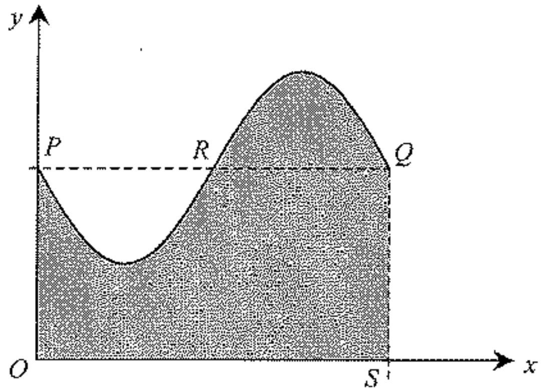
Solution	Marks	Remarks
14.		
		
(a) $CB = OB\cos\angle AOB$		
$= 1(\frac{1}{4})$	1M	
$= \frac{1}{4}$		
$\overrightarrow{CB} = \frac{1}{4}(\frac{1}{2}\overrightarrow{OA})$	1M	For $\overrightarrow{CB} = CB(\frac{1}{2}\overrightarrow{OA})$
$= \frac{1}{8}\mathbf{a}$	1A	
<u>Alternative solution</u>		
Let $\overrightarrow{CB} = \lambda\overrightarrow{OA} = \lambda\mathbf{a}$.		
$\overrightarrow{OB}\cdot\overrightarrow{OA} = \lvert\overrightarrow{OB}\rvert\lvert\overrightarrow{OA}\rvert\cos\angle AOB$		
$(\mathbf{c}+\lambda\mathbf{a})\cdot\mathbf{a} = 1(2)(\frac{1}{4})$	1M	
$0+\lambda(2)^2 = \frac{1}{2}$	1M	For $\mathbf{c}\cdot\mathbf{a}=0$ or $\mathbf{a}\cdot\mathbf{a}=\lvert\mathbf{a}\rvert^2$
$\lambda = \frac{1}{8}$		
$\therefore\ \overrightarrow{CB} = \frac{1}{8}\mathbf{a}$	1A	
$\mathbf{c} = \overrightarrow{OB}+\overrightarrow{BC}$		
$= \mathbf{b}-\frac{1}{8}\mathbf{a}$	1	
	4	

Solution	Marks	Remarks
(b) (i) 		
$DA = 2(OA\cos\angle OAD)$		
$= 2(2)(\frac{1}{4})$		
$= 1$	1A	
$\mathbf{d} = \overrightarrow{OA} + \overrightarrow{AD}$	} 1M	For $\mathbf{d} = \overrightarrow{OA} + \overrightarrow{AD}$ and $\overrightarrow{AD} = k\mathbf{b}$
$= \overrightarrow{OA} + \overrightarrow{BO}$ (since $AD = 1 = BO$)		
$= \mathbf{a} - \mathbf{b}$	1A	
(ii) $\overrightarrow{OP} = \dfrac{\overrightarrow{OC} + r\overrightarrow{OD}}{1+r}$		
$= \dfrac{(\mathbf{b} - \frac{1}{8}\mathbf{a}) + r(\mathbf{a} - \mathbf{b})}{1+r}$	1M	
$= \dfrac{1}{1+r}[(r - \frac{1}{8})\mathbf{a} + (1-r)\mathbf{b}]$	1A	$\dfrac{8r-1}{8(r+1)}\mathbf{a} + \dfrac{1-r}{1+r}\mathbf{b}$
(iii) $\overrightarrow{OM} = \dfrac{\overrightarrow{OA} + \overrightarrow{OB}}{2}$		
$= \dfrac{\mathbf{a} + \mathbf{b}}{2}$	1A	
When P lies on OM, OM // OP.		OR $\overrightarrow{OM} = k\overrightarrow{OP}$
$r - \frac{1}{8} = 1 - r$	1M	
$r = \frac{9}{16}$		
$\therefore$ OM divides CD in the ratio 9 : 16.	1A	Omit vector sign/dot sign in most cases (pp–1)
	8	

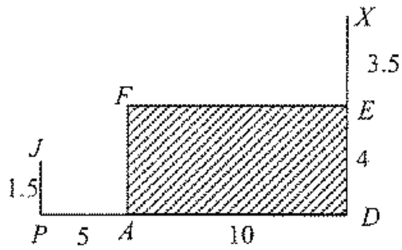
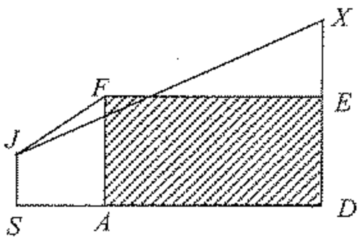
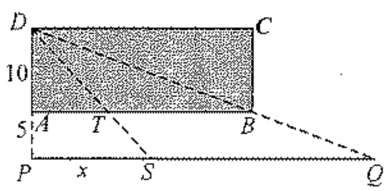
Solution	Marks	Remarks
15. (a) $\begin{cases} x^2+y^2-4x-2y+4=0 \\ y=kx \end{cases}$		
$x^2+(kx)^2-4x-2(kx)+4=0$		
$(k^2+1)x^2-(2k+4)x+4=0$ ...... (*)	1M	For expressing in the form $ax^2+bx+c=($
If C_1 and L intersect,		
$\Delta=(2k+4)^2-4(k^2+1)(4)\ge 0$	1M	
$4k^2+16k+16-16k^2-16\ge 0$		
$3k^2-4k\le 0$		
$k(3k-4)\le 0$		
$0\le k\le \frac{4}{3}$	1A	
Alternative solution (1)		
$x^2+y^2-4x-2y+4=0$		
$(x-2)^2+(y-1)^2=1$		
Coordinates of A are (2, 1) and radius of C_1 is 1.		
Distance from A to L		
$=\left\lvert\frac{k(2)-1}{\sqrt{1+k^2}}\right\rvert$	1M	Accept omitting absolute sign
If C_1 and L intersects,		
$\left\lvert\frac{2k-1}{\sqrt{1+k^2}}\right\rvert\le 1$	1M	
$(2k-1)^2\le 1+k^2$		
$3k^2-4k\le 0$		
$0\le k\le \frac{4}{3}$	1A	

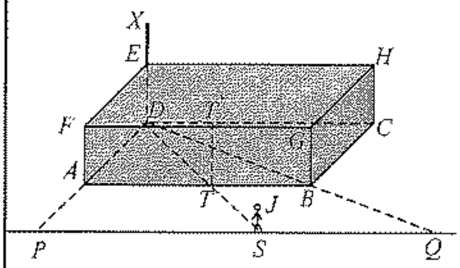
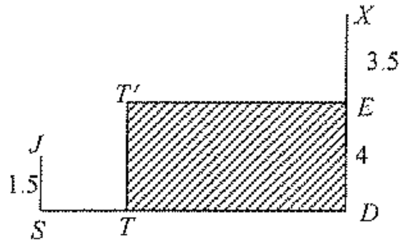
Solution	Marks	Remarks
Alternative Solution (2)		
		
$x^2+y^2-4x-2y+4=0$ Coordinates of A are (2, 1). Let θ be the angle between OA and the x-axis.		
$\tan\theta=\frac{1}{2}$	1M	
$\tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}$		
$=\frac{2(\frac{1}{2})}{1-(\frac{1}{2})^2}$	1M	
$=\frac{4}{3}$		
$\therefore$ the range of values of k is $0\le k\le\frac{4}{3}$.	1A	
	3	
(b) From (a), slope of $L_1=\frac{4}{3}$.	1M	OR Set $\Delta=0$ OR Set distance from A to $L_1=r$
The equation of L_1 is $y=\frac{4}{3}x$.	1A	
	2	
(c) (i) Let the coordinates of P and Q be (x_1, y_1) and (x_2, y_2) respectively. x_1 and x_2 are the roots of the equation (*) in (a).		
$x_1+x_2=\frac{2k+4}{k^2+1}$	1M	
Let (x, y) be the coordinates of M. $x=\frac{x_1+x_2}{2}$		
$=\frac{k+2}{k^2+1}$	1	
$\therefore$ the x coordinate of M is $\frac{k+2}{k^2+1}$.		

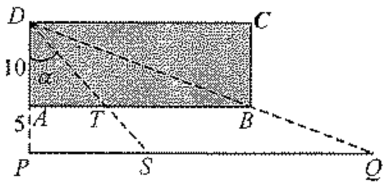
Solution	Marks	Remarks
(ii) (1) $\begin{cases} x = \dfrac{k+2}{k^2+1} \\ y = kx \end{cases}$		
OR $\begin{cases} x = \dfrac{k+2}{k^2+1} \\ y = kx = \dfrac{k(k+2)}{k^2+1} \end{cases}$		
Substitute $k = \dfrac{y}{x}$ into $x = \dfrac{k+2}{k^2+1}$:		OR Substitute into $y = \dfrac{k(k+2)}{k^2+1}$
$x = \dfrac{\frac{y}{x}+2}{(\frac{y}{x})^2+1}$	2M	For eliminating k
$x^2+y^2-2x-y=0$	1A	
$\therefore$ the equation of C_2 is $x^2+y^2-2x-y=0$.		
OR $(x-1)^2+(y-\frac{1}{2})^2=\frac{5}{4}$.		
(2) 	1A 1A	For L_1 For locus of M (passing through A) Axes not labelled (pp–1)
	7	

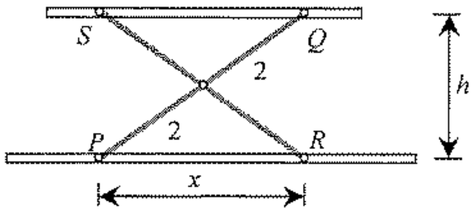
Solution	Marks	Remarks
16. (a) (i) The coordinates of P are $(0, k)$.	1A	
The coordinates of Q are $(\frac{2\pi}{k}, k)$.	1A	
(ii) Let R be the mid-point of PQ, and S be the point $(\frac{2\pi}{k}, 0)$		
		
By the property of the sine function (OR Since $\sin(\pi+\theta) = -\sin\theta$,), the area bounded by the curve and line segment PR is equal to that bounded by the curve and line segment RQ.		
Area of the shaded region		
= area of rectangle $OPQS$		
$= k(\frac{2\pi}{k})$	1M	
$= 2\pi$	1A	
	4	
(b) (i) $V = \pi \int_0^{\frac{2\pi}{k}} y^2 \, dx$		
$= \pi \int_0^{\frac{2\pi}{k}} (k - \sin kx)^2 \, dx$	1A	
$= \pi \int_0^{\frac{2\pi}{k}} (k^2 - 2k\sin kx + \sin^2 kx) \, dx$		
$= \pi \int_0^{\frac{2\pi}{k}} (k^2 - 2k\sin kx + \frac{1-\cos 2kx}{2}) \, dx$	1M	For expressing $\sin^2 kx$ in terms of $\cos 2kx$
$= \pi[k^2 x + 2\cos kx + \frac{x}{2} - \frac{\sin 2kx}{4k}]_0^{\frac{2\pi}{k}}$	1M+1M	1M for evaluating $\int k^2 \, dx$ and $\int \frac{1}{2} \, dx$, 1M for evaluating $\int \cos kx \, dx$ or $\int \sin 2kx \, dx$
$= \pi(2k\pi + 2 + \frac{\pi}{k} - 2)$		
$= \pi^2(2k + \frac{1}{k})$	1	

Solution	Marks	Remarks
(ii) $V = \pi^2(2k + \frac{1}{k})$		
$\frac{dV}{dk} = \pi^2(2 - \frac{1}{k^2})$		
Put $\frac{dV}{dk} = 0$: $2 - \frac{1}{k^2} = 0$	1M	
$k = \frac{1}{\sqrt{2}}$, which lies outside the range $2 \le k \le 3$. So there is no turning point within the range. The greatest value of V occurs at the end-points.	1A	
At $k = 2$, $V = \pi^2(4 + \frac{1}{2})$ $= \frac{9\pi^2}{2}$		
At $k = 3$, $V = \pi^2(6 + \frac{1}{3})$ $= \frac{19\pi^2}{3} > \frac{9\pi^2}{2}$		
$\therefore$ the greatest value of V is $\frac{19\pi^2}{3}$.	1A	
Alternative Solution		
$\frac{dV}{dk} = \pi^2(2 - \frac{1}{k^2})$ $\ge \pi^2(2 - \frac{1}{2^2})$ for $2 \le k \le 3$ > 0 for $2 \le k \le 3$	1M	
$\therefore$ V is strictly increasing in the interval $2 \le k \le 3$. The greatest value of V occurs at $k = 3$.	1A	
At $k = 3$, $V = \pi^2(6 + \frac{1}{3})$ $= \frac{19\pi^2}{3}$		
$\therefore$ the greatest value of V is $\frac{19\pi^2}{3}$.	1A	
	8	

Solution	Marks	Remarks
17. (a) (i)		
		
Let θ_1 be the angle of elevation of F from J.		
$\tan\theta_1 = \frac{FA - JP}{AP}$		
$= \frac{4 - 1.5}{5}$	1M	
$= 0.5$		
$\theta_1 = 27°$ (correct to the nearest degree)	1A	
(ii) Let θ_2 be the angle JX makes with the horizontal.		
$\tan\theta_2 = \frac{XD - JP}{DP}$		
$= \frac{3.5 + 4 - 1.5}{5 + 10}$	1M	
$= 0.4$		
$\theta_2 = 22°$ (correct to the nearest degree)	1A	
Since $\theta_1 > \theta_2$, Philip cannot see the pole.	1	
OR The ray from J to X is blocked by the building, so Philip cannot see the pole.	1	
OR  As shown in the above diagram, Philip cannot see the pole.	1	
	5	
(b)		
		
$\Delta DAT \sim \Delta DPS$		
$\frac{DT}{DS} = \frac{DA}{DP}$		
$= \frac{10}{10 + 5}$	1M	
$= \frac{2}{3}$	1A	

Solution	Marks	Remarks
Marking criteria : 1M – for considering θ_3 and θ_4 1M+1M – for a correct expression of $\tan\theta_3$ and $\tan\theta_4$ 1A – for expressing $\tan\theta_3$ and $\tan\theta_4$ correctly in terms of a common variable 1 – for the conclusion		
		
Let T' be the point on the top of the building which is directly above T.		
Let $PS = x$ m, where $0 \le x \text{ m} \le PQ$. $DS = \sqrt{(10+5)^2 + x^2}$ $= \sqrt{225 + x^2}$		
Since $\frac{DT}{DS} = \frac{2}{3}$, $TS = \frac{1}{3}DS$ $= \frac{1}{3}\sqrt{225+x^2}$		
Let θ_3 be the angle of elevation of T' from J and θ_4 be the angle JX makes with the horizontal.	1M	
$\tan\theta_3 = \frac{T'T - JS}{TS}$ $= \frac{4-1.5}{\frac{1}{3}DS}$ OR $= \frac{4-1.5}{\frac{1}{3}\sqrt{225+x^2}}$ $= \frac{7.5}{DS}$ $= \frac{7.5}{\sqrt{225+x^2}}$	1M	$\frac{5}{DT}, \frac{2.5}{TS}$
$\tan\theta_4 = \frac{XD - JS}{DS}$ $= \frac{3.5+4-1.5}{DS}$	1M	
$= \frac{6}{DS}$ OR $= \frac{6}{\sqrt{225+x^2}}$	1A	For both $\tan\theta_3$ and $\tan\theta_4$ $\frac{4}{DT}, \frac{2}{TS}$
$\therefore \tan\theta_3 > \tan\theta_4$ $\theta_3 > \theta_4$ Philip cannot see the pole no matter where he stands on the road PQ.	1	

Solution	Marks	Remarks
Alternative Solution		
		
Let $\angle PDS = \alpha$, where $0 \le \alpha \le \angle PDQ$.		
$DS = \dfrac{15}{\cos\alpha}$		
$TS = \dfrac{1}{3}DS = \dfrac{5}{\cos\alpha}$		
Let θ_3 be the angle of elevation of T' from J and θ_4 be the angle JX makes with the horizontal.	1M	
$\tan\theta_3 = \dfrac{4-1.5}{5/\cos\alpha}$	1M	
$= \dfrac{\cos\alpha}{2}$		
$\tan\theta_4 = \dfrac{3.5+4-1.5}{15/\cos\alpha}$	1M	
$= \dfrac{2\cos\alpha}{5}$	1A	For both $\tan\theta_3$ and $\tan\theta_4$
$\therefore \tan\theta_3 > \tan\theta_4$		
$\theta_3 > \theta_4$		
Philip cannot see the pole no matter where he stands on the road PQ.	1	
	7	

Solution	Marks	Remarks
18. (a) (i) 		
$h=\sqrt{4^2-x^2}$	1A	(can be awarded in (a) (ii))
When $x=0.8$, $h=\sqrt{4^2-0.8^2}$ ≈ 3.92 When $x=3.6$, $h=\sqrt{4^2-3.6^2}$ ≈ 1.74	1M	
$\therefore$ the range of possible values of h is 1.74 to 3.92 (correct to 3 significant figures).	1A	$1.74 \le h \le 3.92$
(ii) $h=\sqrt{16-x^2}$ Differentiate with respect to t: $\frac{dh}{dt}=\frac{-2x}{2\sqrt{16-x^2}}(\frac{dx}{dt})$	1M	For chain rule
$=\frac{-x}{\sqrt{16-x^2}}(-\frac{1}{2})$ $=\frac{x}{2\sqrt{16-x^2}}$	1	
	5	
(b) (i) From (a) (ii), $\frac{dh}{dt}=\frac{x}{2\sqrt{16-x^2}}$		
As x increases, $\sqrt{16-x^2}$ decreases. $\therefore \frac{dh}{dt}$ is increasing in $0.8 \le x \le 3.6$	1M	For attempting to show that $\frac{dh}{dt}$ is increasing
OR $\frac{d^2h}{dt^2}=\frac{1}{2}\left[\frac{\sqrt{16-x^2}-x(\frac{-2x}{2\sqrt{16-x^2}})}{16-x^2}\right]$ $=\frac{4}{(16-x^2)^{3/2}}>0$ for $0.8 \le x \le 3.6$ $\therefore \frac{dh}{dt}$ is increasing in $0.8 \le x \le 3.6$	1M	
$\therefore$ the greatest value of $\frac{dh}{dt}$ occurs at $x=3.6$.		

Solution	Marks	Remarks
Put $x = 3.6$:		
$\frac{dh}{dt} = \frac{3.6}{2\sqrt{16-3.6^2}}$ $\approx 1.032 < 2$ ∴ the elevating platform complies with the safety regulation.	1A	
(ii) Let y m be the height of the work platform of the scissors-type elevating platform above the rail.		
$y = 3h$	1A	
$\frac{dy}{dt} = 3\frac{dh}{dt}$		
$= \frac{3x}{2\sqrt{16-x^2}}$ (Since $\frac{dx}{dt} = -\frac{1}{2}$)		
From (b) (i), greatest value of $\frac{dh}{dt} \approx 1.032$		
∴ greatest value of $\frac{dy}{dt} \approx 1.032 \times 3$	1M	
$= 3.10 > 2$ ∴ the scissors-type elevating platform does not comply with the safety regulation.	1A	
$\frac{dy}{dt} = 3(\frac{-x}{\sqrt{16-x^2}})(\frac{dx}{dt})$		
$= \frac{-3x}{\sqrt{16-x^2}}(\frac{dx}{dt})$		
$0 < \frac{dy}{dt} \le 2$		
$0 < \frac{-3x}{\sqrt{16-x^2}}(\frac{dx}{dt}) \le 2$		
Put $x = 3.6$:		
$0 < \frac{-3(3.6)}{\sqrt{16-(3.6)^2}}(\frac{dx}{dt}) \le 2$	1M	
$0 > \frac{dx}{dt} \ge -0.3228$		
∴ $\frac{dx}{dt}$ should lie within the range $-0.322 \le \frac{dx}{dt} < 0$ (correct to 3 significant figures).	1A	Accept $-0.323 \le \frac{dx}{dt} < 0$
	7	