Page total

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

* *

Section A (62 marks)
Answer **ALL** questions in this section and write your answers in the spaces provided in this Question-Answer Book.

1. Find $\int (2x-3)^7 \mathrm{d}x$.

(2 marks)

2. (a) Expand $(1+y)^5$.

   (b) Using (a), or otherwise, expand $(1+x+2x^2)^5$ in ascending powers of x up to the term x^2.

(4 marks)

3. Find $\frac{\mathrm{d}}{\mathrm{d}x}(\frac{1}{x})$ from first principles.

(4 marks)

Please do not write in the margin.

Page total

4. Find the general solution of the equation $\sin(\theta + 30°) = \cos\theta$.

(4 marks)

5. Find the range of values of k such that

$$x^2 - x - 1 > k(x - 2)$$

for all real values of x.

(4 marks)

Please do not write in the margin.

Page total

6.

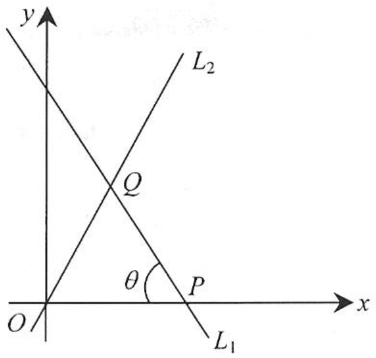


Figure 1

Figure 1 shows the line $L_1 : 2x + y - 6 = 0$ intersecting the x-axis at point P.

(a) Let θ be the acute angle between L_1 and the x-axis. Find $\tan \theta$.

(b) L_2 is a line with positive slope passing through the origin O. If L_1 intersects L_2 at a point Q such that $OP = OQ$, find the equation of L_2.

(4 marks)

7. Solve $| x - x^2 | = -4x$.

(5 marks)

Please do not write in the margin.

Page total

8. Prove, by mathematical induction, that

$$\frac{1\times 2}{2\times 3}+\frac{2\times 2^2}{3\times 4}+\frac{3\times 2^3}{4\times 5}+\cdots+\frac{n\times 2^n}{(n+1)(n+2)}=\frac{2^{n+1}}{n+2}-1$$

for all positive integers n.

(5 marks)

9. (a) Find $\frac{d}{dx}\sin^3(x^2+1)$.

(b) Let $xy+y^2=2005$. Find $\frac{dy}{dx}$.

(6 marks)

Please do not write in the margin.

Page total

10. (a) Show that $\frac{d}{dx}[x(x+1)^n] = (x+1)^{n-1}[(n+1)x+1]$, where n is a rational number.

(b) The slope at any point (x, y) of a curve C is given by $\frac{dy}{dx} = (x+1)^{2004}(2006x+1)$.

If C passes through the point $(-1, 1)$, find the equation of C.

(6 marks)

11.

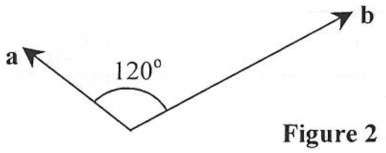


Figure 2

Figure 2 shows two vectors **a** and **b**, where $|\mathbf{a}| = 3$, $|\mathbf{b}| = 5$, and the angle between the two vectors is 120°.

(a) Find $\mathbf{a} \cdot \mathbf{b}$.

(b) Let $\mathbf{c}$ be a vector such that $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$. Find $|\mathbf{c}|$.

(6 marks)

Please do not write in the margin.

Page total

12. $A(-1, 0)$, $B(4, 2)$ and $C(0, 6)$ are three points on a rectangular coordinates system.

(a) Find the area of ΔABC.

(b) P is a point on the line segment BC such that $\dfrac{\text{area of } \Delta APC}{\text{area of } \Delta ABC} = \dfrac{1}{4}$. Find the coordinates of P.

(6 marks)

13. (a) Find $\int \sin \pi x \, dx$.

(b)

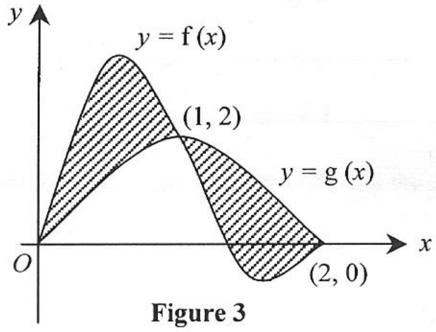


Figure 3

Figure 3 shows two curves $y = f(x)$ and $y = g(x)$ intersecting at three points $(0, 0)$, $(1, 2)$ and $(2, 0)$ for $0 \le x \le 2$. It is given that $f(x) - g(x) = 2\sin \pi x$. Find the area of the shaded region as shown in Figure 3.

(6 marks)

Please do not write in the margin.

SECTION B (48 marks)
Answer any **FOUR** questions in this section. Each question carries 12 marks.
Write your answers in the CE(A)2 answer book.

14.

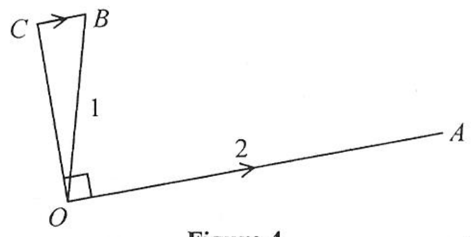


Figure 4

In Figure 4, $OA = 2$, $OB = 1$ and $\cos\angle AOB = \frac{1}{4}$. C is a point such that $CB \,//\, OA$ and $OC \perp OA$. Let $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$.

(a) Find $\overrightarrow{CB}$ in terms of $\mathbf{a}$.

Hence, or otherwise, show that $\mathbf{c} = \mathbf{b} - \frac{1}{8}\mathbf{a}$.

(4 marks)

(b)

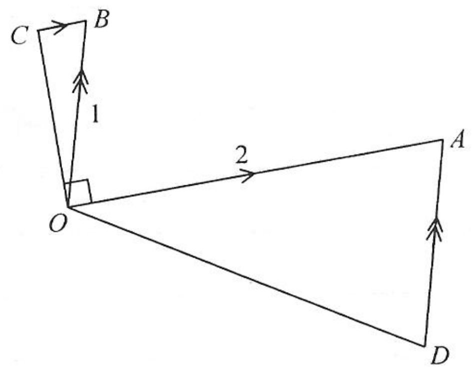


Figure 5

D is a point such that $DA \,//\, OB$ and $OD = OA$ (see Figure 5). Let $\overrightarrow{OD} = \mathbf{d}$.

(i) By finding DA, or otherwise, express $\mathbf{d}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(ii) P is a point on the line segment CD such that $CP : PD = r : 1$. Express $\overrightarrow{OP}$ in terms of r, $\mathbf{a}$ and $\mathbf{b}$.

(iii) If M is the mid-point of AB, find the ratio in which OM divides CD.

(8 marks)

15.

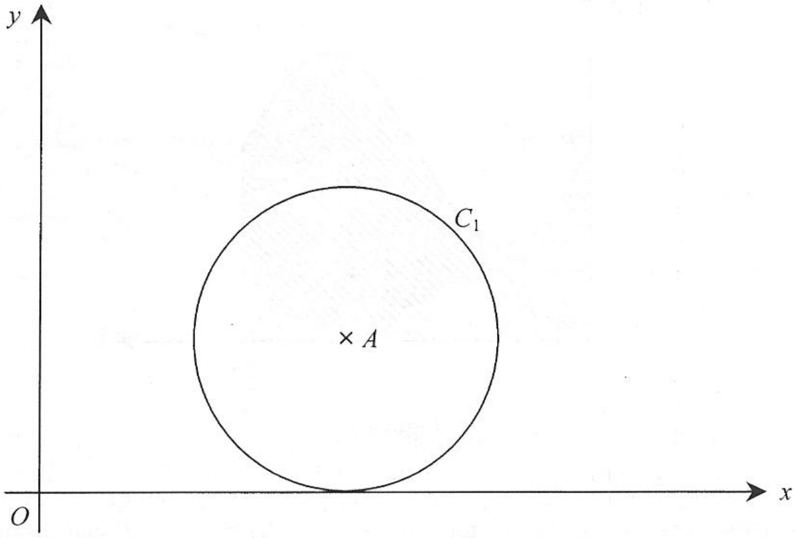


Figure 6

Figure 6 shows a circle $C_1 : x^2 + y^2 - 4x - 2y + 4 = 0$ centred at point A. L is the straight line $y = kx$.

(a) Find the range of values of k such that C_1 and L intersect.

(3 marks)

(b) There are two tangents from the origin O to C_1. Find the equation of the tangent L_1 other than the x-axis.

(2 marks)

(c) Suppose that L and C_1 intersect at two distinct points P and Q. Let M be the mid-point of PQ.

(i) Show that the x-coordinate of M is $\dfrac{k+2}{k^2+1}$.

(ii) It is known that the locus of M, as k varies, lies on a circle C_2.

(1) Find the equation of C_2.

(2) Copy Figure 6 into your answer book and sketch the tangent L_1 found in (b) and the locus of M in the figure.

(7 marks)

16.

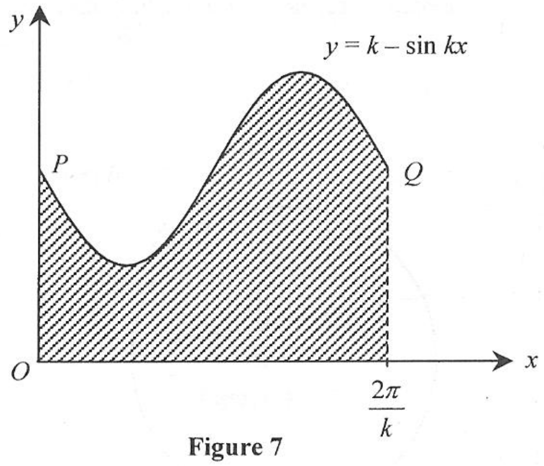


Figure 7

Figure 7 shows the curve $y = k - \sin kx$ for $0 \le x \le \frac{2\pi}{k}$, where $k > 1$. P and Q are the end points of the curve.

(a) (i) Find the coordinates of points P and Q.

(ii) Without using integration, find the area of the shaded region as shown in Figure 7.

(4 marks)

(b) A solid is formed by revolving the shaded region in Figure 7 about the x-axis. Let V be the volume of the solid.

(i) Show that $V = \pi^2 (2k + \frac{1}{k})$.

(ii) Suppose that $2 \le k \le 3$. Find the greatest value of V as k varies.

(8 marks)

17.

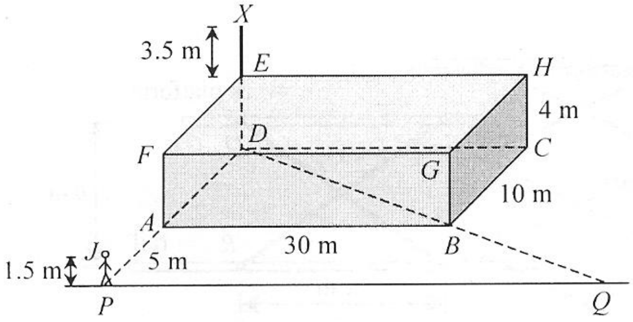


Figure 8

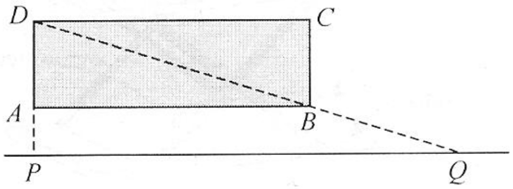


Figure 9

In Figure 8, $ABCDEFGH$ is a building standing on horizontal ground. The building is in the shape of a rectangular block with $AB = 30$ m, $BC = 10$ m and $CH = 4$ m. XE is a pole of length 3.5 m standing vertically at E. There is a straight road on the ground, which is parallel to and at a distance of 5 m from AB. DA produced meets the road at point P and DB produced meets the road at point Q. Philip stands at P. His eyes J are at 1.5 m above the ground. Figure 9 shows the base plane of the building.

(a) **In this part, numerical answers should be correct to the nearest degree.**

(i) Find the angle of elevation of F from J.

(ii) Find the angle the line JX makes with the horizontal.

Hence, or otherwise, explain why Philip cannot see the pole when he stands at P.

(5 marks)

(b) Let S be a point on PQ. DS cuts AB at a point T. Find $\dfrac{DT}{DS}$.

Hence, or otherwise, explain why Philip cannot see the pole no matter where he stands on the road PQ.

(7 marks)

Go on to the next page

18.

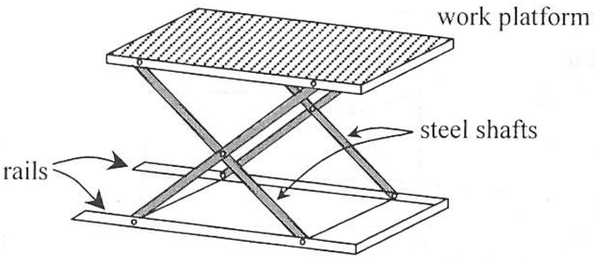


Figure 10

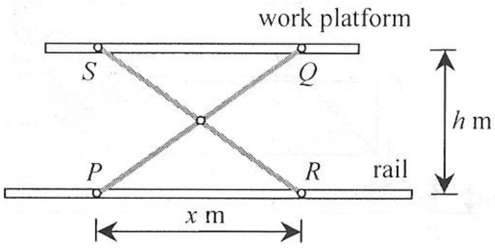


Figure 11

Figure 10 shows an elevating platform for lifting workers to work at different heights. The horizontal work platform is supported by two identical pairs of steel shafts. Figure 11 shows a cross-section of the elevating platform in a vertical plane containing one pair of shafts PQ and RS. The two shafts, each of length 4 m, are hinged at their mid-points. The ends P and R of the shafts can move along a straight horizontal rail with identical uniform speed and in opposite directions. Suppose that the elevating platform is operated under the following conditions :

(*)

Initially, $PR = 3.6$ m . The work platform is then lifted upward by moving the ends P and R of the shafts towards each other such that both PR and SQ decrease at a uniform rate of $\frac{1}{2}$ m s^{-1}. Let $PR = x$ m at time t s. It is given that $0.8 \le x \le 3.6$.

In this question, numerical answers should be correct to three significant figures.

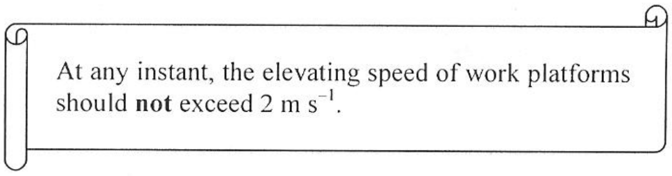
(a) Let h m be the height of the work platform above the rail at time t s.

(i) Find the range of possible values of h .

(ii) Show that $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{x}{2\sqrt{16 - x^2}}$.

(5 marks)

(b) Suppose that the operation of elevating platforms has to comply with the following safety regulation :



(i) Determine whether the operation of the above elevating platform under the conditions (*) will comply with this regulation.

(b) **(continued)**

(ii)

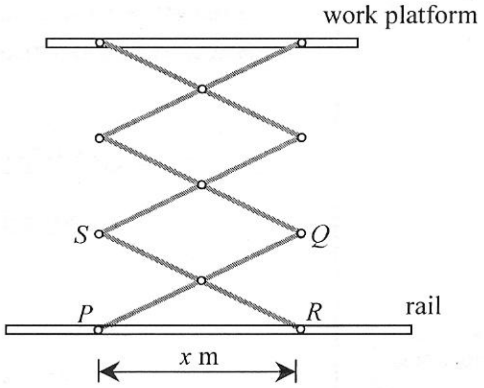


Figure 12

Figure 12 shows a vertical cross-section of a scissors-type elevating platform which can bring workers to a greater height. Two more identical pairs of shafts are installed on each side of the elevating platform as shown. Suppose that this elevating platform is operated under the same conditions (*) as described above. Do you think the operation of this elevating platform will comply with the safety regulation ?

If 'Yes', state your reasoning.

If 'No', find the range of possible values of $\frac{dx}{dt}$ in order for the operation of this elevating platform to comply with the safety regulation.

(7 marks)

END OF PAPER