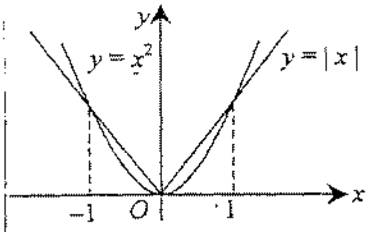
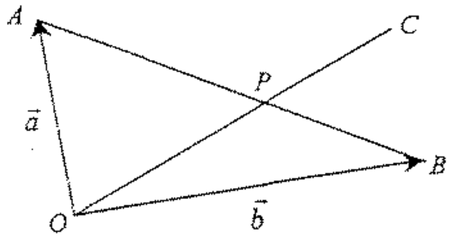
Solution	Marks	Remarks
1. $\int \cos^2\theta \, d\theta$		
$= \int \frac{1}{2}(1-\cos 2\theta)\, d\theta$	1A	
$= \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta + c$ (c is a constant)	1M+1A	1M for $\int \cos k\theta \, d\theta = \frac{1}{k}\sin k\theta$ withhold 1A if c was omitted
	3	
2. $\frac{(x+\Delta x)^3 - x^3}{\Delta x} = \frac{x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x}$	1A	For expanding $(x+\Delta x)^3$
$= 3x^2 + 3x(\Delta x) + (\Delta x)^2$	1A	
$\frac{(x+\Delta x)^3 - x^3}{\Delta x} = \frac{[(x+\Delta x) - x][(x+\Delta x)^2 + (x+\Delta x)x + x^2)]}{\Delta x}$	1A	
$= (x+\Delta x)^2 + (x+\Delta x)x + x^2$	1A	
$\frac{d}{dx}x^3 = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^3 - x^3}{\Delta x}$	1A	no mark if $\lim_{\Delta x \to 0}$ was omitted
$= \lim_{\Delta x \to 0} [3x^2 + 3x(\Delta x) + (\Delta x)^2]$		
$= 3x^2$	1A	
	4	

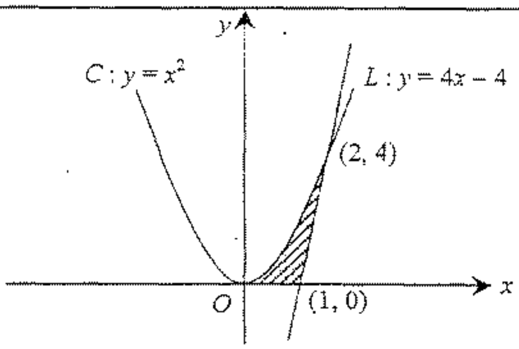
Solution	Marks	Remarks
3. $\begin{cases}\alpha+\beta=5\\ \alpha\beta=k\end{cases}$	1M	
$\lvert\alpha-\beta\rvert=3$		
$(\alpha-\beta)^2=3^2$	1M	
$\alpha^2-2\alpha\beta+\beta^2=9$		
$(\alpha+\beta)^2-4\alpha\beta=9$	1M	
$5^2-4k=9$		
$k=4$	1A	
Alternative Solution (1)		
$x=\dfrac{5\pm\sqrt{25-4k}}{2}$	1M	
$\lvert\alpha-\beta\rvert=3$		
$\left\lvert\dfrac{5+\sqrt{25-4k}}{2}-\dfrac{5-\sqrt{25-4k}}{2}\right\rvert=3$	1M	Accept without absolute sign
$\lvert\sqrt{25-4k}\rvert=3$		
$25-4k=3^2$	1M	
$k=4$	1A	
Alternative Solution (2)		
$\begin{cases}\alpha+\beta=5\\ \alpha\beta=k\end{cases}$	1M	
$\lvert\alpha-\beta\rvert=3$		
$\alpha-\beta=3$ or $\alpha-\beta=-3$	1M	
$\begin{cases}\alpha+\beta=5\\ \alpha-\beta=3\end{cases}$ OR $\alpha+\beta=5$, $\alpha-\beta=-3$, $\alpha=1$ and $\beta=4$	1M	
$\alpha=4$ and $\beta=1$		
$k=\alpha\beta$		
$=4$	1A	
	4	

Solution	Marks	Remarks
4. $3x^2+3y^2-2xy=12$		
$6x+6y\frac{\mathrm{d}y}{\mathrm{d}x}-2y-2x\frac{\mathrm{d}y}{\mathrm{d}x}=0$	1A+1A+1A	1A for $\frac{\mathrm{d}}{\mathrm{d}x}(3x^2+3y^2)$,
$\boxed{\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y-3x}{3y-x}}$		1A for $\frac{\mathrm{d}}{\mathrm{d}x}(2xy)$,
		1A for $\frac{\mathrm{d}}{\mathrm{d}x}(12)$
Put $x=2$, $y=0$:		
$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-6}{-2}$		
$\frac{\mathrm{d}y}{\mathrm{d}x}=3$	1A	
	4	

Solution	Marks	Remarks
5. $x^2 > \lvert x \rvert$		
$\lvert x \rvert^2 > \lvert x \rvert$	1M	
$\boxed{\lvert x \rvert(\lvert x \rvert - 1) > 0}$	1A	
$\boxed{x \neq 0 \text{ and }}$ $\lvert x \rvert - 1 > 0$	1A	$\lvert x \rvert > 1$ or $\lvert x \rvert < 0$
$\lvert x \rvert > 1$		
$x > 1$ or $x < -1$	1A	
Alternative Solution (1)		
$\lvert x \rvert < x^2$		
$\boxed{-x^2 < x < x^2}$	1M	
$x^2 + x > 0$ and $x^2 - x > 0$	1A+1A	
$x(x+1) > 0$ and $x(x-1) > 0$		
$x > 0$ or $x < -1$ and $x > 1$ or $x < 0$		
Combining the two cases, $x > 1$ or $x < -1$.	1A	
Alternative Solution (2)		
Consider the two cases : (1) $x > 0$, (2) $x < 0$.	1M	Accept including $x = 0$.
Case 1 ($x > 0$) : The inequality becomes		
$x^2 > x$	1A	
$x(x-1) > 0$		
Since $x > 0$, so $x > 1$.		
Case 2 ($x < 0$) : The inequality becomes		
$x^2 > -x$	1A	
$x(x+1) > 0$		
Since $x < 0$, so $x < -1$.		
Combining the two cases, $x > 1$ or $x < -1$.	1A	
Alternative Solution (3)		
$x^2 > \lvert x \rvert$		
$x^4 > x^2$	1M	
$\boxed{x^2(x^2 - 1) > 0}$	1A	
$\boxed{x \neq 0 \text{ and }}$ $x^2 - 1 > 0$	1A	$x^2 > 1$ or $x^2 < 0$
$x^2 > 1$		
$x > 1$ or $x < -1$	1A	
Alternative Solution (4)		
	1M+1A 1A	1M for sketching the two graphs For intersecting at $x = 1$ and $x = -1$
From the above graph, $x^2 > \lvert x \rvert$ when $x > 1$ or $x < -1$.	1A	
	4	

Solution	Marks	Remarks
6. 		
(a) $\overrightarrow{OP}=\dfrac{\vec{a}+3\vec{b}}{4}$	1A	
(b) $\overrightarrow{OC}=\dfrac{4}{3}\overrightarrow{OP}$		
$=\dfrac{4}{3}(\dfrac{\vec{a}+3\vec{b}}{4})$	1M	
$=\dfrac{1}{3}\vec{a}+\vec{b}$	1A	
$\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}$		
$=\dfrac{1}{3}\vec{a}+\vec{b}-\vec{b}$	1M	
$=\dfrac{1}{3}\vec{a}$		
$\boxed{=\dfrac{1}{3}\overrightarrow{OA}}$		
∴ OA is parallel to BC.	1	Omitting vector sign in most cases (pp−1)
	5	
7. For $n=1$, LHS $=\dfrac{1}{2}$		
RHS $=2-\dfrac{1+2}{2^1}=\dfrac{1}{2}=$ LHS		
∴ the statement is true for $n=1$.	1	
Assume $\dfrac{1}{2}+\dfrac{2}{2^2}+\dfrac{3}{2^3}+\cdots+\dfrac{k}{2^k}=2-\dfrac{k+2}{2^k}$,	1	
where k is a positive integer.		
$\dfrac{1}{2}+\dfrac{2}{2^2}+\dfrac{3}{2^3}+\cdots+\dfrac{k}{2^k}+\dfrac{k+1}{2^{k+1}}$		
$=2-\dfrac{k+2}{2^k}+\dfrac{k+1}{2^{k+1}}$	1	
$=2-\dfrac{2(k+2)-(k+1)}{2^{k+1}}$		
$=2-\dfrac{(k+1)+2}{2^{k+1}}$	1	
The statement is also true for $n=k+1$ if it is true for $n=k$. By the principle of mathematical induction,		
the statement is true for all positive integers n.	1	Not awarded if any one of the above marks was withheld
	5	

Solution	Marks	Remarks
8. (a) The equation of the family of straight line is $2x-3y+4+\lambda(x+y-3)=0$, where λ is a constant, and $x-y-3=0$	1A	OR $(x+y-3)+\lambda'(2x-3y+4)=0$.
OR $h(2x-3y+4)+k(x+y-3)=0$, where h, k are constants	1A	
Alternative Solution $\begin{cases}2x-3y+4=0\\x+y-3=0\end{cases}$ Solve the equations, $x=1$ and $y=2$. The point of intersection of L_1 and L_2 is $(1, 2)$. The equation of the family of straight line is $y-2=m(x-1)$	1A	
(b) $2x-3y+4+\lambda(x+y-3)=0$ $(2+\lambda)x+(\lambda-3)y+(4-3\lambda)=0$ $\dfrac{\lvert 4-3\lambda\rvert}{\sqrt{(2+\lambda)^2+(\lambda-3)^2}}=1$	1M+1M	1M for distance formula, 1M for substituting (0, 0) & distance = 1 Accept omitting absolute sign
$(4-3\lambda)^2=(2+\lambda)^2+(\lambda-3)^2$ $7\lambda^2-22\lambda-3=0$ $\lambda=3$ or $\dfrac{1}{7}$ Put $\lambda=3$ and $\dfrac{1}{7}$ into (1), the equation of the two lines are $x-1=0$ and $3x-4y+5=0$.	1A+1A	
Alternative Solution $mx-y+(2-m)=0$ $\dfrac{\lvert 2-m\rvert}{\sqrt{m^2+1}}=1$	1M+1M	Same as above
$(2-m)^2=m^2+1$ $m=\dfrac{3}{4}$ ∴ the equation of the line is $y-2=\dfrac{3}{4}(x-1)$ i.e. $3x-4y+5=0$.	1A	
Another line which is of distance 1 from the origin is $x-1=0$.	1A	
	5	

Solution	Marks	Remarks
9. 		
Area $= \int_0^4 (x_2 - x_1)\,\mathrm{d}y$		
$= \int_0^4 (\frac{y+4}{4} - y^{\frac{1}{2}})\,\mathrm{d}y$	1M+1A+1A	1M for $A = \int_a^b x\,\mathrm{d}y$, 1A for integrand, 1A for limit
$= [\frac{y^2}{8} + y - \frac{2}{3}y^{\frac{3}{2}}]_0^4$	1M	For evaluating $\int \frac{y+4}{4}\,\mathrm{d}y$ & $\int y^{\frac{1}{2}}\,\mathrm{d}y$
$= \frac{4^2}{8} + 4 - \frac{2}{3}(4^{\frac{3}{2}}) - 0$		
$= \frac{2}{3}$	1A	
Alternative Solution		
Area $= \int_0^2 y_2\,\mathrm{d}x - \int_1^2 y_1\,\mathrm{d}x$		
$= \int_0^2 x^2\,\mathrm{d}x - \int_1^2 (4x-4)\,\mathrm{d}x$	1M+1A+1A	1M for $A = \int_a^b y\,\mathrm{d}x$, 1A for any correct expression
$\boxed{\text{OR } = \int_0^2 x^2\,\mathrm{d}x - \frac{1}{2}(4)(2-1)}$		
$= [\frac{x^3}{3}]_0^2 - [2x^2 - 4x]_1^2$	1M	For evaluating all primitive function(s)
$= \frac{8}{3} - 2$		
$= \frac{2}{3}$	1A	
	5	

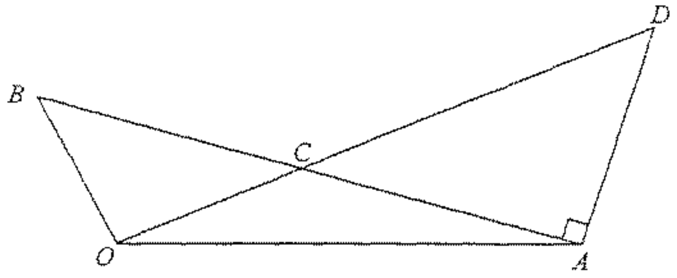
Solution	Marks	Remarks
10. (a) $\frac{\sin\alpha+\sin\beta}{\cos\alpha+\cos\beta}=\frac{2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}}{2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}}$	1A	
$=\tan\frac{\alpha+\beta}{2}$	1	
(b) $3\sin\alpha-4\cos\alpha=4\cos\beta-3\sin\beta$ $3\sin\alpha+3\sin\beta=4\cos\alpha+4\cos\beta$ $\frac{\sin\alpha+\sin\beta}{\cos\alpha+\cos\beta}=\frac{4}{3}$		
Using (a), $\tan(\frac{\alpha+\beta}{2})=\frac{4}{3}$	1M	
$\tan(\alpha+\beta)=\frac{2\tan(\frac{\alpha+\beta}{2})}{1-\tan^2(\frac{\alpha+\beta}{2})}$		
$=\frac{2(\frac{4}{3})}{1-(\frac{4}{3})^2}$	1M	
$=-\frac{24}{7}$	1A	
	5	

Solution	Marks	Remarks
11. $(\frac{1+3i}{1-2i})^{20}$		
$\frac{1+3i}{1-2i}=\frac{1+3i}{1-2i}(\frac{1+2i}{1+2i})$	1M	
$=\frac{1-2i+3i-6}{1+4}$		
$=-1+i$	1A	
$=\sqrt{2}(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4})$	1M	For expressing in polar form
$(\frac{1+3i}{1-2i})^{20}=[\sqrt{2}(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4})]^{20}$		
$=(\sqrt{2})^{20}[\cos(\frac{3\pi}{4}\times 20)+i\sin(\frac{3\pi}{4}\times 20)]$	1M	
$\boxed{=(\sqrt{2})^{20}(\cos\pi+i\sin\pi)}$		
$=(\sqrt{2})^{20}(-1)$		
$=-2^{10}$ $(=-1024)$	1A	Accept $2^{10}(\cos\pi+i\sin\pi)$
Alternative Solution		
$\frac{1+3i}{1-2i}=\cdots=-1+i$	1M+1A	same as above
$(-1+i)^2=(-1)^2-2i+i^2$	1M	
$=-2i$		
$(\frac{1+3i}{1-2i})^{20}=(-2i)^{10}$		
$\boxed{=(-2)^{10}(-1)^5}$	1M	
$=-2^{10}$	1A	
	5	

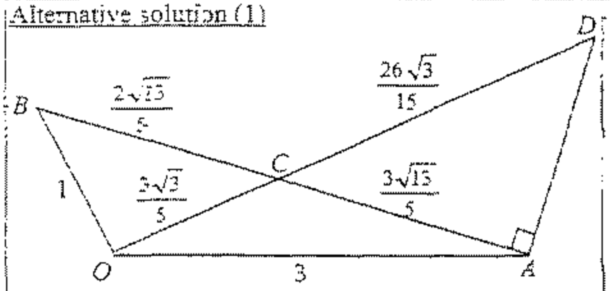
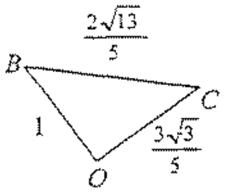
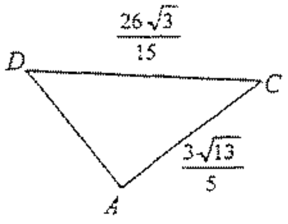
Solution	Marks	Remarks
12. $(2x^2+\frac{1}{x})^9$		
General term $={}_9C_r(2x^2)^{9-r}(\frac{1}{x})^r$	1A	${}_9C_r(2x^2)^r(\frac{1}{x})^{9-r}$
$={}_9C_r(2^{9-r})(x^{18-3r})$	1M	${}_9C_r(2^r)(x^{3r-9})$
(a) Put $18-3r=0$: $r=6$	1M	
$\therefore$ constant term $={}_9C_6(2^{9-6})$ $=84(8)=672$	1A	
(b) Put $18-3r=2$: $r=\frac{16}{3}$ which is not an integer.		
$\therefore$ there is no x^2 term.	1A+1M	
Alternative Solution (1) $(2x^2+\frac{1}{x})^9$ $=(2x^2)^9+{}_9C_1(2x^2)^8(\frac{1}{x})+{}_9C_2(2x^2)^7(\frac{1}{x})^2$ $+{}_9C_3(2x^2)^6(\frac{1}{x})^3+{}_9C_4(2x^2)^5(\frac{1}{x})^4+{}_9C_5(2x^2)^4(\frac{1}{x})^5$ $+{}_9C_6(2x^2)^3(\frac{1}{x})^6+{}_9C_7(2x^2)^2(\frac{1}{x})^7+{}_9C_8(2x^2)^1(\frac{1}{x})^8$ $+(\frac{1}{x})^9$	1M+1A	
$=2^9x^{18}+9(2^8)x^{15}+36(2^7)x^{12}+84(2^6)x^9$ $+126(2^5)x^6+126(2^4)x^3+84(2^3)+36(2^2)x^{-3}$ $+9(2)x^{-6}+x^{-9}$ $=512x^{18}+2304x^{15}+4608x^{12}+5376x^9$ $+4032x^6+2016x^3+672+144x^{-3}+18x^{-6}+x^{-9}$		
Constant term $={}_9C_6(2^3)$	1M	
$=84(8)=672$	1A	
The above expansion indicates that there is no x^2 term.	1A+1M	
Alternative Solution (2) $(2x^2+\frac{1}{x})^9=\frac{(2x^3+1)^9}{x^9}$	1M	
Constant term $=\frac{{}_9C_3(2x^3)^3}{x^9}$	1M+1A	
$={}_9C_3(2^3)$ $=84(8)=672$	1A	
The numerator does not contain an x^{11} term, so there is no x^2 term.	1A+1M	
	6	

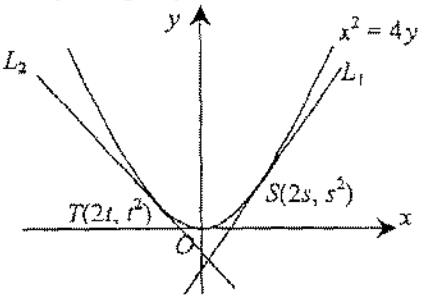
<table>
<tr><th>Solution</th><th>Marks</th><th>Remarks</th></tr>
<tr><td>13. $f(x) = 2\sin x - x$
$f'(x) = 2\cos x - 1$</td><td></td><td></td></tr>
<tr><td>$f'(x) = 0$ $2\cos x - 1 = 0$</td><td>1M</td><td></td></tr>
<tr><td>$\cos x = \frac{1}{2}$</td><td></td><td></td></tr>
<tr><td>$x = \frac{\pi}{3}$</td><td>1A</td><td>Withhold this mark for $x = 60°$</td></tr>
<tr><td>$f''(x) = -2\sin x$
$f''(\frac{\pi}{3})\boxed{= -\sqrt{3}} < 0$
$\therefore$ $f(x)$ attains a maximum at $x = \frac{\pi}{3}$.</td><td>1M+1A</td><td></td></tr>
<tr><td><u>Alternative Solution for checking</u>
$f'(x) = 2\cos x - 1$
<table>
<tr><td>x</td><td>$\boxed{0 \le} x < \frac{\pi}{3}$</td><td>$\frac{\pi}{3}$</td><td>$\frac{\pi}{3} < x \boxed{\le \pi}$</td></tr>
<tr><td>$f'(x) = 0$</td><td>+ve</td><td>0</td><td>−ve</td></tr>
</table>
Since $f'(x)$ changes from +ve to −ve at $x = \frac{\pi}{3}$,
$f(x)$ attains a maximum at $x = \frac{\pi}{3}$.</td><td>1M+1A</td><td></td></tr>
<tr><td>$f(\frac{\pi}{3}) = 2\sin\frac{\pi}{3} - \frac{\pi}{3}$
$= \sqrt{3} - \frac{\pi}{3}$
Since $f(x)$ is continuous and has only one turning point,</td><td></td><td></td></tr>
<tr><td>the greatest value of $f(x)$ is $\sqrt{3} - \frac{\pi}{3}$.</td><td>1A</td><td>Not awarded if checking was incomplete</td></tr>
<tr><td>The least value of $f(x)$ occurs at one of the end-points.</td><td>1M</td><td>(can be omitted)</td></tr>
<tr><td>$\boxed{f(0) = 0}$
$f(\pi) = -\pi$
$\therefore$ the least value of $f(x)$ is $-\pi$.</td><td>1A
7</td><td></td></tr>
</table>

14.

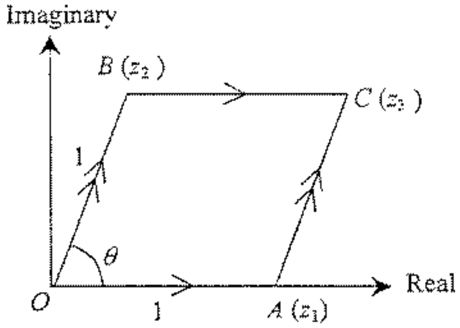


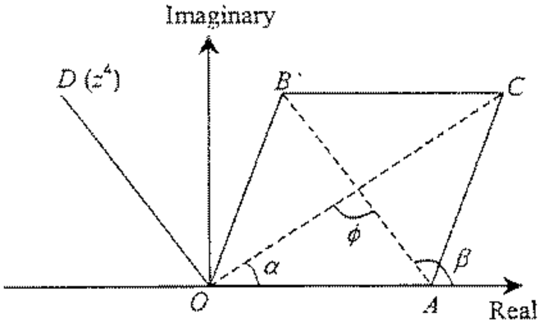
Solution	Marks	Remarks
(a) $\vec{a}\cdot\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\angle AOB$		
$= 3(1)\cos 120°$	1M	
$= -\frac{3}{2}$	1A	
	2	
(b) $\overrightarrow{OC} = \frac{2\vec{a}+3\vec{b}}{2+3} = \frac{2}{5}\vec{a}+\frac{3}{5}\vec{b}$		
$\overrightarrow{OD} = k\overrightarrow{OC}$		
$= \frac{2k}{5}\vec{a}+\frac{3k}{5}\vec{b}$		
$\overrightarrow{AD} = \overrightarrow{OD}-\overrightarrow{OA}$		
$= \frac{2k}{5}\vec{a}+\frac{3k}{5}\vec{b}-\vec{a}$	1M	
$= (\frac{2k}{5}-1)\vec{a}+\frac{3k}{5}\vec{b}$	1	
$\overrightarrow{AD}\cdot\overrightarrow{AB} = 0$		
$[(\frac{2k}{5}-1)\vec{a}+\frac{3k}{5}\vec{b}]\cdot(\vec{b}-\vec{a}) = 0$	1M	
$(\frac{2k}{5}-1)\vec{a}\cdot\vec{b}-(\frac{2k}{5}-1)\vec{a}\cdot\vec{a}+\frac{3k}{5}\vec{b}\cdot\vec{b}-\frac{3k}{5}\vec{b}\cdot\vec{a} = 0$	1M	For distributive law
$(\frac{2k}{5}-1)(-\frac{3}{2})-(\frac{2k}{5}-1)(3)^2+\frac{3k}{5}(1)^2-\frac{3k}{5}(-\frac{3}{2}) = 0$	1M	For $\vec{a}\cdot\vec{a}=9$ or $\vec{b}\cdot\vec{b}=1$
$\frac{105}{2}-\frac{27}{2}k = 0$		
$k = \frac{35}{9}$	1A	
	6	
(c) $\overrightarrow{OC}\cdot\overrightarrow{OB} = (\frac{2}{5}\vec{a}+\frac{3}{5}\vec{b})\cdot\vec{b}$	1M	
$= \frac{2}{5}\vec{a}\cdot\vec{b}+\frac{3}{5}\vec{b}\cdot\vec{b}$		
$= \frac{2}{5}(-\frac{3}{2})+\frac{3}{5}(1)^2 = 0$		
$\therefore \angle BOC = \frac{\pi}{2}$ (OR $\overrightarrow{OC}\perp\overrightarrow{OB}$)	1A	
$\angle BOC = \angle DAC = \frac{\pi}{2}$		
$\angle BCO = \angle DCA$ $\boxed{\text{(vertically opposite } \angle\text{s)}}$	1A	
$\boxed{\angle OBC = \angle ADC \quad (\angle \text{ sum of } \Delta)}$		
$\therefore \Delta OCB \sim \Delta ACD$ $\boxed{(AAA)}$	1	Omitting vector sign or dot product sign in most cases (pp−1)

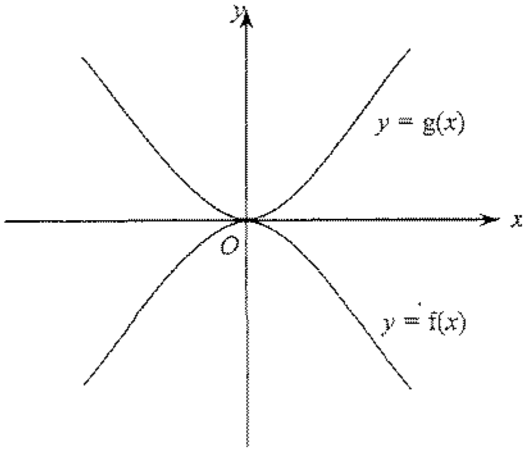
Solution	Marks	Remarks
Alternative solution (1)		
		
$AB^2 = 3^2 + 1^2 - 2(3)(1)\cos 120°$		
$AB = \sqrt{13}$		
$\cos\angle OAB = \dfrac{3^2 + (\sqrt{13})^2 - 1^2}{2(3)(\sqrt{13})} = \dfrac{7}{2\sqrt{13}}$	1M	
$AC = \dfrac{3}{5}(AB) = \dfrac{3\sqrt{13}}{5}$		
$OC^2 = 3^2 + (\dfrac{3\sqrt{13}}{5})^2 - 2(3)(\dfrac{3\sqrt{13}}{5})(\dfrac{7}{2\sqrt{13}}) = \dfrac{27}{25}$		
$OB^2 + OC^2 = 1^2 + \dfrac{27}{25}$		
$= \dfrac{52}{25} = (\dfrac{2\sqrt{13}}{5})^2 = BC^2$		
$\therefore OB \perp OC$ (OR $\angle BOC = \dfrac{\pi}{2}$)	1A	
⋮	1A+1	(same as above)
Alternative solution (2)		
 		
$BC = \dfrac{2\sqrt{13}}{5}$, $AC = \dfrac{3\sqrt{13}}{5}$		
$OC = \dfrac{3\sqrt{3}}{5}$		
$CD = (\dfrac{35}{9} - 1)\,OC = \dfrac{26\sqrt{3}}{15}$		
$\dfrac{OC}{BC} = \dfrac{3\sqrt{3}}{5} \Big/ \dfrac{2\sqrt{13}}{5} = \dfrac{3\sqrt{3}}{2\sqrt{13}}$	1M+1A	
$\dfrac{AC}{CD} = \dfrac{3\sqrt{13}}{5} \Big/ \dfrac{26\sqrt{3}}{15} = \dfrac{3\sqrt{3}}{2\sqrt{13}} = \dfrac{OC}{BC}$		
$\angle BCO = \angle DCA$ (vertically opposite ∠s)	1A	
$\Delta OCB \sim \Delta ACD$ (ratio of 2 sides, incl. ∠)	1	
	4	

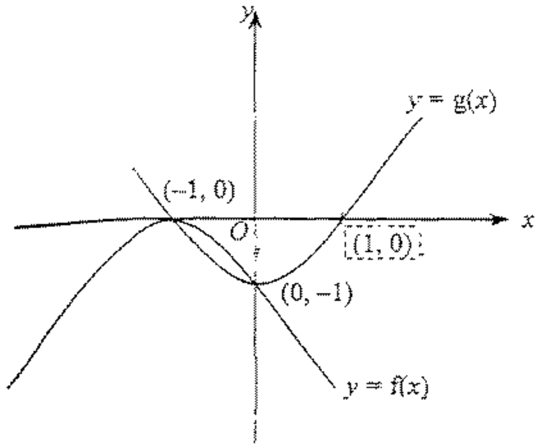
Solution	Marks	Remarks
15. (a) (i) $x^2=4y$ $2x=4\frac{\mathrm{d}y}{\mathrm{d}x}$		
At S, $\frac{\mathrm{d}y}{\mathrm{d}x}=s$ Equation of L_1 $\frac{y-s^2}{x-2s}=s$	1M	
$sx-y=s^2$	1A	
Alternative solution (1) Using the formula $xx_1=2(y+y_1)$, equation of L_1 is		
$x(2s)=2(y+s^2)$	1A	
$sx-y=s^2$	1A	
Alternative solution (2) Let the equation of L_1 be $y-s^2=m(x-2s)$		
$\begin{cases} x^2=4y \\ y-s^2=m(x-2s) \end{cases}$ $x^2-4mx+(8ms-4s^2)=0$ $\Delta=16m^2-4(8ms-4s^2)=0$	1M	
$m=s$ $\therefore$ the equation of L_1 is $y=sx-s^2$.	1A	
(ii) Equation of L_2 is $tx-y=t^2$.	1M	For replacing s by t 
$\begin{cases} sx-y=s^2 \text{ ------ (1)} \\ tx-y=t^2 \text{ ------ (2)} \end{cases}$		
$(1)-(2)$: $(s-t)x=s^2-t^2$ $x=s+t$	1M	
Substitute $x=s+t$ into (1) : $s(s+t)-y=s^2$ $y=st$	1M	
Since the two tangents meet at the point $P(\alpha,\beta)$, $\begin{cases} s+t=\alpha \\ st=\beta \end{cases}$.	1	
Alternative solution Equation of L_2 is $tx-y=t^2$.	1M	
Since $P(\alpha,\beta)$ lies on both tangents, s, t are the roots of the equation $z\alpha-\beta-z^2=0$ in z.	2M	
$z^2-\alpha z+\beta=0$ $\therefore \begin{cases} s+t=\alpha \\ st=\beta \end{cases}$.	1	

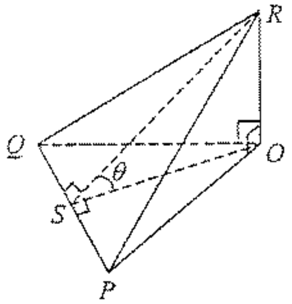
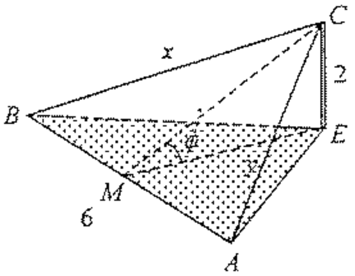
Solution	Marks	Remarks
(iii) Slope of $L_1 = s$ Slope of $L_2 = t$ If the line makes equal angles with L_1 and L_2,		
$\frac{s-1}{1+s} = \frac{1-t}{1+t}$ $\boxed{\text{OR } \left\lvert \frac{1-s}{1+s} \right\rvert = \left\lvert \frac{1-t}{1+t} \right\rvert}$	1M+1M	1M for $\tan\theta = \frac{m_2 - m_1}{1+m_2 m_1}$, 1M for substitution
$(s-1)(1+t) = (1+s)(1-t)$		
$st = 1$	1	
	9	
(b) Let the coordinates of R be (α , β) and the two points of tangency be $(2s , s^2)$ and $(2t , t^2)$. From (a) (ii), $\begin{cases} s+t = \alpha \\ st = \beta \end{cases}$. From (a) (iii), $st = 1$.		
$\therefore\ \beta = st = 1$	1A	
Since R lies on the line $x - y + 4 = 0$,		
$\alpha - 1 + 4 = 0$	1M	
$\alpha = -3$		
$\therefore$ the coordinates of R are $(-3, 1)$.	1A	
	3	

Solution	Marks	Remarks
16. (a) 	1A	Position of A and B
	1M	Position of C
	1A	Quadrilateral $OABC$ (a ∥-gram with $OA = OB$)
	1A	Label θ
	4	Not labelling the axes (pp–1)
(b) (i) $z_4 = z_2 - z_1$		
$= (\cos\theta - 1) + i\sin\theta$		
$z_3 = z_1 + z_2$	1A	
$= (1 + \cos\theta) + i\sin\theta$		
$\frac{z_4}{z_3} = \frac{(\cos\theta - 1) + i\sin\theta}{(1+\cos\theta) + i\sin\theta} \cdot \frac{(1+\cos\theta) - i\sin\theta}{(1+\cos\theta) - i\sin\theta}$	1M	
$= \frac{(\cos\theta - 1)(1 - \cos\theta) - i\sin\theta(\cos\theta - 1) + i\sin\theta(1+\cos\theta) - \sin^2\theta}{(1+\cos\theta)^2 + \sin^2\theta}$	1A	
$= \frac{\cos^2\theta - 1 + i[-\sin\theta\cos\theta + \sin\theta + \sin\theta + \sin\theta\cos\theta] + \sin^2\theta}{1 + 2\cos\theta + \cos^2\theta + \sin^2\theta}$		
$= \frac{i2\sin\theta}{2(1+\cos\theta)}$		
$= \frac{i\sin\theta}{1+\cos\theta}$	1	
<u>Alternative solution (1)</u>		
$z_4 = (\cos\theta - 1) + i\sin\theta$	1A	
$z_3 = (1+\cos\theta) + i\sin\theta$		
$z_4(1+\cos\theta) = [(\cos\theta - 1) + i\sin\theta](1+\cos\theta)$	1M	
$= (\cos\theta - 1)(1+\cos\theta) + i\sin\theta(1+\cos\theta)$	1A	
$= -\sin^2\theta + i(\sin\theta + \sin\theta\cos\theta)$		
$z_3(i\sin\theta) = [(1+\cos\theta) + i\sin\theta]\, i\sin\theta$		
$= i(\sin\theta + \sin\theta\cos\theta) - \sin^2\theta$		
$\therefore\ z_4(1+\cos\theta) = z_3(i\sin\theta)$		
$\frac{z_4}{z_3} = \frac{i\sin\theta}{1+\cos\theta}$	1	

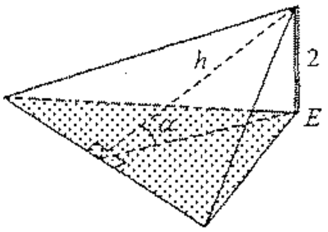
Solution	Marks	Remarks
Alternative solution (2)		
$z_4=(\cos\theta-1)+i\sin\theta\ ,\ z_3=(1+\cos\theta)+i\sin\theta$	1A	
$\frac{z_4}{z_3}=\frac{-1+\cos\theta+i\sin\theta}{1+\cos\theta+i\sin\theta}$		
$=\frac{-2\sin^2\frac{\theta}{2}+2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}+2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$	1M	
$=\frac{2i\sin\frac{\theta}{2}(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2})}{2\cos\frac{\theta}{2}(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2})}$		
$=i\tan\frac{\theta}{2}$	1A	
$=i\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\left(\frac{\cos\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right)$		
$=\frac{i\sin\theta}{1+\cos\theta}$	1	
$\arg(\frac{z_4}{z_3})=\frac{\pi}{2}$	1A	
(ii) Let ϕ be the angle between OC and AB. 		
$\arg(\frac{z_4}{z_3})=\frac{\pi}{2}$		
$\arg(z_4)-\arg(z_3)=\frac{\pi}{2}$	1M	For $\arg(\frac{z_4}{z_3})=\arg(z_4)-\arg(z_3)$
$\arg(z_4)=\beta,$ $\arg(z_3)=\alpha$ (see Figure above) $\arg(z_4)-\arg(z_3)=\beta-\alpha$ $=\phi$	1A	
$\therefore\ \phi=\frac{\pi}{2},$ i.e. the diagonals of $OACB$ are perpendicular to each other.	1	
	8	

Solution	Marks	Remarks
17. (a) The co-ordinates of P are (a, b).	1A	
	1	
(b) (i) $g(x)=(x-b)^2+a$	1A	
Substitute $Q(b, a)$ into $y=f(x)$:		
$a=-(b-a)^2+b$	1A	
$(b-a)^2=b-a \cdots (1)$ $\boxed{\text{OR } a^2-2ab+b^2+a-b=0}$		
$g(a)=(a-b)^2+a$	1M	
$\boxed{=b-a+a}$ $\boxed{\text{OR } =a^2-2ab+b^2+a}$	1M	For using (1)
$=b$		
$\therefore$ $y=g(x)$ passes through P.	1	
Alternative Solution		
Substitute $Q(b, a)$ into $y=f(x)$:		
$a=-(b-a)^2+b$	1A	
$(b-a)^2=b-a$		
$b-a=0$ or $b-a=1$		
Case 1: $(b-a)=0$, i.e. $b=a$		
$g(a)=(a-a)^2+a$	1M+1M	1M for considering $g(a)$,
$=a=b$		1M for using $b-a=0$ or $b-a=1$
Case 2: $(b-a)=1$		
$=g(a)=(a-b)^2+a$		
$=1+a=b$		
$g(a)=b$ in both cases.		
$\therefore$ $y=g(x)$ passes through P.	1	
(ii) Since $y=f(x)$ touches the x-axis,		$-x^2+2ax+(b-a^2)=0$
y-coordinate of vertex $=0$, i.e. $b=0$.	1M	$\Delta=4a^2+4(b-a^2)=0$
Substitute $b=0$ into (1):		$b=0$
$(0-a)^2=0-a$	1M	
$a=0$ or -1		
Case 1: $a=0$, $b=0$		
$\boxed{f(x)=-x^2 \text{ and } g(x)=x^2}$		
	1A+1A	Axes not labelled (pp−1)

Solution	Marks	Remarks
Case 2 $a = -1, b = 0$ $\boxed{f(x) = -(x+1)^2 \text{ and } g(x) = x^2 - 1}$ 	1A+1A	Withhold 1A if (−1, 0) or (0, −1) was not labelled
	11	

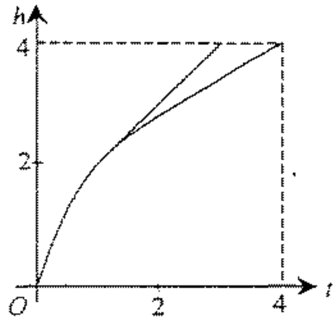
Solution	Marks	Remarks
18. (a) 		
Let S be the point on PQ such that $OS \perp PQ$ and $RS \perp PQ$.	1M	
Area of $\Delta OPQ = \frac{1}{2}(PQ)(OS)$	1A	
Area of $\Delta RPQ = \frac{1}{2}(PQ)(RS)$	1A	
$\frac{\text{Area of } \Delta OPQ}{\text{Area of } \Delta RPQ} = \frac{\frac{1}{2}(PQ)(OS)}{\frac{1}{2}(PQ)(RS)}$ $= \frac{OS}{RS}$ $= \cos\theta$	1 4	
(b) (i) 		
Let M be the foot of perpendicular from C to AB and $\angle CME = \phi$.		
$\frac{1}{2}(AB)(CM) = 12$		
$\frac{1}{2}(6)(CM) = 12$	1M	For finding CM
$CM = 4$ $\sin\phi = \frac{CE}{CM}$		
$= \frac{2}{4} = \frac{1}{2}$	1M	For finding ϕ, $\cos\phi$ or $ME = (\sqrt{12})$
$\phi = \frac{\pi}{6}$		
From (a), area of ΔEAB $=$ (area of ΔCAB) $\cos\phi$		OR $= \frac{AB \times ME}{2}$ OR $= 12\sqrt{1-\sin^2\phi}$
$= 12\cos\frac{\pi}{6}$	1M	$= \frac{6 \times \sqrt{12}}{2}$ $= 12\sqrt{1-(\frac{1}{2})^2}$
$= 6\sqrt{3}$ m^2	1A	$= 6\sqrt{3}$ m^2 $= \sqrt{144-6^2}$
$\therefore$ the area of the shadow is $6\sqrt{3}$ m^2.		$= 6\sqrt{3}$

(ii)



Solution	Marks	Remarks
Let α be the angle between the board and the ground, h be the altitude of the board from the vertex fastened to the top of the pole. From (a), area of shadow = $12\cos\alpha$ In order for the area of the shadow to be the greatest,		
$\cos\alpha$ should be the greatest.	1M	
$\sin\alpha$ should be the least.	1M	<u>OR</u> $\frac{2}{h}$ (or α) should be the least
h should be the greatest. Since $6 > x > y$, the altitude from B to CA is the longest among the 3 altitudes.	1M	
So vertex B should be fastened to the top of the pole.	1A	
<u>Alternative Solution</u> If vertex B is fastened to the top of the pole, $h = \frac{12(2)}{y} = \frac{24}{y}$ $\sin\alpha = \frac{2}{24/y} = \frac{y}{12}$ $\cos\alpha = \sqrt{1-(\frac{y}{12})^2}$	1M	
Area of shadow $= 12\sqrt{1-(\frac{y}{12})^2} = \sqrt{144-y^2}$	1A	
Similarly, if vertex A is fastened to the top of the pole, area of shadow $= \sqrt{144-x^2}$		
Since $6 > x > y$, $\sqrt{144-6^2} < \sqrt{144-x^2} < \sqrt{144-y^2}$ area of the largest shadow $= \sqrt{144-y^2}$.	1M	
So vertex B should be fastened to the top of the pole.	1A	
	8	

Solution	Marks	Remarks
19. (a) $V=\int_0^h \pi x^2\,\mathrm{d}y$		
$=\int_0^h \pi y\,\mathrm{d}y$	1M+1A	1M for $V=\int_a^b \pi x^2\,\mathrm{d}y$
$=[\frac{\pi y^2}{2}]_0^h$		
$V=\frac{1}{2}\pi h^2$	1A	
At $x=2, y=2^2=4$.		
Capacity of the tank $=\frac{1}{2}\pi(4)^2$ $=8\pi$ Time required to fill the tank $=\frac{8\pi}{2\pi}$	1M	
$=4$ (minutes)	1	
	5	
(b) $V=\frac{1}{2}\pi h^2$ Differentiate with respect to t :		
$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{1}{2}\pi(2h)\frac{\mathrm{d}h}{\mathrm{d}t}$	1M	
Put $\frac{\mathrm{d}V}{\mathrm{d}t}=2\pi$:		
$2\pi=\pi h\frac{\mathrm{d}h}{\mathrm{d}t}$	1M	
$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{2}{h}$	1	
	3	
(c) From (b), $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{2}{h}$. The rate of change of h decreases as t increases. So Sketch C best describes the variation of h with t.	1M+1A	1M for a correct argument
Alternative solution (1) Sketch B is incorrect as * $\frac{\mathrm{d}h}{\mathrm{d}t}$ is not a constant. * the x-section of the tank is non-uniform.	1M	
As h increases, the surface area increases. Since $\frac{\mathrm{d}V}{\mathrm{d}t}$ is a constant, h will increase at a lower rate. So Sketch C best describes the variation of h with t.	1A	

Solution	Marks	Remarks
Alternative solution (2)		
$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{2}{h}$		
$\frac{\mathrm{d}^2h}{\mathrm{d}t^2}=\frac{-2}{h^2}\frac{\mathrm{d}h}{\mathrm{d}t}$	1M	
$=\frac{-4}{h^3}<0$		
So Sketch C best describes the variation of h with t.	1A	
Alternative solution (3)		
$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{2}{h}$		
$\int h\,\mathrm{d}h=\int 2\,\mathrm{d}t$	1M	
$\frac{h^2}{2}=2t+c$		
At $t=0,\ h=0\ \ \therefore\ c=0$		
$h^2=4t$		
So Sketch C best describes the variation of h with t.	1A	
	2	
(d)  Sketch C	1M+1A	1M for a straight line with +ve slope for $h>2$ Withhold 1 mark if not drawn on the sketch chosen in (c)
	2	