2003-CE
A MATH

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2003

ADDITIONAL MATHEMATICS

8.30 am – 11.00 am (2½ hours)
This paper must be answered in English

1. Answer **ALL** questions in Section A and any **FOUR** questions in Section B.

2. Write your answers in the answer book provided. **For Section A, there is no need to start each question on a fresh page.**

3. All working must be clearly shown.

4. Unless otherwise specified, numerical answers must be **exact**.

5. In this paper, vectors may be represented by bold-type letters such as **u**, but candidates are expected to use appropriate symbols such as $\vec{u}$ in their working.

6. The diagrams in the paper are not necessarily drawn to scale.

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

* *

Section A (62 marks)
Answer **ALL** questions in this section.

1. Find $\int \cos^2\theta \, d\theta$.

(3 marks)

2. Find $\frac{d}{dx}(x^3)$ from first principles.

(4 marks)

3. α and β are the roots of the quadratic equation $x^2 - 5x + k = 0$ such that $|\alpha - \beta| = 3$. Find the value of k.

(4 marks)

4. Given that $3x^2 + 3y^2 - 2xy = 12$, find $\frac{dy}{dx}$ when $x = 2$, $y = 0$.

(4 marks)

5. Solve the inequality $x^2 > |x|$.

(4 marks)

6.

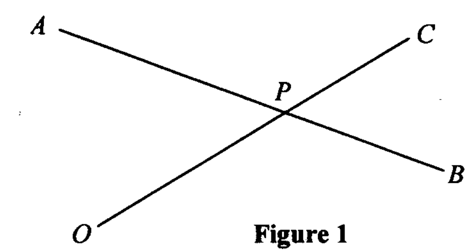

Figure 1

In Figure 1, point P divides both line segments AB and OC in the same ratio 3 : 1. Let $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$.

(a) Express $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(b) Express $\overrightarrow{OC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Hence show that OA is parallel to BC.

(5 marks)

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7. Prove, by mathematical induction, that

$$\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\cdots+\frac{n}{2^n}=2-\frac{n+2}{2^n}$$

for all positive integers n.

(5 marks)

8. Given two lines $L_1: 2x-3y+4=0$ and $L_2: x+y-3=0$.

(a) Write down the equation of the family of straight lines passing through the point of intersection of L_1 and L_2.

(b) Find the equations of two lines in the family in (a) such that the distance from the origin to each line is 1.

(5 marks)

9.

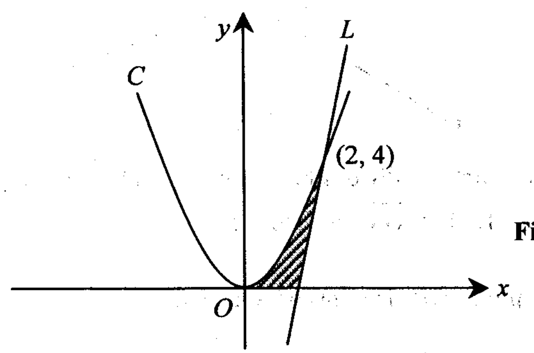

Figure 2

In Figure 2, the curve $C: y=x^2$ and line $L: y=4x-4$ intersect at the point $(2, 4)$. Find the area of the shaded region bounded by C, L and the x-axis.

(5 marks)

10. Given two acute angles α and β.

(a) Show that $\dfrac{\sin\alpha+\sin\beta}{\cos\alpha+\cos\beta}=\tan(\dfrac{\alpha+\beta}{2})$.

(b) If $3\sin\alpha-4\cos\alpha=4\cos\beta-3\sin\beta$, find the value of $\tan(\alpha+\beta)$.

(5 marks)

11. Simplify the complex number $\left(\dfrac{1+3i}{1-2i}\right)^{20}$.

(5 marks)

12. Determine whether the expansion of $(2x^2+\dfrac{1}{x})^9$ consists of

(a) a constant term,

(b) an x^2 term.

In each part, find the term if it exists.

(6 marks)

13. Let $f(x)=2\sin x-x$ for $0\le x\le\pi$. Find the greatest and least values of $f(x)$.

(7 marks)

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Section B (48 marks)
Answer any **FOUR** questions in this section.
Each question carries 12 marks.

14.

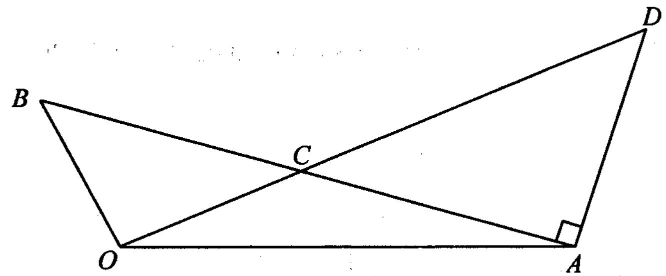

Figure 3

In Figure 3, OAB is a triangle such that $OA = 3$, $OB = 1$ and $\angle AOB = 120°$. C is a point on AB such that $AC : CB = 3 : 2$. D is a point on OC produced such that $\overrightarrow{OD} = k\overrightarrow{OC}$ and AB is perpendicular to AD. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Find $\mathbf{a} \cdot \mathbf{b}$.
(2 marks)

(b) Show that $\overrightarrow{AD} = (\frac{2k}{5} - 1)\mathbf{a} + \frac{3k}{5}\mathbf{b}$.

Hence find the value of k.
(6 marks)

(c) Determine whether the triangles OCB and ACD are similar.
(4 marks)

15. (a) $S(2s, s^2)$ and $T(2t, t^2)$ are two distinct points on the parabola $x^2 = 4y$, where $s > t$. L_1 and L_2 are the tangents to the parabola at S and T respectively.

(i) Find the equation of L_1.

(ii) If L_1 and L_2 meet at a point $P(\alpha, \beta)$, show that

$$\begin{cases} s + t = \alpha \\ st = \beta \end{cases}.$$

(iii) Given a line of slope 1. If this line makes equal angles with L_1 and L_2, show that $st = 1$.
(9 marks)

(b) Two tangents are drawn from a point R to the parabola $x^2 = 4y$. If the line $x - y + 4 = 0$ is an angle bisector of these two tangents, find the coordinates of point R.
(3 marks)

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16. A and B are two points in an Argand diagram representing the complex numbers $z_1 = 1$ and $z_2 = \cos\theta + i\sin\theta$ respectively, where $0 < \theta < \frac{\pi}{2}$. C is the point representing the complex number $z_3 = z_1 + z_2$.

(a) Sketch the quadrilateral $OACB$ in an Argand diagram, where O is the point representing the complex number 0. Mark an angle in the diagram which is equal to θ.

(4 marks)

(b) Let $z_4 = z_2 - z_1$.

(i) Show that $\frac{z_4}{z_3} = i\left(\frac{\sin\theta}{1+\cos\theta}\right)$.

Hence find $\arg\left(\frac{z_4}{z_3}\right)$.

(ii) Using (i), show that the diagonals of the quadrilateral $OACB$ are perpendicular to each other.

(8 marks)

17. Let $f(x) = -(x-a)^2 + b$, where a and b are real. Point P is the vertex of the graph of $y = f(x)$.

(a) Write down the coordinates of point P.

(1 mark)

(b) Let $g(x)$ be a quadratic function such that the coefficient of x^2 is 1 and the vertex of the graph of $y = g(x)$ is the point $Q(b, a)$. It is given that the graph of $y = f(x)$ passes through point Q.

(i) Write down $g(x)$ and show that the graph of $y = g(x)$ passes through point P.

(ii) Furthermore, the graph of $y = f(x)$ touches the x-axis. For each of the possible cases, sketch the graphs of $y = f(x)$ and $y = g(x)$ in the same diagram.

(11 marks)

18. (a)

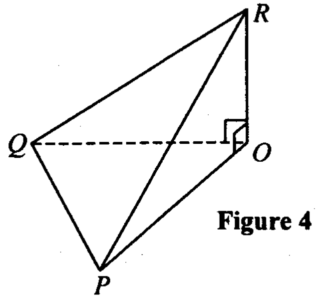

Figure 4

Figure 4 shows a tetrahedron $OPQR$ with RO perpendicular to the plane OPQ. Let θ be the angle between the planes RPQ and OPQ. Show that

$$\frac{\text{Area of } \Delta OPQ}{\text{Area of } \Delta RPQ} = \cos\theta .$$

(4 marks)

(b)

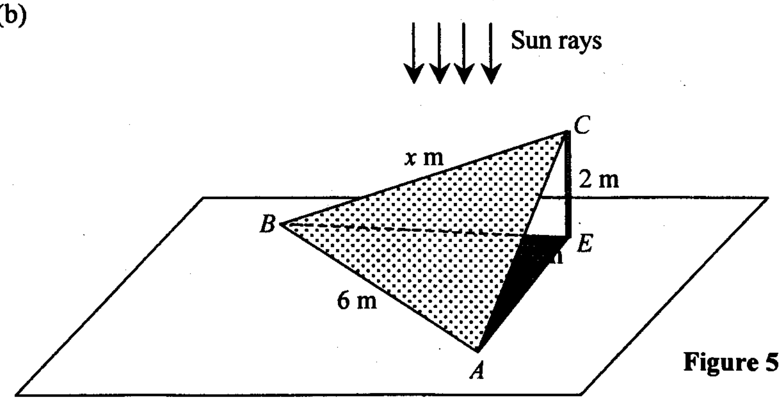

Figure 5

In Figure 5, a pole of length 2 m is erected vertically at a point E on the horizontal ground. A triangular board ABC of area 12 m^2 is supported by the pole such that side AB touches the ground and vertex C is fastened to the top of the pole. $AB = 6$ m, $BC = x$ m and $CA = y$ m, where $6 > x > y$. The sun rays are vertical and cast a shadow of the board on the ground.

(b) **(continued)**

(i) Find the area of the shadow.

(ii) Two other ways of supporting the board with the pole are to fasten vertex A or B to the top of the pole with the opposite side touching the ground. Among these three ways, determine which one will give the largest shadow.

(8 marks)

19.

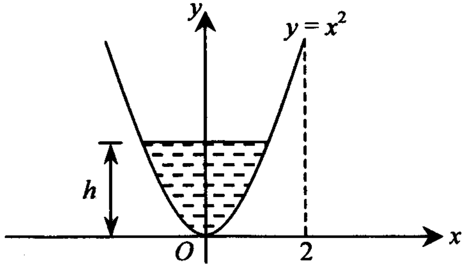

Figure 6

A water tank is formed by revolving the curve $y = x^2$, where $0 \le x \le 2$, about the y-axis (see Figure 6). Starting from time $t = 0$, water is pumped into the tank at a constant rate of 2π cubic units per minute. Let the volume and the depth of water (measured from the lowest point of the tank) in the tank at time t (in minutes) be V cubic units and h units respectively.

(a) Express V in terms of h.

Hence show that it takes 4 minutes to fill up the tank.

(5 marks)

(b) Show that $\frac{dh}{dt} = \frac{2}{h}$.

(3 marks)

(c) Which of the sketches in Figure 7 best describes the relation between h and t? Explain your answer.

(2 marks)

(d) An engineer decides to modify the tank by laying cement on the upper part of its interior wall, so that the interior of the tank becomes cylindrical in shape for $y \ge 2$ as shown in Figure 8. Water is pumped into the empty new tank at a constant rate of 2π cubic units per minute until it is full. On Page 12, sketch the graph of h against t for the new tank in the same sketch you chose in (c).

(2 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page	

If you attempt Question 19, fill in the first three boxes above and tie this sheet to your answer book.

(c) **(continued)**

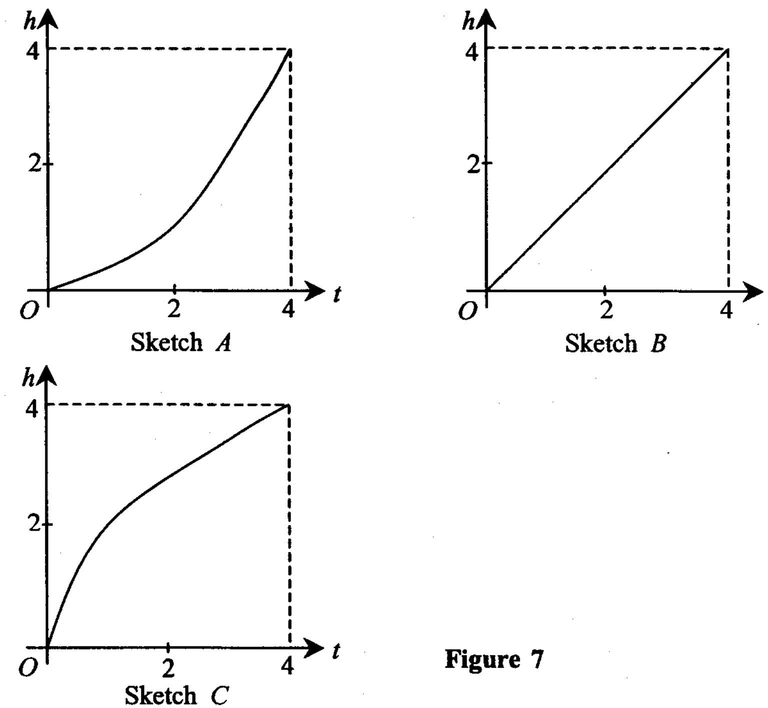

Figure 7

(d) **(continued)**

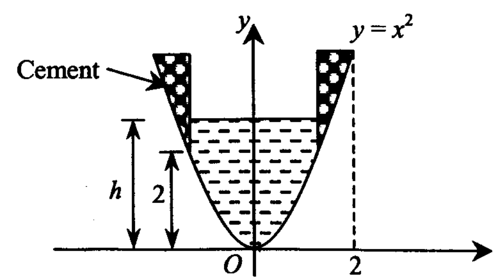

Figure 8

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