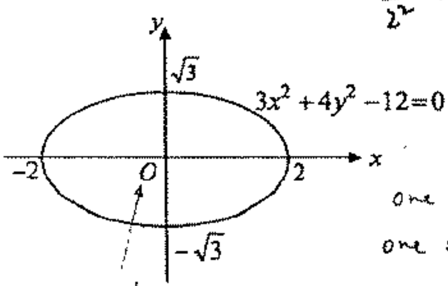
Solution	Marks	Remarks
1. $(1+x)^n + (1+2x)^n$		
$= 1 + {}_nC_1 x + {}_nC_2 x^2 + \cdots + 1 + {}_nC_1(2x) + {}_nC_2(2x)^2 + \cdots$	either	
Coefficient of $x^2 = ({}_nC_2 + 4\,{}_nC_2)x^2$　pp-1	1A+1A	
$= 5\,{}_nC_2$　　$1 + nx + \frac{n(n-1)}{2}x^2 + \cdots$	both	
$5\,{}_nC_2 = 75$　　$1 + 2nx + 2n(n-1)x^2$		
${}_nC_2 = 15$		
$\frac{n(n-1)}{2} = 15$　　accept	1M	For ${}_nC_2 = \frac{n(n-1)}{2}$ ✓ (can be omitted)
$n^2 - n - 30 = 0$		
$n = 6$ or -5 (rejected)		
$n = 6$	1A	
	4	
2. $y = (x-1)^4 + 4$		
$\frac{dy}{dx} = 4(x-1)^3$	1A	
Slope of the line $y = 4x + 8$ is 4.		
$4(x-1)^3 = 4$	1M	For setting $\frac{dy}{dx} = 4$
$(x-1)^3 = 1$		
$x = 2$		
Put $x = 2$, $y = (2-1)^4 + 4$	1M	For finding x, y and equation of tangent
$= 5$		
Equation of tangent is		
$\frac{y-5}{x-2} = 4$		
$y = 4x - 3$	1A	
	4	

只限教師參閱　　FOR TEACHERS' USE ONLY

Solution	Marks	Remarks
3. $x\sin y = 2002$		
$\sin y + x\cos y\dfrac{dy}{dx} = 0$ $\boxed{\text{OR } \sin y\dfrac{dx}{dy} + x\cos y = 0}$ 1A	1M+1A+1A	1M for product rule, 1A for $\dfrac{d}{dx}\sin y = \cos y\dfrac{dy}{dx}$, 1A for $\dfrac{d}{dx}(2002) = 0$
$\dfrac{dy}{dx} = \dfrac{-\tan y}{x}$	1A	Accept $\dfrac{dy}{dx} = \dfrac{-\sin y}{x\cos y}$
Alternative solution (1)		
$\sin y = \dfrac{2002}{x}$	1A	
$\cos y\dfrac{dy}{dx} = \dfrac{-2002}{x^2}$	1M+1A	1M for chain rule 1A for RHS
$\dfrac{dy}{dx} = \dfrac{-2002}{x^2\cos y}$ $= \dfrac{-2002}{\cos y}\cdot\dfrac{\sin y}{2002}\cdot\dfrac{1}{x} = \dfrac{-\tan y}{x}$	1A	
Alternative solution (2)		
$x = \dfrac{2002}{\sin y}$	1A	
$\dfrac{dx}{dy} = -2002\csc y\cot y$ $= -\dfrac{2002\cos y}{\sin^2 y}$	1M+1A	1M for finding $\dfrac{dx}{dy}$ 1A for RHS
$\dfrac{dy}{dx} = \dfrac{-\sin y\tan y}{2002}$	1A	
	4	
4. Let $x = \sin\theta$. $dx = \cos\theta\, d\theta$	1A	
$\displaystyle\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}\,dx = \int_0^{\frac{\pi}{6}} \frac{1}{\sqrt{1-\sin^2\theta}}\cos\theta\, d\theta$	1A+1M	1A for limits, 1M for integrand
$\displaystyle = \int_0^{\frac{\pi}{6}} d\theta$		
$= [\theta]_0^{\frac{\pi}{6}}$		
$= \dfrac{\pi}{6}$	1A	Omit dx, $d\theta$ in most cases (pp-1)
	4	

Solution	Marks	Remarks
5. (a) $\lvert x-4 \rvert = 2\sqrt{(x-1)^2+y^2}$	1M+1A (either one / all)	1M for distance formula Accept omitting absolute sign
$x^2-8x+16=4(x-1)^2+4y^2$ $=4x^2-8x+4+4y^2$		
$3x^2+4y^2-12=0$	1	
(b) $\frac{x^2}{2^2}+\frac{y^2}{\sqrt{3}^2}=1$  accept no origin one for x one for y	1A+1A	1A for shape Axes not labeled (pp−1)
	5	
6. $\boxed{\begin{cases} y=\sin x \\ y=\cos x \end{cases} \quad \sin x=\cos x}$	1M	(can be omitted)
$x=\frac{\pi}{4}$ or 45° (accept)		
Area $=\int_0^{\frac{\pi}{4}}(\cos x-\sin x)\,dx$ accept $\int_0^{\frac{\pi}{4}}(\sin x-\cos x)dx$	1M+1A	1M for $=\int_a^b (y_2-y_1)\,dx$
$=\left[\sin x+\cos x\right]_0^{\frac{\pi}{4}}$	1M	for $\int \cos x\,dx=\sin x$ and $\int \sin x\,dx=-\cos x$
$=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}-1$ $=\sqrt{2}-1$	1A	
	5	

只限教師參閱 FOR TEACHERS' USE ONLY

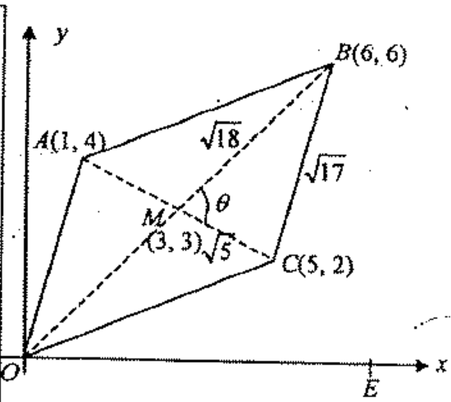
Solution	Marks	Remarks
7. (a) $\lvert x-1\rvert > 2$		
$x-1>2$ or $x-1<-2$	1A	
$x>3$ or $x<-1$	1A	
Alternative solution (1)		
$\lvert x-1\rvert > 2$		
$(x-1)^2>4$	1A	
$x^2-2x-3>0$		
$(x+1)(x-3)>0$		
$x>3$ or $x<-1$	1A	
Alternative solution (2)		
Consider the cases (1) $x \geqslant 1$, (2) $x<1$.		Accept including "=" sign
Case 1 : $x \geqslant 1$		
The inequality becomes		
$x-1>2$	1A	
$x>3$		
Since $x>1$, $x>3$.		
Case 2 : $x<1$ and		
The inequality becomes		
$-x+1>2$		
$x<-1$		
Since $x<1$, $x<-1$.		
Combining the 2 cases, $x>3$ or $x<-1$.	1A	
(b) $\lvert\lvert y\rvert-1\rvert>2$		
Using the result in (a),		
$\lvert y\rvert>3$ or $\lvert y\rvert<-1$ (no solution)	1M+1M	1M for using (a), 1M for 2nd term having no solution
$y>3$ or $y<-3$	1A	
Alternative solution (1)		
$\lvert\lvert y\rvert-1\rvert>2$		
$\lvert y\rvert-1>2$ or $\lvert y\rvert-1<-2$	1A	
$\lvert y\rvert>3$ or $\lvert y\rvert<-1$ (no solution)	1M	For 2nd term having no solution
$y>3$ or $y<-3$	1A	
Alternative solution (2)		
Consider the cases (1) $\lvert y\rvert>1$, (2) $\lvert y\rvert<1$.		
Case 1 : $\lvert y\rvert>1$ ($y>1$ or $y<-1$)		
The inequality becomes		
$\lvert y\rvert-1>2$	1A	
$\lvert y\rvert>3$		
$y>3$ or $y<-3$		
Since $\lvert y\rvert>1$, $y>3$ or $y<-3$.		
Case 2 : $\lvert y\rvert<1$ ($-1<y<1$)		
The inequality becomes		
$-\lvert y\rvert+1>2$		
$\lvert y\rvert<-1$ (no solution)	1M	
Combining the 2 cases, $y>3$ or $y<-3$.	1A	
	5	

只限教師參閱 FOR TEACHERS' USE ONLY

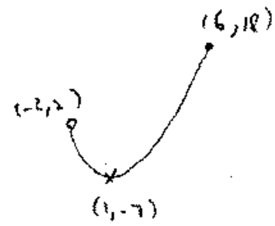
Solution	Marks	Remarks
8. $\dfrac{\tan x-\sin^2 x}{\tan x+\sin^2 x}=\dfrac{\dfrac{\sin x}{\cos x}-\sin^2 x}{\dfrac{\sin x}{\cos x}+\sin^2 x}$		
$=\dfrac{\sin x-\sin^2 x\cos x}{\sin x+\sin^2 x\cos x}$		
$=\dfrac{1-\sin x\cos x}{1+\sin x\cos x}$	1A	
$=\dfrac{1-\frac{1}{2}\sin 2x}{1+\frac{1}{2}\sin 2x}$	1M	For $\sin x\cos x=\frac{1}{2}\sin 2x$
$=\dfrac{2-\sin 2x}{2+\sin 2x}=\dfrac{4-(2+\sin 2x)}{2+\sin 2x}$		
$=\dfrac{4}{2+\sin 2x}-1$	1	
Alternative solution		
$\dfrac{4}{2+\sin 2x}-1$		
$=\dfrac{4}{2+2\sin x\cos x}-1$	1M	For $\sin 2x=2\sin x\cos x$
$=\dfrac{2}{1+\sin x\cos x}-1$		
$=\dfrac{1-\sin x\cos x}{1+\sin x\cos x}$	1A	
$=\dfrac{\tan x(1-\sin x\cos x)}{\tan x(1+\sin x\cos x)}$		
$=\dfrac{\tan x-\sin^2 x}{\tan x+\sin^2 x}$	1	
$\dfrac{\tan x-\sin^2 x}{\tan x+\sin^2 x}$ is the least when $(2+\sin 2x)$ is the greatest, i.e. when $\sin 2x=1$. $x=\dfrac{\pi}{4}$	1M	(can be omitted)
$\therefore$ Least value $=\dfrac{4}{2+1}-1$		
$=\dfrac{1}{3}$	1A	
	5	

Solution	Marks	Remarks
9. $z^2+z+1=0$		
$z=\dfrac{-1\pm\sqrt{1-4}}{2}$		
$=-\dfrac{1}{2}\pm\dfrac{\sqrt{3}}{2}i$	1M	
120° -120°		
α, β are $\cos\dfrac{2\pi}{3}+i\sin\dfrac{2\pi}{3}$, $\cos(-\dfrac{2\pi}{3})+i\sin(-\dfrac{2\pi}{3})$.	1A+1A	Accept degrees, Accept cis $\dfrac{4\pi}{3}$
$\alpha^6+\beta^6=(\cos\dfrac{2\pi}{3}+i\sin\dfrac{2\pi}{3})^6+[\cos(-\dfrac{2\pi}{3})+i\sin(-\dfrac{2\pi}{3})]^6$		
$=\cos 4\pi+i\sin 4\pi+\cos(-4\pi)+i\sin(-4\pi)$		
$=2\cos 4\pi$	1M	For De Moivre Theorem
$=2$	1A	
Alternative solution		
$z^2+z+1=0$		
$(z-1)(z^2+z+1)=0$		
$z^3-1=0$		
As α,β are the roots of $z^3-1=0$,		
$\alpha^3=\beta^3=1$		
$\alpha^6+\beta^6=(\alpha^3)^2+(\beta^3)^2$	1M	
$=1^2+1^2$		
$=2$		

	Solution	Marks	Remarks
10. (a)	$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$		
	$= \overrightarrow{OA} + \overrightarrow{OC}$		
	$= (\vec{i} + 4\vec{j}) + (5\vec{i} + 2\vec{j})$	1M	For finding $\overrightarrow{OB}$ or $\overrightarrow{AC}$
	$= 6\vec{i} + 6\vec{j}$	1A	
	$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$		
	$= (5\vec{i} + 2\vec{j}) - (\vec{i} + 4\vec{j})$		
	$= 4\vec{i} - 2\vec{j}$	1A	
(b)	$\overrightarrow{OB} \cdot \overrightarrow{AC} = \lvert\overrightarrow{OB}\rvert \, \lvert\overrightarrow{AC}\rvert \cos\theta$		
	$(6\vec{i} + 6\vec{j}) \cdot (4\vec{i} - 2\vec{j}) = \sqrt{6^2 + 6^2}\,\sqrt{4^2 + (-2)^2}\cos\theta$	1M	
	$24 - 12 = \sqrt{72}\,\sqrt{20}\cos\theta$	1M	For LHS
	$\cos\theta = \frac{1}{\sqrt{10}}$		
	$\theta = 72°$ (correct to the nearest degree)	1A	Omit vector sign or dot product sign in most cases (pp−1)
	Alternative solution (1)		
	$mOB = 1$ $mAC = -\frac{1}{2}$	1M	
	$\tan\theta = \left\lvert \frac{m_1 - m_2}{1 + m_1 m_2} \right\rvert$ accept absolute value		
	$= \frac{1 - (-\frac{1}{2})}{1 + 1(-\frac{1}{2})}$	1M	
	$= 3$		
	$\theta = 72°$ (correct to the nearest degree)	1A	
	Alternative solution (2)		
	$BC = \sqrt{(6-5)^2 + (6-2)^2} = \sqrt{17}$ $CM = \sqrt{(5-3)^2 + (2-3)^2} = \sqrt{5}$ $MB = \sqrt{(6-3)^2 + (6-3)^2} = \sqrt{18}$	1M	
	$\cos\theta = \frac{5 + 18 - 17}{2\sqrt{5}\,\sqrt{18}}$	1M	
	$\theta = 72°$ (correct to the nearest degree)	1A	
	Alternative solution (3)		
	Slope of $OB = 1$		
	$\therefore \angle BOE = \tan^{-1}(1) = 45°$	1M	
	Slope of $AC = \frac{4-2}{1-5} = -\frac{1}{2}$		
	$\therefore \quad = 180° + \tan^{-1}(-\frac{1}{2}) = 153.4°$		
	$\theta = \phi - \angle BOE$	1M	
	$= 180° - (153.4° - 45°)$		
	$= 72°$ (correct to the nearest degree)	1A	
		6	



只限教師參閱 FOR TEACHERS' USE ONLY

Solution	Marks	Remarks
11. (a) $\begin{cases} y = x^2 - 2x - 6 \\ y = 2x + 6 \end{cases}$		
$x^2 - 2x - 6 = 2x + 6$ $x^2 - 4x - 12 = 0$	1M	
$x = -2$ or 6 When $x = -2$, $y = 2$ When $x = 6$, $y = 18$ $\therefore$ the coordinates of A and B are $(-2, 2)$ and $(6, 18)$ respectively.	1A	
$f(x) = x^2 - 2x - 6$ $= (x-1)^2 - 1 - 6 = (x-1)^2 - 7$	1M	
$\therefore$ the coordinates of C are $(1, -7)$.	1A	
(b) The range of values of x such that $f(x) \le g(x)$ is $-2 \le x \le 6$.	1A	
From Figure 3, $f(x) = k$ has only one real root when $2 < k \le 18$ OR $k = -7$.	1A+1A	
	7	

$x^2 - 2x - 6 = 0$

$(x-1)^2 = 7$

$x = 1 \pm \sqrt{7}$

Solution	Marks	Remarks
12. (a) For $n=1$, LHS $=2(2)=4$		
RHS $=1(2^{1+1})=4=$ LHS	1	
$\therefore$ the statement is true for $n=1$.		
Assume $2(2)+3(2)^2+\cdots+(k+1)(2^k)=k(2^{k+1})$ for any positive integer k.	1	
Then $2(2)+3(2)^2+4(2)^3+\cdots+(k+1)(2^k)+(k+2)(2^{k+1})$		
$=k(2^{k+1})+(k+2)(2^{k+1})$	1	
$=2^{k+1}(k+k+2)$		
$=(k+1)2^{k+2}$	1	
The statement is also true for $n=k+1$ if it is true for $n=k$. By the principle of mathematical induction, the statement is true for all positive integers n.	1	Not awarded if any one of the above marks was withheld.
(b) $1(2)+2(2^2)+3(2^3)+\cdots+98(2^{98})$	1M	
$=2[1+2(2)+3(2^2)+\cdots+98(2^{97})]$		
$=2+2[2(2)+3(2^2)+\cdots+98(2^{97})]$		
$=2+2(97)(2^{98})$	1M	For using (a)
$=2+97(2^{99})$	1	
Alternative solution (1)		
Put $n=97$:		
$2(2)+3(2^2)+4(2^3)+\cdots+98(2^{97})=97(2^{98})$	1M	For using (a)
Add 1 to both sides:		
$1+2(2)+3(2^2)+\cdots+98(2^{97})=97(2^{98})+1$	1M	
Multiply both sides by 2:		
$2+2(2^2)+3(2^3)+\cdots+98(2^{98})=97(2^{99})+2$	1	
Alternative solution (2)		
$1(2)+2(2^2)+3(2^3)+\cdots+98(2^{98})$		
$=(1+1)(2)+(2+1)(2^2)+(3+1)(2^3)+\cdots+(98+1)(2^{98})-(2+2^2+\cdots+2^{98})$	1M	
$=2(2)+3(2^2)+4(2^3)+\cdots+(99)(2^{98})-(2+2^2+2^3+\cdots2^{98})$		For using (a)
$=98(2^{98+1})-\dfrac{2(2^{98}-1)}{2-1}$	√1M	
$=98(2^{99})-2^{99}+2$		
$=97(2^{99})+2$	1	
	8	

$1(2)+2(2^2)+\cdots+n2^n=(n-1)2^{n+1}+2$

$n=1$ = L.H.S $=2$, R.H.S $=2$

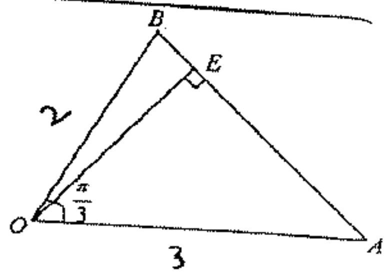
Assume $1(2)+2(2^2)+\cdots+k2^k=(k-1)2^{k+1}+2$

$1(2)+2(2^2)+\cdots+k2^k+(k+1)2^{k+1}$

$=(k-1)2^{k+1}+2+(k+1)2^{k+1}$

$=k2^{k+2}+2$

Set $n=98$

Solution	Marks	Remarks
13. (a) $\vec{a}\cdot\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\angle AOB$		
$= (3)(2)\cos\frac{\pi}{3}$	1M	
$= 3$	1A	
	2	
(b) $\overrightarrow{OE} = \dfrac{t\overrightarrow{OA} + (1-t)\overrightarrow{OB}}{t+(1-t)}$		
$= t\vec{a} + (1-t)\vec{b}$	1A	
Since $OE \perp AB$,		
$\overrightarrow{OE}\cdot\overrightarrow{AB} = 0$		
$[t\vec{a} + (1-t)\vec{b}]\cdot(\vec{b} - \vec{a}) = 0$	1M	
$t\,\vec{a}\cdot\vec{b} - t\,\vec{a}\cdot\vec{a} + (1-t)\,\vec{b}\cdot\vec{b} - (1-t)\,\vec{a}\cdot\vec{b} = 0$	1M	For distributive law
$3t - 9t + 4(1-t) - 3(1-t) = 0$	1A	For $\vec{a}\cdot\vec{a} = 9$ or $\vec{b}\cdot\vec{b} = 4$
$1 - 7t = 0$		
$t = \frac{1}{7}$		
$\therefore \overrightarrow{OE} = \frac{1}{7}\vec{a} + \frac{6}{7}\vec{b}$	1A	
$\overrightarrow{BE} = \overrightarrow{OE} - \overrightarrow{OB}$ $= \frac{1}{7}\vec{a} - \frac{1}{7}\vec{b}$	5	
(c) $\overrightarrow{BA}\cdot\overrightarrow{BF}$		
$= \lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BF}\rvert\cos\angle ABF$	1A	
$= \lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BE}\rvert$	1A	
$=$ a constant since $\lvert\overrightarrow{BA}\rvert$ and $\lvert\overrightarrow{BE}\rvert$ are constants		
$\therefore$ the student is correct.	1	
By Cosine Law,		
$\lvert\overrightarrow{BA}\rvert^2 = \lvert\overrightarrow{OA}\rvert^2 + \lvert\overrightarrow{OB}\rvert^2 - 2\lvert\overrightarrow{OA}\rvert\lvert\overrightarrow{OB}\rvert\cos\angle AOB$		
$= 3^2 + 2^2 - 2(3)(2)\cos\frac{\pi}{3}$		
$= 7$		
$\lvert\overrightarrow{BA}\rvert = \sqrt{7}$		
$\vdots$		
$\lvert\overrightarrow{BE}\rvert = \frac{\sqrt{7}}{7}$	1M	For finding $\lvert\overrightarrow{BA}\rvert$ and $\lvert\overrightarrow{BE}\rvert$
From (b), $\overrightarrow{BE} = \frac{1}{7}\overrightarrow{BA}$		
$\lvert\overrightarrow{BE}\rvert = \frac{1}{7}\lvert\overrightarrow{BA}\rvert$		
$= \frac{\sqrt{7}}{7}$		
$\therefore \overrightarrow{BA}\cdot\overrightarrow{BF} = \lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BE}\rvert$		
$= \sqrt{7}\left(\frac{\sqrt{7}}{7}\right) = 1$	1A	

Solution	Marks	Remarks
<u>Alternative solution for finding $\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BE}\rvert$</u>		
$\overrightarrow{BA}\cdot\overrightarrow{BE}$		
$=(\overrightarrow{OA}-\overrightarrow{OB})\cdot(\overrightarrow{OE}-\overrightarrow{OB})$	1M	
$=(\vec{a}-\vec{b})\cdot(\frac{1}{7}\vec{a}+\frac{6}{7}\vec{b}-\vec{b})$		
$=\frac{1}{7}(\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})$		
$=\frac{1}{7}(\vec{a}\cdot\vec{a}-2\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{b})$		
$=\frac{1}{7}[3^2-2(3)+2^2]$		
$=1$	1A	
$\therefore\ \overrightarrow{BA}\cdot\overrightarrow{BF}=1$		
<u>Alternative solution</u>		
(c) Let $\overrightarrow{OF}=k\,\overrightarrow{OE}$. ($k$ is a real no.) $=k(\frac{1}{7}\vec{a}+\frac{6}{7}\vec{b})$	1A	
$\overrightarrow{BF}=\overrightarrow{OF}-\overrightarrow{OB}$	1M	
$=k\overrightarrow{OE}-\overrightarrow{OB}$		
$=k(\frac{1}{7}\vec{a}+\frac{6}{7}\vec{b})-\vec{b}$		
$\overrightarrow{BA}\cdot\overrightarrow{BF}$		
$=(\vec{a}-\vec{b})\cdot[\frac{k}{7}\vec{a}+(\frac{6k}{7}-1)\vec{b}]$	1M !	
$=\frac{k}{7}\vec{a}\cdot\vec{a}+(\frac{6k}{7}-1)\vec{a}\cdot\vec{b}-\frac{k}{7}\vec{a}\cdot\vec{b}-(\frac{6k}{7}-1)\vec{b}\cdot\vec{b}$		
$=\frac{k}{7}(9)+(\frac{6k}{7}-1)(3)-\frac{k}{7}(3)-(\frac{6k}{7}-1)(4)$		
$=1$	1A	
$\therefore\ \overrightarrow{BA}\cdot\overrightarrow{BF}$ is constant and the student is correct.	1	
	5	Omit vector sign or dot product sign in most cases (pp−1)

$\overrightarrow{BA}\cdot\overrightarrow{BF}=(\vec{a}-\vec{b})\cdot(\overrightarrow{OF}-\overrightarrow{OB})$

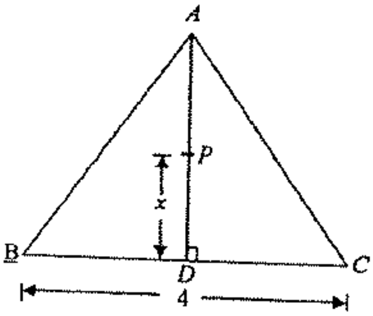
$=(\vec{a}-\vec{b})\cdot\overrightarrow{OF}-(\vec{a}-\vec{b})\cdot\vec{b}$

$=0-\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{b}$

$=-3+2^2$

$=1$

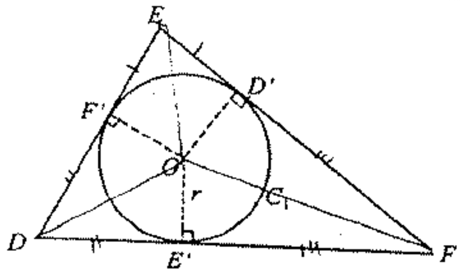
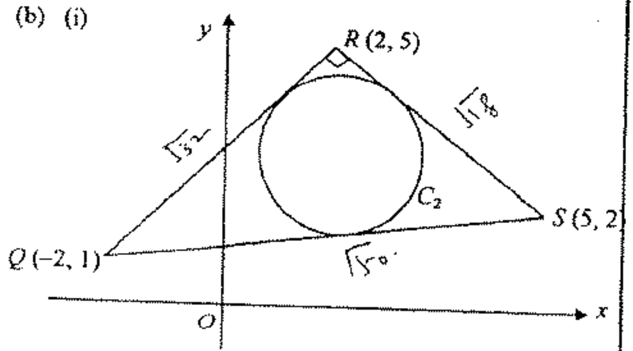
只限教師參閱　　FOR TEACHERS' USE ONLY

Solution	Marks	Remarks
14. (a)		
		
(i) $PB = PC = \sqrt{x^2+4}$		
$PA = (3-x)$		
$r = PA + PB + PC$		
$= 2\sqrt{x^2+4} + (3-x)$	1A	
$\frac{dr}{dx} = 2(\frac{1}{2})\frac{2x}{\sqrt{x^2+4}} - 1$		
$= \frac{2x}{\sqrt{x^2+4}} - 1$	1	
(ii) (1) $\frac{dr}{dx} \geq 0$		Accept $\frac{dr}{dx} > 0$
$\frac{2x}{\sqrt{x^2+4}} - 1 \geq 0$	1M	
$2x \geq \sqrt{x^2+4}$		
$4x^2 \geq x^2+4$		
$x \geq \frac{2}{\sqrt{3}}$		
$\therefore$ r is increasing on $3 \geq$ $x \geq \frac{2}{\sqrt{3}}$.	1A	Accept $x > \frac{2}{\sqrt{3}}$
(2) $\frac{dr}{dx} \leq 0$ $x \geq \frac{2\sqrt{3}}{3}$		
$x \leq \frac{2}{\sqrt{3}}$		
$\therefore$ r is decreasing on $0 \leq$ $x \leq \frac{2}{\sqrt{3}}$. $\frac{2\sqrt{3}}{3}$	1A	Accept $x < \frac{2}{\sqrt{3}}$
r is the least at $x = \frac{2}{\sqrt{3}}$. $\frac{2\sqrt{3}}{3}$	1M	
Least value of $r = 2\sqrt{(\frac{2}{\sqrt{3}})^2 + 4} + (3 - \frac{2}{\sqrt{3}})$		
$= 2\sqrt{3} + 3$	1A	

只限教師參閱 FOR TEACHERS' USE ONLY

Solution	Marks	Remarks
Alternative solution		
(ii) $\frac{\mathrm{d}r}{\mathrm{d}x}=\frac{2x}{\sqrt{x^2+4}}-1=0$	1M	
$x=\frac{2}{\sqrt{3}}$		
$\frac{\mathrm{d}^2r}{\mathrm{d}x^2}=2(x^2+4)^{-\frac{1}{2}}-x(x^2+4)^{-\frac{3}{2}}(2x)$		
$=\frac{8}{(x^2+4)^{\frac{3}{2}}}$	1M	For checking
$\left.\frac{\mathrm{d}^2r}{\mathrm{d}x^2}\right\vert_{x=\frac{2}{\sqrt{3}}}\boxed{=\frac{3\sqrt{3}}{8}}>0$		
$\therefore$ r attains a minimum at $x=\frac{2}{\sqrt{3}}$.		
r is increasing on $\boxed{3\geq}$ $x\geq\frac{2}{\sqrt{3}}$.	1A	Not awarded if checking was incomplete.
and decreasing on $\boxed{0\leq}$ $x\leq\frac{2}{\sqrt{3}}$.	1A	
Least value of $r=2\sqrt{(\frac{2}{\sqrt{3}})^2+4}+(3-\frac{2}{\sqrt{3}})$		
$=2\sqrt{3}+3$	1A	
(iii) The greatest value of r occurs at the end-points.		
At $x=0$, $r=2\sqrt{0+4}+(3-0)=7$	1M	
At $x=3$, $r=2\sqrt{3^2+4}+(3-3)=2\sqrt{13}$		
$\therefore$ the greatest value of r is $2\sqrt{13}$.	1A / 9	when $x=\frac{2}{\sqrt{3}}$
(b) $r=2\sqrt{x^2+4}+(1-x)$		$r_{least}=1+2\sqrt{3}$
$\boxed{\frac{\mathrm{d}r}{\mathrm{d}x}=\frac{2x}{\sqrt{x^2+4}}-1}$	1A	
From (a), r is decreasing on $0\leq x\leq 1$.		
r is the least at $x=1$.	1M	
Least value $=2\sqrt{1+4}+(1-1)$		
$=2\sqrt{5}$	1A / 3	

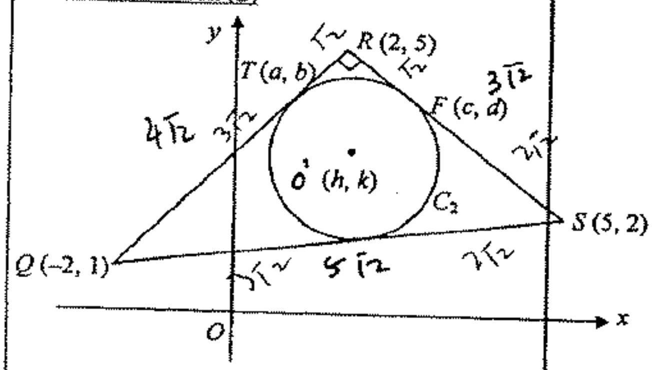
只限教師參閱 FOR TEACHERS' USE ONLY

Solution	Marks	Remarks
15. (a)		
		
A = Area of ΔODE + area of ΔOEF + area of ΔODF (O is centre of C_1)		
$=\frac{1}{2}(DE)(r)+\frac{1}{2}(EF)(r)+\frac{1}{2}(DF)(r)$	2A	
$=\frac{1}{2}(DE+EF+FD)(r)$		
$A=\frac{1}{2}pr$	1M+1	1M for $p=DE+EF+FD$
Alternative solution		
A = Area of $OD'EF'$ + area of $\Delta OE'FD'$ + area of $\Delta OF'DE'$		
$=2(\frac{1}{2})(ED')(r)+2(\frac{1}{2})(FE')(r)+2(\frac{1}{2})(DF')(r)$	2A	
$=\frac{1}{2}(2ED'+2FE'+2DF')(r)$		
$A=\frac{1}{2}pr$	1M+1	1M for $p=2(ED'+FE'+DF')$
	4	
(b) (i)		
		
$QR=\sqrt{(-2-2)^2+(1-5)^2}=\sqrt{32}$		
$RS=\sqrt{(2-5)^2+(5-2)^2}=\sqrt{18}$		
$SQ=\sqrt{(-2-5)^2+(1-2)^2}=\sqrt{50}$		
$p=\sqrt{32}+\sqrt{18}+\sqrt{50}$		
$=12\sqrt{2}$		
Area of $\Delta QRS=\frac{1}{2}(QR)(RS)$		
$=\frac{1}{2}(\sqrt{32})(\sqrt{18})$	1M	
$=12$		

只限教師參閱 FOR TEACHERS' USE ONLY

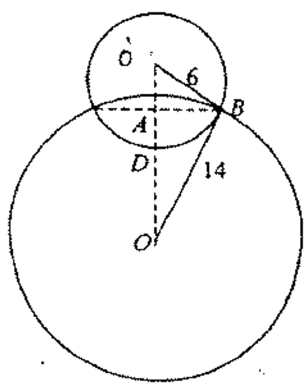
Solution	Marks	Remarks
Alternative solution		
Area of $\Delta QRS = \frac{1}{2}\begin{vmatrix} -2 & 1 \\ 5 & 2 \\ 2 & 5 \\ -2 & 1 \end{vmatrix}$ X		
$= \frac{1}{2}(-4+25+2-5-4+10)$	1M	
$= 12$		
Using (a), $A = \frac{1}{2}pr$		
$12 = \frac{1}{2}(12\sqrt{2})r$	1M	
$r = \sqrt{2}$	1A	
(ii) Let (h, k) be the centre of C_2. From (ii), slope of $QR = 1$ and slope of $RS = -1$, so the angle bisector of QR and RS is a vertical line.	1M	
As the centre of C_2 lies on the angle bisector, so $h = 2$.	1A	
Distance between R and the centre $= \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}$ $= 2$	1M	
$k = 5 - 2 = 3$	1M	
$\therefore$ the coordinates of the centre of C_2 are (2, 3).		
The equation of C_2 is $(x-2)^2 + (y-3)^2 = 2$.	1A	OR $x^2 + y^2 - 4x - 6y + 11 = 0$
Alternative solution (1)		
Equation of QR: $\frac{y-5}{x-2} = \frac{5-1}{2-(-2)}$	1M	1 M for eqt of QR, RS and QS 1M
$x - y + 3 = 0$		
Let (h, k) be the centre of C_2.		
$(\frac{h-k+3}{\sqrt{2}}) = \sqrt{2}$	1M	For distance formula
$h - k + 1 = 0$ ------(1)	1A	For either (1), (2) or (3)
Equation of RS: $\frac{y-5}{x-2} = \frac{5-2}{2-5}$		
$x + y - 7 = 0$		
$-(\frac{h+k-7}{\sqrt{2}}) = \sqrt{2}$		
$h + k - 5 = 0$ ------(2)		
Solve (1) and (2), $h = 2$, $k = 3$.	1M	For solving (1) and (2)
$\therefore$ the equation of C_2 is $(x-2)^2 + (y-3)^2 = 2$. 1A	1A	
Equation of QS: $x - 7y + 9 = 0$		
$\left\lvert\frac{h-7k+9}{-\sqrt{50}}\right\rvert = \sqrt{2}$	1A	
$h - 7k + 19 = 0$ ------(3)		
1M		

只限教師參閱 FOR TEACHERS' USE ONLY

Solution	Marks	Remarks
Alternative solution (2)		
		
$\frac{RT}{TQ}=\frac{\sqrt{2}}{3\sqrt{2}}=\frac{1}{3}$	1M	
$a=\frac{-2+6}{1+3}=1$, $b=\frac{1+15}{1+3}=4$	1M	
$\frac{RF}{FS}=\frac{\sqrt{2}}{2\sqrt{2}}=\frac{1}{2}$		
$c=\frac{5+4}{1+2}=3$, $d=\frac{2+10}{1+2}=4$	1A	For a, b, c, d
$\frac{h+2}{2}=\frac{1+3}{2}$ $\therefore h=2$	1M	
$\frac{k+5}{2}=\frac{4+4}{2}$ $\therefore k=3$		
$\therefore$ the equation of C_2 is $(x-2)^2+(y-3)^2=2$.	1A	
$x^2+y^2-4x-6y+11=0$		

Solution	Marks	Remarks
16. (a) Volume $= \int_h^r \pi x^2 \mathrm{d}y$ accept wrong upper lower limit	1M	For $V = \pi \int_a^b x^2 \mathrm{d}y$,
$= \pi \int_h^r (r^2 - y^2)\mathrm{d}y$	1A	
$= \pi \left[r^2 y - \frac{1}{3} y^3 \right]_h^r$	1A	For primitive function
$= \pi \left[r^3 - \frac{1}{3} r^3 - r^2 h + \frac{1}{3} h^3 \right]$		
$= \frac{\pi}{3}(2r^3 - 3r^2 h + h^3)$	1	
may be wrong	4	
(b) (i) Using the result in (a), substitute $r = 14$, $h = 13$: correct Capacity of the pot		
$= \frac{4}{3}\pi(14)^3 - \frac{\pi}{3}[2(14)^3 - 3(14)^2(13) + (13)^3]$	1M	
$= 3645\pi$	1A	
<u>Alternative solution</u> Capacity of the pot		
$= \frac{4}{3}\pi(14)^3 - \pi \int_{13}^{14} (196 - y^2)\mathrm{d}y$	1M	<u>OR</u> $V = \pi \int_{-14}^{13} (196 - y^2)\mathrm{d}y$ correct, may be wrong
$= \frac{4}{3}\pi(14)^3 - \pi \left[196y - \frac{1}{3} y^3 \right]_{13}^{14}$		$\pi \int_0^{13} (196 - y^2)\mathrm{d}y$
$= \frac{10976\pi}{3} - \frac{41\pi}{3}$		$= 1815\frac{2}{2}\pi$
$= 3645\pi$	1A	$\pi \int_{-14}^{0} (196 - y^2)\mathrm{d}y$ $= 1829\frac{1}{3}\pi$.

(ii)



Solution	Marks	Remarks
Let C be the centre of the stopper. $OB = 14$, $BC = 6$, $OA = 13$		<u>Alternative solution</u>
$AB = \sqrt{OB^2 - OA^2}$ $= \sqrt{14^2 - 13^2} = \sqrt{27}$	1M	Subs. $y = 13$ into $x^2 + y^2 = 196$, $x = \sqrt{27}$
$AC = \sqrt{BC^2 - AB^2}$ $= \sqrt{6^2 - 27} = 3$	1M	Subs. $x = \sqrt{27}$ into $x^2 + y^2 = 36$, $y = 3$
$\boxed{AD = 6 - 3 = 3}$	1A	

Solution	Marks	Remarks
Capacity of the perfume bottle = Capacity of pot found in (i) − Volume of portion of the stopper lying inside the pot	1M	
$= 3645\pi - \frac{\pi}{3}[2(6)^3 - 3(6)^2(3) + (3)^3]$	1M	For 2nd term
$= 3645\pi - 45\pi$		
$= 3600\pi$	1A	
	8	

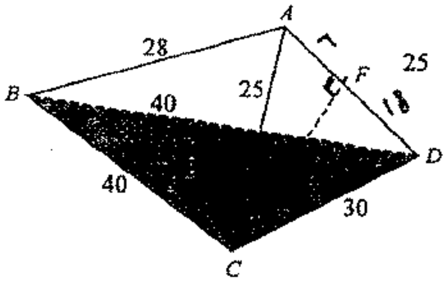
只限教師參閱 FOR TEACHERS' USE ONLY

Solution	Marks	Remarks

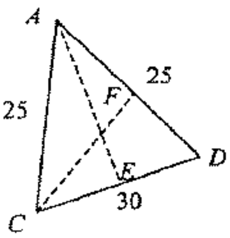
17. (a)

Marking Criteria:

Find CF	2M+1A
Find BF	3M
Find $\angle BFC$	1M+1A



Consider ΔACD:
Let E be the foot of perpendicular from A to CD.



$\cos\angle ADE = \dfrac{DE}{AD} = \dfrac{15}{25} = \dfrac{3}{5}$ — 1M

Remarks: $\angle ACD = \angle ADC = 53.13°$, $\angle CAD = 73.74°$

$CF = CD \sin\angle ADE$

$= 30\left(\dfrac{\sqrt{5^2 - 3^2}}{5}\right)$ (accept round up to 24) — 1M

$= 24$ — 1A

Alternative solution

$AE = \sqrt{25^2 - 15^2}$ — 1M

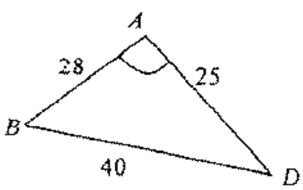
$= 20$

$\dfrac{1}{2}(CD)(AE) = \dfrac{1}{2}(AD)(CF)$

$\dfrac{1}{2}(30)(20) = \dfrac{1}{2}(25)(CF)$ — 1M

$CF = 24$ — 1A

Consider ΔABD:



$\cos\angle BAD = \dfrac{28^2 + 25^2 - 40^2}{2(28)(25)}$ — 1M

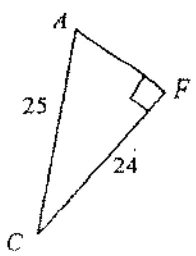
$= -\dfrac{191}{1400}$

Remarks: $\angle BAD = 97.84°$, $\angle ADB = 43.90°$, $\angle ABD = 38.26°$

$625 - AF^2 = 30^2 - (25 - AF)^2$

$AF = 7$

Consider ΔACF:

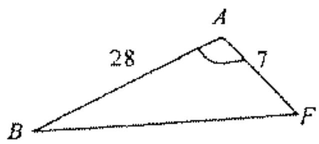


$AF^2 = AC^2 - CF^2$

$= 25^2 - 24^2 = 49$ — 1M

$AF = 7$

Consider ΔABF:



$BF^2 = AB^2 + AF^2 - 2(AB)(AF)\cos\angle BAF$

$= 28^2 + 7^2 - 2(28)(7)\left(-\dfrac{191}{1400}\right)$ — 1M

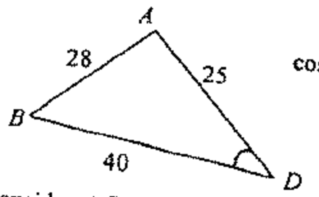
$BF = \sqrt{886.48}$ (≈ 29.77)

只限教師參閱 FOR TEACHERS' USE ONLY

Solution	Marks	Remarks

Alternative solution

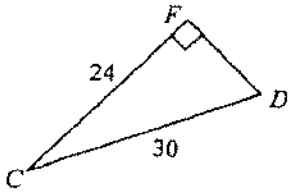
Consider ΔABD :



$$\cos\angle BDA = \frac{40^2+25^2-28^2}{2(40)(25)}$$
$$= \frac{1441}{2000}$$

1M

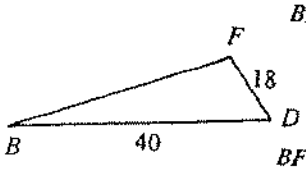
Consider ΔCDF :



$$DF^2 = CD^2 - CF^2$$
$$= 30^2 - 24^2$$
$$DF = 18$$

1M

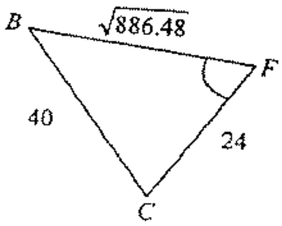
Consider ΔBDF :



$$BF^2 = BD^2 + DF^2 - 2(BD)(DF)\cos\angle BDF$$
$$= 40^2 + 18^2 - 2(40)(18)\left(\cdot\frac{1441}{2000}\right)$$
$$BF = \sqrt{886.48} \quad (\approx 29.77)$$

1M

Consider ΔBCF :



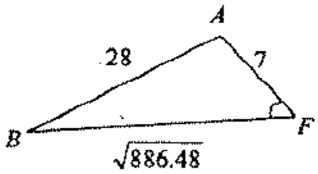
$$\cos\angle BFC = \frac{BF^2+CF^2-BC^2}{2(BF)(CF)}$$
$$= \frac{886.48+24^2-40^2}{2(\sqrt{886.48})(24)}$$

1M

$$= -0.096$$
$\angle BFC = 96°$ (correct to the nearest degree)

1A

8

Solution	Marks	Remarks
(b) Consider ΔABF:  $BF^2 + FA^2 = 886.48 + 49$ $= 935.48$ $\neq AB^2$ $\therefore \angle AFB \neq 90°$	1M+1M+1A	1M for considering $\angle AFB$ (OR $\angle BFD$)
Alternative solution (1) $\cos\angle AFB$ $= \dfrac{7^2 + 886.48 - 28^2}{2(7)(\sqrt{886.48})}$ $= 0.363 \neq 0$ $\therefore \angle AFB \neq 90°$ $\angle AFB \approx 68.72°$	1M+1M+1A	1M for considering ΔAFB
Alternative solution (2) From (a), $\cos\angle BAD = \dfrac{-191}{1400}$. $\angle BAD > 90°$, i.e. $\angle BAF > 90°$. $\therefore \angle AFB < 90°$	1M+1M+1A	1M for considering ΔAFB
From (a), $\angle BFC = 96°$. Since $CF \perp AD$ but BF is not perpendicular to AD, $\angle BFC$ does not represent the angle between the two planes. The student is incorrect.	1A	
	4	

只限教師參閱 FOR TEACHERS' USE ONLY

Solution	Marks	Remarks
18. (a) $z^2 = \cos 2\theta + i\sin 2\theta$ $(\cos^2\theta \cdot \sin^2\theta) + 2i\sin\theta\cos\theta$	1A	$z^2+1 = (\cos^2\theta - \sin^2\theta) + 2i\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta$
$z^2 + 1 = (\cos 2\theta + 1) + i\sin 2\theta$		$= 2\cos^2\theta + 2i\sin\theta\cos\theta$
$\lvert z^2+1\rvert^2 = (\cos 2\theta + 1)^2 + \sin^2 2\theta$	1M	$= 2\cos\theta(\cos\theta + i\sin\theta)$
$= \cos^2 2\theta + 2\cos 2\theta + 1 + \sin^2 2\theta$		$\lvert z^2+1\rvert^2 = 4\cos^2\theta \lvert\cos\theta + i\sin\theta\rvert$
$= 2(1+\cos 2\theta)$	1	$= 2(2\cos^2\theta)$
Since $-\pi < \theta \le \pi$, $-2\pi < 2\theta \le 2\pi$, $\cos 2\theta \le 1$	1M	$= 2(\cos 2\theta + 1)$
$\therefore$ the greatest value of $\lvert z^2+1\rvert = \sqrt{2(1+1)}$ $= 2$	1A	$\lvert z^2+1\rvert = 2\cos\theta$.
	5	
(b) (i) $w = 3z$ (OR $w = 3(\cos\theta + i\sin\theta)$)	1A	
$\lvert w^2+9\rvert = \lvert(3z)^2+9\rvert$ $= 9\lvert z^2+1\rvert$		
From (a), $\lvert z^2+1\rvert \le 2$.		
$\therefore$ greatest value of $\lvert w^2+9\rvert = 9(2)$	1M	For using (a)
$= 18$	1	
(ii) $w^4 - 81 = 100i(w^2 - 9)$		
$(w^2+9)(w^2-9) - 100i(w^2-9) = 0$	1M	For factorization
$(w^2-9)(w^2+9-100i) = 0$		
$w^2 - 9 = 0$ - - - (1) or $w^2 + 9 - 100i = 0$ - - - (2)		
Consider (1): $w = \pm 3$ which satisfies the condition $\lvert w\rvert = 3$	1A	
Consider (2): $w^2 + 9 = 100i$ From (i), $\lvert w^2+9\rvert \le 18$ but $\lvert 100i\rvert = 100$. So equation (2) has no solutions	1M	
$\therefore$ the equation has only two roots.	1	
Alternative solution for explaining why (2) has no solution		
$w^2 = -9 + 100i$		
Let $w = a + bi$. $(a^2 - b^2) + 2abi = -9 + 100i$ $\begin{cases} a^2 - b^2 = -9 & \text{- - - (3)} \\ 2ab = 100 & \text{- - - (4)} \end{cases}$ Since $\lvert w\rvert = 3$, $a^2 + b^2 = 9$ - - - (5) From (3) and (5), $a = 0$.	1M	For attempting to solve (3), (4), (5)
Substitute $a = 0$ into (4): LHS $= 0 \ne$ RHS. So the equation has no solution.	1	
	7	

只限教師參閱　　FOR TEACHERS' USE ONLY