

香港考試局**HONG KONG EXAMINATIONS AUTHORITY****2001年香港中學會考****HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2001****附加數學****ADDITIONAL MATHEMATICS**

本評卷參考乃考試局專為今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視為標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations Authority for markers' reference. The Examinations Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Examinations Authority is counting on the co-operation of markers/teachers in this regard.

考試結束後，各科評卷參考將存放於教師中心，供教師參閱。

After the examinations, marking schemes will be available for reference at the teachers' centre.




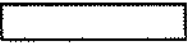
只限教師參閱 FOR TEACHERS' USE ONLY

GENERAL INSTRUCTIONS TO MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates would use alternative methods not specified in the marking scheme. Markers should be patient in marking these alternative solutions. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method is specified in the question.
2. In the marking scheme, marks are classified as follows :

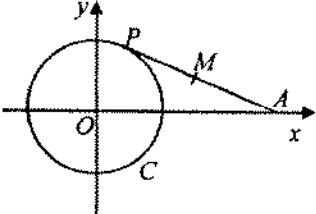
'M' marks – awarded for knowing a correct method of solution and attempting to apply it;

'A' marks – awarded for the accuracy of the answer;

Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.
3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.
4. The symbol $\textcircled{\text{pp-1}}$ should be used to denote marks deducted for poor presentation (p.p.). Note the following points:
 - (a) At most deduct 1 mark for p.p. in each question, up to a maximum of 3 marks for the whole paper.
 - (b) For similar p.p., deduct only 1 mark for the first time that it occurs, i.e. do not penalise candidates twice in the whole paper for the same p.p.
 - (c) In any case, do not deduct any marks for p.p. in those steps where candidates could not score any marks.
 - (d) Some cases in which marks should be deducted for p.p. are specified in the marking scheme. However, the lists are by no means exhaustive. Markers should exercise their professional judgement to give p.p.s in other situations.
5. The symbol $\textcircled{\text{u-1}}$ should be used to denote marks deducted for wrong/no units in the final answers (if applicable). Note the following points:
 - (a) At most deduct 1 mark for wrong/no units for the whole paper.
 - (b) Do not deduct any marks for wrong/no units in case candidate's answer was already wrong.
6. Marks entered in the Page Total Box should be the net total score on that page.
7. In the Marking Scheme, steps which can be omitted are enclosed by dotted rectangles  , whereas alternative answers are enclosed by solid rectangles  .
8.
 - (a) Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.
 - (b) In case a certain degree of accuracy had been specified in the question, answers not accurate up to that degree should not be accepted. For answers with an excess degree of accuracy, deduct 1 mark for the first time if happened. In any case, do not deduct any marks for excess degree of accuracy in those steps where candidates could not score any marks.
9. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.
10. Unless the form of answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they were correct.

Solution	Marks	Remarks
<p>1. $\frac{d}{dx} \left(\frac{x^2}{2x+1} \right)$</p> <div style="border: 1px dashed black; padding: 5px; margin: 5px 0;"> $\frac{(2x+1) \frac{d}{dx} (x^2) - x^2 \frac{d}{dx} (2x+1)}{(2x+1)^2}$ </div> $= \frac{(2x+1)(2x) - x^2(2)}{(2x+1)^2}$ $= \frac{2x(x+1)}{(2x+1)^2}$	<p>1M+1A</p> <p>1A</p>	<p>1M for quotient rule $\left(\frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \right)$</p> <p>Accept equivalent forms</p>
<p>Alternative Solution</p> $\frac{d}{dx} \left(\frac{x^2}{2x+1} \right)$ <div style="border: 1px dashed black; padding: 5px; margin: 5px 0;"> $= \frac{d}{dx} x^2 (2x+1)^{-1}$ </div> $= 2x(2x+1)^{-1} + x^2(-1)(2x+1)^{-2}(2)$ $= \frac{2x}{2x+1} - \frac{2x^2}{(2x+1)^2}$ $= \frac{2x(x+1)}{(2x+1)^2}$	<p>1M+1A</p> <p>1A</p>	<p>1M for product rule</p>
	<p style="text-align: center;"><u>3</u></p>	

Solution	Marks	Remarks
<p>2. Let $u = 3x^2 + 1$.</p> <p>$du = 6x dx$</p> $\int \frac{x}{\sqrt{3x^2 + 1}} dx = \int \frac{1}{u^{\frac{1}{2}}} \left(\frac{du}{6}\right)$ $= \int \frac{1}{6} u^{-\frac{1}{2}} du$ $= \frac{1}{6} \frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c \quad \boxed{c \text{ is a constant}}$ $= \frac{1}{3} u^{\frac{1}{2}} + c$ $= \frac{1}{3} (3x^2 + 1)^{\frac{1}{2}} + c$	<p>1A</p> <p>1A</p> <p>1M</p> <p>1A</p>	<p>Omit du in most cases (pp-1)</p> <p>For $\int u^n du = \frac{1}{n+1} u^{n+1}$ Awarded even if 'c' was omitted</p> <p>Withhold this mark if 'c' was omitted</p>
<p>Alternative Solution (1)</p> <p>Let $v = \sqrt{3x^2 + 1}$.</p> $dv = \frac{3x}{\sqrt{3x^2 + 1}} dx$ $\int \frac{x}{\sqrt{3x^2 + 1}} dx = \int \frac{1}{3} dv$ $= \frac{1}{3} v + c \quad \boxed{c \text{ is a constant}}$ $= \frac{1}{3} (3x^2 + 1)^{\frac{1}{2}} + c$	<p>1A</p> <p>1A</p> <p>1M</p> <p>1A</p>	
<p>Alternative Solution (2)</p> $\int \frac{x}{\sqrt{3x^2 + 1}} dx$ $= \int \frac{1}{\sqrt{3x^2 + 1}} \left[\frac{1}{6} d(3x^2 + 1) \right]$ $= \frac{1}{6} \frac{(3x^2 + 1)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c \quad \boxed{c \text{ is a constant}}$ $= \frac{1}{3} (3x^2 + 1)^{\frac{1}{2}} + c$	<p>1A+1A</p> <p>1M</p> <p>1A</p>	<p>1A for $dx \rightarrow d(3x^2 + 1)$</p>
<p>4</p>		

Solution	Marks	Remarks
3. (a) The coordinates of M are $\left(\frac{h+4}{2}, \frac{k}{2}\right)$.	1A	
(b) Let the coordinates of M be (x, y) . $\begin{cases} x = \frac{h+4}{2} \\ y = \frac{k}{2} \end{cases}$ $\begin{cases} h = 2x - 4 \\ k = 2y \end{cases}$	1M	
Since (h, k) is a point on C , $(2x-4)^2 + (2y)^2 = 4$	1M	For substituting into C
$4x^2 - 16x + 16 + 4y^2 = 4$		
$x^2 + y^2 - 4x + 3 = 0$	1A	Accept equivalent forms
	4	

Solution	Marks	Remarks
<p>4. General term $= {}_8C_r(2x^3)^{8-r}\left(\frac{1}{x}\right)^r$ $= {}_8C_r(2^{8-r})x^{24-4r}$ $24 - 4r = 0$ $r = 6$ Constant term $= {}_8C_6(2^{8-6})$ $= 28(4)$ $= 112$</p>	<p>2A 1M 1A</p>	<p>OR $= {}_8C_r\left(\frac{1}{x}\right)^{8-r}(2x^3)^r$ $= {}_8C_r(2^r)x^{4r-8}$ $4r - 8 = 0$ $r = 2$</p>
<p>Alternative Solution (1) $(2x^3 + \frac{1}{x})^8 = (2x^3)^8 + {}_8C_1(2x^3)^7\left(\frac{1}{x}\right) + {}_8C_2(2x^3)^6\left(\frac{1}{x}\right)^2 +$ ${}_8C_3(2x^3)^5\left(\frac{1}{x}\right)^3 + {}_8C_4(2x^3)^4\left(\frac{1}{x}\right)^4 +$ ${}_8C_5(2x^3)^3\left(\frac{1}{x}\right)^5 + \boxed{{}_8C_6(2x^3)^2\left(\frac{1}{x}\right)^6} + \dots$ $= (2x^3)^8 + 8(2x^3)^7\left(\frac{1}{x}\right) + 28(2x^3)^6\left(\frac{1}{x}\right)^2 +$ $56(2x^3)^5\left(\frac{1}{x}\right)^3 + 70(2x^3)^4\left(\frac{1}{x}\right)^4 +$ $56(2x^3)^3\left(\frac{1}{x}\right)^5 + \boxed{28(2x^3)^2\left(\frac{1}{x}\right)^6} + \dots$ Constant term $= {}_8C_6(2)^2$ $= 28(4)$ $= 112$</p>	<p>1A+1A 1M 1A</p>	<p>1A for ${}_8C_6(2x^3)^2\left(\frac{1}{x}\right)^6$ 1A for other terms (can be omitted) For choosing the correct term</p>
<p>Alternative Solution (2) $(2x^3 + \frac{1}{x})^8 = \left(\frac{2x^4 + 1}{x}\right)^8$ $= \frac{(1 + 2x^4)^8}{x^8}$ $= \frac{1}{x^8} [1 + {}_8C_1(2x^4) + {}_8C_2(2x^4)^2 + \dots]$ Constant term $= {}_8C_2(2)^2$ $= 28(4)$ $= 112$</p>	<p>1A+1A 1M 1A</p>	<p>1A for $\frac{1}{x^8} {}_8C_2(2x^4)^2$ 1A for other terms (can be omitted) For choosing the correct term</p>
	<p>4</p>	

Solution	Marks	Remarks
5. (a) $\frac{1+i}{1-i} = \frac{1+i}{1-i} \left(\frac{1+i}{1+i}\right)$ $= \frac{1+2i+i^2}{2}$ $= i$	1A 1A	
Alternative Solution $\frac{1+i}{1-i} = \frac{\sqrt{2}(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})}{\sqrt{2}[\cos(-\frac{\pi}{4}) + i \sin(-\frac{\pi}{4})]}$ $= \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$	1A 1A	
(b) $\frac{(1+i)^{16}}{(1-i)^{15}} = \left(\frac{1+i}{1-i}\right)^{16} (1-i)$ $= i^{16} (1-i)$ $= 1-i$	1A 1M 1A	For using (a)
Alternative Solution (1) $\frac{(1+i)^{16}}{(1-i)^{15}} = \left(\frac{1+i}{1-i}\right)^{15} (1+i)$ $= i^{15} (1+i)$ $= i^3 (1+i)$ $= -i(1+i)$ $= 1-i$	1A 1M 1A	For using (a)
Alternative Solution (2) $\frac{(1+i)^{16}}{(1-i)^{15}} = \frac{[\sqrt{2}(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})]^{16}}{[\sqrt{2}(\cos(-\frac{\pi}{4}) + i \sin(-\frac{\pi}{4}))]^{15}}$ $= \frac{(\sqrt{2})^{16} (\cos 4\pi + i \sin 4\pi)}{(\sqrt{2})^{15} (\cos(-\frac{15\pi}{4}) + i \sin(-\frac{15\pi}{4}))}$ $= \sqrt{2} [\cos(4\pi + \frac{15\pi}{4}) + i \sin(4\pi + \frac{15\pi}{4})]$ $= \sqrt{2} [\cos(-\frac{\pi}{4}) + i \sin(-\frac{\pi}{4})]$	1A 1M 1A	Accept $\sqrt{2}(\cos \frac{\pi}{4} - i \sin \frac{\pi}{4})$, $\sqrt{2}(\cos(\frac{7\pi}{4}) + i \sin \frac{7\pi}{4})$
	<u>5</u>	

Solution	Marks	Remarks
6. (a) $\sin x + \cos x = \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right)$ $= \sqrt{2}\left(\sin x \sin \frac{\pi}{4} + \cos x \cos \frac{\pi}{4}\right)$ $= \sqrt{2} \cos\left(x - \frac{\pi}{4}\right)$ $\therefore r = \sqrt{2}, \theta = -\frac{\pi}{4} (-45^\circ)$	1A+1A	OR $\begin{cases} r \cos \theta = 1 \\ r \sin \theta = -1 \end{cases}$
(b) $\sin x + \cos x = \sqrt{2}$ $\sqrt{2} \cos\left(x - \frac{\pi}{4}\right) = \sqrt{2}$ $\cos\left(x - \frac{\pi}{4}\right) = 1$	1M	
$x - \frac{\pi}{4} = 2n\pi$ n is an integer	1M	For $x = 2n\pi \pm \alpha$
$x = 2n\pi + \frac{\pi}{4}$	1A	Accept degrees $x = 2n\pi + 45^\circ$ etc ($n-1$)
<u>Alternative Solution (1)</u> $\sin x + \cos x = \sqrt{2}$ $\sqrt{2} \sin\left(x + \frac{\pi}{4}\right) = \sqrt{2}$ $\sin\left(x + \frac{\pi}{4}\right) = 1$ $x + \frac{\pi}{4} = 2n\pi + \frac{\pi}{2}$ n is an integer $x = 2n\pi + \frac{\pi}{4}$	1M 1M 1A	OR $x + \frac{\pi}{4} = n\pi + (-1)^n \left(\frac{\pi}{2}\right)$ $x = \left(n - \frac{1}{4}\right)\pi + (-1)^n \left(\frac{\pi}{2}\right)$
<u>Alternative Solution (2)</u> $\sin x + \cos x = \sqrt{2}$ $(\sin x + \cos x)^2 = 2$ $\sin^2 x + 2 \sin x \cos x + \cos^2 x = 2$ $\sin 2x = 1$ $2x = 2n\pi + \frac{\pi}{2}$ Since $\sin x + \cos x$ is positive, $x = n\pi + \frac{\pi}{4}$, where n is even.	1M 1M 1A	For $\sin^2 x + \cos^2 x = 1$ & $2 \sin x \cos x = \sin 2x$ OR $2x = n\pi + (-1)^n \left(\frac{\pi}{2}\right)$ $x = \frac{n\pi}{2} + (-1)^n \left(\frac{\pi}{4}\right)$ (n is multiples of 4)
	5	

Solution	Marks	Remarks
7. $x - (1 + \sin y)^5 = 1$ Differentiate with respect to x : $1 - 5(1 + \sin y)^4 \cos y \frac{dy}{dx} = 0$	1A+2A	1A for $\frac{d}{dx}(x)$ and $\frac{d}{dx}(1)$, 2A for $\frac{d}{dx}[-(1 + \sin y)^5]$
Alternative Solution (1) $x - (1 + \sin y)^5 = 1$ Differentiate with respect to y : $\frac{dx}{dy} - 5(1 + \sin y)^4 \cos y = 0$	1A+2A	1A for $\frac{d}{dy}(x)$ and $\frac{d}{dy}(1)$, 2A for $\frac{d}{dy}[-(1 + \sin y)^5]$
Alternative Solution (2) $\sin y = (x - 1)^{\frac{1}{5}} - 1$ Differentiate with respect to x : $\cos y \frac{dy}{dx} = \frac{1}{5}(x - 1)^{-4/5}$	1A 1A+1A	1A for LHS, 1A for RHS
Put $x = 2, y = 0$: $1 - 5 \frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{1}{5}$ Equation of tangent is $\frac{y - 0}{x - 2} = \frac{1}{5}$ $x - 5y - 2 = 0$	} 1M 1A 5	Accept equivalent forms
8. (a) $\vec{a} \cdot \vec{b} = \vec{a} \vec{b} \cos \theta$ $= 4(3) \cos 60^\circ$ $= 6$ (b) $(\vec{a} + k\vec{b}) \cdot (\vec{a} - 2\vec{b}) = 0$ $\vec{a} \cdot \vec{a} - 2\vec{a} \cdot \vec{b} + k\vec{b} \cdot \vec{a} - 2k\vec{b} \cdot \vec{b} = 0$ $4^2 + (k - 2)(6) - 2k(3)^2 = 0$ $4 - 12k = 0$ $k = \frac{1}{3}$	1A 1A 1M 1A+1M 1A 6	1A For $\vec{a} \cdot \vec{a} = 16$ or $\vec{b} \cdot \vec{b} = 9$ 1M For $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} = \text{Ans. in (a)}$ Omit vector sign in most cases (pp-1) Omit dot product sign more than once (pp-1) $\vec{a}^2, \frac{\vec{a}}{\vec{b}}$ etc. (pp-1)

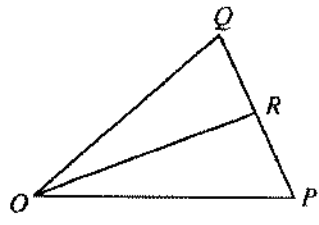
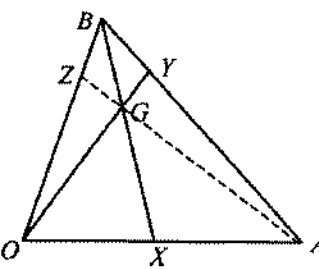
Solution	Marks	Remarks
<p>9. $p = \frac{x^2 + 2x + 8}{x - 2}$</p> $px - 2p = x^2 + 2x + 8$ $x^2 + (2 - p)x + (8 + 2p) = 0$ <p>Since x is real, discriminant $= (2 - p)^2 - 4(8 + 2p) \geq 0$</p> $p^2 - 12p - 28 \geq 0$ $(p + 2)(p - 14) \geq 0$ $p \geq 14 \text{ or } p \leq -2.$ $\therefore \frac{x^2 + 2x + 8}{x - 2} \geq 14 \text{ or } \frac{x^2 + 2x + 8}{x - 2} \leq -2$ <div style="border: 1px dashed black; padding: 2px; display: inline-block;"> $p \geq 14 \text{ or } p \geq 2$ </div> $\therefore \left \frac{x^2 + 2x + 8}{x - 2} \right \geq 2$	<p>1M+1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <hr/> <p>6</p>	<p>1M for transforming into an quadratic equation in x</p> <p>$\Delta > 0$ – no mark</p>

Solution	Marks	Remarks
<p><u>Alternative Solution (1)</u></p> $\begin{cases} x+y-5=0 \\ 2x-3y=0 \end{cases}$ <p>Solve the 2 equations : $x=3, y=2$.</p> <p>Let the slope of the line be m.</p> <p>Equation of a line passing through A is $\frac{y-2}{x-3} = m$ $mx - y + (2 - 3m) = 0$</p> <p>Distance from the origin to the line</p> $= \frac{ m(0) - 0 + (2 - 3m) }{\sqrt{m^2 + 1}}$ $= \frac{ 2 - 3m }{\sqrt{m^2 + 1}}$ $\frac{ 2 - 3m }{\sqrt{m^2 + 1}} = 2$ $(2 - 3m)^2 = 4(m^2 + 1)$ $9m^2 - 12m + 4 = 4m^2 + 4$ $5m^2 - 12m = 0$ $m = 0 \text{ or } \frac{12}{5}$ <p>The equations of the two lines are $12x - 5y - 26 = 0$ and $y - 2 = 0$.</p>	<p>1M</p> <p>1M</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>1A</p>	<p>Omitting absolute value sign (pp-1)</p> <p>Accept equivalent forms</p>
<p><u>Alternative Solution (2)</u></p> <p>$x=3, y=2$</p> <p>Let the equation of the line be $Ax + By + C = 0$.</p> $\begin{cases} 3A + 2B + C = 0 \\ \frac{ C }{\sqrt{A^2 + B^2}} = 2 \\ \frac{ -3A - 2B }{\sqrt{A^2 + B^2}} = 2 \end{cases}$ $9A^2 + 12AB + 4B^2 = 4(A^2 + B^2)$ $A(5A + 12B) = 0$ $A = 0 \text{ or } A = -\frac{12B}{5}$ <p>∴</p> <p>The equation of the two lines are $12x - 5y - 26 = 0$ and $y - 2 = 0$.</p>	<p>1M</p> <p>1M</p> <p>1M</p> <p>1A</p> <p>1A+1A</p>	<p>Same as above</p> <p>For a correct equation in 2 unknowns</p>
	<p>6</p>	

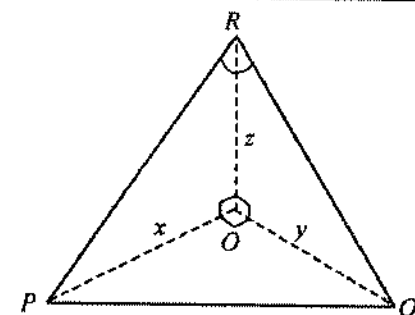
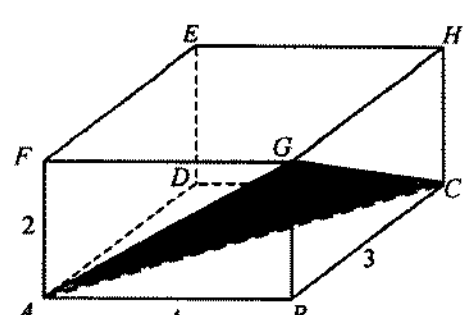
Solution	Marks	Remarks
11. $\frac{y}{y-2} \leq 2$ $\frac{y}{y-2} - 2 \leq 0$ $\frac{-y+4}{y-2} \leq 0$ $\frac{y-4}{y-2} \geq 0$ $(y-4)(y-2) \geq 0$, where $y \neq 2$ $y \geq 4$ or $y < 2$	1M 1A 1A	 $y \geq 4$ or $y < 2$ - no mark
Alternative Solution (1) $\frac{y}{y-2} \leq 2$ Multiplying both sides by $(y-2)^2$, $y(y-2) \leq 2(y-2)^2$ where $y \neq 2$ $y^2 - 2y \leq 2y^2 - 8y + 8$ $y^2 - 6y + 8 \geq 0$ $(y-2)(y-4) \geq 0$ $y \geq 4$ or $y < 2$	1M 1A 1A	
Alternative Solution (2) Consider the two cases : (i) $y > 2$, (ii) $y < 2$. Case 1 : $y > 2$ The inequality becomes $y \leq 2(y-2)$ $y \geq 4$ Since $y > 2$, $\therefore y \geq 4$. Case 2 : $y < 2$ The inequality becomes $y \geq 2(y-2)$ $y \leq 4$ Since $y < 2$, $\therefore y < 2$. Combining the two cases, $y \geq 4$ or $y < 2$.	1M 1A 1A	Accepting including '=' sign Both were correct
$\frac{2^x}{2^x-2} \leq 2$ Put $2^x = y$. Using the result in (a), $2^x \geq 4$ or $2^x < 2$ $x \geq 2$ or $x < 1$	1M $\frac{1A+1A}{6}$	

Solution	Marks	Remarks
12. For $n = 1$, LHS = $1(2) = 2$		
$\text{RHS} = \frac{1}{3}(1)(1+1)(1+2) = 2 = \text{LHS}$	1	
\therefore the statement is true for $n = 1$.		
Assume $1 \times 2 + 2 \times 3 + 3 \times 4 + \dots + k(k+1) = \frac{1}{3}k(k+1)(k+2)$	1	
$\text{for some positive integer } k.$		
Then $1 \times 2 + 2 \times 3 + 3 \times 4 + \dots + k(k+1) + (k+1)[(k+1)+1]$		
$= \frac{1}{3}k(k+1)(k+2) + (k+1)(k+2)$	1	
$= \frac{1}{3}(k+1)(k+2)(k+3)$	1	
$\text{The statement is also true for } n = k + 1 \text{ if it is true for } n = k.$		
$\text{By the principle of mathematical induction,}$		
$\text{the statement is true for all positive integers } n.$	1	Not awarded if any one of the above marks was withheld
$1 \times 3 + 2 \times 4 + 3 \times 5 + \dots + 50 \times 52$		
$= 1 \times (2+1) + 2 \times (3+1) + 3 \times (4+1) + \dots + 50 \times (51+1)$		
$= (1 \times 2 + 2 \times 3 + 3 \times 4 + \dots + 50 \times 51) + (1 + 2 + 3 + \dots + 50)$	1A	
$= \frac{1}{3}(50)(51)(52) + \frac{1}{2}(1+50)(50)$	1M	For using the above result
$= 44200 + 1275$		
$= 45475$	1A	
$\text{Alternative Solution}$		
$1 \times 3 + 2 \times 4 + 3 \times 5 + \dots + 50 \times 52$		
$= (2-1) \times 3 + (3-1) \times 4 + (4-1) \times 5 + \dots + (51-1) \times 52$	1A	
$= (2 \times 3 + 3 \times 4 + 4 \times 5 + \dots + 51 \times 52) - (3 + 4 + 5 + \dots + 52)$	1M	For using the above result
$= \left[\frac{1}{3}(51)(52)(53) - 1 \times 2 \right] - \frac{1}{2}(3+52)(50)$	1A	
$= 46850 - 1375$		
$= 45475$	8	

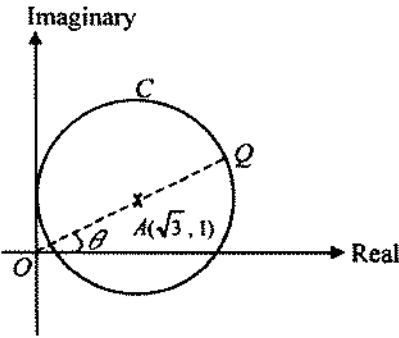
Solution	Marks	Remarks
<p>Alternative Solution</p> <p>Slope of $L = -\frac{1}{m}$</p> <p>Equation of L is</p> $\frac{y+3}{x} = -\frac{1}{m}$ $x + my + 3m = 0$ <p>Substitute $x = \frac{1-4m}{2m^2}$, $y = \frac{1}{2m}$:</p> $\frac{1-4m}{2m^2} + m\left(\frac{1}{2m}\right) + 3m = 0$ $(1-4m) + m^2 + 3(2m^3) = 0$ $6m^3 + m^2 - 4m + 1 = 0$	<p>1M</p> <p>1</p>	$\frac{y - \frac{1}{2m}}{x - \frac{1-4m}{2m^2}} = -\frac{1}{m}$ <p>Substitute $x = 0$, $y = -3$.</p>
$(m+1)(6m^2 - 5m + 1) = 0$ $(m+1)(3m-1)(2m-1) = 0$ $m = -1, \frac{1}{3} \text{ or } \frac{1}{2}$ <p>From (a), $m < \frac{1}{8} \therefore m = -1$</p> <p>The equation of L is</p> $y - \frac{1}{2(-1)} = \frac{-1}{(-1)} \left[x - \frac{1-4(-1)}{2(-1)^2} \right]$ $y = x - 3$	<p>1A</p> <p>1A</p> <p>1M</p> <p><u>1A</u></p> <p><u>6</u></p>	<p>For finding one factor</p> <p>For using (a)</p> $\frac{y - (-3)}{x - 0} = \frac{1}{(-1)}$

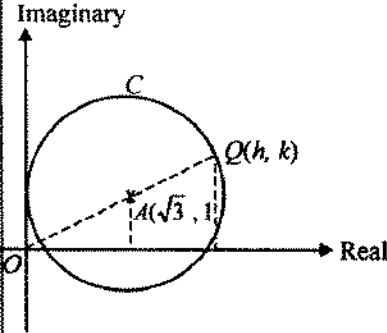
Solution	Marks	Remarks
<p>14. (a) $\overrightarrow{OR} = \frac{s\overrightarrow{OP} + r\overrightarrow{OQ}}{r+s}$</p> <p>If $\overrightarrow{OR} = m\overrightarrow{OP} + n\overrightarrow{OQ}$, comparing with the above expression,</p> $m = \frac{s}{r+s} \text{ and } n = \frac{r}{r+s}$ $m+n = \frac{s}{r+s} + \frac{r}{r+s} = 1$	<p>1A</p> <p>1M</p> <hr/> <p>$\frac{1}{3}$</p>	
<p>(b) (i) $\overrightarrow{OG} = \frac{\frac{1}{2}\vec{a} + 3\vec{b}}{4}$</p> $= \frac{1}{8}\vec{a} + \frac{3}{4}\vec{b}$ <p>(ii) $\overrightarrow{OY} = k\overrightarrow{OG}$</p> $= \frac{k}{8}\vec{a} + \frac{3k}{4}\vec{b}$ <p>Using (a), $\frac{k}{8} + \frac{3k}{4} = 1$</p> $k = \frac{8}{7}$ $\therefore \overrightarrow{OY} = \frac{1}{8}\left(\frac{8}{7}\right)\vec{a} + \frac{3}{4}\left(\frac{8}{7}\right)\vec{b}$ $= \frac{1}{7}\vec{a} + \frac{6}{7}\vec{b}$	<p>1A</p> <p>2M</p> <p>1A</p>	
<p>(iii) (1) From (b) (i), $\overrightarrow{OG} = \frac{1}{8}\vec{a} + \frac{3}{4}\vec{b}$</p> $= \frac{1}{8}\vec{a} + \frac{3}{4h}\overrightarrow{OZ}$ <p>Using (a), $\frac{1}{8} + \frac{3}{4h} = 1$</p> $h = \frac{6}{7}$	<p>1M</p> <p>1M</p> <p>1A</p>	<p>For expressing \vec{b} in terms of h, \overrightarrow{OZ}.</p>

Solution	Marks	Remarks
<p>Alternative Solution (1) Let $\vec{AZ} = \lambda \vec{AG}$. $\vec{OZ} = \vec{OA} + \vec{AZ}$ $= \vec{a} + \lambda \left(\frac{1}{8} \vec{a} + \frac{3}{4} \vec{b} - \vec{a} \right)$ $= \left(1 - \frac{7\lambda}{8} \right) \vec{a} + \frac{3\lambda}{4} \vec{b}$ Since $\vec{OZ} = h\vec{b}$,</p> $\begin{cases} 1 - \frac{7\lambda}{8} = 0 \\ \frac{3\lambda}{4} = h \end{cases}$ $\lambda = \frac{8}{7}$ $\therefore h = \frac{3}{4} \left(\frac{8}{7} \right) = \left(\frac{6}{7} \right)$	<p>1M</p> <p>1M</p> <p>1A</p>	
<p>Alternative Solution (2) $\vec{AG} = \frac{3\vec{AB} + \vec{AX}}{4}$ $= \frac{3}{4} \vec{AB} + \frac{1}{8} \vec{AO}$ Let $\vec{AZ} = \lambda \vec{AG}$. $\vec{AZ} = \frac{3\lambda}{4} \vec{AB} + \frac{\lambda}{8} \vec{AO}$ Using (a), $\frac{3\lambda}{4} + \frac{\lambda}{8} = 1$ $\lambda = \frac{8}{7}$ $\therefore \vec{AZ} = \frac{6}{7} \vec{AB} + \frac{1}{7} \vec{AO}$ $\therefore OZ : ZB = 6 : 1$ i.e. $h = \frac{6}{7}$</p>	<p>1M</p> <p>1M</p> <p>1A</p>	
<p>(2) $\vec{ZY} = \vec{OY} - \vec{OZ}$ $= \frac{1}{7} \vec{a} + \frac{6}{7} \vec{b} - \frac{6}{7} \vec{b}$ $= \frac{1}{7} \vec{a}$ As $\vec{ZY} = \lambda \vec{OA}$, so ZY is parallel to OA</p>	<p>1M</p> <p>1</p>	<p>Omit vector sign in most cases (pp-1)</p>

Solution	Marks	Remarks
<p>15. (a)</p>  <p>(i) $RP = \sqrt{x^2 + z^2}$, $PQ = \sqrt{x^2 + y^2}$, $QR = \sqrt{y^2 + z^2}$</p> $\cos \angle PRQ = \frac{RP^2 + QR^2 - PQ^2}{2(RP)(QR)}$ $= \frac{(x^2 + z^2) + (y^2 + z^2) - (x^2 + y^2)}{2\sqrt{x^2 + z^2}\sqrt{y^2 + z^2}}$ $= \frac{z^2}{\sqrt{x^2 + z^2}\sqrt{y^2 + z^2}}$ <p>(ii) $S_1 = \frac{1}{2}xz$, $S_2 = \frac{1}{2}xy$, $S_3 = \frac{1}{2}yz$</p> $\sin \angle PRQ = \frac{\sqrt{(x^2 + z^2)(y^2 + z^2) - z^4}}{\sqrt{x^2 + z^2}\sqrt{y^2 + z^2}}$ $S_4 = \frac{1}{2}(QR)(RP)\sin \angle PRQ$ $= \frac{1}{2}\sqrt{y^2 + z^2}\sqrt{x^2 + z^2} \frac{\sqrt{(x^2 + z^2)(y^2 + z^2) - z^4}}{\sqrt{x^2 + z^2}\sqrt{y^2 + z^2}}$ $= \frac{1}{2}\sqrt{(x^2 + z^2)(y^2 + z^2) - z^4}$ $S_4^2 = \frac{1}{4}(x^2y^2 + y^2z^2 + z^2x^2)$ $= \left(\frac{xy}{2}\right)^2 + \left(\frac{yz}{2}\right)^2 + \left(\frac{zx}{2}\right)^2$ $= S_1^2 + S_2^2 + S_3^2$	<p>1M</p> <p>1A</p> <p>1M</p> <p>1M</p> <p>1A</p> <hr/> <p>1</p> <hr/> <p>6</p>	
<p>(b)</p>  <p>(i) Volume of $ABCG = \frac{1}{3} \times \text{base area} \times \text{height}$</p> $= \frac{1}{3} \times \left(\frac{1}{2} \times 4 \times 3\right) \times 2$ $= 4$	<p>1A</p>	

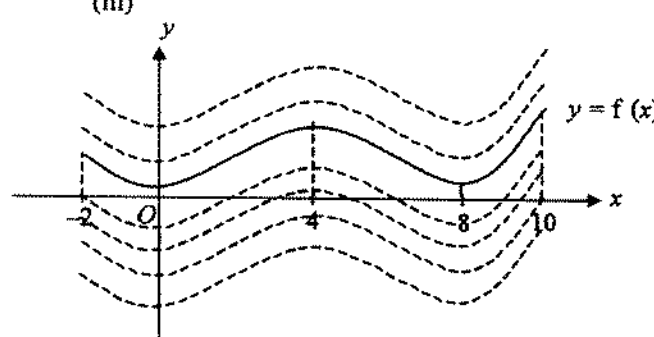
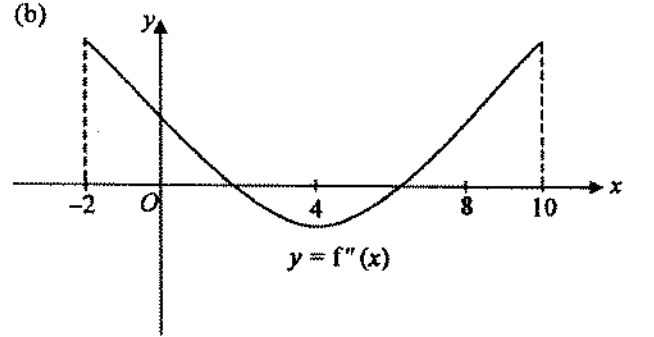
Solution	Marks	Remarks
<p>(ii) Using (a), area of $\triangle GAC = \sqrt{\left(\frac{4 \times 2}{2}\right)^2 + \left(\frac{2 \times 3}{2}\right)^2 + \left(\frac{4 \times 3}{2}\right)^2}$ $= \sqrt{61}$</p> <p>Let h be the perpendicular distance from B to plane GAC. Considering the volume of $ABCG$,</p> $\frac{1}{3}(\sqrt{61})h = 4$ $h = \frac{12}{\sqrt{61}}$ <p>Let θ be the angle between AB and plane GAC.</p> $\sin \theta = \frac{h}{AB}$ $= \frac{12 / \sqrt{61}}{4}$ $\theta = 23^\circ \text{ (correct to the nearest degree)}$	<p>1M</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <hr/> <p>6</p>	<p>For using (a) (ii)</p> <p>For identifying the angle (can be omitted)</p>

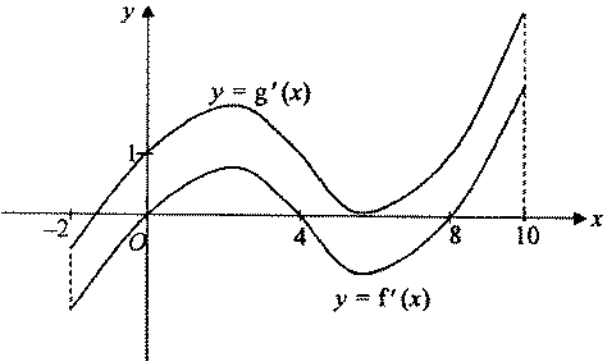
Solution	Marks	Remarks
17. (a) The centre is $(\sqrt{3}, 1)$ (OR $\sqrt{3} + i$). The radius is $\sqrt{3}$.	1A <hr/> 1A <hr/> 2	
(b) The position of Q is shown below :		
	1A	For identifying position of Q
Let A denote the centre of C and θ be the angle between OA and the real axis.		
$OA = \sqrt{(\sqrt{3})^2 + 1^2} = 2$		
$OQ = OA + AQ$		
$= 2 + \sqrt{3}$	1M	For evaluating OQ
$\tan \theta = \frac{1}{\sqrt{3}}$	1M	For evaluating argument
$\theta = \frac{\pi}{6}$		
\therefore the complex number represented by Q is		
$(2 + \sqrt{3}) \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) = \left(\sqrt{3} + \frac{3}{2} \right) + \left(1 + \frac{\sqrt{3}}{2} \right) i$	1M+1A	Accept degrees

Solution	Marks	Remarks
<p>Alternative Solution (1)</p>  <p>$OA = \sqrt{(\sqrt{3})^2 + 1^2} = 2$ $OQ = 2 + \sqrt{3}$ Let the coordinates of Q be (h, k). By similar triangles, $\frac{OA}{OQ} = \frac{\sqrt{3}}{h} = \frac{1}{k}$ $\frac{2}{2 + \sqrt{3}} = \frac{\sqrt{3}}{h}$ and $\frac{2}{2 + \sqrt{3}} = \frac{1}{k}$ $h = \sqrt{3} + \frac{3}{2}$, $k = 1 + \frac{\sqrt{3}}{2}$ \therefore the complex number represented by Q is $(\sqrt{3} + \frac{3}{2}) + (1 + \frac{\sqrt{3}}{2})i$.</p>	<p>1A</p> <p>1M</p> <p>1M+1M</p> <p>1A</p>	<p>For identifying position of Q</p>
<p>Alternative Solution (2)</p> <p>Position of Q</p> <p>Equation of C: $(x - \sqrt{3})^2 + (y - 1)^2 = 3$</p> <p>Equation of OA (OR OQ): $y = \frac{1}{\sqrt{3}}x$</p> $\begin{cases} (x - \sqrt{3})^2 + (y - 1)^2 = 3 \\ y = \frac{1}{\sqrt{3}}x \end{cases}$ <p>$(x - \sqrt{3})^2 + (\frac{1}{\sqrt{3}}x - 1)^2 = 3$</p> <p>$4x^2 - 8\sqrt{3}x + 3 = 0$</p> <p>$x = \frac{8\sqrt{3} \pm \sqrt{144}}{8}$</p> <p>$= \sqrt{3} + \frac{3}{2}$ or $\sqrt{3} - \frac{3}{2}$ (rejected)</p> <p>$y = \frac{1}{\sqrt{3}}(\sqrt{3} + \frac{3}{2})$</p> <p>$= 1 + \frac{\sqrt{3}}{2}$</p> <p>$\therefore$ the complex number represented by Q is $(\sqrt{3} + \frac{3}{2}) + (1 + \frac{\sqrt{3}}{2})i$.</p>	<p>1A</p> <p>1M</p> <p>1M</p> <p>1M</p> <p>1A</p>	<p>Same as above</p> <p>For setting up the equations of C and OA</p> <p>For solving x</p> <p>For finding y</p>

Solution	Marks	Remarks
<p>Alternative Solution (3) Position of Q Since O, A, Q lie on the same line let the complex number represented by Q be $r(\sqrt{3} + i)$. $r(\sqrt{3} + i) - (\sqrt{3} + i) = \sqrt{3}$ $(r-1)(\sqrt{3} + i) = \sqrt{3}$ $(r-1)^2 (4) = 3$ $r = 1 + \frac{\sqrt{3}}{2}$ or $1 - \frac{\sqrt{3}}{2}$ (rejected) \therefore the complex number represented by Q is $(1 + \frac{\sqrt{3}}{2})(\sqrt{3} + i)$.</p>	<p>1A 1M 1M 1M 1A</p>	<p>Same as above For substituting into the equation</p>
<p><u>5</u></p>		
<p>(c) The position of R is shown below (OR is a tangent to C):</p>		
	<p>1A</p>	<p>For identifying position of R</p>
<p>Let S be the point of contact between C and the imaginary axis. $OR = OS = 1$</p>	<p>1M</p>	<p>For evaluating OR</p>
<p>$\angle AOR = \angle AOS = \frac{\pi}{2} - \frac{\pi}{6}$ $= \frac{\pi}{3}$</p>	<p>1M</p>	
<p>Angle between OR and the real axis $= \frac{\pi}{3} - \frac{\pi}{6}$ $= \frac{\pi}{6}$</p>	<p>1A</p>	
<p>\therefore the complex number represented by R is $\cos(-\frac{\pi}{6}) + i \sin(-\frac{\pi}{6}) = \frac{\sqrt{3}}{2} - \frac{1}{2}i$</p>	<p>1A</p>	

Solution	Marks	Remarks
<p>Alternative Solution Position of R Let the equation of OR be $y = mx$.</p> $\begin{cases} (x - \sqrt{3})^2 + (y - 1)^2 = 3 \\ y = mx \end{cases}$ $(x - \sqrt{3})^2 + (mx - 1)^2 = 3$ $(1 + m^2)x^2 - (2m + 2\sqrt{3})x + 1 = 0 \text{ ----- (*)}$ <p>Since OR is a tangent to C,</p> $\Delta = (2m + 2\sqrt{3})^2 - 4(1 + m^2) = 0$ $4m^2 + 8\sqrt{3}m + 12 - 4 - 4m^2 = 0$ $m = -\frac{1}{\sqrt{3}}$ $\tan \alpha = -\frac{1}{\sqrt{3}}$ $\alpha = -\frac{\pi}{6}$ <p>$OR = OS = 1$</p> <p>\therefore the complex number represented by R is</p> $\cos\left(-\frac{\pi}{6}\right) + i \sin\left(-\frac{\pi}{6}\right).$	<p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p>	<p>Same as above</p> <p>OR Substitute $m = -\frac{1}{\sqrt{3}}$ into (*)</p> $\frac{4}{3}x^2 - \frac{4\sqrt{3}}{3}x + 1 = 0 \quad \text{1M}$ $x = \frac{\sqrt{3}}{2}, y = -\frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{2}\right) = -\frac{1}{2}$
	<p>5</p>	

Solution	Marks	Remarks
<p>18. (a) (i) $f(x)$ is increasing when $0 \leq x \leq 4$ and $8 \leq x \leq 10$.</p> <p>(ii) From Figure 5 (a), $f'(x) = 0$ at $x = 0, 4$ and 8. As $f'(x)$ changes from -ve to +ve as x increases through 0 and 8, $f(x)$ attains a minimum at $x = 0$ and 8. As $f'(x)$ changes from +ve to -ve as x increases through 4, $f(x)$ attains a maximum at $x = 4$.</p> <p>(iii)</p> 	<p>2A</p> <p>1A+1</p> <p>1A+1A</p> <hr/> <p>6</p>	<p>OR $0 < x < 4$ or $8 < x < 10$ Any one correct - 1A All correct - 2A</p> <p>1A for the 3 x coordinates 1 for the explanation</p> <p>1A for the 3 turning points 1A for shape</p>
<p>(b)</p> 	<p>1A+1A</p> <hr/> <p>2</p>	<p>1A for $f''(4) < 0$ and $f''(0) \& f''(8) > 0$ 1A for shape</p>

Solution	Marks	Remarks
<p>(c) (i) $g(x) = f(x) + x.$ $g'(x) = f'(x) + 1. \dots\dots (1)$</p>  <p>(ii) Differentiate (1) with respect to x : $g''(x) = f''(x).$ Since $f''(x) = g''(x)$, the graphs of $y = f''(x)$ and $y = g''(x)$ are identical. The student is incorrect.</p>	<p>IM+1A</p> <hr/> <p>2A</p> <hr/> <p>4</p>	<p>1M for translating vertically upwards</p>