香港考試局
HONG KONG EXAMINATIONS AUTHORITY

一九九九年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1999

附加數學 試卷一
ADDITIONAL MATHEMATICS PAPER I

本評卷參考乃考試局專爲今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視爲標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations Authority for markers' reference. The Examinations Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Examinations Authority is counting on the co-operation of markers/teachers in this regard.

考試結束後，各科評卷參考將存放於教師中心，供教師參閱。
After the examinations, marking schemes will be available for reference at the Teachers' Centres.

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GENERAL INSTRUCTIONS TO MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates would use alternative methods not specified in the marking scheme. Markers should be patient in marking these alternative solutions. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method is specified in the question.

2. In the marking scheme, marks are classified as follows :

   'M' marks – awarded for knowing a correct method of solution and attempting to apply it;

   'A' marks – awarded for the accuracy of the answer;

   Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. The symbol (pp–1) should be used to denote marks deducted for poor presentation (p.p.). Note the following points:

   (a) At most deduct 1 mark for p.p. in each question, up to a maximum of 3 marks for the whole paper.

   (b) For similar p.p., deduct only 1 mark for the first time that it occurs, i.e. do not penalise candidates twice in the whole paper for the same p.p.

   (c) In any case, do not deduct any marks for p.p. in those steps where candidates could not score any marks.

   (d) Note : if the final answers are not expressed in the simplest form, deduct 1 mark for p.p.

   (e) Some cases in which marks should be deducted for p.p. are specified in the marking scheme. However, the lists are by no means exhaustive. Markers should exercise their professional judgement to give p.p.s in other situations.

5. The symbol (u–1) should be used to denote marks deducted for wrong/no units in the final answers (if applicable). Note the following points:

   (a) At most deduct 1 mark for wrong/no units for the whole paper.

   (b) Do not deduct any marks for wrong/no units in case candidate's answer was already wrong.

6. Marks entered in the Page Total Box should be the net total score on that page.

7. In the Marking Scheme, steps which can be omitted are enclosed by dotted rectangles [dotted rectangle],

   whereas alternative answers are enclosed by solid rectangles [solid rectangle].

8. Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.

9. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

10. Unless the form of answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they were correct.

Solution	Marks	Remarks
1. (a) $\frac{d}{dx}\sin(x^2+1)$		
$= \frac{d}{d(x^2+1)}\sin(x^2+1)\,\frac{d}{dx}(x^2+1)$	1M	For chain rule (can be omitted)
$= 2x\cos(x^2+1)$	1A	
(b) $\frac{d}{dx}\frac{\sin(x^2+1)}{x}$		
$= \dfrac{x\frac{d}{dx}\sin(x^2+1) - \sin(x^2+1)\frac{d}{dx}(x)}{x^2}$	1M	For quotient rule (can be omitted)
$= \dfrac{2x^2\cos(x^2+1) - \sin(x^2+1)}{x^2}$	1A	
Alternative solution		
$\frac{d}{dx}\frac{\sin(x^2+1)}{x}$		
$= \frac{1}{x}\frac{d}{dx}\sin(x^2+1) + \sin(x^2+1)\frac{d}{dx}\frac{1}{x}$	1M	For product rule (can be omitted)
$= \frac{1}{x}2x\cos(x^2+1) + \sin(x^2+1)(-\frac{1}{x^2})$		
$= 2\cos(x^2+1) - \frac{1}{x^2}\sin(x^2+1)$	1A	
	4	

Solution	Marks	Remarks
2. $\frac{x}{x-1}>2$		
$\frac{x}{x-1}-2>0$	1M	
$\frac{-x+2}{x-1}>0$	1A	
$1<x<2$	2A	
Alternative solution (1)		
Consider the following cases : (i) $x>1$, (ii) $x<1$	1M	Awarded even if equality sign is included
Case 1 : $x>1$		
$x>2(x-1)$		
$x<2$		
Since $x>1$, $\therefore$ $1<x<2$.	→ 1A	
Case 2 : $x<1$		
$x<2(x-1)$		
$x>2$		
Since $x<1$, $\therefore$ there is no solution.		
Combining the 2 cases, $1<x<2$.	2A	
Alternative solution (2)		
$\frac{x}{x-1}>2$		
$x(x-1)>2(x-1)^2$	1M	
$x^2-3x+2<0$		
$(x-1)(x-2)<0$	1A	(can be omitted)
$1<x<2$	2A	
	4	

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Solution	Marks	Remarks
3. $\lvert x-3\rvert = \lvert x^2-4x+3\rvert$		
$\lvert x-3\rvert = \lvert x-1\rvert\,\lvert x-3\rvert$	2A	
$\lvert x-3\rvert = 0$ or $\lvert x-1\rvert = 1$		
$x=3$ $\quad$ $x-1=\pm 1$		
$x=0$ or 2		
$\therefore x=0, 2$ or 3.	1A+1A+1A	Can be awarded even if the above '2A' were not given.
Alternative solutions		
(1) $x-3=x^2-4x+3$ or $x-3=-(x^2-4x+3)$	2A	
$x-3=(x-1)(x-3)$ $\quad$ $(x-3)=-(x-1)(x-3)$		
$x=3$ or $x-1=1$ $\quad$ $x=3$ or $x-1=-1$		
$x=2$ $\quad$ $x=0$		
$\therefore x=0, 2$ or 3.	1A+1A+1A	
(2) $(x-3)^2=(x^2-4x+3)^2$	2A	
$(x-3)^2[(x-1)^2-1]=0$		
$(x-3)^2(x-2)(x)=0$		
$x=0, 2$ or 3.	1A+1A+1A	
(3) Consider the following cases : $x\ge 3, 1<x<3, x\le 1$.	1A	
Case 1 : $x\ge 3$		
$x-3=x^2-4x+3$		
$x^2-5x+6=0$		
$x=2$ (rejected) or 3		
Case 2 : $1<x<3$		
$-(x-3)=-(x^2-4x+3)$		
$x=2$ or 3 (rejected)		
Case 3 : $x\le 1$		
$-(x-3)=x^2-4x+3$	1A	Awarded only if the 3 equations were all correct.
$x^2-3x=0$		
$x=0$ or 3 (rejected)		
Combining the 3 cases, $x=0, 2$ or 3.	1A+1A+1A	
	5	
4. (a) Discriminant $=4(k-4)^2-4(2)(k)$	1M	
$=4k^2-40k+64$	1A	
(b) Discriminant <0		
$4k^2-40k+64<0$	2M	1M for $\Delta\le 0$
$k^2-10k+16<0$		
$(k-2)(k-8)<0$		
$2<k<8$.	1A	
	5	

Solution	Marks	Remarks
5. $1+i=\sqrt{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})$	1A+1A	1A for modulus, 1A for argument (Accept degrees, Accept other equivalent values for argument)
$(1+i)^{\frac{1}{3}}=[\sqrt{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})]^{\frac{1}{3}}$		
$=(\sqrt{2})^{\frac{1}{3}}(\cos\frac{2k\pi+\frac{\pi}{4}}{3}+i\sin\frac{2k\pi+\frac{\pi}{4}}{3})$ $k=-1,0,1$	1M+1A	1M for De Moivre's Theorem, 1A for modulus
$=2^{\frac{1}{6}}[\cos(\frac{2k\pi}{3}+\frac{\pi}{12})+i\sin(\frac{2k\pi}{3}+\frac{\pi}{12})]$ $k=-1,0,1$	1A	1A for argument (accept degrees) (accept k = any 3 consecutive integers)
OR $=2^{\frac{1}{6}}(\cos\frac{-7\pi}{12}+i\sin\frac{-7\pi}{12}),\ 2^{\frac{1}{6}}(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}),\ 2^{\frac{1}{6}}(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4})$.		
	5	
6. (a) $3x^2-xy-y^2-a^2=0$		
$6x-y-x\frac{dy}{dx}-2y\frac{dy}{dx}=0$	1A+1A	1A for $\frac{d}{dx}(xy)$, 1A for other terms
$\frac{dy}{dx}=\frac{6x-y}{x+2y}$		
Substitute $x=a$, $y=a$:		
$\frac{dy}{dx}=\frac{6a-a}{a+2a}$	1M	} (can be awarded in (b))
$=\frac{5}{3}$	1A	
(b) Equation of tangent is		
$\frac{y-a}{x-a}=\frac{5}{3}$	1M	
$5x-3y-2a=0$.	1A	Accept equivalent forms
	6	

Solution	Marks	Remarks
7. (a) $\lvert\vec{a}\rvert=\sqrt{3^2+4^2}=5$	1A	
(b) $\vec{a}\cdot\vec{b}=\lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos 60°$		
$=5(4)\cos 60°$	1M	
$=10$	1A	
(c) $(m\vec{a}+\vec{b})\cdot\vec{b}=0$	1M	
$m\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{b}=0$	1M	For distribution law only
$m(10)+4^2=0$		
$m=-1.6$	1A	Omit vectors sign in most cases (pp–1) Omit dot product sign more than once (pp–1)
	6	
8. (a) (i) $\tan\theta=\frac{h+40}{55}$		
$=\frac{20t-5t^2+40}{55}$		
$\tan\theta=\frac{4t-t^2+8}{11}$ ------(1)	1A	
(ii) At $t=3$, $\tan\theta=\frac{4(3)-3^2+8}{11}=1$		
$\theta=\frac{\pi}{4}$ (OR $=45°$)	1A	
(b) Differentiate (1) with respect to t:		
$\sec^2\theta\frac{d\theta}{dt}=\frac{4-2t}{11}$	1M+1A+1A	1M for chain rule, 1A for LHS, 1A for RHS
At $t=3$, $\sec^2\frac{\pi}{4}\frac{d\theta}{dt}=\frac{4-2(3)}{11}$	1M	For substituting t and θ
$2\frac{d\theta}{dt}=\frac{-2}{11}$		
$\frac{d\theta}{dt}=\frac{-1}{11}$		
$\therefore$ the rate of change of θ with respect to time at $t=3$ is $\frac{-1}{11}s^{-1}$.	1A	Omit/wrong unit ($u-1$)
(OR θ decreases at a rate of $\frac{1}{11}s^{-1}$ at $t=3$.)		
	7	

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Solution	Marks	Remarks
9. (a) (i) $f'(x) = 2a\cos 2x - b\sin x$	1A	
(ii) From figure 2 (a), $f'(0) = -4$.		OR $f'(\pi) = -4$
$2a\cos 0 - b\sin 0 = -4$	1M	
$2a = -4$		
$a = -2$	1	
$f'(\frac{\pi}{6}) = 0$ $\quad 2(-2)\cos\frac{\pi}{3} - b\sin\frac{\pi}{6} = 0$		OR $f'(\frac{5\pi}{6}) = 0$
$b = -4$	1	
$\therefore f(x) = -2\sin 2x - 4\cos x$		
	4	
(b) (i) $f(0) = -4$ $\therefore$ the y-intercept is -4.	1A	(pp−1) for $(0, -4)$
Put $f(x) = 0$: $-2\sin 2x - 4\cos x = 0$	1M	
$-4\sin x\cos x - 4\cos x = 0$	1A	
$-4\cos x(1 + \sin x) = 0$		
$\cos x = 0$ or $\sin x = -1$ (rejected)		
$x = \frac{\pi}{2}$	1A	No mark for $x = 90°$
$\therefore$ the x-intercept is $\frac{\pi}{2}$.		(pp−1) for $(\frac{\pi}{2}, 0)$
(ii) From Figure 2 (a), $f'(x) = 0$ when $x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$.		
As $f'(x)$ changes from −ve to +ve as x increases through $\frac{\pi}{6}$, so $(\frac{\pi}{6}, -3\sqrt{3})$ is a minimum point.	1A+1M	Withhold 1M if explanation was omitted
As $f'(x)$ changes from +ve to −ve as x increases through $\frac{5\pi}{6}$, so $(\frac{5\pi}{6}, 3\sqrt{3})$ is a maximum point.	1A	
<u>Alternative solution</u>		
$f'(x) = 0$ at $x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$.		
$f''(x) = 4\cos x + 8\sin 2x$		
$f''(\frac{\pi}{6})$ $= 6\sqrt{3}$ > 0.		
$\therefore (\frac{\pi}{6}, -3\sqrt{3})$ is a minimum point.	1M+1A	Withhold 1M if checking was omitted
$f''(\frac{5\pi}{6})$ $= -6\sqrt{3}$ < 0		
$\therefore (\frac{5\pi}{6}, 3\sqrt{3})$ is a maximum point.	1A	
	7	

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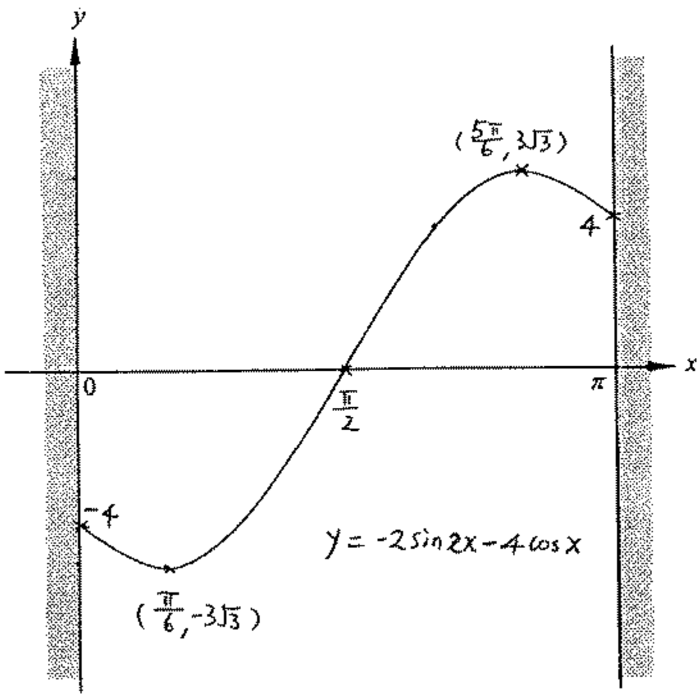
Solution	Marks	Remarks
(c)		(Awarded even if checking was omitted in (b))
	1A	For shape
	1A	For x-intercepts and turning points
	1A	For end-points
	3	
(d) $6-3\sqrt{3} \le g(x) \le 6+3\sqrt{3}$	1A+1A	1A for LHS, 1A for RHS
		Award 1A for $6-3\sqrt{3} < g(x) < 6+3\sqrt{3}$
	2	

Solution	Marks	Remarks
10. (a) $\overrightarrow{OC}=\dfrac{7\vec{a}+8\vec{b}}{15}$	1A	
$\overrightarrow{AD}=\overrightarrow{OD}-\overrightarrow{OA}$	1M	Accept other correct methods
$=\dfrac{16}{21}\vec{b}-\vec{a}$	1A+1A	1A for $\overrightarrow{OD}=\dfrac{16}{21}\vec{b}$
Alternative solution		
$\overrightarrow{AD}=\dfrac{16\overrightarrow{AB}+5\overrightarrow{AO}}{21}$	1M	
$=\dfrac{16(\vec{b}-\vec{a})+5(-\vec{a})}{21}$	1A	1A for $\overrightarrow{AB}=\vec{b}-\vec{a}$
$=\dfrac{16}{21}\vec{b}-\vec{a}$	1A	
	4	
(b) (i) $\overrightarrow{OE}=r\overrightarrow{OC}$		
$=\dfrac{7r}{15}\vec{a}+\dfrac{8r}{15}\vec{b}$	1A	
$=\dfrac{r}{15}(7\vec{a}+8\vec{b})$		
(ii) $\overrightarrow{OE}=\overrightarrow{OA}+\overrightarrow{AE}$		OR $=\overrightarrow{OD}+\overrightarrow{DE}$
$=\vec{a}+k\overrightarrow{AD}$		$=\dfrac{16}{21}\vec{b}+(1-k)\overrightarrow{DA}$
$=\vec{a}+k(\dfrac{16\vec{b}}{21}-\vec{a})$	1M	$=\dfrac{16}{21}\vec{b}+(1-k)(\vec{a}-\dfrac{16}{21}\vec{b})$
$=(1-k)\vec{a}+\dfrac{16k}{21}\vec{b}$	1A	$=(1-k)\vec{a}+\dfrac{16k}{21}\vec{b}$
Alternative solution		
$\overrightarrow{OE}=\dfrac{(1-k)\overrightarrow{OA}+k\overrightarrow{OD}}{(1-k)+k}$	1M	
$=(1-k)\vec{a}+\dfrac{16k}{21}\vec{b}$	1A	
Comparing the two expressions :		
$\begin{cases}\dfrac{7r}{15}=1-k & \text{------(1)}\\ \dfrac{8r}{15}=\dfrac{16}{21}k & \text{------(2)}\end{cases}$	} 1M	For comparing coefficients
(1) ÷ (2) $\dfrac{7}{8}=\dfrac{21(1-k)}{16k}$		
$14k=21-21k$		
$k=\dfrac{3}{5}$	1	

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Solution	Marks	Remarks
Substitute $k=\frac{3}{5}$ into (1) : $\frac{7}{15}r=1-\frac{3}{5}$		
$r=\frac{6}{7}$	1	
$\therefore k=\frac{3}{5}$ and $r=\frac{6}{7}$.		
	6	
(c) (i) Let $EC=x$.		Let $EC=1$ etc. (pp–1)
Since $EC:ED=1:2$, $ED=2x$.		
From (b), $\overrightarrow{OE}=\frac{6}{7}\overrightarrow{OC}$.		
$\therefore EO:EC=6:1$, i.e. $EO=6x$.		
From (b), $\overrightarrow{AE}=\frac{3}{5}\overrightarrow{AD}$	}1M+1A	1A for $EO:EC=6:1$ or $EA:ED=3:2$ etc.
$\therefore EA:ED=3:2$, i.e. $EA=3x$.		
$\therefore EA:EO=3x : 6x$		
$=1 : 2$.	1A	
(ii) In ΔEAC and ΔEOD,		
$\angle AEC=\angle OED$		1A for naming a pair of similar Δs
From (b), $\frac{EA}{EO}=\frac{1}{2}=\frac{EC}{ED}$	1A+1	1 for completing the proof
$\therefore \Delta EAC \sim \Delta EOD$.		
$\angle EAC=\angle EOD$ (Corr ∠s of similar Δs)	}1	
$\therefore OACD$ is a cyclic quadrilateral. (Converse of ∠s in the same segment)		Omit vector sign in most cases (pp–1)
	6	

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Solution	Marks	Remarks
11. (a) $z_1 = 2(\cos 60° + i\sin 60°)$	1A	
$= 1+\sqrt{3}i$	1A	
$z_3 = (\sqrt{3}i)\, z_1$		
$= \sqrt{3}i(1+\sqrt{3}i)$		
$= -3+\sqrt{3}i$	1A	
<u>Alternative solution</u>		
$OC = 2\sqrt{3}$		
$\arg(z_3) = 60° + 90° = 150°$		
$z_3 = 2\sqrt{3}(\cos 150° + i\sin 150)$		
$= -3+\sqrt{3}i$	1A	
	3	
(b) $\frac{z_2}{z_1} = \frac{z_1+z_3}{z_1}$	1M	For $z_2 = z_1 + z_3$
$= 1+(\frac{z_3}{z_1})$	1A	
$= 1+\sqrt{3}i \quad (\because\ z_3 = (\sqrt{3}i)z_1)$	1	
<u>Alternative solution</u>		
$z_2 = z_1 + z_3$	1M	
$= (1+\sqrt{3}i)+(-3+\sqrt{3}i)$		
$= -2+2\sqrt{3}i$	1A	
$\frac{z_2}{z_1} = \frac{-2+2\sqrt{3}i}{1+\sqrt{3}i}(\frac{1-\sqrt{3}i}{1-\sqrt{3}i})$		
$= \frac{-2+2\sqrt{3}i+2\sqrt{3}i+6}{4}$		
$= 1+\sqrt{3}i$	1	
$\angle AOB = \arg(\frac{z_2}{z_1})$	1M	
$= \arg(1+\sqrt{3}i)$		
$= 60°$	1A	
<u>Alternative solution (1)</u>		
$\arg(z_2) = 180° + \tan^{-1}(\frac{2\sqrt{3}}{-2})$		
$= 120°$		
$\angle AOB = \arg(z_2) - \arg(z_1)$	1M	
$= 120° - 60°$		
$= 60°$	1A	
<u>Alternative solution (2)</u>		
$\tan\angle AOB = \frac{AB}{OA}$	1M	
$= \frac{2\sqrt{3}}{2}$		
$= \sqrt{3}$		
$\therefore\ \angle AOB = 60°$	1A	
	5	

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Solution	Marks	Remarks
(c) (i) $z_3 = 2\sqrt{3}(\cos 150° + i\sin 150°)$		
$\arg(z_3) = 150°$	1A	can be awarded in (ii)
Let u be the complex number represented by E.		
$u = wz_3$ $= (\cos\theta + i\sin\theta)2\sqrt{3}(\cos 150° + i\sin 150°)$ $= 2\sqrt{3}[\cos(150° + \theta) + i\sin(150° + \theta)]$		
$\arg(u) = 150° + \theta$	1M	
$\bar{z}_3 = 2\sqrt{3}[\cos(-150°) + i\sin(-150°)]$		
$\arg(\bar{z}_3) = -150°$	1A	(OR = 210°)
If E represents the complex number $\bar{z}_3$,		
$150° + \theta = -150° + 360°$	1M	OR $150° + \theta = 210°$
$\theta = 60°$	1A	
Alternative solution $wz_3 = \bar{z}_3$		
$w(-3 + \sqrt{3}i) = -3 - \sqrt{3}i$	1M	
$w = \dfrac{-3 - \sqrt{3}i}{-3 + \sqrt{3}i}\left(\dfrac{-3 - \sqrt{3}i}{-3 - \sqrt{3}i}\right)$		
$\cos\theta + i\sin\theta = \dfrac{6 + 6\sqrt{3}i}{12}$		
$\cos\theta = \dfrac{1}{2}$ and $\sin\theta = \dfrac{\sqrt{3}}{2}$ (OR $\tan\theta = \sqrt{3}$)	1A	
$\theta = 60°$	1A	
(ii) If E, O and A lie on a straight line,		
$150° + \theta = 60° + 360k°$ or $150° + \theta = -120° + 360k°$	1M	Awarded if either one was correct
OR $150° + \theta = 60° + 360°$ or $150° + \theta = -120° + 360°$ or $= 60° + 180°$		
$\theta = 270°$ or $90°$.	1A+1A 8	

Solution	Marks	Remarks
12. (a) S = Area of $ABCD$ – Area of ΔABE – Area of ΔCEF – Area of ΔADF		
$=2(2k)-\frac{1}{2}(2)(2x)-\frac{1}{2}(x)(2k-2x)-\frac{1}{2}(2k)(2-x)$	1M+1A	
$=4k-2x-(kx-x^2)-(2k-kx)$		
$=x^2-2x+2k$	1	
	3	
(b) (i) As E lies on BC, so $0\le 2x\le 2k$		
$0\le x\le\frac{3}{2}$.	1A	
As F lies on CD, so $0\le x\le 2$.	}1	
Combining the two inequalities, $0\le x\le\frac{3}{2}$.		
(ii) $S=x^2-2x+2k$		
$=x^2-2x+3$		
$S=(x-1)^2+2$	1M+1A	1M for method of completing squares
As $x=1$ lies in the range of possible value of $x\,(0\le x\le\frac{3}{2})$,		
$\therefore$ the least value of $S=2$, which occurs when $x=1$.	1A+1A	$S=2\text{ cm}^2, x=1\text{ cm}$ $(u-1)$
Alternative solution		
$S=x^2-2x+3$		
$\frac{dS}{dx}=2x-2$		
$\frac{dS}{dx}=0$ when $x=1$.	1A	
$\frac{d^2S}{dx^2}=2>0$ $\therefore S$ is a minimum at $x=1$.	1M	For checking
As $x=1$ lies in the range of possible values of x,		
$\therefore$ the least value of $S=1^2-2(1)+3$		
$=2$ which occurs when $x=1$.	1A+1A	
(iii) Since $S=x^2-2x+3$ is a parabola and there is only a minimum in the range $0\le x\le\frac{3}{2}$, so greatest value of S occurs at the end points.	1M	(can be omitted)
At $x=0, S=3$.	1M	For evaluating the end-values
At $x=\frac{3}{2}$, $S=(\frac{3}{2})^2-2(\frac{3}{2})+3=\frac{9}{4}$.		
$\therefore$ the greatest value of S is 3.	1A	
	9	

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Solution	Marks	Remarks
(c) (i) Put $k=\frac{3}{8}$, $S=x^2-2x+\frac{3}{4}$.		
The range of possible values of x is $0\le x\le\frac{3}{8}$.	1A	(can be awarded in (ii))
As $x=1$ does not lie in the above interval, the least value of S will not happen when $x=1$. $\therefore$ the student is incorrect.	} 1M+1	
<u>Alternative solution</u> Put $x=1$: $S=1^2-2(1)+\frac{3}{4}=-\frac{1}{4}$	1M	
As $S<0$ at $x=1$, so the least value of S will not happen when $x=1$.	1	
(ii) As S is monotonic decreasing on $0\le x\le\frac{3}{8}$, least value of S occurs when $x=\frac{3}{8}$. $\therefore$ least value of $S=(\frac{3}{8})^2-2(\frac{3}{8})+\frac{3}{4}$		
$=\frac{9}{64}$	1A	
<u>Alternative solution</u> (i) Put $k=\frac{3}{8}$, $S=x^2-2x+\frac{3}{4}$.		
The range of possible values of x is $0\le x\le\frac{3}{8}$.	1A	
$\frac{dS}{dx}=2x-2$ As $\frac{dS}{dx}<0$ for $0\le x\le\frac{3}{8}$, the least value of S occurs when $x=\frac{3}{8}$. (S is monotonic decreasing on $0\le x\le\frac{3}{8}$.) $\therefore$ the student is incorrect.	} 1M+1	
(ii) Least value of $S=(\frac{3}{8})^2-2(\frac{3}{8})+\frac{3}{4}$		
$=\frac{9}{64}$	1A	
	4	

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Solution	Marks	Remarks
13. (a) $V = \pi x^2 h$		
$h = \dfrac{V}{\pi x^2}$	1A	
$C = (2\pi x h) + k(\pi x^2)2$	1A	
$= 2\pi x(\dfrac{V}{\pi x^2}) + 2\pi x^2 k$		
$= \dfrac{2V}{x} + 2\pi k x^2$	1	
	3	
(b) $\dfrac{dC}{dx} = -\dfrac{2V}{x^2} + 4\pi k x$	1A	
$\dfrac{dC}{dx} = 0 \quad -\dfrac{2V}{x^2} + 4\pi k x = 0$		
$x^3 = \dfrac{V}{2\pi k}$	1A	
$\dfrac{d^2C}{dx^2} = \dfrac{4V}{x^3} + 4\pi k$	1A	
Put $x^3 = (\dfrac{V}{2\pi k})$: $\dfrac{d^2C}{dx^2}$ $= 12\pi k$ > 0.	1M	For checking
$\therefore$ C is a minimum.		OR $\dfrac{d^2C}{dx^2} > 0$ for all x.
Alternative solution		
$\dfrac{dC}{dx} = \dfrac{4\pi k}{x^2}(x^3 - \dfrac{V}{2\pi k})$	1A	
When $x > (\dfrac{V}{2\pi k})^{\frac{1}{3}}$, $\dfrac{dC}{dx} > 0$	1M	For checking
When $0 < x < (\dfrac{V}{2\pi k})^{\frac{1}{3}}$, $\dfrac{dC}{dx} < 0$		
$\therefore$ C is a minimum at $x = (\dfrac{V}{2\pi k})^{\frac{1}{3}}$.		
$\dfrac{x}{h} = \dfrac{x}{V/\pi x^2}$	1M	
$= \dfrac{\pi x^3}{V}$		
$= \dfrac{\pi}{V}(\dfrac{V}{2\pi k})$		
$= \dfrac{1}{2k}$	1	
	6	

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Solution	Marks	Remarks
(c) (i) From (b), $x^3 = (\frac{V}{2\pi k})$		
$= (\frac{256\pi}{2\pi(2)})$	1M	For substitution
$= 64$		
$x = 4$	1A	
Since $\frac{x}{h} = \frac{1}{2k}$, $\frac{4}{h} = \frac{1}{2(2)}$		
$h = 16$	1A	
(ii) Since $x^3 = \frac{V}{2\pi k}$, so x decreases when k increases.	1A	
As $h = \frac{V}{\pi x^2}$, so h increases when x decreases.	1A	
∴ the base radius of the can decreases and the height of the can increases.		
	5	
(d) The costs of the curved and plane surfaces remain unchanged.		
From (b), the ratio $\frac{x}{h} = \frac{1}{2k}$ is independent of the volume of the can.	1	
∴ the ratio $\frac{\text{base radius}}{\text{height}}$ of the bigger can should remain identical to that of the smaller can <u>OR</u> need not be twice that of the smaller can in order to minimise the cost. So the worker is incorrect.	1	'Incorrect' without explanation – no mark
	2	

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香港考試局
HONG KONG EXAMINATIONS AUTHORITY

一九九九年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1999

附加數學 試卷二
ADDITIONAL MATHEMATICS PAPER 2

本評卷參考乃考試局專爲今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視爲標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations Authority for markers' reference. The Examinations Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Examinations Authority is counting on the co-operation of markers/teachers in this regard.

考試結束後，各科評卷參考將存放於教師中心，供教師參閱。
After the examinations, marking schemes will be available for reference at the Teachers' Centres.

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GENERAL INSTRUCTIONS TO MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates would use alternative methods not specified in the marking scheme. Markers should be patient in marking these alternative solutions. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method is specified in the question.

2. In the marking scheme, marks are classified as follows :

   'M' marks – awarded for knowing a correct method of solution and attempting to apply it;

   'A' marks – awarded for the accuracy of the answer;

   Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. The symbol (pp–1) should be used to denote marks deducted for poor presentation (p.p.). Note the following points:

   (a) At most deduct 1 mark for p.p. in each question, up to a maximum of 3 marks for the whole paper.

   (b) For similar p.p., deduct only 1 mark for the first time that it occurs, i.e. do not penalise candidates twice in the whole paper for the same p.p.

   (c) In any case, do not deduct any marks for p.p. in those steps where candidates could not score any marks.

   (d) Note : if the final answers are not expressed in the simplest form, deduct 1 mark for p.p.

   (e) Some cases in which marks should be deducted for p.p. are specified in the marking scheme. However, the lists are by no means exhaustive. Markers should exercise their professional judgement to give p.p.s in other situations.

5. The symbol (u–1) should be used to denote marks deducted for wrong/no units in the final answers (if applicable). Note the following points:

   (a) At most deduct 1 mark for wrong/no units for the whole paper.

   (b) Do not deduct any marks for wrong/no units in case candidate's answer was already wrong.

6. Marks entered in the Page Total Box should be the net total score on that page.

7. In the Marking Scheme, steps which can be omitted are enclosed by dotted rectangles ⬚ , whereas alternative answers are enclosed by solid rectangles ☐ .

8. Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.

9. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

10. Unless the form of answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they were correct.

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Solution	Marks	Remarks
1. $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$		
$= \int_0^{\frac{\pi}{2}} \frac{1}{2}(1+\cos 2x) \, dx$	1A	
$= \left[\frac{1}{2}(x + \frac{1}{2}\sin 2x) \right]_0^{\frac{\pi}{2}}$	1A	
$= \frac{\pi}{4}$	1A	
	3	
2. Let $u = x + 2$.	1M	Accept other suitable substitutions
$\int x(x+2)^{99} \, dx$		
$= \int (u-2)\, u^{99} \, du$	1A	Omit du in most cases (pp–1)
$= \int (u^{100} - 2u^{99}) \, du$		
$= \frac{u^{101}}{101} - \frac{u^{100}}{50} + c$ (c is a constant)	1A	Awarded even if c was omitted
$= \frac{(x+2)^{101}}{101} - \frac{(x+2)^{100}}{50} + c$	1A	Withhold this mark if c was omitted
Alternative solution		
$\int x(x+2)^{99} \, dx$		
$= \int x \sum_{i=0}^{99} {}_{99}C_i (2^{99-i})(x^i) \, dx$	1M	For using binomial expansion
$= \sum_{i=0}^{99} {}_{99}C_i (2^{99-i}) \int x^{i+1} \, dx$		
$= \sum_{i=0}^{99} \frac{{}_{99}C_i (2^{99-i})\, x^{i+2}}{(i+2)} + c$	3A	
(OR) $= \frac{x^{101}}{101} + {}_{99}C_1 \frac{x^{100}}{50} + {}_{99}C_2 \frac{4x^{99}}{99} + \ldots + 2^{98}x^2 + c$	3A	Should at least contain first two terms and last term. Deduct 1A for each wrong term, up to zero.
	4	

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Solution	Marks	Remarks
3. (a) The y-intercept of L_1 is $\frac{1}{2}$.	1A	(pp−1) for $(0, \frac{1}{2})$
(b) Distance between L_1 and L_2		
$= \left\lvert \frac{2(0)+2(\frac{1}{2})-13}{\sqrt{2^2+2^2}} \right\rvert$	1M	Accept omitting absolute sign
$= 3\sqrt{2}$.	1A	
Alternative solution (1)		
Distance $= \left\lvert \frac{-13-(-1)}{\sqrt{2^2+2^2}} \right\rvert$	1M	For using the formula $d = \left\lvert \frac{c_1 - c_2}{\sqrt{a^2+b^2}} \right\rvert$
$= 3\sqrt{2}$	1A	
Alternative solution (2)		
y-intercept of $L_2 = \frac{13}{2}$		
Distance $= (\frac{13}{2} - \frac{1}{2})\sin 45°$	1M	
$= 3\sqrt{2}$	1A	
(c) Let the equation of L_3 be $2x+2y+c=0$.		
$\left\lvert \frac{2(0)+2(\frac{1}{2})+c}{\sqrt{2^2+2^2}} \right\rvert = 3\sqrt{2}$	1M	OR $c-(-1) = -1-(-13)$ $c = 11$
$\lvert 1+c \rvert = 12$		
$c = 11$ or -13 (rejected)		
$\therefore$ the equation of L_3 is $2x+2y+11=0$.	1A	Accept equivalent forms
Alternative solution (1)		
Let the equation of L_3 be $2x+2y+c=0$.		
$\left\lvert \frac{c-(-1)}{\sqrt{2^2+2^2}} \right\rvert = 3\sqrt{2}$	1M	
$\lvert 1+c \rvert = 12$		
$c = 11$ or -13 (rejected)		
$\therefore$ the equation of L_3 is $2x+2y+11=0$.	1A	
Alternative solution (2)		
y-intercept of $L_1 = \frac{1}{2}$ and y-intercept of $L_2 = \frac{13}{2}$		
$\therefore$ y-intercept of $L_3 = \frac{1}{2} - (\frac{13}{2} - \frac{1}{2})$ $= -\frac{11}{2}$		
Slope of L_3 = slope of $L_1 = -1$	1M	
$\therefore$ Equation of L_3 is $\frac{y-(-\frac{11}{2})}{x} = -1$		
$2x+2y+11=0$	1A	

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Solution	Marks	Remarks
Alternative solution (3) Let (x, y) be a point on L_3.		
$\left\|\frac{2x+2y-1}{\sqrt{2^2+2^2}}\right\| = 3\sqrt{2}$	1M	
$\|2x+2y-1\| = 12$		
$2x+2y+11=0$ or $2x+2y-13=0$ (rejected)	1A	
$\therefore$ the equation of L_3 is $2x+2y+11=0$.		
	5	

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Solution	Marks	Remarks
4. Marking criteria		
– Area $A=\int_a^b y\,\mathrm{d}x$	1M	
– A correct expression for the shaded area	2A	A correct expression for the area under a certain portion of a curve – award 1A
– One correct primitive function	1A	
– Answer	1A	
Area $=\int_0^1(6x^2-3x^2)\,\mathrm{d}x+\int_1^2(6x-3x^2)\,\mathrm{d}x$	1M+2A	Omit $\mathrm{d}x$ in most cases (pp–1)
$=\int_0^1 3x^2\,\mathrm{d}x+\int_1^2(6x-3x^2)\,\mathrm{d}x$		
$=\left[x^3\right]_0^1+\left[3x^2-x^3\right]_1^2$	1A	For primitive function, awarded if one was correct
$=1+(12-8)-(3-1)$		
$=3$	1A	
Alternative solution (1)		
Area $=\int_0^2(6x-3x^2)\mathrm{d}x-\int_0^1(6x-6x^2)\mathrm{d}x$	1M+2A	
$=\left[3x^2-x^3\right]_0^2-\left[3x^2-2x^3\right]_0^1$	1A	
$=4-1$		
$=3$	1A	
Alternative solution (2)		
Area $=\int_0^1 6x^2\mathrm{d}x+\int_1^2 6x\mathrm{d}x-\int_0^2 3x^2\mathrm{d}x$	1M+2A	
$=\left[2x^3\right]_0^1+\left[3x^2\right]_1^2-\left[x^3\right]_0^2$	1A	
$=2+9-8$		
$=3$	1A	
Alternative solution (3)		
Area $=\int_0^6\left(\sqrt{\frac{y}{3}}-\sqrt{\frac{y}{6}}\right)\mathrm{d}y+\int_6^{12}\left(\sqrt{\frac{y}{3}}-\frac{y}{6}\right)\mathrm{d}y$	1M+2A	
$=\left[\frac{2}{3\sqrt{3}}y^{\frac{3}{2}}-\frac{2}{3\sqrt{6}}y^{\frac{3}{2}}\right]_0^6+\left[\frac{2}{3\sqrt{3}}y^{\frac{3}{2}}-\frac{y^2}{12}\right]_6^{12}$	1A	
$=4\sqrt{2}-4+(16-12)-(4\sqrt{2}-3)$		
$=3$	1A	
	5	

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Solution	Marks	Remarks
5. (a) Put $x=5$, $y=0$:		
$-3+k(5+1)=0$	1M	
$k=\frac{1}{2}$		
$\therefore$ the equation of L_1 is $y-3+\frac{1}{2}(x-y+1)=0$		
$x+y-5=0$.	1A	Accept equivalent forms
Alternative solution (1)		
$\begin{cases} y-3=0 \\ x-y+1=0 \end{cases}$		
Solve the equations, $x=2$ and $y=3$.	1M	
The equation of L_1 is		
$\frac{y-3}{x-2}=\frac{0-3}{5-2}$		
$x+y-5=0$	1A	
Alternative solution (2)		
$y-3+k(x-y+1)=0$		
$kx+(1-k)y+(k-3)=0$		
Put $y=0$: $x=\frac{3-k}{k}=5$	1M	
$3-k=5k$		
$k=\frac{1}{2}$		
$\therefore$ the equation of L_1 is $x+y-5=0$.	1A	
(b) The equation of L_2 is $y-3=0$.	2A	
Alternative solution		
$kx+(1-k)y+(k-3)=0$		
Slope $=\frac{-k}{1-k}=0$	1M	
$k=0$		
$\therefore$ the equation of L_2 is $y-3=0$.	1A	
(c) Slope of $L_1=-1$		
Let θ be the acute angle between L_1 and L_2.		
$\tan\theta=\lvert -1\rvert$	1M	OR $\tan\theta=\left\lvert\frac{-1-0}{1+(-1)(0)}\right\rvert$
$\theta=45°$ (OR $\frac{\pi}{4}$)	1A	Accept omitting absolute sign
	6	

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Solution	Marks	Remarks
6. (a) $\left.\dfrac{dy}{dx}\right\|_{(2,0)}=0$		
$3(2)^2-2(2)+k=0$	1M	
$k=-8$	1A	
(b) $y=\int(3x^2-2x-8)\,dx$	1M	Withhold this mark if "$y=$" is omitted
$=x^3-x^2-8x+c$ (c is a constant)	1M	Awarded even if c is omitted
Put $x=2$, $y=0$: $0=2^3-2^2-8(2)+c$	1M	For finding c
$c=12$	1A	
$\therefore$ the equation of the curve is $y=x^3-x^2-8x+12$.	6	
7. (a) $(1+2x)^n=1+{}_nC_1(2x)+{}_nC_2(2x)^2+{}_nC_3(2x)^3+\cdots$	1A	Omit dots (pp–1) in all cases
$=1+2{}_nC_1x+4{}_nC_2x^2+8{}_nC_3x^3+\cdots$	1A	Deduct 1A for each wrong term, up to zero
$=1+2nx+2n(n-1)x^2+\dfrac{4}{3}n(n-1)(n-2)x^3+\cdots$		
(b) $(x-\dfrac{3}{x})^2(1+2x)^n$		
$=(x^2-6+\dfrac{9}{x^2})(1+2{}_nC_1x+4{}_nC_2x^2+\cdots)$	1A	For expanding $(x-\dfrac{3}{x})^2$
$-6+36{}_nC_2=210$	1M	
${}_nC_2=6$		
$\dfrac{n(n-1)}{2}=6$	1M	For ${}_nC_2=\dfrac{n(n-1)}{2}$ (OR $=\dfrac{n!}{2!(n-2)!}$) (can be awarded in (a))
$n^2-n-12=0$		
$(n+3)(n-4)=0$		
$n=4$ or $=-3$ (rejected)		
$\therefore n=4$	1A	
	6	

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Solution	Marks	Remarks
8. (a) $\cos 3\theta = \cos(\theta + 2\theta)$		
$= \cos\theta\cos 2\theta - \sin\theta\sin 2\theta$	1A	For expanding $\cos(a+b)$
$= \cos\theta(2\cos^2\theta - 1) - \sin\theta(2\sin\theta\cos\theta)$	} 1M	For expressing in terms of $\cos\theta$
$= \cos\theta(2\cos^2\theta - 1) - 2\cos\theta(1-\cos^2\theta)$		
$= 4\cos^3\theta - 3\cos\theta$	1	
Alternative solution (1)		
$4\cos^3\theta - 3\cos\theta$		
$= \cos\theta(4\cos^2\theta - 3)$		
$= \cos\theta[2(1+\cos 2\theta) - 3]$	1M	For $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$
$= 2\cos\theta\cos 2\theta - \cos\theta$		
$= \cos 3\theta + \cos\theta - \cos\theta$	1A	
$= \cos 3\theta$	1	
Alternative solution (2)		
$(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta$	} 1A	
$(\cos\theta + i\sin\theta)^3 = \cos^3\theta + 3\cos^2\theta(i\sin\theta) + 3\cos\theta(i\sin\theta)^2 + (i\sin\theta)^3$		
Equate real parts :		
$\cos 3\theta = \cos^3\theta - 3\cos\theta(\sin^2\theta)$	1M	
$= \cos^3\theta - 3\cos\theta(1-\cos^2\theta)$		
$= 4\cos^3\theta - 3\cos\theta$	1	
(b) $\cos 6x + 4\cos 2x = 0$		
$4\cos^3 2x - 3\cos 2x + 4\cos 2x = 0$	1A	
$4\cos^3 2x + \cos 2x = 0$		
$\cos 2x = 0$ or $4\cos^2 2x + 1 = 0$	1A	
$\cos 2x = 0$ [or $\cos^2 2x = -\frac{1}{4}$ (rejected)]		
$2x = 2k\pi \pm \frac{\pi}{2}$ [(k is an integer)]	1M	For $2x = 2n\pi \pm \alpha$
$x = k\pi \pm \frac{\pi}{4}$ [OR $x = \frac{1}{4}(2n+1)\pi$ (n is an integer)]	1A	Accept degrees $k\pi \pm 45°$ etc. (pp − 1)
Alternative solution		
$\cos 6x + 4\cos 2x = 0$		
$\cos 6x + \cos 2x + 3\cos 2x = 0$	1A	
$2\cos 4x\cos 2x + 3\cos 2x = 0$		
$\cos 2x = 0$ or $2\cos 4x + 3 = 0$	1A	
$\cos 2x = 0$ [or $\cos 4x = -\frac{3}{2}$ (rejected)]		
$2x = 2k\pi \pm \frac{\pi}{2}$	1M	For $2x = 2n\pi \pm \alpha$
$x = k\pi \pm \frac{\pi}{4}$ [OR $x = \frac{1}{4}(2n+1)\pi$]	1A	
	7	

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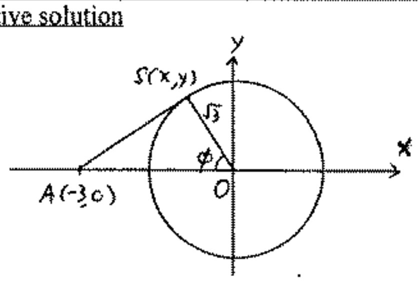
Solution	Marks	Remarks
9. (a) The equation of L is $y = mx + 1$.	1A	
Substitute $y = mx+1$ into $x^2 = 4y$:		
$x^2 = 4(mx+1)$	1M	
$x^2 - 4mx - 4 = 0$		
$\therefore x_1, x_2$ are the roots of the equation $x^2 - 4mx - 4 = 0$.	1	
	3	
(b) $\begin{cases} x_1 + x_2 = 4m \\ x_1 x_2 = -4 \end{cases}$	1A	
$(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2$	1M	
$= (4m)^2 - 4(-4)$		
$= 16(m^2 + 1)$	1A	
$AB^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$		
$= (x_1 - x_2)^2 + (mx_1 + 1 - mx_2 - 1)^2$	1M	For expressing y_1, y_2 in terms of x_1, x_2 OR $= (x_1 - x_2)^2 + (\frac{1}{4}x_1^2 - \frac{1}{4}x_2^2)^2$
$= (x_1 - x_2)^2 + (mx_1 - mx_2)^2$		
$= (1 + m^2)(x_1 - x_2)^2$		
$= (1 + m^2)[16(m^2 + 1)]$	1M	For using the above result
$AB = 4(1 + m^2)$	1	
	6	
(c) (i) x-coordinate of centre of $C = \frac{x_1 + x_2}{2}$		
$= 2m$	1A	
y-coordinate of centre of $C = \frac{y_1 + y_2}{2}$		OR $y = m(2m) + 1$ $= 2m^2 + 1$
$= \frac{mx_1 + 1 + mx_2 + 1}{2}$		
$= \frac{m}{2}(4m) + 1$		
$= 2m^2 + 1$	1A	
$\therefore$ the coordinates of the centre are $(2m, 2m^2 + 1)$.		
Radius of $C = \frac{AB}{2}$		
$= 2(1 + m^2)$	1A	
(ii) Distance from centre of C to $y + 1 = 0$		
$= \lvert 2m^2 + 1 - (-1) \rvert$ OR $= \left\lvert \frac{(2m^2 + 1) + 1}{\sqrt{1^2 + 0^2}} \right\rvert$	1M	Accept omitting absolute sign
$= 2(m^2 + 1)$	1A	
As the distance from centre of C to $y + 1 = 0$ is equal to the radius of C, the line $y + 1 = 0$ is a tangent to C. (OR The line $y + 1 = 0$ and C meet at one point.)	1M+1A	1M for comparing the radius and the distance
	7	

Solution	Marks	Remarks
10. (a) $PA=\sqrt{3}PB$		
$\sqrt{(x+3)^2+y^2}=\sqrt{3}\sqrt{(x+1)^2+y^2}$	1A	(can be omitted)
$x^2+6x+9+y^2=3(x^2+2x+1+y^2)$	1M	For squaring and expanding both sides
$x^2+y^2=3$	1	
	3	
(b) Differentiate $x^2+y^2=3$ with respect to x:		
$2x+2y\frac{dy}{dx}=0$ $\frac{dy}{dx}=-\frac{x}{y}$ Equation of tangent at $T(a,b)$ is $\frac{y-b}{x-a}=\frac{-a}{b}$	1M	
$by-b^2=-ax+a^2$		
$ax+by=a^2+b^2$ OR $ax+by=3$	1A	
Alternative solution Using the formula $xx_1+yy_1=3$, the equation of the tangent at T is $ax+by=3$.	2A	
	2	
(c) Substitute $A(-3,0)$ into the equation of tangent:		
$a(-3)+b(0)=3$	1M	
$a=-1$	1A	
$b=\sqrt{3-(-1)^2}$ ($\because$ S lies in the 2nd quadrant.)		
$=\sqrt{2}$	1A	
$\therefore$ the coordinates of S are $(-1,\sqrt{2})$.		
Alternative solution		
		
Let ϕ be the angle between OS and the negative x-axis.		
$\angle OSA=\frac{\pi}{2}$		
$-x=OS\cos\phi$	1M	
$=\sqrt{3}(\frac{\sqrt{3}}{3})$		
$x=-1$	1A	
$y=OS\sin\phi$		
$=\sqrt{3}(\frac{\sqrt{6}}{3})$		
$=\sqrt{2}$	1A	
$\therefore$ the coordinates of S are $(-1,\sqrt{2})$.		
	3	

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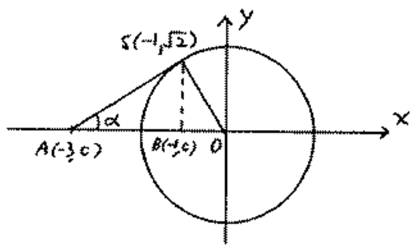
Solution	Marks	Remarks
(d) (i) The coordinates of Q are $(-3+r\cos\theta, r\sin\theta)$.	1A+1A	
(ii) (1) Substitute $(-3+r\cos\theta, r\sin\theta)$ into C:		
$(-3+r\cos\theta)^2+(r\sin\theta)^2=3$	1M	
$9-6r\cos\theta+r^2\cos^2\theta+r^2\sin^2\theta=3$		
$r^2-6r\cos\theta+6=0$ ----(*)		
Since $AH=r_1$, $AK=r_2$, r_1 and r_2 are the roots of (*).	1	
(2) Since ℓ cuts C at two distinct points, (*) has two distinct real roots.		
$(6\cos\theta)^2-4(6)>0$	1M	Can be awarded even if considering $\Delta=0$ or $\Delta\ge 0$
$\cos^2\theta>\frac{2}{3}$	1A	
$\cos\theta>\sqrt{\frac{2}{3}}$ or $\cos\theta<-\sqrt{\frac{2}{3}}$ (rejected $\because -\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$)	1A	
$\therefore -0.615<\theta<0.615$ (correct to 3 sig. figures)	1A	(OR $-35.3°<\theta<35.3°$)
Alternative solution Let α be the angle between AS and the x-axis.		
		
$\tan\alpha=\frac{SB}{AB}$ OR $\sin\alpha=\frac{OS}{OA}$ OR $\sin\alpha=\frac{SB}{SA}$	1M	OR S is a point on the locus $\therefore SA=\sqrt{3}\,SB$ $\therefore \sin\alpha=\frac{SB}{SA}=\frac{1}{\sqrt{3}}$
$=\frac{\sqrt{2}}{2}$; $=\frac{\sqrt{3}}{3}$; $=\frac{\sqrt{2}}{\sqrt{6}}=\frac{1}{\sqrt{3}}$	1A	
$\alpha=35.3°$ (correct to 3 sig. figures)	1A	
Since ℓ cuts the circle at two distinct points, $-0.615<\theta<0.615$.	1A	
	8	

Solution	Marks	Remarks
11. (a) Consider ΔABD :		
By Sine Law, $\frac{AD}{\sin\angle ABD}=\frac{\ell}{\sin\angle ADB}$	1M	
$\frac{AD}{\sin(180^\circ-\alpha)}=\frac{\ell}{\sin(\alpha-10^\circ)}$		
$AD=\frac{\ell\sin\alpha}{\sin(\alpha-10^\circ)}$ m	1	
(i) Consider ΔACD : $CD=AD\sin 10^\circ$ $=\frac{\ell\sin\alpha\sin 10^\circ}{\sin(\alpha-10^\circ)}$ m	1A	
(ii) Consider ΔADH : $\frac{AD}{\sin(\alpha-\beta)}=\frac{DH}{\sin(\beta-10^\circ)}$	1M	
$DH=AD\frac{\sin(\beta-10^\circ)}{\sin(\alpha-\beta)}=\frac{\ell\sin\alpha\sin(\beta-10^\circ)}{\sin(\alpha-10^\circ)\sin(\alpha-\beta)}$		
Consider ΔDHG : $h=DH\sin\alpha$	1M	
$=\frac{\ell\sin^2\alpha\sin(\beta-10^\circ)}{\sin(\alpha-10^\circ)\sin(\alpha-\beta)}$	1	
	6	
(b) (i) (1) Using (a) (ii) : height of pole $=\frac{97\sin^2 15^\circ\sin(10.2^\circ-10^\circ)}{\sin(15^\circ-10^\circ)\sin(15^\circ-10.2^\circ)}$ $=3.1100$		
$=3.1$ m (correct to 2 sig. fig.)	1A	Omit/wrong unit $(u-1)$
(2) Using (a) (i) : height of tower $CD=\frac{97\sin 15^\circ\sin 10^\circ}{\sin(15^\circ-10^\circ)}$ $=50.020$		
$=50$ m (correct to 2 sig. fig.)	1A	
radius of tower $=\frac{h}{\tan 15^\circ}$		
$=\frac{3.1100}{\tan 15^\circ}$	1M	
$=11.607$ $=12$ m (correct to 2 sig. fig.)	1A	

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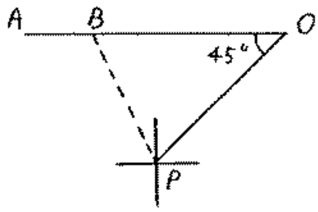
Solution	Marks	Remarks
(ii) (1)		
		
Consider ΔHPO :		
$\tan \angle HPO = \frac{OH}{OP}$		
$OP = \frac{OH}{\tan 15°}$	2M	
$= \frac{3.1100 + 50.020}{\tan 15°}$	1M	For $OH = h + CD$
$= 198.28$		
$= 200$ m (correct to 2 sig. fig.)	1A	
<u>Alternative solution</u>		
$OP = OB$	1M	(can be omitted)
$OP = OC + CB$	1M	
$= r + \frac{CD}{\tan \alpha}$		
$= 11.607 + \frac{50.020}{\tan 15°}$	1M	
$= 198.28$		
$= 200$ m (correct to 2 sig. fig.)	1A	
(2) $\angle BPO = \frac{1}{2}(180° - 45°)$	1A	(Awarded 1M for other correct methods)
$= 67.5°$		
Bearing of B from P is N(67.5° − 45°)W,		
i.e. N22.5°W.	1A	337.5° (<u>OR</u> 340°)
(OR N23°W correct to 2 sig. fig.)	10	

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Solution	Marks	Remarks
12. (a) For $n=1$, LHS $=\cos\theta$.		
RHS $=\dfrac{\sin 2\theta}{2\sin\theta}$		
$=\dfrac{2\sin\theta\cos\theta}{2\sin\theta}=\cos\theta=$ LHS.	1	
$\therefore$ the statement is true for $n=1$.		
Assume $\cos\theta+\cos3\theta+\cos5\theta+\cdots+\cos(2k-1)\theta=\dfrac{\sin2k\theta}{2\sin\theta}$	1	
for some +ve integer k.		
Then $\cos\theta+\cos3\theta+\cos5\theta+\cdots+\cos(2k-1)\theta+\cos[2(k+1)-1]\theta$		
$=\dfrac{\sin 2k\theta}{2\sin\theta}+\cos(2k+1)\theta$	1	
$=\dfrac{\sin 2k\theta+2\sin\theta\cos(2k+1)\theta}{2\sin\theta}$		
$=\dfrac{\sin 2k\theta+\sin(2k+2)\theta-\sin 2k\theta}{2\sin\theta}$	1	(cannot be omitted)
$=\dfrac{\sin 2(k+1)\theta}{2\sin\theta}$	1	
The statement is also true for $n=k+1$ if it is true for $n=k$. By the principle of mathematical induction,		
the statement is true for all positive integers n.	1	
	6	
(b) Using (a) : $\cos\theta+\cos3\theta+\cos5\theta=\dfrac{\sin6\theta}{2\sin\theta}$, where $\sin\theta\neq0$.	1A	(can be omitted)
Put $\theta=\dfrac{\pi}{2}-x$:		
$\cos(\frac{\pi}{2}-x)+\cos3(\frac{\pi}{2}-x)+\cos5(\frac{\pi}{2}-x)=\dfrac{\sin6(\frac{\pi}{2}-x)}{2\sin(\frac{\pi}{2}-x)}$		
$\cos(\frac{\pi}{2}-x)+\cos(\frac{3\pi}{2}-3x)+\cos(\frac{5\pi}{2}-5x)=\dfrac{\sin(3\pi-6x)}{2\sin(\frac{\pi}{2}-x)}$		
$\sin x-\sin3x+\sin5x=\dfrac{\sin6x}{2\cos x}$	1	
(where $\sin(\frac{\pi}{2}-x)=\cos x\neq0$).		
Alternative solution		
Consider $2\cos x(\sin x-\sin3x+\sin5x)$		
$=\sin2x-(\sin4x+\sin2x)+(\sin6x+\sin4x)$	1A	
$=\sin6x$		
$\therefore\ \sin x-\sin3x+\sin5x=\dfrac{\sin6x}{2\cos x}$, where $\cos x\neq0$.	1	
	2	

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Solution	Marks	Remarks
(c) $\int_{0.1}^{0.5}\left(\frac{\sin x-\sin 3x+\sin 5x}{\cos x+\cos 3x+\cos 5x}\right)^2 dx$		
$=\int_{0.1}^{0.5}\left[\frac{\sin 6x}{2\cos x}/\frac{\sin 6x}{2\sin x}\right]^2 dx$	1A	For integrand only
$=\int_{0.1}^{0.5}\tan^2 x\,dx$		
$=\int_{0.1}^{0.5}(\sec^2 x-1)\,dx$	1M	For $\tan^2 x=\sec^2 x-1$
$=[\tan x-x]_{0.1}^{0.5}$	1A	For primitive function only
$=0.046$ (correct to 2 sig. fig.)	1A	
	4	
(d) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}(\sin x+3\sin 3x+5\sin 5x+7\sin 7x+\cdots+1999\sin 1999x)dx$		
$=[-\cos x-\cos 3x-\cos 5x-\cos 7x-\cdots-\cos 1999x]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$	1M+1A	1M for integrating each term (At least two terms)
$=-\frac{1}{2}\left[\frac{\sin 2000x}{\sin x}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$	1A	
$=-\frac{1}{2}\left(\frac{\sin 1000\pi}{\sin\frac{\pi}{2}}-\frac{\sin\frac{2000\pi}{3}}{\sin\frac{\pi}{3}}\right)$		
$=\frac{1}{2}$	1A	
Alternative solution		
$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}(\sin x+3\sin 3x+5\sin 5x+7\sin 7x+\cdots+1999\sin 1999x)dx$		
$=[-\cos x-\cos 3x-\cos 5x-\cos 7x-\cdots-\cos 1999x]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$	1M+1A	1M for integrating each term
$=[-\cos\frac{\pi}{2}-\cos\frac{3\pi}{2}-\cos\frac{5\pi}{2}-\cdots-\cos\frac{1999\pi}{2}]-[-\cos\frac{\pi}{3}-\cos\pi-\cos\frac{5\pi}{3}-\cos\frac{7\pi}{3}-\cdots-\cos\frac{1999\pi}{3}]$		
$=0-[(-\frac{1}{2}+1-\frac{1}{2})+(-\frac{1}{2}+1-\frac{1}{2})+\ldots+(-\frac{1}{2}+1-\frac{1}{2})-\frac{1}{2}]$	1A	(can be omitted)
$=\frac{1}{2}$	1A	
	4	

Solution	Marks	Remarks
13. (a) Substitute $(r, 2)$ into $x=\sqrt{4+3y^2}$:		
$r=\sqrt{4+3(2)^2}$	1M	
$=4$	1A	
	2	
(b) V = Volume of lower cylindrical part + volume of upper part		
Volume of lower cylindrical part $=\pi r^2 h$		
$=\pi(4)^2(2)$	1M	
$=32\pi$	1A	
Alternative solution		
Volume of lower cylindrical part $=\pi\int_0^2 x^2\,\mathrm{d}y$		
$=\pi\int_0^2 4^2\,\mathrm{d}y$	1M	
$=32\pi$	1A	
Volume of upper part $=\pi\int_2^h x^2\,\mathrm{d}y$	1M	
$=\pi\int_2^h (4+3y^2)\,\mathrm{d}y$	1A	
$=\pi[4y+y^3]_2^h$	1A	For $\pi[4y+y^3]$ only
$=(h^3+4h-16)\pi$	1A	
$\therefore V=32\pi+(h^3+4h-16)\pi$		
$=(h^3+4h+16)\pi$ [cubic units]	1	
	7	
(c) (i) Let h units be the depth of water at time t.		
$\frac{\mathrm{d}V}{\mathrm{d}t}=\pi(3h^2+4)\frac{\mathrm{d}h}{\mathrm{d}t}$	1M+1A	1 M for chain rule
Put $\frac{\mathrm{d}V}{\mathrm{d}t}=-2\pi$ and $h=3$:		
$-2\pi=\pi[3(3)^2+4]\frac{\mathrm{d}h}{\mathrm{d}t}$	1M	For substitution
		(Accept substitute $\frac{\mathrm{d}V}{\mathrm{d}t}=2\pi$)
$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{-2}{31}$ units per sec. (OR s^{-1})	1A	omit/wrong unit $(u-1)$
(OR The depth decreases at a rate $\frac{2}{31}$ units per sec.)		

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Solution	Marks	Remarks
(ii) When $h=1$, the water remained is in the cylindrical part only.		
$\frac{dh}{dt}=\frac{\frac{dV}{dt}}{\text{base area of cylilnder}}$	1M	(can be omitted)
$=\frac{-2\pi}{\pi(4)^2}$	1M	For substitution
		(Accept substitute $\frac{dV}{dt}=2\pi$)
$=-\frac{1}{8}$ units per sec. (OR s^{-1})	1A	
(OR The depth decreases at a rate $\frac{1}{8}$ units per sec.)		
Alternative solution		
$V=\pi(4)^2h$ $=16\pi h$	1M	
$\frac{dV}{dt}=16\pi\frac{dh}{dt}$	1M	
$-2\pi=16\pi\frac{dh}{dt}$		
$\therefore\ \frac{dh}{dt}=-\frac{1}{8}$ units per sec. (OR s^{-1})	1A	
	7	

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