香港考試局
HONG KONG EXAMINATIONS AUTHORITY

一九九八年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1998

附加數學 試卷一
ADDITIONAL MATHEMATICS PAPER I

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GENERAL INSTRUCTIONS TO MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates would use alternative methods not specified in the marking scheme. Markers should be patient in marking these alternative solutions. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method is specified in the question.

2. In the marking scheme, marks are classified as follows :

   'M' marks – awarded for knowing a correct method of solution and attempting to apply it;

   'A' marks – awarded for the accuracy of the answer;

   Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. The symbol (pp–1) should be used to denote marks deducted for poor presentation (p.p.). Note the following points:

   (a) At most deduct 1 mark for p.p. in each question, up to a maximum of 3 marks for the whole paper.

   (b) For similar p.p., deduct only 1 mark for the first time that it occurs, i.e. do not penalise candidates twice in the whole paper for the same p.p.

   (c) In any case, do not deduct any marks for p.p. in those steps where candidates could not score any marks.

   (d) Some cases in which marks should be deducted for p.p. are specified in the marking scheme. However, the lists are by no means exhaustive. Markers should exercise their professional judgement to give p.p.s in other situations.

5. The symbol (u–1) should be used to denote marks deducted for wrong/no units in the final answers (if applicable). Note the following points:

   (a) At most deduct 1 mark for wrong/no units for the whole paper.

   (b) Do not deduct any marks for wrong/no units in case candidate's answer was already wrong.

6. Marks entered in the Page Total Box should be the net total score on that page.

7. In the Marking Scheme, steps which can be omitted are enclosed by dotted rectangles [dotted rectangle], whereas alternative answers are enclosed by solid rectangles [solid rectangle].

8. Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.

9. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

10. Unless the form of answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they were correct.

Solution	Marks	Remarks
1. $\frac{d}{dx}(\sqrt{x}) = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x}-\sqrt{x}}{\Delta x}$	1A	Withhold this mark if $\lim_{\Delta x \to 0}$ is omitted
$= \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x}-\sqrt{x}}{\Delta x}\left(\frac{\sqrt{x+\Delta x}+\sqrt{x}}{\sqrt{x+\Delta x}+\sqrt{x}}\right)$	1A	For simplification only
$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x}+\sqrt{x}}$	1A	
$= \frac{1}{2\sqrt{x}}$	1A	
	4	
2. $\begin{cases}\alpha+\beta=2\\ \alpha\beta=7\end{cases}$	1A	
Let the equation be $x^2 - Sx + P = 0$, where $S=(\alpha+2)+(\beta+2)$ and $P=(\alpha+2)(\beta+2)$.	1M	
$S=(\alpha+2)+(\beta+2)$ $=(\alpha+\beta)+4$ $=6$ $P=(\alpha+2)(\beta+2)$ $=\alpha\beta+2(\alpha+\beta)+4$ $=7+2(2)+4$ $=15$	1M	
$\therefore$ The equation is $x^2-6x+15=0$.	1A	
Alternative solution (1) $x^2-2x+7=0$ $x=\frac{2\pm\sqrt{4-4(7)}}{2}$ $=1\pm\sqrt{6}i$	1A	
The roots of the required equation are $1+\sqrt{6}i+2=3+\sqrt{6}i$ and $1-\sqrt{6}i+2=3-\sqrt{6}i$.	1M	
$\therefore$ The equation is $[x-(3+\sqrt{6}i)][x-(3-\sqrt{6}i)]=0$	1M	
$[(x-3)-\sqrt{6}i][(x-3)+\sqrt{6}i]=0$ $(x-3)^2-(\sqrt{6}i)^2=0$ $x^2-6x+9+6=0$ $x^2-6x+15=0$.	1A	
Alternative solution (2) Put $y=x+2$.	1A	(can be omitted)
The equation is $(y-2)^2-2(y-2)+7=0$	2M	
$(y^2-4y+4)-2y+4+7=0$ $y^2-6y+15=0$	1A	
	4	

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Solution	Marks	Remarks
3. $\alpha^2-6\alpha+2k=0$ ------ (1)		
$\alpha^2-5\alpha+k=0$ ------ (2)		
(2) − (1) $\alpha-k=0$	1M	
$\alpha=k$.	1	
Alternative solution for 1st part		
Let α, β_1 be the roots of $x^2-6x+2k=0$		
$\alpha+\beta_1=6$ ----- (3)		
$\alpha\beta_1=2k$ ----- (4)		
Let α, β_2 be the roots of $x^2-5x+k=0$		
$\alpha+\beta_2=5$ ----- (5)		
$\alpha\beta_2=k$ ----- (6)		
(3) − (5) : $\beta_1-\beta_2=1$ ----- (7)	1M	For attempt to find α
(4) − (6) : $\alpha(\beta_1-\beta_2)=k$ ----- (8)		
Substitute (7) into (8), $\alpha=k$.	1	
Substitute $\alpha=k$ into (1).		OR substitute into (2)
$k^2-6k+2k=0$	1M	
$k^2-4k=0$		
$k=0$ or 4.	1A	
Alternative solution for 2nd part		
Substitute $\alpha=k$ into (4).		
$k\beta_1=2k$	1M	
$k=0$ or $\beta_1=2$		
From (3), $k+2=6$		
$k=4$		
$\therefore k=0$ or 4.	1A	
Alternative solution for Q.3		
$\alpha^2-6\alpha+2k=0$ ----- (1)		
$\alpha^2-5\alpha+k=0$ ----- (2)		
From (1) : $\dfrac{6\alpha-\alpha^2}{2}=k$	} 1M	(can be omitted)
From (2) : $5\alpha-\alpha^2=k$		
$\therefore \dfrac{6\alpha-\alpha^2}{2}=5\alpha-\alpha^2$	1M	
$\alpha^2-4\alpha=0$		
$\alpha=0$ or 4		
When $\alpha=0$, $k=0$.		
When $\alpha=4$, $k=4$.		
In both cases, $\alpha=k$.	1A	
$k=0$ or 4.	1A	
	4	

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Solution	Marks	Remarks
4. (a) $r=\sqrt{(\frac{\sqrt{3}}{2})^2+(\frac{1}{2})^2}=1$ $\tan\theta=\frac{1/2}{\sqrt{3}/2}=\frac{1}{\sqrt{3}}$ $\theta=\frac{\pi}{6}$	1M	(can be omitted)
$\therefore \frac{\sqrt{3}}{2}+\frac{1}{2}i=\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}$ (OR $\cos 30°+i\sin 30°$)	1A	
(b) $(\frac{\sqrt{3}}{2}+\frac{1}{2}i)^n=1$ $(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6})^n=1$		
$\cos\frac{n\pi}{6}+i\sin\frac{n\pi}{6}=1$	1M	For De Moivre's Theorem
$\cos\frac{n\pi}{6}+i\sin\frac{n\pi}{6}=\cos 2k\pi+i\sin 2k\pi$ $\frac{n\pi}{6}=2k\pi$	1M	(can be omitted)
$n=12k$, where k is a positive integer. (OR $n=12, 24, 36, \ldots$)	1A	k not defined – no marks Include 0 – no marks
	5	
5. (a) $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$ $=(4\vec{i}+4\vec{j})-(\vec{i}-\vec{j})$	1A	Awarded if either one was correct
$=3\vec{i}+5\vec{j}$	1A	
$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}$ $=-2\vec{i}+7\vec{j}-(\vec{i}-\vec{j})$ $=-3\vec{i}+8\vec{j}$	1A	
(b) $\overrightarrow{AB}\cdot\overrightarrow{AC}=(3\vec{i}+5\vec{j})\cdot(-3\vec{i}+8\vec{j})$ $=3(-3)+5(8)$	1M	
$=31$ $\cos\angle BAC=\frac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{\lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{AC}\rvert}$		
$=\frac{31}{\sqrt{3^2+5^2}\sqrt{(-3)^2+8^2}}$	1M	
$\angle BAC=52°$ (correct to the nearest degree)	1A	Omit vector sign or dot sign in most cases (pp–1)
	6	

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Solution	Marks	Remarks
6. (a) $x^2-6x-16>0$		
$(x+2)(x-8)>0$	1A	(can be omitted)
$x>8$ or $x<-2$	1A	No mark for using 'and' or ','
(b) $(y+1)^2-6\|y+1\|-16>0$		
$\|y+1\|^2-6\|y+1\|-16>0$	1M	no mark if absolute sign was omitted (can be omitted)
Put $x=\|y+1\|$ in (a).		
$\|y+1\|>8$ or $\|y+1\|<-2$ (no solution)	1M+1A	1A for $\|y+1\|<-2$ has no solution
$y+1>8$ or $y+1<-8$		omit $\|y+1\|<-2$ (pp–1)
$y>7$ or $y<-9$	1A	
Alternative solution (1)		
Consider the following cases : (i) $y+1\geq 0$; (ii) $y+1<0$	1M	Accept omitting equality sign
Case 1 : $y\geq -1$		
$(y+1)^2-6(y+1)-16>0$	1A	
$y^2-4y-21>0$		
$(y+3)(y-7)>0$		
$y>7$ or $y<-3$		
Since $y\geq -1$, $y>7$.		
Case 2 : $y<-1$		
$(y+1)^2+6(y+1)-16>0$	1A	
$y^2+8y-9>0$		
$(y-1)(y+9)>0$		
$y>1$ or $y<-9$		
Since $y<-1$, $y<-9$.		
$\therefore y>7$ or $y<-9$	1A	
Alternative solution (2)		
$(y+1)^2-6\|y+1\|-16>0$		
$6\|y+1\|<(y+1)^2-16$		
$6(y+1)<(y+1)^2-16$ and $6(y+1)>-[(y+1)^2-16]$	1M	
$y^2-4y-21>0$ and $y^2+8y-9>0$	1A+1A	
$(y+3)(y-7)>0$ and $(y-1)(y+9)>0$		
$(y>7$ or $y<-3)$ and $(y>1$ or $y<-9)$		
Combining the 2 solutions, $y>7$ or $y<-9$.	1A	
	6	

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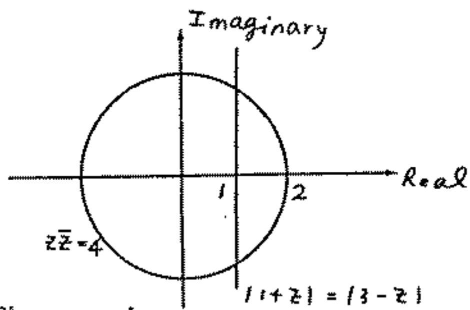
Solution	Marks	Remarks
7. Let $z=a+bi$.		
$\|1+z\|=\|3-z\|$		
$\sqrt{(1+a)^2+b^2}=\sqrt{(3-a)^2+(-b)^2}$	1M	For the modulus
$a^2+2a+1+b^2=a^2-6a+9+b^2$		
$a=1$	1A	
$z\bar{z}=4$		
$(a+bi)(a-bi)=4$ (OR $\|z\|^2=4$)	1M	(can be omitted)
$a^2+b^2=4$	1A	
Substitute $a=1$, $1^2+b^2=4$		
$b=\pm\sqrt{3}$		
$\therefore z=1+\sqrt{3}i$ or $1-\sqrt{3}i$.	1A + 1A	
Alternative solution		
Let $z=a+bi$ be the complex number satisfying both equations.		
The locus of the 2 equations are shown below.		
	1M 1M 1A	For drawing a straight line For drawing a circle Awarded if both were correct (pp−1) for not labelling the axes
From the Figure, $a=1$.	1A	
$b^2=4-a^2=3$		
$b=\pm\sqrt{3}$		
$\therefore z=1+\sqrt{3}i$ or $1-\sqrt{3}i$.	1A+1A	
	6	
8. (a) $x^2-xy+3y^2=12$		
$2x-(y+x\frac{dy}{dx})+6y\frac{dy}{dx}=0$	1A+1A	1A for $\frac{d}{dx}(xy)$, 1A for other terms
$\frac{dy}{dx}=\frac{y-2x}{6y-x}$		
Substitute $x=0$, $y=2$.		
$0-(2+0)+6(2)\frac{dy}{dx}=0$	1M	} can be awarded in part (b)
$\frac{dy}{dx}=\frac{1}{6}$	1A	
(b) Slope of the normal $=-6$	1M	
Equation of normal is		
$\frac{y-2}{x-0}=-6$	1M	
$6x+y-2=0$	1A	
	7	

Solution	Marks	Remarks
9. (a) (i) $\vec{a}\cdot\vec{b}=\lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos 60°$		Omit vector sign or dot sign in most cases (pp–1)
$=2(3)\cos 60°$	1M	
$=3$	1A	
(ii) $\overrightarrow{OC}=t\overrightarrow{OB}+(1-t)\overrightarrow{OA}$		
$=(1-t)\vec{a}+t\vec{b}$	1A	
(iii) $\vec{a}\cdot\overrightarrow{OC}=\vec{a}\cdot[(1-t)\vec{a}+t\vec{b}]$		
$=(1-t)\vec{a}\cdot\vec{a}+t\vec{a}\cdot\vec{b}$	1M	For distribution
$=(1-t)(4)+t(3)$	1A	For either $\vec{a}.\vec{a}=4$ or $\vec{b}.\vec{b}=9$
$=4-t$	1A	
$\vec{b}\cdot\overrightarrow{OC}=\vec{b}\cdot[(1-t)\vec{a}+t\vec{b}]$		
$=(1-t)\vec{a}\cdot\vec{b}+t\vec{b}\cdot\vec{b}$		
$=(1-t)(3)+t(9)$		
$=3+6t$	1A	
	7	
(b) (i) $\vec{a}\cdot\overrightarrow{OD}=\vec{a}\cdot(\overrightarrow{OC}+\overrightarrow{CD})$		
$=\vec{a}\cdot\overrightarrow{OC}+\vec{a}\cdot\overrightarrow{CD}$		
$=4-t+0$	1A	For either $\vec{a}\cdot\overrightarrow{CD}=0$ or $\vec{b}\cdot\overrightarrow{CE}=0$
$=4-t$	1	
Alternative solution		
$\vec{a}\cdot\overrightarrow{OC}=\lvert\vec{a}\rvert\lvert\overrightarrow{OC}\rvert\cos\angle AOC$		
$=\lvert\vec{a}\rvert\lvert\overrightarrow{OD}\rvert$		
$=\vec{a}\cdot\overrightarrow{OD}$	1A	
$\therefore\ \vec{a}\cdot\overrightarrow{OD}=4-t$	1	
$\vec{b}.\overrightarrow{OE}=\vec{b}.(\overrightarrow{OC}+\overrightarrow{CE})$		
$=\vec{b}\cdot\overrightarrow{OC}+\vec{b}\cdot\overrightarrow{CE}$		
$=\vec{b}\cdot\overrightarrow{OC}+0$		
$=3+6t$	1	
Alternative solution		
$\vec{b}\cdot\overrightarrow{OC}=\lvert\vec{b}\rvert\lvert\overrightarrow{OC}\rvert\cos\angle BOC$		
$=\lvert\vec{b}\rvert\lvert\overrightarrow{OE}\rvert$		
$=\vec{b}\cdot\overrightarrow{OE}$		
$\therefore\ \vec{b}.\overrightarrow{OE}=3+6t$	1	
(ii) $\overrightarrow{OD}=k\vec{a}$		
$\vec{a}\cdot(k\vec{a})=4-t$	1M	
$4k=4-t$		
$\therefore k=\frac{4-t}{4}$	1A	

Solution	Marks	Remarks
$\overrightarrow{OE} = s\vec{b}$		
$\vec{b}\cdot(s\vec{b}) = 3+6t$		
$9s = 3+6t$		
$\therefore s = \dfrac{1+2t}{3}$	1A	
Alternative solution		
$\vec{a}\,.\,\overrightarrow{OD} = 4-t$		
$\lvert\vec{a}\rvert\lvert\overrightarrow{OD}\rvert = 4-t$	} 1M	
$\lvert\overrightarrow{OD}\rvert = \dfrac{4-t}{2}$		
$\overrightarrow{OD} = (\dfrac{4-t}{2})(\dfrac{1}{2}\vec{a})$		
$\therefore k = \dfrac{4-t}{4}$	1A	
$\vec{b}\,.\,\overrightarrow{OE} = 3+6t$		
$\lvert\vec{b}\rvert\lvert\overrightarrow{OE}\rvert = 3+6t$		
$\lvert\overrightarrow{OE}\rvert = \dfrac{3+6t}{3} = 1+2t$		
$\overrightarrow{OE} = (1+2t)(\dfrac{1}{3}\vec{b})$		
$\therefore s = \dfrac{1+2t}{3}$	1A	
	6	
(c) $\overrightarrow{AB} = \vec{b}-\vec{a}$	} 1M	For finding $\overrightarrow{AB}$ and $\overrightarrow{DE}$
$\overrightarrow{DE} = \overrightarrow{OE}-\overrightarrow{OD}$		
$= \dfrac{1+2t}{3}\vec{b} - \dfrac{4-t}{4}\vec{a}$		
If $\overrightarrow{DE} \mathbin{/\mkern-6mu/} \overrightarrow{AB}$, $\dfrac{1+2t}{3} = \dfrac{4-t}{4}$	1M	
$t = \dfrac{8}{11}$	1A	
	3	

Solution	Marks	Remarks
10. (a) (i) $f(x) = 2\cos 2x + 4\sin x - 3$		
$f(0) = -1$ ∴ The y-intercept is -1.	1A	(pp–1) for giving (0, –1)
Put $f(x) = 0$. $2\cos 2x + 4\sin x - 3 = 0$		
$2(1 - 2\sin^2 x) + 4\sin x - 3 = 0$	1M	For $\cos 2x = 1 - 2\sin^2 x$
$4\sin^2 x - 4\sin x + 1 = 0$		
$\sin x = \frac{1}{2}$	1A	
$x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$		
∴ The x-intercepts are $\frac{\pi}{6}$ and $\frac{5\pi}{6}$.	1A	(pp–1) for giving $(\frac{\pi}{6}, 0)$ etc. No marks for degrees
(ii) $f'(x) = -4\sin 2x + 4\cos x$		
$f'(x) = 0$ $-4\sin 2x + 4\cos x = 0$	1M	
$4\cos x(1 - 2\sin x) = 0$		
$\cos x = 0$ or $\sin x = \frac{1}{2}$	1A+1A	
$x = \frac{\pi}{2}$ or $-\frac{\pi}{2}$ or $x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$		
$f''(x) = -8\cos 2x - 4\sin x$		
$f''(\frac{\pi}{2})$ $= 4$ > 0 ∴ $(\frac{\pi}{2}, -1)$ is a minimum point.	1M + 2A	1M for checking All correct – 2A 2-3 points correct – 1A ≤ 1 point correct – 0A no marks if checking was omitted
$f''(\frac{-\pi}{2})$ $= 12$ > 0 ∴ $(-\frac{\pi}{2}, -9)$ is a minimum point		
$f''(\frac{\pi}{6})$ $= -6$ < 0 ∴ $(\frac{\pi}{6}, 0)$ is a maximum point.		
$f''(\frac{5\pi}{6})$ $= -6$ < 0 ∴ $(\frac{5\pi}{6}, 0)$ is a maximum point.		

Alternative solution for checking

$f'(x) = 4\cos x(1 - 2\sin x)$

x	$x < -\frac{\pi}{2}$	$-\frac{\pi}{2}$	$-\frac{\pi}{2} < x < \frac{\pi}{6}$	$\frac{\pi}{6}$	$\frac{\pi}{6} < x < \frac{\pi}{2}$	$\frac{\pi}{2}$	$\frac{\pi}{2} < x < \frac{5\pi}{6}$	$\frac{5\pi}{6}$	$x > \frac{5\pi}{6}$
$f'(x)$	–ve	0	+ve	0	–ve	0	+ve	0	–ve

1M — For checking

So $(\frac{\pi}{2}, -1)$ and $(-\frac{\pi}{2}, -9)$ are minimum points,

and $(\frac{\pi}{6}, 0)$ and $(\frac{5\pi}{6}, 0)$ are maximum points. — 2A

All correct – 2A
2-3 points correct – 1A
≤ 1 point correct – 0A
no marks if checking was omitted

Total: 10

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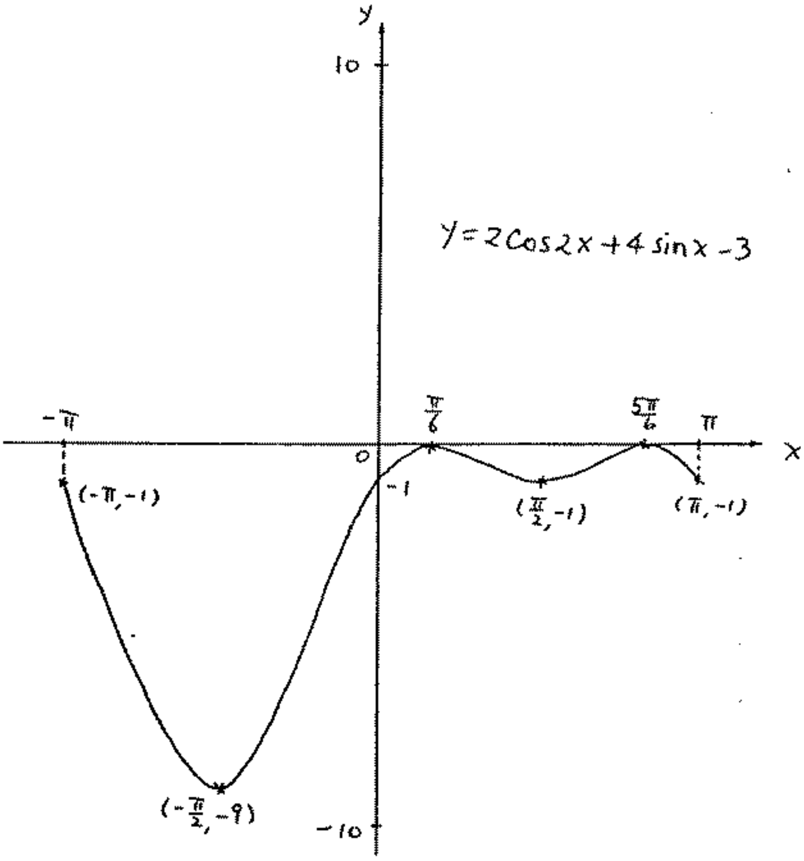
Solution	Marks	Remarks
(b)		
		(Awarded even if checking was omitted in (a) (ii))
	1A	For shape
	1A	For intercepts and turning points
	1A	For end points
The greatest value of $\lvert 2\cos 2x + 4\sin x \rvert$ is 6.	1M+1A	1M for finding the greatest value of $\lvert f(x)+3 \rvert$ from the graph.
The least value of $\lvert 2\cos 2x + 4\sin x \rvert$ is 0.	1A	
	6	

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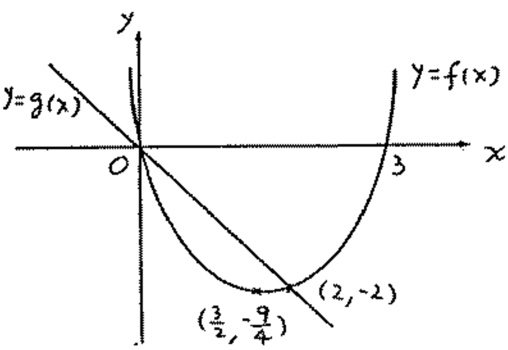
Solution	Marks	Remarks
11. (a) $f(x)=x^2-kx$		
$=(x^2-kx+\frac{k^2}{4})-\frac{k^2}{4}$	1M	For completing square
$=(x-\frac{k}{2})^2-\frac{k^2}{4}$		
$\therefore$ Least value of $f(x)=-\frac{k^2}{4}$,	1	
which occurs at $x=\frac{k}{2}$.	1A	
Alternative solution		
$f'(x)=2x-k$		
$f'(x)=0$ at $x=\frac{k}{2}$.	1A	
$f''(x)=2>0$	1M	For checking
$\therefore$ $f(x)$ is a minimum at $x=\frac{k}{2}$.		
As $f(x)$ has only one turning points,	1	
$\therefore$ least value of $f(x)=(\frac{k}{2})^2-k(\frac{k}{2})=-\frac{k^2}{4}$		
	3	
(b) $\begin{cases} y=x^2-kx \\ y=-x \end{cases}$		
$x^2-kx=-x$	1M	For solving the 2 equations
$x^2-(k-1)x=0$		
$x=0$ or $x=k-1$		
$y=0$ $\quad$ $y=1-k$		
$\therefore$ The coordinates of the intersecting points are (0, 0) and $(k-1, 1-k)$.	1A+1A	
	3	
(c) (i) 	1A	For the line $y=g(x)$
	1A	For the curve $y=f(x)$
	1A	For labelling the intercepts and turning point of $y=f(x)$
	1A	For labelling the intersecting points
		(pp−1) for not labelling the axes
(ii) From the figure, the range of values of x such that $f(x)\le g(x)$ is $0\le x\le 2$.	1A	
Alternative solution		
$f(x)\le g(x)$		
$x^2-3x\le -x$		
$x(x-2)\le 0$		
$0\le x\le 2$.	1A	
From the graph in (i), the least value of $f(x)=-\frac{9}{4}$.	1A	
	6	

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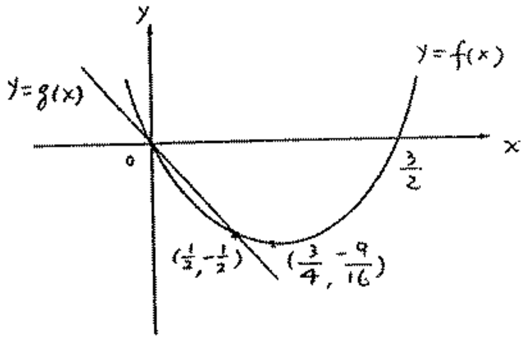
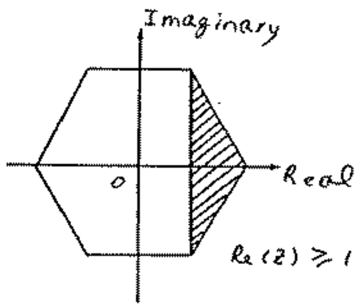
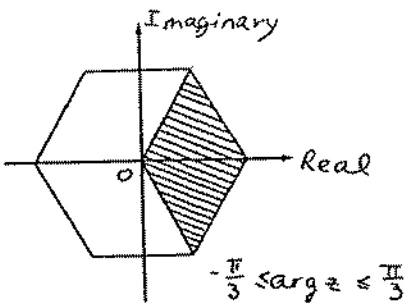
Solution	Marks	Remarks
(d) $k = \frac{3}{2}$		
	1A 1M 1A	For the curve $y = f(x)$ For turning point of f(x) lying outside the range $0 \le x \le \frac{1}{2}$. For labelling the intersecting points
From the graph, the least value of f(x) is $-\frac{1}{2}$.	1A	
Alternative solution $f(x) \le g(x)$ $x^2 - \frac{3}{2}x \le -x$ $2x^2 - x \le 0$ $0 \le x \le \frac{1}{2}$	1A	
f(x) is strictly decreasing in this range.	1	Withhold these 2 marks if explanation was not given
So the least value of f(x) occurs at $x = \frac{1}{2}$.	1A	
$\therefore$ Least value of $f(x) = -\frac{1}{2}$.	1A	
	4	

Solution	Marks	Remarks
12. (a) $z^6 = 64$		
$z = 64^{\frac{1}{6}}(\cos\frac{2n\pi}{6} + i\sin\frac{2n\pi}{6})$ (n is an integer)	1M	For De Moivre's Theorem
$z = 2(\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3})$		
$z_0 = 2(\cos 0 + i\sin 0) = 2$		
$z_1 = 2\cos(\frac{\pi}{3} + i\sin\frac{\pi}{3}) = 1 + \sqrt{3}i$	1A	For z_0 and z_3
$z_2 = 2\cos(\frac{2\pi}{3} + i\sin\frac{2\pi}{3}) = -1 + \sqrt{3}i$	1A+1A	1A for two other correct answers
$z_3 = 2(\cos\pi + i\sin\pi) = -2$		
$z_4 = 2(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}) = -1 - \sqrt{3}i$		OR $z_4 = \bar{z}_2 = -1 - \sqrt{3}i$
$z_5 = 2(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}) = 1 - \sqrt{3}i$		OR $z_5 = \bar{z}_1 = 1 - \sqrt{3}i$
	4	
(b) (i) 	1A+1A	1A for the line $Re(z) = 1$ 1A for shading the region (pp−1) for not labelling the axes
(ii) 	1M+1A+1A	1M for 2 lines through O 1A for 2 correct lines 1A for shading the region (pp−1) for not labelling the axes
	5	
(c) (i) $w = 3z_1$		
$= 3(1+\sqrt{3}i)$	1M	
$= 3 + 3\sqrt{3}i$	1A	
(ii) $w - z_2 = 3 + 3\sqrt{3}i - (-1 + \sqrt{3}i)$		
$= 4 + 2\sqrt{3}i$		
$\tan\arg(w - z_2) = \frac{2\sqrt{3}}{4}$	1M	
$\arg(w - z_2) = 0.714$ (correct to 3 sig. figures)	1A	OR 40.9°
$\arg(w - z_2)$ = Angle between CP and the real axis	1M	(can be omitted)
$\angle OPC = \arg z_1 - \arg(w - z_2)$	1M	
$= \frac{\pi}{3} - 0.714$		
$= 0.333$ (correct to 3 significant figures)	1A	OR 19.1°
	7	

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Solution	Marks	Remarks
13. (a) By Sine Law,		
$\frac{1}{\sin\frac{\pi}{6}}=\frac{y}{\sin\theta}=\frac{x}{\sin(\pi-\frac{\pi}{6}-\theta)}$	1M	For sine law
$\therefore y=2\sin\theta$	1A	
$x=2\sin(\frac{5\pi}{6}-\theta)$ [OR $\cos\theta+\sqrt{3}\sin\theta$, $2\sin(\frac{\pi}{6}+\theta)$]	1A	Accept degrees
	3	
(b) $S=\frac{1}{2}x(1)\sin\theta$ [OR $=\frac{1}{2}xy\sin\frac{\pi}{6}$]		
$=\sin\theta\sin(\frac{5\pi}{6}-\theta)$	1A	OR $=\frac{1}{2}\sin\theta\cos\theta+\frac{\sqrt{3}}{2}\sin^2\theta$
$\frac{dS}{d\theta}=\cos\theta\sin(\frac{5\pi}{6}-\theta)-\sin\theta\cos(\frac{5\pi}{6}-\theta)$	1M	For product rule
$=\sin(\frac{5\pi}{6}-\theta-\theta)$		
$=\sin(\frac{5\pi}{6}-2\theta)$	1	
Alternative solution		
$S=\sin\theta\sin(\frac{5\pi}{6}-\theta)$	1A	
$=\frac{1}{2}[\cos(\frac{5\pi}{6}-2\theta)-\cos\frac{5\pi}{6}]$		
$\frac{dS}{d\theta}=-\frac{1}{2}\sin(\frac{5\pi}{6}-2\theta)(-2)$	1M	
$=\sin(\frac{5\pi}{6}-2\theta)$	1	
$\frac{dS}{d\theta}=0$ $\quad\sin(\frac{5\pi}{6}-2\theta)=0$	1M	
$\frac{5\pi}{6}=2\theta$		
$\theta=\frac{5\pi}{12}$	1A	No mark for degrees
$\frac{d^2S}{d\theta^2}=-2\cos(\frac{5\pi}{6}-2\theta)$		
At $\theta=\frac{5\pi}{12}$, $\frac{d^2S}{d\theta^2}$ [$=-2$] <0.	1	For checking
$\therefore$ S is a maximum at $\theta=\frac{5\pi}{12}$.		
Alternative solution for checking		
$\frac{dS}{d\theta}=\sin(\frac{5\pi}{6}-2\theta)$		
$\frac{dS}{d\theta}>0$ when $0\le\theta<\frac{5\pi}{12}$	1	For checking
$\frac{dS}{d\theta}<0$ when $\theta>\frac{5\pi}{12}$		
$\therefore$ S is a maximum at $\theta=\frac{5\pi}{12}$.		
	6	

Solution	Marks	Remarks
(c) Differentiate x and y with respect to t,		
$\frac{dx}{dt} = -2\cos(\frac{5\pi}{6}-\theta)\frac{d\theta}{dt}$ ------(1)	1M+1A	1M for differentiating w.r.t. t.
$\frac{dy}{dt} = 2\cos\theta\frac{d\theta}{dt}$ ------(2)		
Eliminating $\frac{d\theta}{dt}$,		
$\frac{dx}{dt} = -2\cos(\frac{5\pi}{6}-\theta)(\frac{1}{2\cos\theta}\frac{dy}{dt})$	1M	
$\frac{dx}{dt} = -\frac{\cos(\frac{5\pi}{6}-\theta)}{\cos\theta}\frac{dy}{dt}$	1	
<u>Alternative solution</u>		
$\frac{dx}{d\theta} = -2\cos(\frac{5\pi}{6}-\theta)$	1A	
$\frac{dy}{d\theta} = 2\cos\theta$		
$\frac{dx}{dy} = \frac{dx}{d\theta}/\frac{dy}{d\theta}$	1M	
$= -\frac{\cos(\frac{5\pi}{6}-\theta)}{\cos\theta}$		
$\frac{dx}{dt} = \frac{dx}{dy}\frac{dy}{dt}$	1M	
$\therefore \frac{dx}{dt} = -\frac{\cos(\frac{5\pi}{6}-\theta)}{\cos\theta}\frac{dy}{dt}$	1	
	4	
(d) As the rod moves from its initial position to O, $\frac{dy}{dt} < 0$.	1	For $\frac{dy}{dt} < 0$
$\cos(\frac{5\pi}{6}-\theta) > 0$ when $\frac{\pi}{3} < \theta < \frac{4\pi}{9}$, and < 0 when $0 < \theta < \frac{\pi}{3}$.	1	No need to specify the ranges of values of θ
<u>OR</u> $\cos(\frac{5\pi}{6}-\theta)$ is first positive and later becomes negative.	1	
$\frac{dx}{dt} = \frac{-\cos(\frac{5\pi}{6}-\theta)}{\cos\theta}\frac{dy}{dt} > 0$ for $\frac{\pi}{3} < \theta < \frac{4\pi}{9}$, and < 0 for $0 < \theta < \frac{\pi}{3}$.		<u>OR</u> is first positive and later becomes negative
Hence end A first moves away from O (as $\frac{dx}{dt} > 0$) and then moves towards O (as $\frac{dx}{dt} < 0$). So the student is correct.	1	'Yes' without explanation – no marks
	3	

香港考試局
HONG KONG EXAMINATIONS AUTHORITY

一九九八年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1998

附加數學 試卷二
ADDITIONAL MATHEMATICS PAPER II

本評卷參考乃考試局專爲今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視爲標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations Authority for markers' reference. The Examinations Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Examinations Authority is counting on the co-operation of markers/teachers in this regard.

考試結束後，各科評卷參考將存放於教師中心，供教師參閱。
After the examinations, marking schemes will be available for reference at the Teachers' Centres.

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GENERAL INSTRUCTIONS TO MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates would use alternative methods not specified in the marking scheme. Markers should be patient in marking these alternative solutions. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method is specified in the question.

2. In the marking scheme, marks are classified as follows :

   'M' marks – awarded for knowing a correct method of solution and attempting to apply it;

   'A' marks – awarded for the accuracy of the answer;

   Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. The symbol (pp–1) should be used to denote marks deducted for poor presentation (p.p.). Note the following points:

   (a) At most deduct 1 mark for p.p. in each question, up to a maximum of 3 marks for the whole paper.

   (b) For similar p.p., deduct only 1 mark for the first time that it occurs, i.e. do not penalise candidates twice in the whole paper for the same p.p.

   (c) In any case, do not deduct any marks for p.p. in those steps where candidates could not score any marks.

   (d) Some cases in which marks should be deducted for p.p. are specified in the marking scheme. However, the lists are by no means exhaustive. Markers should exercise their professional judgement to give p.p.s in other situations.

5. The symbol (u–1) should be used to denote marks deducted for wrong/no units in the final answers (if applicable). Note the following points:

   (a) At most deduct 1 mark for wrong/no units for the whole paper.

   (b) Do not deduct any marks for wrong/no units in case candidate's answer was already wrong.

6. Marks entered in the Page Total Box should be the net total score on that page.

7. In the Marking Scheme, steps which can be omitted are enclosed by dotted rectangles [dotted rectangle], whereas alternative answers are enclosed by solid rectangles [solid rectangle].

8. Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.

9. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

10. Unless the form of answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they were correct.

Solution	Marks	Remarks
1. General term $= {}_6C_r\,(x)^{6-r}(\frac{-2}{x})^r$	2A	
$= {}_6C_r\,(-2)^r\,x^{6-2r}$		
$6-2r=2$	1M	
$r=2$		
$\therefore$ coefficient of $x^2 = {}_6C_2\,(-2)^2$		
$=60$	1A	
Alternative solution		
$(x-\frac{2}{x})^6 =$ [$x^6 + {}_6C_1\,x^5(-\frac{2}{x}) +$] ${}_6C_2\,x^4(-\frac{2}{x})^2 + \ldots$	1A	For ${}_6C_2\,x^4(-\frac{2}{x})^2$
	1A	For other terms (can be omitted) Omit dots (pp–1)
Coefficient of $x^2 = {}_6C_2\,(-2)^2$	1M	For choosing the correct term
$= 60$	1A	
	4	
2. (a) The centre is (2, –5).		
Distance $= \left\lvert \frac{2-7(-5)+3}{\sqrt{1^2+(-7)^2}} \right\rvert$	1M	Accept omitting absolute sign
$=4\sqrt{2}$	1A	Accept equivalent forms
(b) If L is a tangent to C,		
$4\sqrt{2}=\sqrt{a}$	1M	
$a=32$.	1A	
Alternative solution		
Substitute $x=7y-3$ into C.		
$(7y-3-2)^2+(y+5)^2=a$		OR
$50y^2-60y+(50-a)=0$		$50x^2-120x+(1640-49a)=0$
$\Delta=(-60)^2-4(50)(50-a)=0$	1M	$\Delta=(-120)^2-4(50)(1640-49a)=0$
$a=32$.	1A	
	4	

Solution	Marks	Remarks
3. For $n=1$, LHS $= 1\times 2 = 2$.		
RHS $= 2^1 \times 1 = 2 =$ LHS.	1	
$\therefore$ the statement is true for $n=1$.		
Assume $1\times 2 + 2\times 3 + 2^2\times 4 + \ldots + 2^{k-1}(k+1) = 2^k(k)$	1	
for some +ve integer k.		
Then $1\times 2 + 2\times 3 + 2^2\times 4 + \ldots + 2^{k-1}(k+1) + 2^k(k+2)$		
$= 2^k(k) + 2^k(k+2)$	1	
$= 2^k(k+k+2)$		
$= 2^{k+1}(k+1)$	1	
The statement is also true for $n=k+1$ if it is true for $n=k$. By the principle of mathematical induction,		
the statement is true for all positive integers n.	1	
	5	
4. $\frac{dy}{dx} = \cos^2 x$		
$y = \int \cos^2 x \, dx$	1A	Withhold this mark if "$y =$" is omitted. Omit dx (pp−1)
$= \int \frac{1}{2}(1+\cos 2x)\, dx$	1A	
$= \frac{x}{2} + \frac{\sin 2x}{4} + c$ c is a constant.	1A	Awarded even if c is omitted
Put $x = \frac{\pi}{2}$, $y = \pi$. $\pi = \frac{\pi}{4} + \frac{\sin\pi}{4} + c$	1M	For finding c
$c = \frac{3\pi}{4}$.	1A	
$\therefore$ the equation of the curve is $y = \frac{x}{2} + \frac{\sin 2x}{4} + \frac{3\pi}{4}$.		
	5	

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Solution	Marks	Remarks
5. (a) The equation is $(2x+y-3)+k(x-3y+1)=0$. (k is real)	1A	OR $(2+k)x+(1-3k)y-(3-k)=0$
(b) (i) Substitute (0, 0) into the equation in (a). $-3+k(1)=0$	1M	
$k=3$		
$\therefore$ the equation of L is $2x+y-3+3(x-3y+1)=0$		
$5x-8y=0$.	1A	
(ii) Slope of $L=\frac{5}{8}$. Slope of $L_1=-2$.	1A	For either one of them
Let θ be the acute angle between L and L_1.		
$\tan\theta=\left\|\dfrac{\frac{5}{8}-(-2)}{1+(\frac{5}{8})(-2)}\right\|$	1M	Accept omitting absolute sign
$=\left\|-\frac{21}{2}\right\|$		
$\theta=85°$ (correct to the nearest degree)	1A	
Alternative solution (1)		
(a) $(x-3y+1)+\lambda(2x+y-3)=0$ (λ is real)	1A	OR $(1+2\lambda)x-(3-\lambda)y+(1-3\lambda)=0$
(b) (i) Substitute (0, 0) into the equation in (a). $1+\lambda(-3)=0$	1M	
$\lambda=\frac{1}{3}$		
The equation of L is $(x-3y+1)+\frac{1}{3}(2x+y-3)=0$		
$5x-8y=0$.	1A	
(ii) Same as above		
Alternative solution (2)		
(a) $\begin{cases}2x+y-3=0\\x-3y+1=0\end{cases}$		
Solving the 2 equations, $x=\frac{8}{7}$, $y=\frac{5}{7}$.		
$\therefore$ The coordinates of P are $(\frac{8}{7},\frac{5}{7})$.		
The equation of the family of straight lines through P is $\dfrac{y-\frac{5}{7}}{x-\frac{8}{7}}=m$ (m is real)	1A	
$7mx-7y+5-8m=0$.		

Solution	Marks	Remarks
(b) (i) Substitute (0, 0) into the equation in (a).		
$0-0+5-8m=0$	1M	
$m=\frac{5}{8}$		
$\therefore$ the equation of L is		
$7(\frac{5}{8})x-7y+5-8(\frac{5}{8})=0$		
$5x-8y=0$.	1A	
(ii) Same as above		
	6	
6. $du=\cos\theta\, d\theta$	1A	
$\int_0^{\frac{\pi}{2}}\cos^5\theta\sin^2\theta\, d\theta = \int_0^{\frac{\pi}{2}}(1-\sin^2\theta)^2\sin^2\theta\cos\theta\, d\theta$		
$=\int_0^1(1-u^2)^2u^2\, du$	1A	For the integrand Omit du(pp−1)
$=\int_0^1(u^2-2u^4+u^6)\, du$	1A	For the integrand
$=\left[\frac{u^3}{3}-\frac{2u^5}{5}+\frac{u^7}{7}\right]_0^1$	1A+1A	1A for the primitive function, 1A for limits
$=\frac{1}{3}-\frac{2}{5}+\frac{1}{7}$		
$=\frac{8}{105}$	1A	
	6	

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Solution	Marks	Remarks
7. $\sin(3x+\frac{\pi}{4})\cos(3x-\frac{\pi}{4})$		
$=\frac{1}{2}\{\sin[(3x+\frac{\pi}{4})+(3x-\frac{\pi}{4})]+\sin[(3x+\frac{\pi}{4})-(3x-\frac{\pi}{4})]\}$	1A	
$=\frac{1}{2}(\sin 6x+\sin\frac{\pi}{2})$	1A	
$=\frac{1}{2}(1+\sin 6x)$	1	
<u>Alternative solution</u>		
$\sin(3x+\frac{\pi}{4})\cos(3x-\frac{\pi}{4})$		
$=(\sin 3x\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\cos 3x)(\cos 3x\cos\frac{\pi}{4}+\sin 3x\sin\frac{\pi}{4})$	1A	
$=\frac{\sqrt{2}}{2}(\cos 3x+\sin 3x)\frac{\sqrt{2}}{2}(\cos 3x+\sin 3x)$	1A	
$=\frac{1}{2}(\cos^2 3x+2\sin 3x\cos 3x+\sin^2 3x)$		
$=\frac{1}{2}(1+\sin 6x)$	1	
$\sin(3x+\frac{\pi}{4})\cos(3x-\frac{\pi}{4})=\frac{3}{4}$		
$\frac{1}{2}(1+\sin 6x)=\frac{3}{4}$		
$\sin 6x=\frac{1}{2}$	1A	
$6x=n\pi+(-1)^n\frac{\pi}{6}$ n is an integer	1M	For $6x=n\pi+(-1)^n\alpha$
$x=\frac{n\pi}{6}+(-1)^n\frac{\pi}{36}$ (<u>OR</u> $x=30n°+(-1)^n 5°$)	1A	$\frac{n\pi}{6}+(-1)^n 5°$ etc. $(u-1)$
	6	
8. (a) $S_1=\int_1^2(3x-2-x^2)\,dx$	1M+1A	1M for area $=\int_a^b(y_1-y_2)\,dx$, 1A for limits. Omit dx(pp−1)
$=[\frac{3x^2}{2}-2x-\frac{x^3}{3}]_1^2$	1A	For primitive function only
$=(6-4-\frac{8}{3})-(\frac{3}{2}-2-\frac{1}{3})$		
$=\frac{1}{6}$	1A	
<u>Alternative solution</u>		
$S_1=\int_1^4(y^{\frac{1}{2}}-\frac{y+2}{3})\,dy$	1M+1A	1M for area $=\int_a^b(x_1-x_2)dy$, 1A for limits. Omit dy (pp−1)
$=[\frac{2}{3}y^{\frac{3}{2}}-\frac{1}{6}y^2-\frac{2}{3}y]_1^4$	1A	For primitive function only
$=\frac{1}{6}$	1A	
(b) Expressions (II) and (III) represent the total area S_1+S_2.	1A+1A	Deduct 1 mark for each wrong answer, up to zero
	6	

Solution	Marks	Remarks
9. (a) (i) Let $x=-y$.	1A	
$dx=-dy$		
$\int_{-a}^{0} f(x)dx=\int_{a}^{0} f(-y)(-dy)$		
$=\int_{0}^{a} f(-y)dy=\int_{0}^{a} f(-x)dx$	1	
(ii) $\int_{-a}^{a} f(x)dx=\int_{-a}^{0} f(x)dx+\int_{0}^{a} f(x)dx$	1A	
$=\int_{0}^{a} f(-x)dx+\int_{0}^{a} f(x)dx$ (using (i))	1A	For the 1st term
$=\int_{0}^{a} f(x)dx+\int_{0}^{a} f(x)dx$ $\because f(x)=f(-x)$		
$=2\int_{0}^{a} f(x)dx$	1	
	5	
(b) $dt=\frac{\sqrt{3}}{3}\sec^2\theta\, d\theta$		
$\int_0^1 \frac{dt}{1+3t^2}=\int_0^{\frac{\pi}{3}} \frac{\sqrt{3}}{3}\frac{\sec^2\theta d\theta}{(1+\tan^2\theta)}$	1A+1A	1A for integrand, 1A for limits
$=\frac{\sqrt{3}}{3}\int_0^{\frac{\pi}{3}} d\theta$		
$=\frac{\sqrt{3}\pi}{9}$	1	
	3	
(c) (i) (1) $I_1+4I_2=\int_0^1 \frac{1-t^2}{1+3t^2}dt+4\int_0^1 \frac{t^2}{1+3t^2}dt$		
$=\int_0^1 dt$		
$=1$	1	
(2) $I_1+I_2=\int_0^1 \frac{1}{1+3t^2}dt$		
$=\frac{\sqrt{3}\pi}{9}$ (by result of (b))	1A	
(ii) $\begin{cases} I_1+4I_2=1 & \text{------ (1)} \\ I_1+I_2=\frac{\sqrt{3}\pi}{9} & \text{------ (2)} \end{cases}$		
(1) − (2) $3I_2=1-\frac{\sqrt{3}\pi}{9}$	1M	For eliminating I_1
$I_2=\frac{1}{3}(1-\frac{\sqrt{3}\pi}{9})$	1A	

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Solution	Marks	Remarks
Alternative solution		
$I_2 = \int_0^1 \frac{t^2}{1+3t^2}\,dt$		
$= \int_0^1 \frac{1}{3}\left(\frac{1+3t^2-1}{1+3t^2}\right)dt$		
$= \frac{1}{3}\int_0^1 dt - \frac{1}{3}\int_0^1 \frac{1}{1+3t^2}\,dt$	1M	
$= \frac{1}{3} - \frac{1}{3}\left(\frac{\sqrt{3}\pi}{9}\right) = \frac{1}{3} - \frac{\sqrt{3}\pi}{27}$	1A	
	4	
(d) $\int_{-1}^1 \frac{1+t^2}{1+3t^2}dt = 2\int_0^1 \frac{1+t^2}{1+3t^2}dt$ $\left[\because \frac{1+t^2}{1+3t^2} = \frac{1+(-t)^2}{1+3(-t)^2} \text{ for all } t\right]$	1M	For using (a) (ii)
$= 2\int_0^1 \frac{1}{1+3t^2}dt + 2\int_0^1 \frac{t^2}{1+3t^2}dt$	1M+1A	
$= 2\left(\frac{\sqrt{3}\pi}{9}\right) + 2\left(\frac{1}{3}\right)\left(1 - \frac{\sqrt{3}\pi}{9}\right)$		
$= \frac{2}{3} + \frac{4\sqrt{3}\pi}{27}$	1A	
Alternative solution		
$\int_{-1}^1 \frac{1+t^2}{1+3t^2}dt = \int_{-1}^1 \left(1 - \frac{2t^2}{1+3t^2}\right)dt$		
$= \int_{-1}^1 dt - 2\int_{-1}^1 \frac{t^2}{1+3t^2}dt$	1M	
$= 2 - 4\int_0^1 \frac{t^2}{1+3t^2}dt$ $\left[\because \frac{t^2}{1+3t^2} = \frac{(-t)^2}{1+3(-t)^2} \text{ for all } t\right]$	1M+1A	1M for using (a) (ii)
$= 2 - 4\left(\frac{1}{3}\right)\left(1 - \frac{\sqrt{3}\pi}{9}\right)$		
$= \frac{2}{3} + \frac{4\sqrt{3}\pi}{27}$	1A	
	4	

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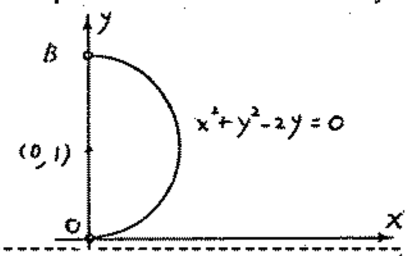
Solution	Marks	Remarks
10. (a) Coordinates of A are $(2p, 0)$. Coordinates of B are $(0, 2)$.	1A	
Since $BC : CA = 1 : p^2$,		
$x_0 = \frac{2p+0}{1+p^2} = \frac{2p}{1+p^2}$	1M+1A	1M for division formula
$y_0 = \frac{2p^2+0}{1+p^2} = \frac{2p^2}{1+p^2}$	1A	
	4	
(b) $\frac{y_0}{x_0} = \frac{2p^2 / p^2+1}{2p / p^2+1} = p$	1	
Put $p = \frac{y_0}{x_0}$,		
$x_0 = \frac{2(\frac{y_0}{x_0})}{(\frac{y_0}{x_0})^2+1}$ OR $y_0 = \frac{2(\frac{y_0}{x_0})^2}{(\frac{y_0}{x_0})^2+1}$	2M	OR $x_0 + (\frac{y_0}{x_0})y_0 - 2(\frac{y_0}{x_0}) = 0$
${x_0}^2 + {y_0}^2 - 2y_0 = 0$	1A	
$\therefore$ the equation of the locus of D is $x^2 + y^2 - 2y = 0$.		
	2A	Accept including O and B (Circle : 1A only)
	1A	For labelling the centre
	7	(pp–1) for not labelling the axes
(c) Area $S = \frac{1}{2}(2)(x_0)$	1A	
Area of ΔOBC is greatest when $x_0 = 1$.	1M	
$\therefore \frac{2p}{p^2+1} = 1$	1M	
$(p-1)^2 = 0$		
$p = 1$	1A	
$\therefore$ the coordinates of A are (2, 0).	1A	
Alternative solution		
Area $S = \frac{1}{2}(2)(x_0)$ $= \frac{2p}{p^2+1}$	1A	
$\frac{dS}{dp} = \frac{2(p^2+1) - 2p(2p)}{(p^2+1)^2}$ $= \frac{2(1-p^2)}{(p^2+1)^2}$	1M	OR differentiating x_0
$\frac{dS}{dp} = 0$ when $p = 1$.	1A	
As $\frac{dS}{dp} > 0$ when $p < 1$ and $\frac{dS}{dp} < 0$ when $p > 1$,	1M	For checking
$\therefore$ S is greatest at $p = 1$. $\therefore$ the coordinates of A are (2, 0).	1A	Withhold this mark if checking was omitted.
	5	

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Solution	Marks	Remarks
11. (a) (i) Since S lies on E,		
$\frac{a^2}{4}+\frac{b^2}{3}=1$		
$3a^2+4b^2=12$	1	
(ii) $\frac{x^2}{4}+\frac{y^2}{3}=1$		
Differentiating w.r.t. x,		
$\frac{x}{2}+\frac{2y}{3}\frac{dy}{dx}=0$	1M	
$\frac{dy}{dx}=\frac{-3x}{4y}$		
$m_1=-\frac{3a}{4b}$	1A	
$m_2=-\frac{1}{m_1}=\frac{4b}{3a}$	1A	
Alternative solution		
Using the formula $\frac{xx_1}{4}+\frac{yy_1}{3}=1$, equation of tangent to E at S is		
$\frac{a}{4}x+\frac{b}{3}y=1$	1A	
$3ax+4by=12$		
$m_1=-\frac{3a}{4b}$	1A	
$m_2=-\frac{1}{m_1}=\frac{4b}{3a}$	1A	
	4	
(b) (i) Substituting $y=mx+\frac{c}{m}$ into P,		OR $x=\frac{y^2}{4c}$
$(mx+\frac{c}{m})^2=4cx$	1M	$y=m(\frac{y^2}{4c})+\frac{c}{m}$
$m^2x^2-2cx+\frac{c^2}{m^2}=0$		$m^2y^2-4mcy+4c^2=0$
$\Delta=(2c)^2-4m^2(\frac{c^2}{m^2})$ $=0$	1M	$\Delta=(4mc)^2-4m^2(4c^2)$ $=0$
$\therefore\ y=mx+\frac{c}{m}$ is a tangent to P.	1	
(ii) If $y=mx+\frac{c}{m}$ passes through S,		
$b=ma+\frac{c}{m}$		
$am^2-bm+c=0$ ----(*)	1	

Solution	Marks	Remarks
(iii) (1) m_1, m_2 are the roots of the equation (*)		
$\therefore m_1 + m_2 = \frac{b}{a}$	1A	
$m_1 m_2 = \frac{c}{a}$	1A	
From (a) (ii), $m_1 = -\frac{3a}{4b}$, $m_2 = \frac{4b}{3a}$.		
$\frac{-3a}{4b} + \frac{4b}{3a} = \frac{b}{a}$	1M	
$\frac{-9a^2 + 16b^2}{12ab} = \frac{b}{a}$		
$-9a^2 + 16b^2 = 12b^2$		
$9a^2 = 4b^2$	1	
Since L_1 and L_2 are perpendicular,		
$m_1 m_2 = \frac{c}{a} = -1$		
$\therefore c = -a$	1	
(2) $\begin{cases} 3a^2 + 4b^2 = 12 \\ 9a^2 = 4b^2 \end{cases}$		
$3a^2 + 4(\frac{9a^2}{4}) = 12$	1M	For eliminating b
$12a^2 = 12$		
$a = -1$ ($\because a < 0$)	1A	
$\therefore c = -a = 1$		
$\therefore$ the equation of P is $y^2 = 4x$.	1A	
	12	

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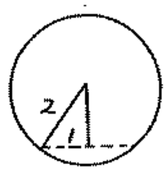
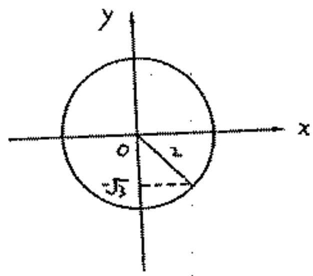
Solution	Marks	Remarks
12. (a) Volume $= \int_{-k}^{k} \pi x^2 \mathrm{d}y$	1M	For $V = \pi \int_a^b x^2 \mathrm{d}y$ Omit dy(pp−1)
$= \int_{-k}^{k} 4\pi(1 - \frac{y^2}{a^2})\mathrm{d}y$	1A	OR $= 2\int_0^k 4\pi(1 - \frac{y^2}{a^2})\mathrm{d}y$
$= 4\pi\left[y - \frac{y^3}{3a^2}\right]_{-k}^{k}$	1A	For primitive function only
$= 4\pi\left[k - \frac{k^3}{3a^2} + k - \frac{k^3}{3a^2}\right]$		
$= 8k(1 - \frac{k^2}{3a^2})\pi$	1	
	4	
(b) (i) Put $x = 1, y = k$ into $\frac{x^2}{4} + \frac{y^2}{a^2} = 1$.		OR Put $x = -1, y = k$.
$\frac{1}{4} + \frac{k^2}{a^2} = 1$	1A	
$k^2 = \frac{3a^2}{4}$		
$k = \frac{\sqrt{3}a}{2}$	1A	$k = \pm\frac{\sqrt{3}a}{2}$ (pp−1)
Height of $S_1 = 2k$ $= \sqrt{3}a$	1	
(ii) Put $k = \frac{\sqrt{3}a}{2}$,		
Volume of $S_1 = 8(\frac{\sqrt{3}a}{2})[1 - \frac{1}{3a^2}(\frac{\sqrt{3}a}{2})^2]\pi$	1M	
$= 3\sqrt{3}a\pi$.	1A	
	5	
(c) (i) Height of S_2		
$= 2 + \sqrt{2^2 - 1^2}$		
$= 2 + \sqrt{3}$	1A	
∴ height of toy $\sqrt{3}a + 2 + \sqrt{3} = 2 + (a+1)\sqrt{3}$	1	
(ii) The ellipse becomes a circle of radius 2 when $a = 2$ Using (b) (ii) and put $a = 2$, Volume of portion of the sphere from $y = -\sqrt{3}$ to $y = \sqrt{3}$		
$= 3\sqrt{3}(2)\pi$	1M+1A	
$= 6\sqrt{3}\pi$		

Solution	Marks	Remarks
Volume of $S_2 = \frac{1}{2}(\frac{4}{3}\pi(2)^3) + \frac{1}{2}(6\sqrt{3})\pi$	1M	OR $= \frac{4}{3}\pi(2)^3 - \frac{1}{2}[\frac{4}{3}\pi(2)^3 - 6\sqrt{3}\pi]$
$= \frac{16\pi}{3} + 3\sqrt{3}\pi$	1A	
Alternative solution (1)		
Volume of S_2		
$= \pi \int_{-\sqrt{3}}^{2} x^2\,dy$, where $x^2 + y^2 = 4$	1M+1A	1M for $V = \int_a^b \pi x^2 dy$ 1A for $x^2 + y^2 = 4$
$= \pi \int_{-\sqrt{3}}^{2} (4 - y^2)\,dy$	1A	
$= \pi \left[4y - \frac{y^3}{3}\right]_{-\sqrt{3}}^{2}$		
$= \pi (8 - \frac{8}{3} + 4\sqrt{3} - \sqrt{3})$		
$= \pi (\frac{16}{3} + 3\sqrt{3})$	1A	
Alternative solution (2)		
Volume of S_2		
$= \frac{2}{3}\pi(2)^3 + \pi \int_{-\sqrt{3}}^{0} x^2\,dy$, where $x^2 + y^2 = 4$	1M+1A	1M for $V = \int_a^b \pi x^2 dy$ 1A for $x^2 + y^2 = 4$
$= \frac{16\pi}{3} + \pi \int_{-\sqrt{3}}^{0} (4 - y^2)\,dy$	1A	
$= \frac{16\pi}{3} + \pi \left[4y - \frac{y^3}{3}\right]_{-\sqrt{3}}^{0}$		
$= \frac{16\pi}{3} + \pi (4\sqrt{3} - \sqrt{3})$		
$= \frac{16\pi}{3} + 3\sqrt{3}\pi$	1A	
Volume of toy = Volume of S_1 + Volume of S_2		
$= 3\sqrt{3}a\pi + (\frac{16\pi}{3} + 3\sqrt{3})\pi$		
$= \frac{16\pi}{3} + 3\sqrt{3}(a+1)\pi$	1A	
	7	

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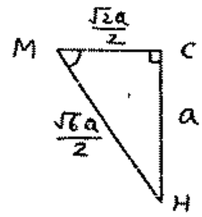
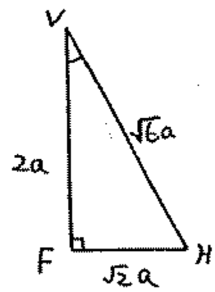
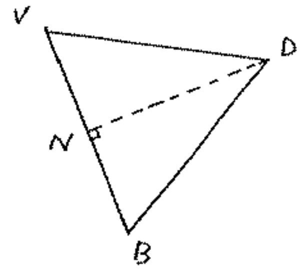
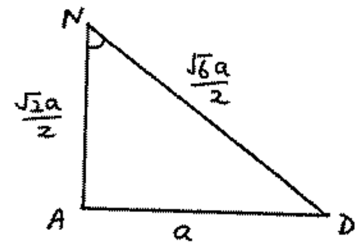
Solution	Marks	Remarks
13. (a) (i) $CM=\frac{1}{2}\sqrt{a^2+a^2}$		
$=\frac{\sqrt{2}}{2}a$ (OR $\frac{a}{\sqrt{2}}$)	1A	
(ii) The angle between the 2 lines is $\angle CMH$.	1A	(can be omitted)
$\tan\angle CMH=\frac{CH}{CM}$		
$=\frac{a}{\sqrt{2}a/2}$	1M	
$=\sqrt{2}$		
$\angle CMH=55°$ (correct to the nearest degree)	1A	
	4	
(b) (i) $\sin\angle FVH=\frac{FH}{VH}$		
$=\frac{\sqrt{2}a}{\sqrt{(2a)^2+(\sqrt{2}a)^2}}$		
$=\frac{\sqrt{3}}{3}$	1	
Perpendicular distance from F to $BVDH$		
$=VF\sin\angle FVH$		
$=2a(\frac{\sqrt{3}}{3})$	1M	
$=\frac{2\sqrt{3}a}{3}$ (OR $=\frac{2a}{\sqrt{3}}$)	1A	
<u>Alternative solution</u>		
Consider area of ΔVFH. Let h be the perpendicular distance.		
$\frac{1}{2}(VF)(FH)=\frac{1}{2}(VH)h$		
$\frac{1}{2}(2a)(\sqrt{2}a)=\frac{1}{2}(\sqrt{6}a)(h)$	1M	
$h=\frac{2\sqrt{3}}{3}a$	1A	

Solution	Marks	Remarks
(ii) (1) $BN=\frac{1}{2}BV$		
$=\frac{\sqrt{2}a}{2}$	1A	
$DN=\sqrt{BD^2-BN^2}$		
$=\sqrt{(\sqrt{2}a)^2-(\frac{\sqrt{2}a}{2})^2}$	1M	
$=\frac{\sqrt{6}a}{2}$	1A	
Alternative solution		
$BD=VD=VB=\sqrt{2}\,a$ $\therefore\ \angle VBD=60°$	1A	
$DN=\sqrt{2}\,a\sin 60°$	1M	
$=\frac{\sqrt{6}\,a}{2}$	1A	
(2) The angle between the 2 faces is $\angle AND$	1A	(can be omitted)
$AN=a\sin 45°=\frac{\sqrt{2}\,a}{2}$	1A	
$\cos\angle AND=\frac{(AN)^2+(ND)^2-(AD)^2}{2(AN)(ND)}$		
$=\frac{(\sqrt{2}a/2)^2+(\sqrt{6}a/2)^2-a^2}{2(\sqrt{2}a/2)(\sqrt{6}a/2)}$	1M	OR $\angle AND=\angle CMH$
$=\frac{1}{\sqrt{3}}$		
$\angle AND=55°$ (correct to the nearest degree)	1A	
Alternative solution		
(2) The angle between the 2 faces is $\angle AND$	1A	
$\angle NAD=90°$	1A	
$\sin\angle AND=\frac{AD}{ND}$		
$=\frac{a}{\sqrt{6}a/2}$	1M	
$=\frac{\sqrt{6}}{3}$		
$\angle AND=55°$ (correct to the nearest degree)	1A	

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Solution	Marks	Remarks
(iii) As the faces *BHD* and *BVD* lie on the same plane and the faces *ABGF* and *BVA* lie on the same plane, the angle between the two faces equals to the angle between the faces *BVA* and *BVD*, i.e. $\angle AND$. So the student is correct.	2	'Correct' without explanation – no mark
Alternative solution As *BV* is a line of intersection of the two faces, and *AN* and *DN* are both perpendicular to *BV*, so the angle between the two faces is $\angle AND$. So the student is correct.	2	
	12	