

只限教師參閱

FOR TEACHERS' USE ONLY

香港考試局

HONG KONG EXAMINATIONS AUTHORITY

一九九七年香港中學會考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1997

附加數學 試卷一

ADDITIONAL MATHEMATICS PAPER I

本評卷參考乃考試局專為今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視為標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations Authority for markers' reference. The Examinations Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Examinations Authority is counting on the co-operation of markers/teachers in this regard.

考試結束後，各科評卷參考將存放於教師中心，供教師參閱。

After the examinations, marking schemes will be available for reference at the Teachers' Centres.

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97-CE-A MATHS I-1



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GENERAL INSTRUCTIONS TO MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method is specified in the question.
2. In the marking scheme, marks are classified as follows :

'M' marks – awarded for knowing a correct method of solution and attempting to apply it;

'A' marks – awarded for the accuracy of the answer;

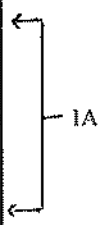
Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

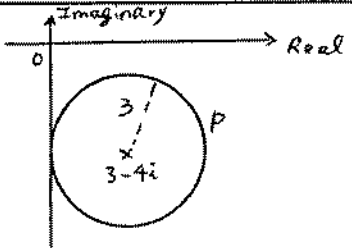
In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answer should **NOT** be awarded. Unless otherwise specified, no marks in the marking scheme are subdivisible.
3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.
4. The symbol $\textcircled{\text{pp-1}}$ should be used to denote marks deducted for poor presentation (p.p.). Note the following points:
 - (a) At most deduct 1 mark for p.p. in each question, up to a maximum of 3 marks for the whole paper.
 - (b) For similar p.p., deduct only 1 mark for the first time that it occurs, i.e. do not penalise candidates twice in the whole paper for the same p.p.
 - (c) In any case, do not deduct any marks for p.p. in those steps where candidates could not score any marks.
 - (d) Some cases in which marks should be deducted for p.p. are specified in the marking scheme. However, the lists are by no means exhaustive. Markers should exercise their professional judgement to give p.p.s in other situations.
5. The symbol $\textcircled{\text{u-1}}$ should be used to denote marks deducted for wrong/no units in the final answers (if applicable). Note the following points :
 - (a) At most deduct 1 mark for wrong/no units for the whole paper.
 - (b) Do not deduct any marks for wrong/no units in case candidate's answer was already wrong.
6. Marks entered in the Page Total Box should be the net total score on that page.
7. In the Marking Scheme, steps which can be omitted are enclosed by dotted rectangles , whereas alternative answers are enclosed by solid rectangles
8. Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.
9. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

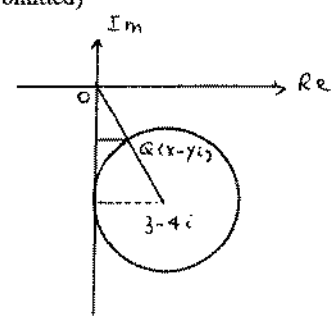
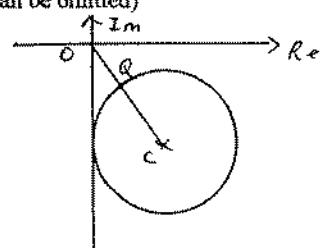
97-CE-A MATHS I-2

Solution .	Marks	Remarks
<p>3. (a) $\frac{1+i}{1-i} = \frac{1+i}{1-i} \cdot \frac{1+i}{1+i}$ $= i$</p> <p>(b) $(1+i)^{2n} = (1-i)^{2n}$ $\left(\frac{1+i}{1-i}\right)^{2n} = 1$ $i^{2n} = 1$ $n = 2k$, where k is a positive integer. (OR $n = 2, 4, 6, \dots$; OR n is an even positive integer.)</p>	<p>1M</p> <p>1A</p> <p>1A</p> <p>1M</p> <p>1A</p>	<p>k not defined – no marks Include 0 – no marks</p>
<p><u>Alternative solution</u></p> <p>(b) $(1+i)^{2n} = [\sqrt{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})]^{2n} = (\sqrt{2})^{2n}(\cos\frac{n\pi}{2} + i\sin\frac{n\pi}{2})$ $(1-i)^{2n} = (\sqrt{2})^{2n}[\cos(-\frac{n\pi}{2}) + i\sin(-\frac{n\pi}{2})] = (\sqrt{2})^{2n}(\cos\frac{n\pi}{2} - i\sin\frac{n\pi}{2})$ $(1+i)^{2n} = (1-i)^{2n}$ $(\sqrt{2})^{2n}(\cos\frac{n\pi}{2} + i\sin\frac{n\pi}{2}) = (\sqrt{2})^{2n}(\cos\frac{n\pi}{2} - i\sin\frac{n\pi}{2})$ $\sin\frac{n\pi}{2} = 0$ $n = 2k$, where k is a positive integer.</p>	<p>1M</p> <p>1A</p> <p>1A</p>	
	<p>5</p>	
<p>4. $h = 30 \tan \theta$</p> <p>Differentiate with respect to time t,</p> <p>$\frac{dh}{dt} = 30 \sec^2 \theta \frac{d\theta}{dt}$</p> <p>when $h = 30\sqrt{3}$, $\tan \theta = \sqrt{3}$ $\theta = \frac{\pi}{3}$ (OR 60°)</p> <p>$15 = 30 \sec^2(\frac{\pi}{3}) \frac{d\theta}{dt}$</p> <p>$\frac{d\theta}{dt} = \frac{1}{80} \text{s}^{-1}$ (OR rad s^{-1})</p> <p>\therefore The rate of change of θ is $\frac{1}{80} \text{s}^{-1}$.</p>	<p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>5</p>	<p>For chain rule</p> <p>OR $\sec^2 \theta = 1 + (\sqrt{3})^2 = 4$</p> <p>For substitution</p> <p>OR 0.0125 s^{-1}</p>

Solution	Marks	Remarks
5. $ 3x-4 < 2$ $-2 < 3x-4 < 2$ $\frac{2}{3} < x < 2$	1A 1A	
<u>Alternative solution</u> $ 3x-4 < 2$ $(3x-4)^2 < 4$ $3x^2 - 8x + 4 < 0$ $(3x-2)(x-2) < 0$ $\frac{2}{3} < x < 2$	1A 1A	
$\frac{1}{2x-1} \leq 1$ $\frac{1}{2x-1} - 1 \leq 0$ $\frac{2-2x}{2x-1} \leq 0$ $x \geq 1$ or $x < \frac{1}{2}$	1M 1A 1A	
<u>Alternative solution (1)</u> $\frac{1}{2x-1} \leq 1$ $\frac{1}{2x-1} (2x-1)^2 \leq (2x-1)^2$ $2x-1 \leq 4x^2 - 4x + 1$ $2x^2 - 3x + 1 \geq 0$ $(2x-1)(x-1) \geq 0$ $x \geq 1$ or $x < \frac{1}{2}$	<div style="border: 1px dashed black; padding: 5px; display: inline-block;"> and $x \neq \frac{1}{2}$ and $x \neq \frac{1}{2}$ </div> 1M 1A 1A	

Solution	Marks	Remarks
<p><u>Alternative solution (2)</u></p> <p>Consider the following cases (i) $x > \frac{1}{2}$, (ii) $x < \frac{1}{2}$.</p> <p>Case 1: $x > \frac{1}{2}$ $1 \leq 2x - 1$ $x \geq 1$ Since $x > \frac{1}{2}$, $\therefore x \geq 1$.</p> <p>Case 2: $x < \frac{1}{2}$ $1 \geq 2x - 1$ $x \leq 1$ Since $x < \frac{1}{2}$, $\therefore x < \frac{1}{2}$.</p> <p>Combining the 2 cases, $x \geq 1$ or $x < \frac{1}{2}$.</p>	<p>1M</p>  <p>1A</p>	<p>Awarded even if equality sign is included.</p>
<p><u>Alternative solution (3)</u></p> $\frac{1}{2x-1} \leq 1$ <p>$2x-1 \geq 1$ or $2x-1 < 0$ $x \geq 1$ or $x < \frac{1}{2}$</p>	<p>2A</p> <p>1A</p>	
<p>Combining the two solutions, $1 \leq x < 2$.</p>	<p>1A</p> <hr/> <p>6</p>	

	Solution	Marks	Remarks
6. (a)		1A 1A 1A	For a circle For centre at $(3 - 4i)$ For radius = 3 (pp-1) for not labelling the axes
(b)	<p>The point Q and the centre of the circle C lie on the same line through O.</p> $OC = \sqrt{3^2 + (-4)^2} = 5$ $OQ = 5 - 3 = 2$ $\therefore OQ : QC = 2 : 3$ <p>The complex number represented by Q</p> $= \frac{2}{5}(3 - 4i)$ $= \frac{6}{5} - \frac{8}{5}i$	1A 1M 1A	(can be omitted)
	<p><u>Alternative solution (1)</u></p> <p>The point Q and the centre of the circle lie on the same line through O.</p> <p>Let the point Q represent the complex number $x - yi$.</p> <p>By similar triangles, $\frac{x}{3} = \frac{\sqrt{3^2 + (-4)^2} - 3}{\sqrt{3^2 + (-4)^2}}$</p> $x = \frac{6}{5}$ <p>Similarly, $\frac{y}{4} = \frac{\sqrt{3^2 + (-4)^2} - 3}{\sqrt{3^2 + (-4)^2}}$</p> $y = \frac{8}{5}$ <p>\therefore The complex number represented by Q is $\frac{6}{5} - \frac{8}{5}i$.</p>	1A 1A	(can be omitted)
	<p><u>Alternative solution (2)</u></p> <p>The point Q and the centre of the circle lie on the same line through O.</p> <p>Modulus of $Q = \sqrt{3^2 + (-4)^2} - 3$</p> $= 2$ <p>Let θ be the argument of Q.</p> $\tan \theta = -\frac{4}{3}$ <p>The complex number represented by Q</p> $= 2(\cos \theta + i \sin \theta)$ $= 2\left(\frac{3}{5} - \frac{4}{5}i\right)$ $= \frac{6}{5} - \frac{8}{5}i$	1A 1A	(can be omitted)

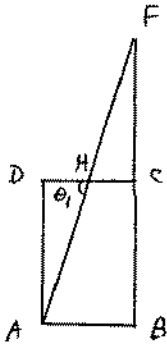


Solution	Marks	Remarks
<p>Alternative solution (3)</p> <p>The point Q and the centre of the circle lie on the same line through O.</p> <p>Equation of circle is $(x-3)^2 + (y+4)^2 = 3^2$ --- (1)</p> <p>Equation of line through O and Q is $y = -\frac{4}{3}x$ --- (2)</p> <p>Solving (1) and (2),</p> $(x-3)^2 + \left(-\frac{4}{3}x+4\right)^2 = 3^2$ $25x^2 - 150x + 144 = 0$ $x = \frac{6}{5} \text{ or } \frac{24}{5} \text{ (rejected)}$ <p>when $x = \frac{6}{5}, y = -\frac{8}{5}$.</p> <p>$\therefore$ The complex number represented by Q is $\frac{6}{5} - \frac{8}{5}i$.</p>	<p>1A</p> <p>1M</p> <p>1A</p>	<p>(can be omitted)</p>
	<p>6</p>	

	Solution	Marks	Remarks
7.	<p>(a) $\vec{a} = \sqrt{2^2 + 4^2} = 2\sqrt{5}$</p> <p>(b) $\vec{a} \cdot \vec{b} = \vec{a} \vec{b} \cos\theta$ $= 2\sqrt{5}(\sqrt{5})(\frac{4}{5})$ $= 8$</p> <p>(c) $\vec{a} \cdot \vec{b} = (2\vec{i} + 4\vec{j}) \cdot (m\vec{i} + n\vec{j})$ $= 2m + 4n$ $\therefore 2m + 4n = 8 \text{ --- (1)}$ As $\vec{b} = 5, \sqrt{m^2 + n^2} = \sqrt{5}$ $m^2 + n^2 = 5 \text{ --- (2)}$ Substitute (1) into (2), $(4 - 2n)^2 + n^2 = 5$ $5n^2 - 16n + 11 = 0$ $n = 1 \text{ or } \frac{11}{5}$ When $n = 1, m = 2.$ $n = \frac{11}{5}, m = -\frac{2}{5}.$</p>	<p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>7</p>	<p>Accept $\sqrt{20}$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $m^2 + (\frac{4-m}{2})^2 = 5$ $5m^2 - 8m - 4 = 0$ $m = 2 \text{ or } -\frac{2}{5}$ </div> <p>Omitting vector sign or dot sign in most cases (pp-1).</p>
8.	<p>(a) Discriminant $= (k+2)^2 - 4(2)(k-1)$ $= k^2 - 4k + 12$ $= (k-2)^2 + 8 > 0$ $\therefore \alpha$ and β are real and distinct.</p>	<p>1A</p> <p>1M+1</p>	
	<p><u>Alternative solution (1)</u> Discriminant $= (k+2)^2 - 4(2)(k-1)$ $= k^2 - 4k + 12$ As discriminant of $k^2 - 4k + 12 = 0$ is $(4)^2 - 4(12)$, which is less than zero, so the graph of $y = k^2 - 4k + 12$ always lies above the x-axis and $k^2 - 4k + 12$ is always positive. $\therefore \alpha$ and β are real and distinct.</p>	<p>1A</p> <p>1M+1</p>	
	<p><u>Alternative solution (2)</u> Discriminant $= (k+2)^2 - 4(2)(k-1)$ $= k^2 - 4k + 12$ Let $f(k) = k^2 - 4k + 12.$ $f'(k) = 2k - 4 = 0$ when $k = 2$ $f''(k) = 2 > 0$ \therefore Minimum value of $f(k) = 2^2 - 4(2) + 12$ $= 8 > 0$ $\therefore f(k) > 0$ for all values of $k.$ $\therefore \alpha$ and β are real and distinct.</p>	<p>1A</p> <p>1M+1</p>	

Solution	Marks	Remarks
(b) $ \alpha - \beta > 3$ $(\alpha - \beta)^2 > 9$ $(\alpha + \beta)^2 - 4\alpha\beta > 9$ $(k+2)^2 - 4(2)(k-1) > 9$ $k^2 - 4k + 3 > 0$ $(k-1)(k-3) > 0$ $k > 3$ or $k < 1$	1A 1M 1A <hr/> 1A <hr/> 7	OR $\alpha - \beta > 3$ or $\alpha - \beta < -3$ For $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

Solution	Marks	Remarks
9. (a) (i) $A\vec{F} = A\vec{B} + B\vec{F}$ $= \vec{a} + 2\vec{b}$ (ii) $D\vec{P} = D\vec{A} + A\vec{P}$ $= (m+1)\vec{a} - \vec{b}$	1M 1A <hr/> 1A <hr/> 3	Can be awarded in (ii)
(b) (i) $A\vec{F} \cdot D\vec{P} = 0$ $(\vec{a} + 2\vec{b}) \cdot \{(m+1)\vec{a} - \vec{b}\} = 0$ $(m+1)\vec{a} \cdot \vec{a} - \vec{a} \cdot \vec{b} + 2(m+1)\vec{a} \cdot \vec{b} - 2\vec{b} \cdot \vec{b} = 0$ <div style="border: 1px solid black; padding: 2px; display: inline-block;">OR $(m+1)\vec{a} \cdot \vec{a} - 2\vec{b} \cdot \vec{b} = 0$</div> $(m+1) - 2(4) = 0$ $m = 7$	1M 1M 1A 1	For distribution law For $\vec{a} \cdot \vec{a} = 1$, $\vec{a} \cdot \vec{b} = 0$, $\vec{b} \cdot \vec{b} = 4$
(ii) (1) $A\vec{E} = \frac{1}{r+1}A\vec{F}$ $= \frac{1}{r+1}(\vec{a} + 2\vec{b})$ (2) $A\vec{E} = \frac{A\vec{P} + kA\vec{D}}{k+1}$ $= \frac{(m+1)\vec{a} + k\vec{b}}{k+1}$ $= \frac{8\vec{a} + k\vec{b}}{k+1}$	1A 1M 1A	
<div style="border: 1px solid black; padding: 5px;"> <u>Alternative solution</u> (2) $A\vec{E} = A\vec{D} + D\vec{E}$ $= \vec{b} + \frac{1}{k+1}D\vec{P}$ $= \vec{b} + \frac{1}{k+1}\{(m+1)\vec{a} - \vec{b}\}$ $= \frac{8\vec{a} + k\vec{b}}{k+1}$ </div>	1M 1A	
Comparing the two expressions, $\begin{cases} \frac{1}{r+1} = \frac{8}{k+1} \text{---(1)} \\ \frac{2}{r+1} = \frac{k}{k+1} \text{---(2)} \end{cases}$ $\therefore \frac{1}{2} = \frac{8}{k}$ $k = 16$ $\frac{1}{r+1} = \frac{8}{16+1}$ $r = \frac{9}{8}$	} 1M 1A <hr/> 1A <hr/> 10	

Solution	Marks	Remarks
<p>(c) As m tends to infinity, the limiting position of E lies on DC.</p> <p>Let this point be H.</p> $\tan \theta_1 = \frac{AD}{DH}$ $= \frac{2}{1/2} = 4$ <p>$\therefore \theta_1 = 76^\circ$ correct to the nearest degree.</p>	<p>1A</p> <p>1M</p> <p>1A</p>	<p>(can be omitted)</p> 
<p><u>Alternative solution (1)</u></p> <p>As m tends to infinity, the limiting position of E lies on DC.</p> $\vec{AF} \cdot \vec{DC} = (\vec{a} + 2\vec{b}) \cdot \vec{a}$ $ \vec{AF} \vec{DC} \cos \theta_1 = (\vec{a} + 2\vec{b}) \cdot \vec{a}$ $\sqrt{4^2 + 1^2} (1) \cos \theta_1 = 1$ $\cos \theta_1 = \frac{1}{\sqrt{17}}$ <p>$\therefore \theta_1 = 76^\circ$ correct to the nearest degree.</p>	<p>1A</p> <p>1M</p> <p>1A</p>	<p>(can be omitted)</p>
<p><u>Alternative solution (2)</u></p> $\vec{AF} \cdot \vec{DP} = \vec{AF} \vec{DP} \cos \theta$ $(\vec{a} + 2\vec{b}) \cdot [(m+1)\vec{a} - \vec{b}] = \sqrt{17} \sqrt{(m+1)^2 + 2^2} \cos \theta$ $\cos \theta = \frac{m-7}{\sqrt{17} \sqrt{m^2 + 2m + 5}}$ $= \frac{1 - \frac{7}{m}}{\sqrt{17} \sqrt{1 + \frac{2}{m} + \frac{5}{m^2}}}$ <p>As $m \rightarrow \infty$, $\cos \theta \rightarrow \frac{1}{\sqrt{17}}$.</p> <p>$\therefore \theta_1 = 76^\circ$ correct to the nearest degree.</p>	<p>1A</p> <p>1M</p> <p>1A</p>	<p>For taking limits</p>
	<p>3</p>	<p>Omitting vector sign or dot sign in most cases (pp-1)</p>

Solution	Marks	Remarks
<p>10. (a) $f'(x) = \frac{(x^2+1)(2x+k) - (x^2+kx+9)(2x)}{(x^2+1)^2}$ $= \frac{-kx^2 - 16x + k}{(x^2+1)^2}$ $f'(3) = 0$ $\frac{-k(3)^2 - 16(3) + k}{(3^2+1)^2} = 0$ $k = -6$</p>	<p>1M 1A 1M <hr/>$\frac{1}{4}$</p>	<p>For quotient rule</p>
<p>(b) (i) $f(x) = \frac{x^2 - 6x + 9}{x^2 + 1}$ $f(0) = 9 \therefore$ The y-intercept is 9. $f(x) = 0$ at $x = 3 \therefore$ The x-intercept is 3.</p>	<p>1A 1A</p>	<p>(pp-1) for giving (0, 9) (pp-1) for giving (3, 0)</p>
<p>(ii) $f'(x) = \frac{-kx^2 - 16x + k}{(x^2+1)^2} = \frac{6x^2 - 16x - 6}{(x^2+1)^2}$ $f'(x) = 0$ $\frac{6x^2 - 16x - 6}{(x^2+1)^2} = 0$ $6x^2 - 16x - 6 = 0$ $(3x+1)(x-3) = 0$ $x = -\frac{1}{3}$ or 3 \therefore The turning points are (3, 0) and $(-\frac{1}{3}, 10)$.</p>	<p>1M 1A</p>	
<p>$f''(x) = \frac{(x^2+1)^2(12x-16) - (6x^2-16x-6)2(x^2+1)2x}{(x^2+1)^4}$ $= \frac{4(-3x^3 + 12x^2 + 9x - 4)}{(x^2+1)^3}$ $f''(3) = \frac{1}{5} > 0$ \therefore (3, 0) is a minimum point. $f''(-\frac{1}{3}) = -\frac{81}{5} < 0$ $\therefore (-\frac{1}{3}, 10)$ is a maximum point.</p>	<p>} 1M 1A 1A</p>	<p>For checking no marks if checking is omitted</p>

Solution	Marks	Remarks
<p>Alternative solution for checking</p> $f'(x) = \frac{2(3x+1)(x-3)}{(x^2+1)^2}$ <p>When $x > 3$, $f'(x) > 0$.</p> <p>When $-\frac{1}{3} < x < 3$, $f'(x) < 0$.</p> <p>$\therefore (3, 0)$ is a minimum point.</p> <p>When $-\frac{1}{3} < x < 3$, $f'(x) < 0$.</p> <p>When $x < -\frac{1}{3}$, $f'(x) > 0$.</p> <p>$\therefore (-\frac{1}{3}, 10)$ is a maximum point.</p>	<p>1M</p> <p>1A</p> <p>1A</p>	<p>For checking</p>
<hr/> <p>7</p> <hr/>		
<p>(c)</p>	<p>1A+1A+1A</p>	<p>$y = f(x)$ (awarded even if checking was omitted in (b))</p> <p>1A for shape</p> <p>1A for intercepts and turning points</p> <p>1A for end-points</p>
<p>Figure 3</p>	<p>1M+1A</p>	<p>$y = -f(x) - 1$</p> <p>- 1M for shape</p> <p>- 1A if the curve is correctly drawn (At least one point on the curve should be labelled)</p>
<hr/> <p>5</p> <hr/>		

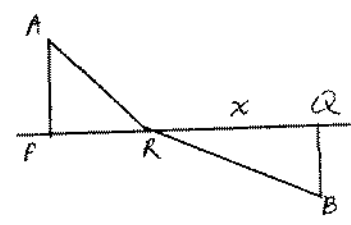
Solution	Marks	Remarks
<p>11. (a) $\cos 5\theta = 0$ $5\theta = 360n^\circ \pm 90^\circ$ n is an integer OR $5\theta = 90^\circ, 270^\circ, 450^\circ, 630^\circ, 810^\circ$ $\theta = 18^\circ, 54^\circ, 90^\circ, 126^\circ, 162^\circ$ OR $\theta = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{\pi}{2}, \frac{7\pi}{10}, \frac{9\pi}{10}$</p>	<p>2A</p>	<p>OR $(2n+1)90^\circ$, $90n^\circ$ (n is odd)</p> <div style="border: 1px solid black; padding: 5px; width: fit-content;"> <p>All correct – 2A 2-4 correct – 1A only ≤ 1 correct – no mark</p> </div>
<p>(b) $(\cos \theta + i \sin \theta)^5 = \cos 5\theta + i \sin 5\theta$ $(\cos \theta + i \sin \theta)^5 = \cos^5 \theta + 5\cos^4 \theta (i \sin \theta) + 10\cos^3 \theta (i \sin \theta)^2$ $\quad + 10\cos^2 \theta (i \sin \theta)^3 + 5\cos \theta (i \sin \theta)^4 + (i \sin \theta)^5$ Comparing the real parts, $\cos 5\theta = \cos^5 \theta - 10\cos^3 \theta \sin^2 \theta + 5\cos \theta \sin^4 \theta$ $= \cos^5 \theta - 10\cos^3 \theta (1 - \cos^2 \theta) + 5\cos \theta (1 - \cos^2 \theta)^2$ $= \cos^5 \theta - 10\cos^3 \theta + 10\cos^5 \theta + 5\cos \theta - 10\cos^3 \theta + 5\cos^5 \theta$ $= 16\cos^5 \theta - 20\cos^3 \theta + 5\cos \theta$</p>	<p>1A 1A 1M $\frac{1}{4}$</p>	<p>For using De Moivre's Theorem For using Binomial Theorem</p>
<p>(c) (i) $f(\cos \theta) = 16\cos^4 \theta - 20\cos^2 \theta + 5$ $= \frac{\cos 5\theta}{\cos \theta}$ (using (b)) From (a) $18^\circ, 54^\circ, 126^\circ, 162^\circ$ are the roots of $\frac{\cos 5\theta}{\cos \theta} = 0$. ($90^\circ$ is rejected $\because \cos \theta \neq 0$.) $\therefore \cos 18^\circ, \cos 54^\circ, \cos 126^\circ, \cos 162^\circ$ are the roots of $f(x) = 0$. \therefore The 4 values of θ are $18^\circ, 54^\circ, 126^\circ$ and 162°.</p>	<p>2M 1A</p>	<p>Award 1M if partially correct</p>
<p><u>Alternative solution</u> (i) From (b), $\cos 5\theta = 16\cos^5 \theta - 20\cos^3 \theta + 5\cos \theta$ Using (a), $18^\circ, 54^\circ, 90^\circ, 126^\circ$ & 162° are the roots of $16\cos^5 \theta - 20\cos^3 \theta + 5\cos \theta = 0$ Since $16\cos^5 \theta - 20\cos^3 \theta + 5\cos \theta$ $= \cos \theta (16\cos^4 \theta - 20\cos^2 \theta + 5)$, $\therefore 18^\circ, 54^\circ, 126^\circ$ & 162° are the roots of $16\cos^4 \theta - 20\cos^2 \theta + 5 = 0$. ($\theta = 90^\circ$ is a root of $\cos \theta = 0$.) As $f(\cos \theta) = 16\cos^4 \theta - 20\cos^2 \theta + 5$, the 4 values of θ for which $\cos \theta$ is a root of $f(x) = 0$ are $18^\circ, 54^\circ, 126^\circ$ and 162°.</p>	<p>2M 1A</p>	<p>Award 1M if partially correct</p>
<p>$f(x) = 16(x - \cos 18^\circ)(x - \cos 54^\circ)(x - \cos 126^\circ)$ $\quad (x - \cos 162^\circ)$ $= 16(x - \cos 18^\circ)(x - \cos 54^\circ)(x + \cos 54^\circ)(x + \cos 18^\circ)$ $= 16(x^2 - \cos^2 18^\circ)(x^2 - \cos^2 54^\circ)$ (ii) From (*), $\cos^2 18^\circ$ and $\cos^2 54^\circ$ are the roots of the equation $16x^2 - 20x + 5 = 0$ in x^2. $\begin{cases} \cos^2 18^\circ + \cos^2 54^\circ = \frac{5}{4} \\ \cos^2 18^\circ \cos^2 54^\circ = \frac{5}{16} \end{cases}$</p>	<p>1M 1 1A 1A</p>	

Solution	Marks	Remarks
$\sin^2 18^\circ + \sin^2 54^\circ = (1 - \cos^2 18^\circ) + (1 - \cos^2 54^\circ)$ $= 2 - \frac{5}{4}$ $= \frac{3}{4}$	1M ↑	For expressing $\sin^2 \theta$ in terms of $\cos^2 \theta$ (can be awarded in either step)
$\sin^2 18^\circ \sin^2 54^\circ = (1 - \cos^2 18^\circ)(1 - \cos^2 54^\circ)$ $= 1 - (\cos^2 18^\circ + \cos^2 54^\circ) + \cos^2 18^\circ \cos^2 54^\circ$ $= 1 - \frac{5}{4} + \frac{5}{16}$ $= \frac{1}{16}$	1M	For expressing in terms of sum and product
$\therefore \text{The equation is } x^2 - \frac{3}{4}x + \frac{1}{16} = 0$ $16x^2 - 12x + 1 = 0$	1A	
<u>Alternative solution (1)</u>		
From (*), $16x^4 - 20x^2 + 5 = 16(x^2 - \cos^2 18^\circ)(x^2 - \cos^2 54^\circ)$		
Put $x = 1$: $16 - 20 + 5 = 16(1 - \cos^2 18^\circ)(1 - \cos^2 54^\circ)$	1M	
$\sin^2 18^\circ \sin^2 54^\circ = \frac{1}{16}$	1A	
Put $x = 0$: $5 = 16 \cos^2 18^\circ \cos^2 54^\circ$	1M	
$= 16(1 - \sin^2 18^\circ)(1 - \sin^2 54^\circ)$		
$\frac{5}{16} = 1 - (\sin^2 18^\circ + \sin^2 54^\circ) + \sin^2 18^\circ \sin^2 54^\circ$		
$\sin^2 18^\circ + \sin^2 54^\circ = \frac{3}{4}$	1A	
$\therefore \text{The equation is } x^2 - \frac{3}{4}x + \frac{1}{16} = 0$		
$16x^2 - 12x + 1 = 0$	1A	
<u>Alternative solution (2)</u>		
$16x^4 - 20x^2 + 5 = 0$		
$x^2 = \frac{20 \pm \sqrt{20^2 - 4(16)(5)}}{2 \times 16}$		
$= \frac{5 \pm \sqrt{5}}{8}$	1A	
From (*), $16x^4 - 20x^2 + 5 = 16(x^2 - \cos^2 18^\circ)(x^2 - \cos^2 54^\circ)$		
$\therefore \cos^2 18^\circ = \frac{5 + \sqrt{5}}{8} \text{ and } \cos^2 54^\circ = \frac{5 - \sqrt{5}}{8}$	1M	
$\sin^2 18^\circ = 1 - \frac{5 + \sqrt{5}}{8} \quad \sin^2 54^\circ = 1 - \frac{5 - \sqrt{5}}{8}$		
$= \frac{3 - \sqrt{5}}{8} \quad = \frac{3 + \sqrt{5}}{8}$	1A	
$\therefore \text{The equation is } (x - \frac{3 - \sqrt{5}}{8})(x - \frac{3 + \sqrt{5}}{8}) = 0$	1M	
$x^2 - (\frac{3 - \sqrt{5}}{8} + \frac{3 + \sqrt{5}}{8})x + \frac{(3 - \sqrt{5})(3 + \sqrt{5})}{64} = 0$		

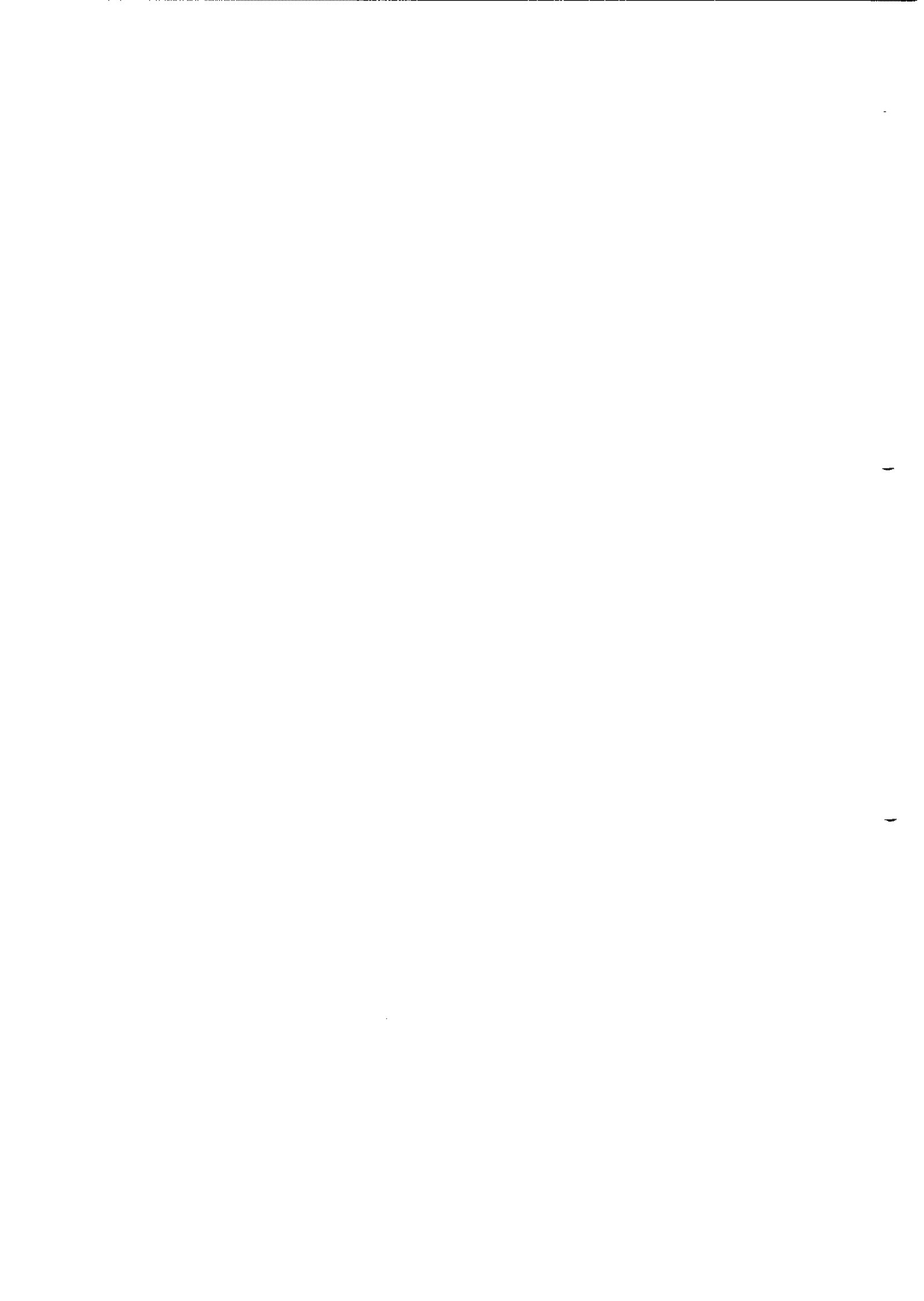
Solution	Marks	Remarks
$x^2 - \frac{3}{4}x + \frac{1}{16} = 0$ $16x^2 - 12x + 1 = 0.$	1A	
<p><u>Alternative solution (3)</u></p> <p>From (*), $\cos^2 18^\circ$ and $\cos^2 54^\circ$ are the roots of the equation $16(x^2)^2 - 20x^2 + 5 = 0$ in x^2.</p> <p>Put $y = 1 - x^2$.</p> $16(1-y)^2 - 20(1-y) + 5 = 0$ $16(1-2y+y^2) - 20(1-y) + 5 = 0$ $16y^2 - 12y + 1 = 0$ which is the equation with roots $\sin^2 18^\circ$ and $\sin^2 54^\circ$.	<p>2M</p> <p>3A</p>	
	<hr/> <p>10</p> <hr/>	

	Solution	Marks	Remarks
12. (a)	$\sin \theta = \frac{GE}{OE}$ $= \frac{s}{1-s}$ $\sin \theta - s \sin \theta = s$ $s = \frac{\sin \theta}{1 + \sin \theta}$ $\frac{ds}{d\theta} = \frac{(1 + \sin \theta) \cos \theta - \sin \theta \cos \theta}{(1 + \sin \theta)^2}$ $= \frac{\cos \theta}{(1 + \sin \theta)^2}$	1A 1 1M <u>1A</u> <u>4</u>	For quotient rule
(b)	$\frac{r}{s} = \frac{OF}{OE}$ $\frac{r}{s} = \frac{1-2s-r}{1-s}$ $r - rs = s - 2s^2 - rs$ $r = s - 2s^2$ $\frac{dr}{d\theta} = \frac{dr}{ds} \frac{ds}{d\theta}$ $= (1-4s) \frac{ds}{d\theta}$ $= (1-4 \frac{\sin \theta}{1 + \sin \theta}) \frac{\cos \theta}{(1 + \sin \theta)^2}$ $= \frac{\cos \theta (1 - 3 \sin \theta)}{(1 + \sin \theta)^3}$	1A 1A 1M 1A 1	For chain rule
	<p><u>Alternative solution for 2nd part</u></p> $r = s - 2s^2$ $= \frac{\sin \theta}{1 + \sin \theta} - 2 \frac{\sin^2 \theta}{(1 + \sin \theta)^2}$ $= \frac{\sin \theta - \sin^2 \theta}{(1 + \sin \theta)^2}$ $\frac{dr}{d\theta} = \frac{(1 + \sin \theta)^2 (\cos \theta - 2 \sin \theta \cos \theta) - (\sin \theta - \sin^2 \theta) 2(1 + \sin \theta) \cos \theta}{(1 + \sin \theta)^4}$ $= \frac{\cos \theta (1 + \sin \theta) (1 - 2 \sin \theta) - 2 \cos \theta (\sin \theta - \sin^2 \theta)}{(1 + \sin \theta)^3}$ $= \frac{\cos \theta (1 - 2 \sin \theta + \sin \theta - 2 \sin^2 \theta - 2 \sin \theta + 2 \sin^2 \theta)}{(1 + \sin \theta)^3}$ $= \frac{\cos \theta (1 - 3 \sin \theta)}{(1 + \sin \theta)^3}$	1A 1M 1	For expressing r in terms of θ and finding $\frac{dr}{d\theta}$
		<u>5</u>	

Solution	Marks	Remarks
<p>(c) (i) r is increasing when $\frac{dr}{d\theta} \geq 0$.</p> $\frac{\cos\theta(1-3\sin\theta)}{(1+\sin\theta)^3} \geq 0$ $1-3\sin\theta \geq 0 \quad (\because 0 < \theta < \frac{\pi}{2})$ $\sin\theta \leq \frac{1}{3}$ <div style="border: 1px dashed black; padding: 2px; display: inline-block;"> $(0 < \theta \leq 0.340)$ </div>	1M	<p>OR $\frac{dr}{d\theta} > 0$</p>
<p>(ii) r is decreasing when $\frac{dr}{d\theta} \leq 0$.</p> $1-3\sin\theta \leq 0$ $\sin\theta \geq \frac{1}{3}$ <div style="border: 1px dashed black; padding: 2px; display: inline-block;"> $(\frac{\pi}{2} > \theta \geq 0.340)$ </div>	1A	<p>OR $\frac{dr}{d\theta} < 0$</p> <p>OR $\sin\theta < \frac{1}{3}$ Accept degrees (19.5°)</p>
<p>$\therefore r$ attains a maximum at $\theta = 0.340$.</p> <p>Maximum area of C_2</p> $= \pi r^2$ $= \pi[s(1-2s)]^2$ $= \pi \left[\frac{1}{3} - \frac{2(\frac{1}{3})}{1+\frac{1}{3}} \right]^2$ $= \frac{\pi}{64} \quad (\text{OR } 0.0491)$	1M <hr/> 1A <hr/> 5	<p>Accept degrees (19.5°)</p> <p>Exact value or correct to at least 3 sig. figures.</p>
<p>(d) At $\sin\theta = \frac{1}{3}$, $\frac{ds}{d\theta} (= \frac{\sqrt{8}/3}{(1+\frac{1}{3})^2}) \neq 0$.</p> <p>Since $\frac{ds}{d\theta} \neq 0$ at $\sin\theta = \frac{1}{3}$, s (and hence the area of C_1) does not attain a minimum when the area of C_2 attains its maximum.</p>	} 2	
<p><u>Alternative solution</u></p> <p>From (a), $\frac{ds}{d\theta} = \frac{\cos\theta}{(1+\sin\theta)^2} > 0 \quad \forall 0 < \theta < \frac{\pi}{2}$.</p> <p>$\therefore s$ increases with θ.</p> <p>\therefore The area of C_1 does not attain a minimum at $\sin\theta = \frac{1}{3}$.</p>	} 2	
	<hr/> 2	

Solution	Marks	Remarks
<p>13. (a) (i) Let $QR = x$.</p> $AR = RB$ $\sqrt{4^2 + (15-x)^2} = \sqrt{2^2 + x^2}$ $16 + x^2 - 30x + 225 = 4 + x^2$ $x = 7.9 \text{ km}$ <div style="border: 1px dashed black; padding: 2px; width: fit-content;"> Checking: Put $x = 7.9$ L.H.S. = R.H.S. = $\sqrt{66.41}$ </div> <p>(ii) Since $AR = RB$ and they walk with equal speeds, they reach R at the same time. If they meet at other points, either one of them have to walk a longer distance than AR (OR RB) and the other has to wait for him/her. Hence the total time for them to meet must be longer.</p> <p>Shortest time = $\frac{AR}{4}$ (OR = $\frac{RB}{4}$)</p> $= \frac{\sqrt{4^2 + (15-7.9)^2}}{4}$ $= 2.04 \text{ hr (OR 2 hr 2.24 min, 2 hr 2 min 14.3 s)}$	<p>1M</p> <p>1A</p> <p>2</p> <p>1M</p> <p>1A</p> <p>6</p>	 <p>Award 1 mark if the answer was partially correct</p>
<p>(b) (i) Let S be the meeting point and $QS = y$. To meet within the shortest time, they should arrive S at the same time.</p> $\frac{\sqrt{4^2 + (15-y)^2}}{4} = \frac{\sqrt{2^2 + y^2}}{8}$ $64 + 4(y^2 - 30y + 225) = 4 + y^2$ $3y^2 - 120y + 960 = 0$ $y = \frac{120 \pm \sqrt{120^2 - 4(3)(960)}}{6}$ $\approx 28.9 \text{ or } 11.1$ <div style="border: 1px dashed black; padding: 2px; width: fit-content;"> Checking: Put $y = 28.9$ $(15-y) < 0 \therefore y = 28.9$ is rejected Put $y = 11.1$ L.H.S. = R.H.S. = 1.404 </div> <p>\therefore They should meet at a distance 11.1 km from Q.</p>	<p>1M+1A</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p>	<p>(can be omitted)</p>
<p><u>Alternative solution</u></p> <p>Let $PS = y$.</p> $\frac{\sqrt{4^2 + y^2}}{4} = \frac{\sqrt{2^2 + (15-y)^2}}{8}$ $64 + 4y^2 = 4 + 225 - 30y + y^2$ $3y^2 + 30y - 165 = 0$ $y = 3.94 \text{ or } -13.9 \text{ (rejected)}$ <p>\therefore They should meet at a distance 3.94 km from P.</p>	<p>1M+1A</p> <p>1A</p> <p>1A</p> <p>1A</p>	
<p>(ii) Let T be the meeting point where $QT = t$.</p> $\frac{\sqrt{2^2 + t^2}}{16} = \frac{\sqrt{4^2 + (15-t)^2}}{4}$ $4 + t^2 = 256 + 16(t^2 - 30t + 225)$ $15t^2 - 480t + 3852 = 0$ $\Delta = (480)^2 - 4(15)(3852)$ $= -720 < 0$ <p>\therefore There are no real roots.</p>	<p>1A</p> <p>1A</p> <p>1A</p>	

Solution	Marks	Remarks
<p>OR Let $PT = s$.</p> $\frac{\sqrt{4+s^2}}{4} = \frac{\sqrt{2^2+(15-s)^2}}{16}$ $16(16+s^2) = 4+225-30s+s^2$ $15s^2+30s+27=0$ $\Delta = -720 < 0$ <p>\therefore There are no real roots.</p>	<p>1A</p> <p>1A</p> <p>1A</p>	
<p>i.e. Billy has already reached P before Amy reaches P.</p> <p>\therefore They should meet at point P.</p>	<p>2A</p>	
<p><u>Alternative solution</u></p> <p>Time for Amy to reach $P = \frac{4}{4} = 1$ hr</p> <p>Time for Billy to reach $P = \frac{\sqrt{2^2+(15)^2}}{16}$ $= 0.95$ hr</p> <p>\therefore Billy has already reach P before Amy reaches P.</p> <p>So they should meet at point P.</p>	<p>} 1M+1A +1A</p> <p>2A</p>	
	<p>10</p>	



只限教師參閱 FOR TEACHERS' USE ONLY

香港考試局

HONG KONG EXAMINATIONS AUTHORITY

一九九七年香港中學會考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1997

附加數學 試卷二

ADDITIONAL MATHEMATICS PAPER II

本評卷參考乃考試局專為今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視為標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations Authority for markers' reference. The Examinations Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Examinations Authority is counting on the co-operation of markers/teachers in this regard.

考試結束後，各科評卷參考將存放於教師中心，供教師參閱。

After the examinations, marking schemes will be available for reference at the Teachers' Centres.



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GENERAL INSTRUCTIONS TO MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method is specified in the question.
2. In the marking scheme, marks are classified as follows :

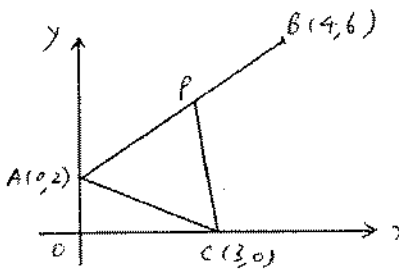
'M' marks – awarded for knowing a correct method of solution and attempting to apply it;

'A' marks – awarded for the accuracy of the answer;

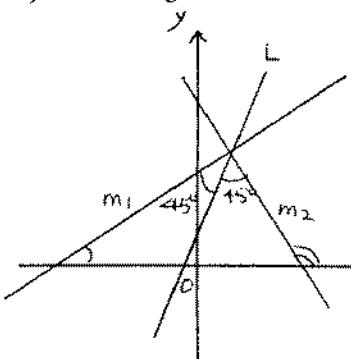
Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answer should **NOT** be awarded. Unless otherwise specified, no marks in the marking scheme are subdivisible.
3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.
4. The symbol $\textcircled{\text{pp-1}}$ should be used to denote marks deducted for poor presentation (p.p.). Note the following points:
 - (a) At most deduct 1 mark for p.p. in each question, up to a maximum of 3 marks for the whole paper.
 - (b) For similar p.p., deduct only 1 mark for the first time that it occurs, i.e. do not penalise candidates twice in the whole paper for the same p.p.
 - (c) In any case, do not deduct any marks for p.p. in those steps where candidates could not score any marks.
 - (d) Some cases in which marks should be deducted for p.p. are specified in the marking scheme. However, the lists are by no means exhaustive. Markers should exercise their professional judgement to give p.p.s in other situations.
5. The symbol $\textcircled{\text{u-1}}$ should be used to denote marks deducted for wrong/no units in the final answers (if applicable). Note the following points :
 - (a) At most deduct 1 mark for wrong/no units for the whole paper.
 - (b) Do not deduct any marks for wrong/no units in case candidate's answer was already wrong.
6. Marks entered in the Page Total Box should be the net total score on that page.
7. In the Marking Scheme, steps which can be omitted are enclosed by dotted rectangles , whereas alternative answers are enclosed by solid rectangles
8. Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.
9. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

Solution	Marks	Remarks
<p>1. $\frac{\sin 3\theta + \cos 3\theta}{\sin \theta + \cos \theta}$</p> $\frac{\sin 3\theta \cos \theta + \sin \theta \cos 3\theta}{\sin \theta \cos \theta}$ $= \frac{\sin 4\theta}{\sin \theta \cos \theta}$ <div style="border: 1px dashed black; padding: 2px; display: inline-block;"> $\frac{2 \sin 2\theta \cos 2\theta}{\sin \theta \cos \theta}$ </div> $= \frac{4 \sin \theta \cos \theta \cos 2\theta}{\sin \theta \cos \theta}$ $= 4 \cos 2\theta$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-top: 10px;"> $\text{OR} = \frac{2 \sin 2\theta \cos 2\theta}{\frac{1}{2} \sin 2\theta}$ </div>	<p>1A 1A 1A 1</p>	<p>} For numerator only</p>
<p><u>Alternative solution</u></p> $\frac{\sin 3\theta + \cos 3\theta}{\sin \theta + \cos \theta}$ <div style="border: 1px dashed black; padding: 2px; display: inline-block;"> $= \frac{3 \sin \theta - 4 \sin^3 \theta}{\sin \theta} + \frac{4 \cos^3 \theta - 3 \cos \theta}{\cos \theta}$ </div> $= 3 - 4 \sin^2 \theta + 4 \cos^2 \theta - 3$ <div style="border: 1px dashed black; padding: 2px; display: inline-block;"> $= 4(\cos^2 \theta - \sin^2 \theta)$ </div> $= 4 \cos 2\theta$	<p>1A+1A 1A 1</p>	<p>For numerator only</p>
<hr style="border: 1px solid black;"/> <p style="text-align: center;">4</p> <hr style="border: 1px solid black;"/>		
<p>2. Let $u = x - 1, du = dx$</p> $\int x \sqrt{x-1} dx$ $= \int (u+1) u^{\frac{1}{2}} du$ $= \int (u^{\frac{3}{2}} + u^{\frac{1}{2}}) du$ $= \frac{2}{5} u^{\frac{5}{2}} + \frac{2}{3} u^{\frac{3}{2}} + c$ <div style="border: 1px dashed black; padding: 2px; display: inline-block;"> $c \text{ is a constant}$ </div> $= \frac{2}{5} (x-1)^{\frac{5}{2}} + \frac{2}{3} (x-1)^{\frac{3}{2}} + c$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-top: 10px;"> $\text{OR} = \frac{2}{15} (3x+2)(x-1)^{\frac{3}{2}} + c$ </div>	<p>1A 1A 1A 1A</p>	<p>Omit du (pp-1)</p> <p>For primitive function only</p> <p>no mark if 'c' is omitted</p>
<p><u>Alternative solution</u></p> <p>Let $u = \sqrt{x-1}, du = \frac{1}{2\sqrt{x-1}} dx$</p> $\int x \sqrt{x-1} dx$ $= \int (u^2+1)u(2u du)$ $= \int (2u^4+2u^2) du$ $= \frac{2}{5} u^5 + \frac{2}{3} u^3 + c$ <div style="border: 1px dashed black; padding: 2px; display: inline-block;"> $c \text{ is a constant}$ </div> $= \frac{2}{5} (x-1)^{\frac{5}{2}} + \frac{2}{3} (x-1)^{\frac{3}{2}} + c$	<p>1A 1A 1A 1A</p>	<p><u>OR</u> Let $u^2 = x - 1$.</p>
<hr style="border: 1px solid black;"/> <p style="text-align: center;">4</p> <hr style="border: 1px solid black;"/>		

Solution	Marks	Remarks
<p>3. (a) The coordinates of P are $(\frac{4\lambda}{1+\lambda}, \frac{6\lambda+2}{1+\lambda})$.</p> <p>(b) $\frac{1}{2} \begin{vmatrix} 0 & 2 \\ 4\lambda & 6\lambda+2 \\ 1+\lambda & 1+\lambda \end{vmatrix} = 6$</p> <p>$\frac{1}{2} (\frac{18\lambda+6}{1+\lambda} + \frac{8\lambda}{1+\lambda} - 6) = 6$</p> <p>$\lambda = \frac{3}{2}$</p>	<p>1A+1A</p> <p>1M+1A</p> <p>1A</p>	<p>1M for LHS</p> 
<p>(b) <u>Alternative solution (1)</u></p> <p>Area of $\triangle ABC = \frac{1}{2} \begin{vmatrix} 0 & 2 \\ 3 & 0 \\ 4 & 6 \end{vmatrix} = 10$</p> <p>Area of $\triangle PAC = \frac{\lambda}{\lambda+1}$ (Area of $\triangle ABC$)</p> <p>$6 = \frac{\lambda}{\lambda+1}(10)$</p> <p>$\lambda = \frac{3}{2}$</p>	<p>1M</p> <p>1A</p> <p>1A</p>	
<p><u>Alternative solution (2)</u></p> <p>$AC = \sqrt{(0-3)^2 + (2-0)^2} = \sqrt{13}$</p> <p>Equation of AC is $\frac{x}{3} + \frac{y}{2} = 1$ $2x+3y-6=0$</p> <p>Distance from P to AC</p> <p>$= \frac{1}{\sqrt{13}} \left 2\left(\frac{4\lambda}{1+\lambda}\right) + 3\left(\frac{6\lambda+2}{1+\lambda}\right) - 6 \right$</p> <p>$= \frac{20\lambda}{\sqrt{13}(1+\lambda)}$</p> <p>$\therefore \frac{1}{2} \left(\frac{20\lambda}{\sqrt{13}(1+\lambda)} \right) (\sqrt{13}) = 6$</p> <p>$\frac{10\lambda}{1+\lambda} = 6$</p> <p>$\lambda = \frac{3}{2}$</p>	<p>1M+1A</p> <p>1A</p>	<p>1M for LHS</p>
	<p>5</p>	

Solution	Marks	Remarks
<p>4. $6\sin x + 8\cos x$</p> $= 10\left(\frac{6}{10}\sin x + \frac{8}{10}\cos x\right)$ $\approx 10\sin(x + 53.13^\circ)$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px 0;"> $OR = 10\sin(x + \alpha) \text{ where } \cos\alpha = \frac{6}{10}$ </div> $6\sin x + 8\cos x = 5$ $10\sin(x + \alpha) = 5$ $x + \alpha = 180n^\circ + (-1)^n 30^\circ \quad n \text{ is an integer}$ $x = 180n^\circ + (-1)^n 30^\circ - 53^\circ \text{ correct to the nearest degree}$	<p>1M</p> <p>1A+1A</p> <p>1M</p> <p>1A</p> <p>5</p>	<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 5px 0;"> $\begin{cases} r\cos\alpha = 6 \\ r\sin\alpha = 8 \end{cases}$ $r = 10, \alpha = 53.13^\circ$ </div> <p>Accept $\alpha = 53^\circ$</p> <p>For $180n^\circ + (-1)^n \theta$</p> $n\pi + (-1)^n \left(\frac{\pi}{6}\right) - 53^\circ \text{ etc. } (n-1)$
<p>5. $\frac{dy}{dx} = 6x + \frac{1}{x^2}$</p> <div style="border: 1px dashed black; padding: 5px; width: fit-content; margin: 5px 0;"> $y = \int \left(6x + \frac{1}{x^2}\right) dx$ </div> $y = 3x^2 - \frac{1}{x} + c \quad c \text{ is a constant}$ <p>Put $x=1, y=0$ $0 = 3 - 1 + c$ $c = -2$</p> <div style="border: 1px dashed black; padding: 5px; width: fit-content; margin: 5px 0;"> $\therefore \text{The equation of the curve is } y = 3x^2 - \frac{1}{x} - 2.$ </div>	<p>1M</p> <p>1A+1A</p> <p>1M</p> <p>1A</p> <p>5</p>	<p>Withhold this mark if "y = " is omitted Omit 'dx' (pp-1)</p> <p>1A for $3x^2$, 1A for $-\frac{1}{x}$</p> <p>Withhold 1 mark if c is omitted.</p>

Solution	Marks	Remarks
6. (a) $\left \frac{m-2}{1+2m} \right = \tan 45^\circ$ $\frac{m-2}{1+2m} = \pm 1$ $m = \frac{1}{3}$ or -3	1A 1A+1A	(pp-1) if absolute sign is omitted 
<u>Alternative solution</u> $\frac{1+m_1}{1-m_1} = 2$ $m_1 = \frac{1}{3}$ $m_2 = -3$ (\because The 2 lines are perpendicular.)	1A 1A 1A	OR $\frac{1+2}{1-2} = m_2$ $m_2 = -3$
(b) $2x - 3y + 2 + k(x - y - 1) = 0$ $(2+k)x - (3+k)y + (2-k) = 0$ Slope = $\frac{2+k}{3+k}$ From (a), $\frac{2+k}{3+k} = \frac{1}{3}$ $k = -\frac{3}{2}$ \therefore The equation of the line is $2x - 3y + 2 - \frac{3}{2}(x - y - 1) = 0$ $x - 3y + 7 = 0$	1A 1M 1A	
<u>Alternative solution</u> $\begin{cases} 2x - 3y + 2 = 0 \\ x - y - 1 = 0 \end{cases}$ Solving the two equations, $x = 5, y = 4$. From (a), the equation of the line is $\frac{y-4}{x-5} = \frac{1}{3}$ $x - 3y + 7 = 0$	1A 1M 1A	
	6	

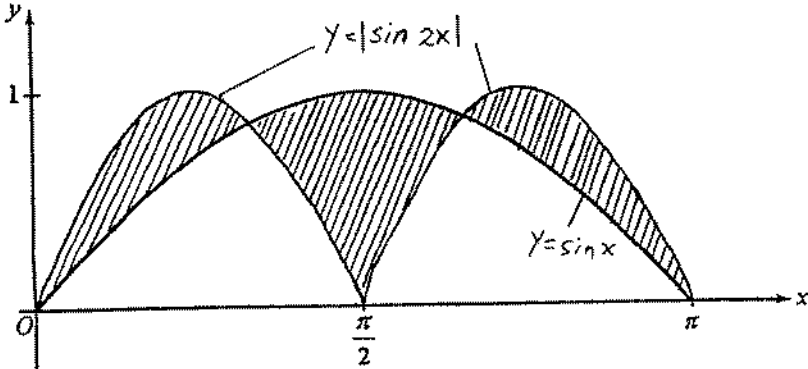
Solution	Marks	Remarks
<p>7. For $n=1, T_1=(1+1)(1!)=2$</p> $=1[(1+1)!]$ <p>\therefore The statement is true for $n=1$.</p> <p>Assume $T_1+T_2+\dots+T_k=k[(k+1)!]$ for some positive integer k.</p> <p>Then $T_1+T_2+\dots+T_k+T_{k+1}$</p> $=k[(k+1)!]+[(k+1)^2+1][(k+1)!]$ $=(k+1)[k+k^2+2k+2]$ $=(k+1)(k^2+3k+2)$ $=(k+1)(k+1)(k+2)$ $=(k+1)[(k+2)!]$ <p>\therefore The statement is also true for $n=k+1$ if it is true for $n=k$. By the principle of mathematical induction, the statement is true for all positive integers n.</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p><u>1</u></p> <p><u>6</u></p>	
<p>8. $(1+x)^n(1-2x)^4$</p> $=(1+{}_nC_1x+{}_nC_2x^2+\dots)[1+{}_4C_1(-2x)+{}_4C_2(-2x)^2+\dots]$ $=(1+{}_nC_1x+{}_nC_2x^2+\dots)(1-8x+24x^2+\dots)$ $=1+({}_nC_1-8)x+({}_nC_2-8{}_nC_1+24)x^2+\dots$ <div style="border: 1px solid black; padding: 5px; width: fit-content;"> <p>OR $=1+(n-8)x+\frac{n(n-1)}{2}-8n+24x^2+\dots$</p> </div> <p>If the coefficient of $x^2=54$,</p> ${}_nC_2-8{}_nC_1+24=54$ $\frac{n(n-1)}{2}-8n+24=54$ $n^2-17n-60=0$ $(n+3)(n-20)=0$ <p>$n=20$ $n=-3$ is rejected</p> <p>Coefficient of $x={}_{20}C_1-8$</p> $=12$	<p>1A+1A</p> <p>1M+1A</p> <p>1M</p> <p>1A</p> <p><u>1A</u></p> <p><u>7</u></p>	<p>(pp-1) for omitting dots</p>

	Solution	Marks	Remarks
9. (a)	$\sqrt{(x-1)^2 + y^2} = x+1 $ OR $(x-1)^2 + y^2 = (x+1)^2$ $(x^2 - 2x + 1) + y^2 = x^2 + 2x + 1$ $y^2 = 4x$	1A+1A <hr/> <u>1</u> <hr/> <u>3</u>	1A for LHS, 1A for RHS (pp-1) if absolute sign is omitted
(b)	$y^2 = 4x$ $2y \frac{dy}{dx} = 4$ At the point $(t^2, 2t)$, $\frac{dy}{dx} = \frac{4}{2(2t)} = \frac{1}{t}$ Equation of tangent is $\frac{y-2t}{x-t^2} = \frac{1}{t}$ $x - ty + t^2 = 0$	} 1M 1	
	<u>Alternative solution</u> Using the formula $yy_1 = \frac{1}{2} \cdot 4(x+x_1)$, the equation of the tangent is $y(2t) = 2(x+t^2)$ $x - ty + t^2 = 0$	1A 1	
	Equation of normal $\frac{y-2t}{x-t^2} = -t$ $tx + y - 2t - t^3 = 0$	<hr/> <u>1A</u> <hr/> <u>3</u>	
(c) (i)	Since P and C have common tangent at R , the normal to P at R passes through $Q(k, 0)$. $0 = -t(k) + 2t + t^3$ $t^3 = kt - 2t$ $t^2 = k - 2$ $\because t \neq 0$	1M 1M 1	(can be omitted)
	<u>Alternative solution (1)</u> RQ is perpendicular to the tangent to P at R . Slope of $RQ = \frac{2t}{t^2 - k}$ Slope of tangent to P at $R = \frac{1}{t}$ $\frac{2t}{t^2 - k} \cdot \left(\frac{1}{t}\right) = -1$ $t^2 = k - 2$	1M 1M 1	(can be omitted)

Solution	Marks	Remarks
<p><u>Alternative solution (2)</u> Equation of C is $(x-k)^2 + y^2 = (t^2 - k)^2 + 4t^2$ — (1) Equation of tangent to P at R is $x - ty + t^2 = 0$ — (2) Substitute (2) into (1), $(ty - t^2 - k)^2 + y^2 = (t^2 - k)^2 + 4t^2$ $(t^2 + 1)y^2 - 2t(t^2 + k)y + 4t^2(k - 1) = 0$ Discriminant $= 4t^2(t^2 + k)^2 - 4(t^2 + 1)4t^2(k - 1) = 0$ $t^4 + (4 - 2k)t^2 + (k - 2)^2 = 0$ $[t^2 - (k - 2)]^2 = 0$ $t^2 = k - 2$</p>	<p>1M 1M 1</p>	
<p><u>Alternative solution (3)</u> Equation of C is $(x-k)^2 + y^2 = (t^2 - k)^2 + 4t^2$. $2(x-k) + 2y \frac{dy}{dx} = 0$ $\frac{dy}{dx} = -\frac{(x-k)}{y}$ At point R, $\frac{dy}{dx} = -\frac{(t^2 - k)}{2t}$ Equation of tangent to C at R $\frac{y - 2t}{x - t^2} = -\frac{(t^2 - k)}{2t}$ $(k - t^2)x - 2ty + (t^4 + 4t^2 - t^2k) = 0$ Compare with the equation of tangent to R at R $(x - ty + t^2 = 0)$ $\frac{k - t^2}{2} = 1$ OR $\frac{t^4 + 4t^2 - t^2k}{2} = t^2$ $t^4 + 2t^2 = t^2k$ $t^2 = k - 2$ $t^2 = k - 2$</p>	<p>1M 1M 1</p>	
<p><u>Alternative solution (4)</u> Distance between R and Q $= \sqrt{(t^2 - k)^2 + (2t)^2}$ Distance from Q to the tangent to P at R $= \left \frac{k + t^2}{\sqrt{1 + t^2}} \right$ Since the circle touches P at R, $\sqrt{(t^2 - k)^2 + 4t^2} = \left \frac{k + t^2}{\sqrt{1 + t^2}} \right$ $[(t^2 - k)^2 + 4t^2](1 + t^2) = (t^2 + k)^2$ $t^2(t^4 - 2kt^2 + 4t^2 + k^2 - 4k + 4) = 0$ $t^4 + (4 - 2k)t^2 + (k - 2)^2 = 0$ $[t^2 - (k - 2)]^2 = 0$ $t^2 = k - 2$</p>	<p>1M 1M 1</p>	

Solution	Marks	Remarks
<p>(ii) (1) Put (0, 2) into $x - ty + t^2 = 0$.</p> $0 - 2t + t^2 = 0$ $t = 2 \quad \boxed{\text{OR } t = 0 \text{ or } 2}$ <p>At point S, $t = -2$.</p> <p>The equation of the tangent to P at S is</p> $x - (-2)y + (-2)^2 = 0.$ $x + 2y + 4 = 0$	<p>1A</p> <p>1A</p> <p>1A</p> <p>1M</p> <p>1A</p>	<p>$\boxed{\text{OR The point } S \text{ is } (4, -4).}$</p>
<p><u>Alternative solution</u></p> <p>$t = 2$</p> <p>Equation of tangent to P at R is</p> $x - 2y + 4 = 0$ <p>By symmetry, slope of tangent to P at S $= -(\text{slope of tangent to } P \text{ at } R)$ $= -\frac{1}{2}$</p> <p>\therefore Equation of tangent to P at S is $x + 2y + 4 = 0.$</p>	<p>} 1A+1A</p> <p>1A</p> <p>1M</p> <p>1A</p>	<p>Same as above</p> <p>$\boxed{\text{OR slope of tangent at } R = \frac{1}{2}}$</p>
<p>(2) $k = t^2 + 2 = 6$</p> <p>Point Q is (6, 0).</p> $\text{radius} = \sqrt{(6-4)^2 + (0-4)^2} = \sqrt{20}$ <p>\therefore The equation of C is $(x-6)^2 + y^2 = 20.$</p>	<p>1M</p> <p><u>1A</u></p> <p><u>10</u></p>	<p>$x^2 + y^2 - 12x + 16 = 0$</p>

Solution	Marks	Remarks
<p>10. (a) $\sin x = \sin 2x$ $\sin x = 2 \sin x \cos x$ $\cos x = \frac{1}{2}$ $x = \frac{\pi}{3}$</p> <p>Put $x = \frac{\pi}{3}, y = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$.</p> <p>$\therefore$ The coordinates of A are $(\frac{\pi}{3}, \frac{\sqrt{3}}{2})$.</p>	<p>} 1M</p> <p>1</p>	
<p><u>Alternative solution</u></p> <p>Put $x = \frac{\pi}{3}, \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$</p> <p>$\sin 2(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$</p> <p>$\therefore$ The coordinates of A are $(\frac{\pi}{3}, \frac{\sqrt{3}}{2})$.</p>	<p>} 1M</p> <p>1</p>	
<hr/> <p>2</p> <hr/>		
<p>(b) Area = $\int_0^{\pi} \sin 2x - \sin x dx$</p> <p>$= \int_0^{\frac{\pi}{3}} (\sin 2x - \sin x) dx + \int_{\frac{\pi}{3}}^{\pi} (\sin x - \sin 2x) dx$</p> <p>$= \left[-\frac{1}{2} \cos 2x + \cos x \right]_0^{\frac{\pi}{3}} + \left[-\cos x + \frac{1}{2} \cos 2x \right]_{\frac{\pi}{3}}^{\pi}$</p> <p>$= \left[\frac{1}{4} + \frac{1}{2} - (-\frac{1}{2} + 1) \right] + \left[1 + \frac{1}{2} - (-\frac{1}{2} - \frac{1}{4}) \right]$</p> <p>$= \frac{1}{4} + 2 \frac{1}{4}$</p> <p>$= 2 \frac{1}{2}$</p>	<p>1M+1A+1A</p> <p>1A</p> <p>1A</p>	<p>1M for Area = $\int_a^b (y_2 - y_1) dx$, omit 'dx' (pp-1) 1A for each expression</p> <p>For primitive function, awarded if one of them is correct</p>
<p><u>Alternative solution</u></p> <p>Area A_1 between $x=0$ to $\frac{\pi}{3}$</p> <p>$A_1 = \int_0^{\frac{\pi}{3}} (\sin 2x - \sin x) dx$ OR $= \left \int_0^{\frac{\pi}{3}} (\sin 2x - \sin x) dx \right$</p> <p>$= \left[-\frac{\cos 2x}{2} + \cos x \right]_0^{\frac{\pi}{3}}$</p> <p>$= \left[\frac{1}{4} + \frac{1}{2} - (-\frac{1}{2} + 1) \right]$</p> <p>$= \frac{1}{4}$</p>	<p>1M+1A</p> <p>1A</p>	<p>1M for Area = $\int_a^b (y_2 - y_1) dx$.</p> <p>For primitive function</p>

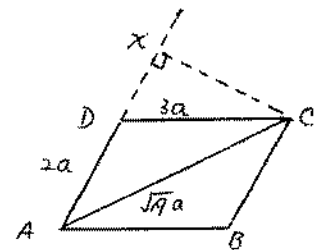
Solution	Marks	Remarks
<p>Area A_2 between $x = \frac{\pi}{3}$ to π</p> $A_2 = \int_{\frac{\pi}{3}}^{\pi} (\sin x - \sin 2x) dx \quad \text{OR} \quad \int_{\frac{\pi}{3}}^{\pi} (\sin 2x - \sin x) dx$ $= \left[-\cos x + \frac{1}{2} \cos 2x \right]_{\frac{\pi}{3}}^{\pi}$ $= \left[1 + \frac{1}{2} - \left(-\frac{1}{2} - \frac{1}{4} \right) \right]$ $= 2\frac{1}{4}$ <p>\therefore Total area = $A_1 + A_2$</p> $= 2\frac{1}{2}$	1A	
	1A	
	5	
<p>(c) Volume = $\pi \int_0^{\frac{\pi}{3}} \sin^2 x dx + \pi \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin^2 2x dx$</p> $= \pi \int_0^{\frac{\pi}{3}} \frac{1}{2} (1 - \cos 2x) dx + \pi \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{2} (1 - \cos 4x) dx$ $= \frac{\pi}{2} \left[x - \frac{1}{2} \sin 2x \right]_0^{\frac{\pi}{3}} + \frac{\pi}{2} \left[x - \frac{1}{4} \sin 4x \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ $= \frac{\pi}{2} \left[\frac{\pi}{3} - \frac{\sqrt{3}}{4} - 0 \right] + \frac{\pi}{2} \left[\frac{\pi}{2} - 0 - \left(\frac{\pi}{3} + \frac{\sqrt{3}}{8} \right) \right]$ $= \left(\frac{\pi^2}{6} - \frac{\sqrt{3}\pi}{8} \right) + \left(\frac{\pi^2}{12} - \frac{\sqrt{3}\pi}{16} \right)$ $= \frac{\pi}{16} (4\pi - 3\sqrt{3})$	1M+1A	1M for $V = \pi \int_a^b y^2 dx$
	1M	
	1A	For either of the 2 primitive functions
	1A	
	5	
(d)	2A	For the graph
	2A	For shading the area
	4	

Solution	Marks	Remarks
11. (a) $du = -\csc^2 \theta d\theta$ $\int \cot^n \theta \csc^2 \theta d\theta = -\int u^n du$ $= -\frac{u^{n+1}}{n+1} + c$ c is a constant $= -\frac{\cot^{n+1} \theta}{n+1} + c$	1A 1A 1A <hr/> 3	no mark if 'c' is omitted
(b) $\int \cot^{n+2} \theta d\theta = \int \cot^n \theta \cot^2 \theta d\theta$ $= \int \cot^n \theta (\csc^2 \theta - 1) d\theta$ $= \int \cot^n \theta \csc^2 \theta d\theta - \int \cot^n \theta d\theta$ $= -\frac{\cot^{n+1} \theta}{n+1} - \int \cot^n \theta d\theta$	2A 1M <hr/> 1 <hr/> 4	For separating into 2 terms
(c) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cot^2 \theta d\theta$ $= \left[-\cot \theta \right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} - \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} d\theta$ $= \left[-\cot \theta \right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} - \left[\theta \right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ $= 1 - \frac{\sqrt{3}}{3} - \frac{\pi}{12}$	1A 1A 1	Omitting limits (pp-1)
<u>Alternative solution</u> $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cot^2 \theta d\theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\csc^2 \theta - 1) d\theta$ $= \left[-\cot \theta - \theta \right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ $= 1 - \frac{\sqrt{3}}{3} - \frac{\pi}{12}$	1A 1A 1	
	<hr/> 3	

Solution	Marks	Remarks
<p>(d) $dx = \sec\theta \tan\theta d\theta$</p> $\int_{\sqrt{2}}^2 \frac{dx}{x\sqrt{(x^2-1)^5}} = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sec\theta \tan\theta d\theta}{\sec\theta \sqrt{(\sec^2\theta-1)^5}}$ $= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cot^4\theta d\theta$ $= \left[-\frac{\cot^3\theta}{3} \right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} - \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cot^2\theta d\theta$ $= \frac{1}{3} \left[\frac{1}{(\sqrt{3})^3} - 1 \right] - \left(1 - \frac{1}{\sqrt{3}} - \frac{\pi}{12} \right)$ $= \frac{2}{3} + \frac{8\sqrt{3}}{27} + \frac{\pi}{12}$	<p>1A</p> <p>1A+1A</p> <p>1A</p> <p>1M</p> <hr/> <p>1A</p> <hr/> <p>6</p>	<p>1A for integrand, 1A for limits</p> <p>For using (b)</p> <p>Omit du, $d\theta$ etc. (pp-1)</p>

Solution	Marks	Remarks
12. (a) (i) By Cosine Law, $(AC)^2 = (AB)^2 + (BC)^2 - 2(AB)(BC)\cos\angle ABC$ $= (3a)^2 + (2a)^2 - 2(3a)(2a)\cos 120^\circ$ $= 19a^2$ $AC = \sqrt{19}a$	1A 1A	
(ii) $MC = \frac{1}{2}AC = \frac{\sqrt{19}}{2}a$ $\tan\angle HMC = \frac{HC}{MC}$ $= \frac{a}{\frac{1}{2}\sqrt{19}a}$ $\angle HMC \approx 25^\circ$ \therefore The angle of elevation is 25° correct to the nearest degree.	1M 1A	
4		
(b) (i) $(BD)^2 = (3a)^2 + (2a)^2 - 2(3a)(2a)\cos 60^\circ$ $= 7a^2$ $BD = \sqrt{7}a$ Consider the area of $\triangle BCD$. $\frac{1}{2}(BD)(CE) = \frac{1}{2}(BC)(CD)\sin 60^\circ$ $\frac{1}{2}(\sqrt{7}a)(CE) = \frac{1}{2}(2a)(3a)\frac{\sqrt{3}}{2}$ $CE = \frac{3\sqrt{21}}{7}a$	1A 1M 1M 1A	For substitution Accept $\frac{3\sqrt{3}}{\sqrt{7}}a$
<u>Alternative solution</u> $BD = \sqrt{7}a$ By Sine Law, $\frac{BD}{\sin 60^\circ} = \frac{BC}{\sin\angle BDC}$ $\sin\angle BDC = \frac{(2a)\sin 60^\circ}{\sqrt{7}a}$ $= \frac{\sqrt{3}}{\sqrt{7}}$ $CE = CD\sin\angle CDE$ $= 3a\left(\frac{\sqrt{3}}{\sqrt{7}}\right)$ $= \frac{3\sqrt{21}}{7}a$	1A 1M 1M 1A	$\sin\angle DBC = \frac{(3a)\sin 60^\circ}{\sqrt{7}a}$ $= \frac{3\sqrt{3}}{2\sqrt{7}}$ $CE = BC\sin\angle DBC$ $= 2a\left(\frac{3\sqrt{3}}{2\sqrt{7}}\right)$ $= \frac{3\sqrt{21}}{7}a$

Solution	Marks	Remarks
$(HE)^2 = a^2 + (CE)^2$ $(EB)^2 = (BC)^2 - (CE)^2$ $(HE)^2 + (EB)^2 = [a^2 + (CE)^2] + [(BC)^2 - (CE)^2]$ $= a^2 + (BC)^2$ $= (HB)^2$ $\therefore HE$ is perpendicular to BD (Converse of Pythagora's Theorem).	$= a^2 + (\frac{3\sqrt{3}}{\sqrt{7}}a)^2 = \frac{34}{7}a^2$ $= (2a)^2 - (\frac{3\sqrt{3}}{\sqrt{7}}a)^2 = \frac{1}{7}a^2$ $= \frac{34}{7}a^2 + \frac{1}{7}a^2$ $= a^2 + (2a)^2$	1M 1M 1
<p><u>Alternative solution</u></p> $(HE)^2 = a^2 + (CE)^2$ $(DE)^2 = (CD)^2 - (CE)^2$ $(HE)^2 + (DE)^2 = [a^2 + (CE)^2] + [(CD)^2 - (CE)^2]$ $= a^2 + (CD)^2$ $= (DH)^2$ $\therefore HE$ is perpendicular to BD (Converse of Pythagora's Theorem).	$= a^2 + (\frac{3\sqrt{3}}{\sqrt{7}}a)^2 = \frac{34}{7}a^2$ $= (3a)^2 - (\frac{3\sqrt{3}}{\sqrt{7}}a)^2 = \frac{36}{7}a^2$ $= \frac{34}{7}a^2 + \frac{36}{7}a^2$ $= a^2 + (3a)^2$	1M 1M 1
<p>The angle between the planes HBD and $ABCD$ is $\angle HEC$.</p> $\tan \angle HEC = \frac{HC}{EC}$ $= \frac{a}{3\sqrt{21}a/7}$ $= \frac{7}{3\sqrt{21}}$ $\angle HEC \approx 27^\circ$ correct to the nearest degree.		1M 1A 9
<p>(c) Point X lies on AD produced such that $CX \perp AX$.</p> $AX = AD + DX$ $= 2a + CD \cos 60^\circ$ $= 2a + \frac{3a}{2}$ $= \frac{7a}{2}$		1A 1M 1A
<p><u>Alternative solution (1)</u></p> <p>Point X lies on AD produced such that $CX \perp AX$.</p> $\cos \angle CAD = \frac{(2a)^2 + (\sqrt{19}a)^2 - (3a)^2}{2(2a)(\sqrt{19}a)}$ $= \frac{7}{2\sqrt{19}}$ $AX = AC \cos \angle CAD$ $= \sqrt{19}a (\frac{7}{2\sqrt{19}})$ $= \frac{7a}{2}$		1A 1M 1A



Solution	Marks	Remarks
<p><u>Alternative solution (2)</u></p> <p>Point X lies on AD produced such that $CX \perp AX$.</p> $AC^2 - AX^2 = DC^2 - DX^2$ <p>Let $AX = d$.</p> $(\sqrt{19}a)^2 - d^2 = (3a)^2 - (d - 2a)^2$ $14a^2 = 4ad$ $d = \frac{7a}{2}$	<p>1A</p> <p>1M</p> <p>1A</p>	
	<p style="text-align: center;"><u>3</u></p>	

Solution	Marks	Remarks
<p>13. (a) $x^2 + y^2 - 6x - 2y + k(2x - 4y + 3) = 0$ $x^2 + y^2 - (6 - 2k)x - (2 + 4k)y + 3k = 0$ The centre is $(3 - k, 1 + 2k)$. Radius $= \sqrt{(3 - k)^2 + (1 + 2k)^2} - 3k$ $= \sqrt{5k^2 - 5k + 10}$ $= \sqrt{5(k^2 - k + 2)}$</p>	<p>1M 1A 1M</p>	<p>For collecting terms $\frac{1}{2}\sqrt{(2k - 6)^2 + (4k + 2)^2} - 4(3k)$</p>
<p>(b) Radius $= \sqrt{5(k^2 - k + \frac{1}{4})} - \frac{5}{4} + 10$ $= \sqrt{5(k - \frac{1}{2})^2 + \frac{35}{4}}$ \therefore Radius of smallest circle in $F = \frac{\sqrt{35}}{2}$. Since AB is a diameter of the smallest circle in F, $AB = 2 \times \frac{\sqrt{35}}{2} = \sqrt{35}$.</p>	<p>1M+1A 1A 1A</p>	
<p>Alternative solution (1) Let $r = \sqrt{5k^2 - 5k + 10}$ $\frac{dr}{dk} = \frac{10k - 5}{2\sqrt{5k^2 - 5k + 10}} = \frac{5(2k - 1)}{2\sqrt{5k^2 - 5k + 10}}$ $\frac{dr}{dk} = 0$ when $k = \frac{1}{2}$.</p>	<p>1M 1A</p>	<p>OR differentiating $(5k^2 - 5k + 10)$</p>
<p>Since $\frac{dr}{dk} > 0$ when $k > \frac{1}{2}$ and $\frac{dr}{dk} < 0$ when $k < \frac{1}{2}$, r is smallest when $k = \frac{1}{2}$.</p>		<p>OR using 2nd derivative test</p>
<p>Smallest radius $= \sqrt{5(\frac{1}{2})^2 - 5(\frac{1}{2}) + 10}$ $= \frac{\sqrt{35}}{2}$ Since AB is a diameter of the smallest circle in F, $AB = 2 \times \frac{\sqrt{35}}{2} = \sqrt{35}$.</p>	<p>1A 1A</p>	
<p>Alternative solution (2) The circle is smallest when the centre lies on AB. Put $(3 - k, 1 + 2k)$ into $2x - 4y + 3 = 0$, $2(3 - k) - 4(1 + 2k) + 3 = 0$ $k = \frac{1}{2}$ \therefore Smallest radius $= \sqrt{5(\frac{1}{2})^2 - 5(\frac{1}{2}) + 10}$ $= \frac{\sqrt{35}}{2}$ Since AB is a diameter of the smallest circle in F, $\therefore AB = 2 \times \frac{\sqrt{35}}{2} = \sqrt{35}$.</p>	<p>1M 1A 1A 1A</p>	

Solution	Marks	Remarks
<p>Alternative solution (3)</p> $\begin{cases} x^2 + y^2 - 6x - 2y = 0 \\ 2x - 4y + 3 = 0 \end{cases}$ $\left(\frac{4y-3}{2}\right)^2 + y^2 - 6\left(\frac{4y-3}{2}\right) - 2y = 0$ $20y^2 - 80y + 45 = 0$ $y = \frac{4+\sqrt{7}}{2} \text{ or } \frac{4-\sqrt{7}}{2}$ $x = \frac{5+2\sqrt{7}}{2} \quad \frac{5-2\sqrt{7}}{2}$ <p>\therefore The coordinates of A and B are $\left(\frac{5+2\sqrt{7}}{2}, \frac{4+\sqrt{7}}{2}\right)$ and $\left(\frac{5-2\sqrt{7}}{2}, \frac{4-\sqrt{7}}{2}\right)$.</p> <p>Length of AB</p> $= \sqrt{\left[\left(\frac{5+2\sqrt{7}}{2}\right) - \left(\frac{5-2\sqrt{7}}{2}\right)\right]^2 + \left[\left(\frac{4+\sqrt{7}}{2}\right) - \left(\frac{4-\sqrt{7}}{2}\right)\right]^2}$ $= \sqrt{35}$	<p>1M</p> <p>1A</p> <p>1A</p> <p>1A</p>	<p>OR $4x^2 - 20x - 3 = 0$</p>
<p>(c) (i) Distance = $\left \frac{4(3-k) + 2(1+2k) - 9}{\sqrt{4^2 + 2^2}} \right$</p> $= \frac{\sqrt{5}}{2} \quad \text{which is a constant.}$ <p>The locus of the centres of the circles in F is parallel to the line L.</p> <p>OR The locus of the centres of the circles in F and the line has no intersection point.</p>	<p>1M</p> <p>1</p> <p>1A</p>	<p>Accept omitting absolute sign</p>
<p>Alternative solution</p> $M_{AB} = \frac{1}{2}, M_L = -2$ $M_{AB} \cdot M_L = -1 \quad \therefore AB \text{ and } L \text{ are perpendicular.}$ <p>As the locus of the centres of circles in F is the perpendicular bisector of AB, so the locus of the centre of circles in F and L are parallel.</p> <p>So the distance from the centre of a circle in F to L is always a constant.</p> <p>The locus of the centres of circles in F and L are parallel.</p>	<p>1</p> <p>1</p> <p>1A</p>	
<p>(ii) $\left(\frac{CD}{2}\right)^2 + \left(\frac{\sqrt{5}}{2}\right)^2 = (\text{radius})^2$</p> $\left(\frac{\sqrt{35}}{2}\right)^2 + \left(\frac{\sqrt{5}}{2}\right)^2 = (\sqrt{5k^2 - 5k + 10})^2$ $5k^2 - 5k = 0$ $k = 0 \text{ or } 1$ <p>\therefore The equations of the two circles are</p> $x^2 + y^2 - 6x - 2y = 0$ <p>and $x^2 + y^2 - 4x - 6y + 3 = 0$.</p>	<p>1M</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p>	<p>$(x-3)^2 + (y-1)^2 = 10$</p> <p>$(x-2)^2 + (y-3)^2 = 10$</p>

Solution	Marks	Remarks
<p><u>Alternative solution</u></p> <p>Distance from centre to AB = Distance from centre to CD</p> $\left \frac{2(3-k) - 4(1+2k) + 3}{\sqrt{4^2 + 2^2}} \right = \frac{\sqrt{5}}{2}$ $\frac{5-10k}{\sqrt{20}} = \pm \frac{\sqrt{5}}{2}$ <p>$k = 0$ or 1</p> <p>\therefore The equations of the two circles are</p> $x^2 + y^2 - 6x - 2y = 0$ <p>and $x^2 + y^2 - 4x - 6y + 3 = 0.$</p>	<p>1M</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>1A</p>	<p>Omit absolute sign (pp-1) .</p> <p>For LHS</p>
	<p>8</p>	

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