Section A (42 marks)

Answer ALL questions in this Section.

1. Let $f(x)=\sqrt{3+x^2}$. Find $f'(-1)$.

(3 marks)

2. $P(8,1)$ is a point on the curve $y^2+\sqrt[3]{x}\,y-3=0$. Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at P.

(3 marks)

3. (a) Express $\frac{1+i}{1-i}$ in standard form.

(b) Using (a), or otherwise, find the value(s) of n such that $(1+i)^{2n}=(1-i)^{2n}$, where n is a positive integer.

(5 marks)

4.

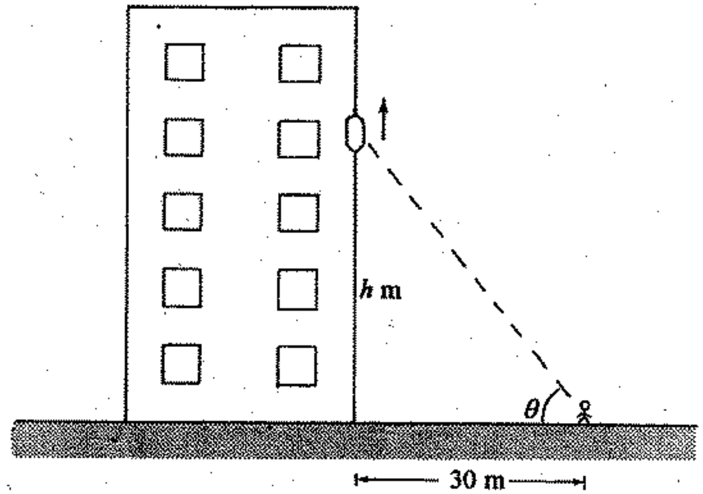

Figure 1

A man stands at a horizontal distance of 30 m from a sight-seeing elevator of a building as shown in Figure 1. The elevator is rising vertically with a uniform speed of 1.5 m s^{-1}. When the elevator is at a height h m above the ground, its angle of elevation from the man is θ. Find the rate of change of θ with respect to time when the elevator is at a height $30\sqrt{3}$ m above the ground. [Note : You may assume that the sizes of the elevator and the man are negligible.]

(5 marks)

5. Solve $\begin{cases} |3x-4|<2 \\ \frac{1}{2x-1}\le 1 \end{cases}$

(6 marks)

Go on to the next page

6. In an Argand diagram, P is the point representing the complex number z which satisfies the equation

$$|z-(3-4i)|=3.$$

(a) Sketch the locus of P.

(b) Q is the point on the locus of P such that the modulus of the complex number represented by Q is the smallest. Find the complex number represented by Q in standard form.

(6 marks)

7. Let $\mathbf{a}$ and $\mathbf{b}$ be two vectors such that $\mathbf{a}=2\mathbf{i}+4\mathbf{j}$, $|\mathbf{b}|=\sqrt{5}$ and $\cos\theta=\frac{4}{5}$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$.

(a) Find $|\mathbf{a}|$.

(b) Find $\mathbf{a}\cdot\mathbf{b}$.

(c) If $\mathbf{b}=m\mathbf{i}+n\mathbf{j}$, find the values of m and n.

(7 marks)

8. Let α and β be the roots of the equation $x^2+(k+2)x+2(k-1)=0$, where k is real.

(a) Show that α and β are real and distinct.

(b) If $|\alpha-\beta|>3$, find the range of possihle values of k.

(7 marks)

Section B (48 marks)

Answer any THREE questions in this Section.
Each question carries 16 marks.

9.

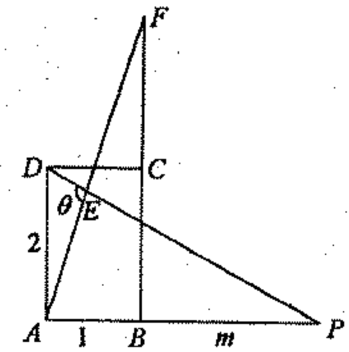

Figure 2

In Figure 2, $ABCD$ is a rectangle with $AB=1$ and $AD=2$. F is a point on BC produced with $BC=CF$. P is a variable point on AB produced such that $BP=m$. AF and DP intersect at a point E. Let $\overrightarrow{AB}=\mathbf{a}$, $\overrightarrow{AD}=\mathbf{b}$ and $\angle AED=\theta$.

(a) (i) Express $\overrightarrow{AF}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(ii) Express $\overrightarrow{DP}$ in terms of m, $\mathbf{a}$ and $\mathbf{b}$. (3 marks)

(b) Suppose $\theta=90°$.

(i) Show that $m=7$.

(ii) Let $AE:EF=1:r$ and $DE:EP=1:k$.

(1) Express $\overrightarrow{AE}$ in terms of r, $\mathbf{a}$ and $\mathbf{b}$.

(2) Express $\overrightarrow{AE}$ in terms of k, $\mathbf{a}$ and $\mathbf{b}$.

Hence find the values of r and k. (10 marks)

(c) As m tends to infinity, θ approaches a certain value θ_1. Find θ_1 correct to the nearest degree. (3 marks)

Go on to the next page

10. The function $f(x)=\dfrac{x^2+kx+9}{x^2+1}$, where k is a constant, attains a stationary value at $x=3$.

(a) Find $f'(x)$ in terms of k and x.

Hence show that $k=-6$.

(4 marks)

(b) (i) Find the x- and y- intercepts of the curve $y=f(x)$.

(ii) Find the maximum and minimum points of the curve $y=f(x)$.

(7 marks)

(c) Sketch the graph of $y=f(x)$ for $-6\le x\le 6$ in Figure 3.

Hence sketch the graph of $y=-f(x)-1$ for $-6\le x\le 6$ in the same figure.

(5 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page

If you attempt Question 10, fill in the details in the first three boxes above and tie this sheet into your answer book.

10. (c) **(contioued)**

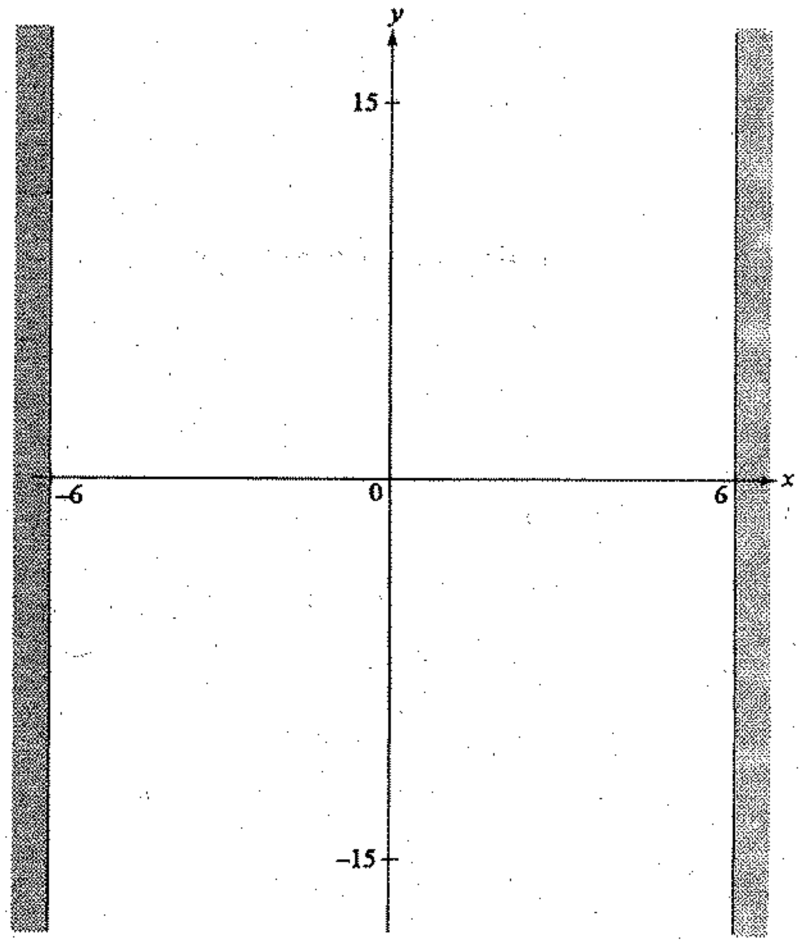

Figure 3

Go on to the next page

This is a blank page.

11. (a) Solve the equation $\cos 5\theta = 0$ for $0° \le \theta \le 180°$.

(2 marks)

(b) Using De Moivre's Theorem, show that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta.$$

(4 marks)

(c) Let $f(x) = 16x^4 - 20x^2 + 5$.

(i) By putting $x = \cos\theta$ and using the results of (a) and (b), find the four values of θ for which $\cos\theta$ is a root of $f(x) = 0$.

Hence show that

$$f(x) = 16(x^2 - \cos^2 18°)(x^2 - \cos^2 54°) \quad \ldots (*).$$

(ii) Using (*), form a quadratic equation with integral coefficients whose roots are $\sin^2 18°$ and $\sin^2 54°$. [Hint : You may treat $f(x)$ as a quadratic function of x^2.]

(10 marks)

Go on to the next page

12.

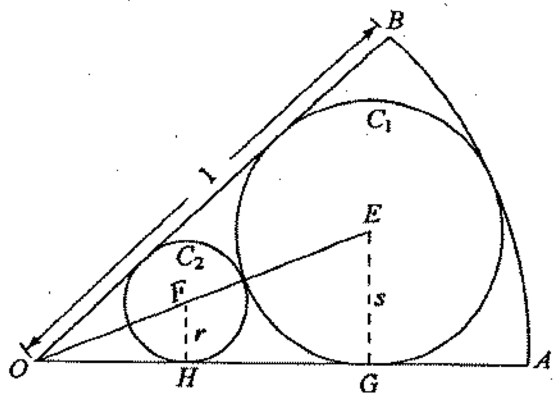

Figure 4

In Figure 4, OAB is a sector of unit radius and $\angle AOB = 2\theta$, where $0<\theta<\frac{\pi}{2}$. C_1 is an inscribed circle of radius s in the sector. C_2 is another circle of radius r touching OA, OB and C_1. Let E and F be the centres of C_1 and C_2 respectively. OA touches C_1 and C_2 at G and H respectively.

(a) Show that $s=\dfrac{\sin\theta}{1+\sin\theta}$.

Hence find $\dfrac{ds}{d\theta}$.

(4 marks)

(b) By considering ΔOFH and ΔOEG, express r in terms of s.

Hence show that $\dfrac{dr}{d\theta}=\dfrac{\cos\theta(1-3\sin\theta)}{(1+\sin\theta)^3}$.

(5 marks)

(c) By considering the ranges of values of θ for which r is

(i) increasing, and

(ii) decreasing,

find the maximum area of circle C_2. [Note : You may give your answers correct to three significant figures.]

(5 marks)

(d) Does the area of circle C_1 attain a minimum when the area of circle C_2 attains its maximum ? Explain your answer.

(2 marks)

13. **In this question, numerical answers may be given correct to three significant figures.**

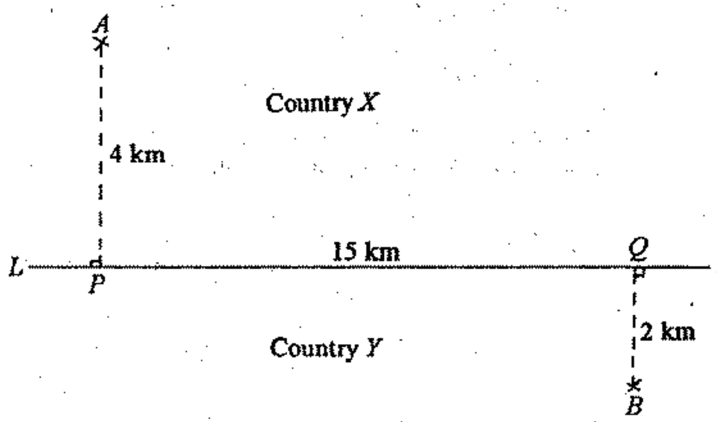

Figure 5

In Figure 5, line L represents the border of two countries X and Y. Amy lives at place A in Country X while Billy lives at place B in Country Y. P and Q are respectively the feet of perpendicular from A and B to the border and $AP = 4$ km, $PQ = 15$ km, $QB = 2$ km. Amy and Billy want to meet each other as early as possible at a certain point **on the border**. They start walking from home to that point at the same time. If one arrives earlier, he/she has to wait for the other.

(a) Let R be a point on the border such that $AR = RB$.

(i) Find the distance of R from Q.

(ii) Suppose Amy and Billy walk at equal speeds of 4 km h^{-1}. Explain briefly why they should walk to R in order to meet each other within the shortest time. Find this shortest time. (6 marks)

(b) (i) Suppose Billy runs at a speed of 8 km h^{-1} instead and Amy still walks at a speed of 4 km h^{-1}. To which point on the border should they go in order to meet each other within the shortest time ?

(ii) Suppose Billy rides on a bicycle at a speed of 16 km h^{-1} instead and Amy still walks at a speed of 4 km h^{-1}. To which point on the border should they go in order to meet each other within the shortest time ? (10 marks)

END OF PAPER

Section A (42 marks)

Answer ALL questions in this Section.

1. Show that $\dfrac{\sin 3\theta}{\sin\theta} + \dfrac{\cos 3\theta}{\cos\theta} = 4\cos 2\theta$.

(4 marks)

2. Find $\int x\sqrt{x-1}\,dx$.

[Hint : Let $u = x-1$.]

(4 marks)

3. Given three points $A(0, 2)$, $B(4, 6)$ and $C(3, 0)$. P is a point on AB such that $AP:PB = \lambda:1$, where $\lambda > 0$.

(a) Find the coordinates of P in terms of λ.

(b) If the area of ΔPAC is 6, find the value(s) of λ.

(5 marks)

4. By expressing $6\sin x + 8\cos x$ in the form $r\sin(x+\alpha)$, find the general solution of the equation

$$6\sin x + 8\cos x = 5,$$

and give your answer correct to the nearest degree.

(5 marks)

5. The slope at any point (x, y) of a curve is given by

$$\frac{dy}{dx} = 6x + \frac{1}{x^2},$$

where $x > 0$. If the curve cuts the x-axis at the point (1, 0), find its equation.

(5 marks)

6. L is the line $y = 2x+3$.

(a) A line with slope m makes an angle of 45° with L. Find the value(s) of m.

(b) A family of straight lines is given by the equation

$$2x - 3y + 2 + k(x-y-1) = 0,$$

where k is real. Using (a), find the equation of the line in the family with positive slope which makes an angle of 45° with L.

(6 marks)

Go on to the next page

7. Let $T_n = (n^2+1)(n!)$ for any positive integer n.

Prove, by mathematical induction, that

$$T_1 + T_2 + \dots + T_n = n[(n+1)!]$$

for any positive integer n.

[Note : $n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$.]

(6 marks)

8. Expand $(1+x)^n(1-2x)^4$ in ascending powers of x up to the term x^2, where n is a positive integer.

If the coefficient of x^2 is 54, find the coefficient of x.

(7 marks)

Section B (48 marks)

Answer any THREE questions in this Section.
Each question carries 16 marks.

9. $A(x, y)$ is a variable point such that the distance between A and the point (1, 0) is always equal to the distance from A to the line $x+1=0$. Let $\mathcal{P}$ be the locus of A.

(a) Show that the equation of $\mathcal{P}$ is $y^2 = 4x$. (3 marks)

(b) Show that the equation of the tangent to $\mathcal{P}$ at the point $(t^2, 2t)$, where $t \neq 0$, is $x - ty + t^2 = 0$.

Find the equation of the normal to $\mathcal{P}$ at that point. (3 marks)

(c)

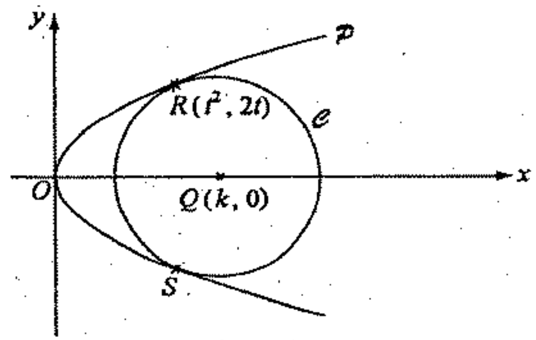

Figure 1

A circle $\mathcal{C}$ centred at point $Q(k, 0)$ touches $\mathcal{P}$ at two distinct points $R(t^2, 2t)$ and S as shown in Figure 1 (i.e. $\mathcal{P}$ and $\mathcal{C}$ have common tangents at these two points), where $t \neq 0$.

(i) Show that $t^2 = k - 2$.

(ii) It is known that the tangent to $\mathcal{P}$ at point R cuts the y-axis at the point (0, 2). Find

(1) the equation of the tangent to $\mathcal{P}$ at point S,

(2) the equation of $\mathcal{C}$. (10 marks)

Go on to the next page

10.

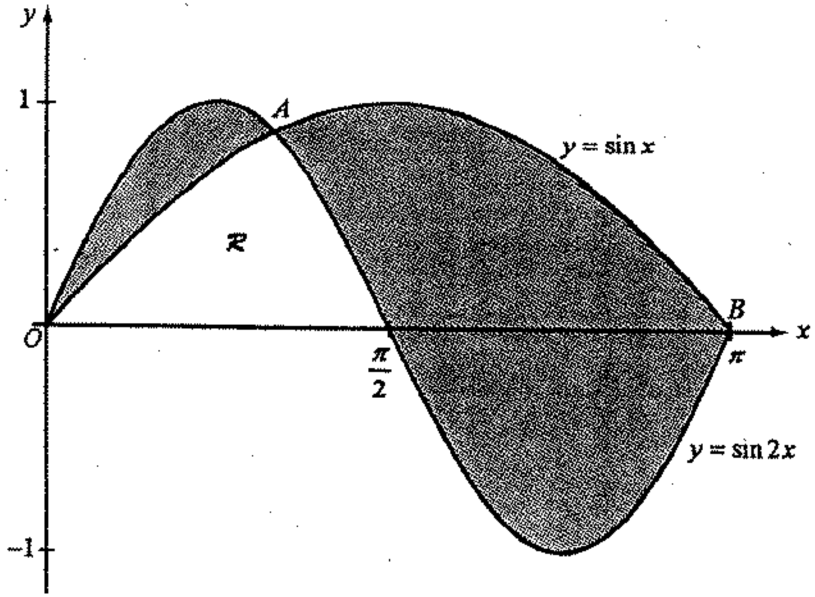

Figure 2(a)

Figure 2(a) shows the curves $y=\sin x$ and $y=\sin 2x$ for $0 \le x \le \pi$. The two curves intersect at the origin O, point A and point $B(\pi, 0)$.

(a) Show that the coordinates of A are $(\frac{\pi}{3}, \frac{\sqrt{3}}{2})$. (2 marks)

(b) Find the area of the shaded region in Figure 2(a). (5 marks)

(c) $\mathcal{R}$ is the region bounded by the two curves and the x-axis from $x=0$ to $\frac{\pi}{2}$. (See Figure 2(a).) If the region $\mathcal{R}$ is revolved about the x-axis, find the volume of the solid of revolution generated.

(5 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page	

If you attempt Question 10, fill in the details in the first three boxes above and tie this sheet into your answer book.

10. (d) Figure 2(b) shows the curve $y=\sin x$ for $0 \le x \le \pi$.

In Figure 2(b), sketch the curve $y = |\sin 2x|$ for $0 \le x \le \pi$.

In the same figure, shade the region whose area is represented by the expression

$$\int_0^{\pi} \big|\, |\sin 2x| - \sin x \,\big| \, dx .$$ (4 marks)

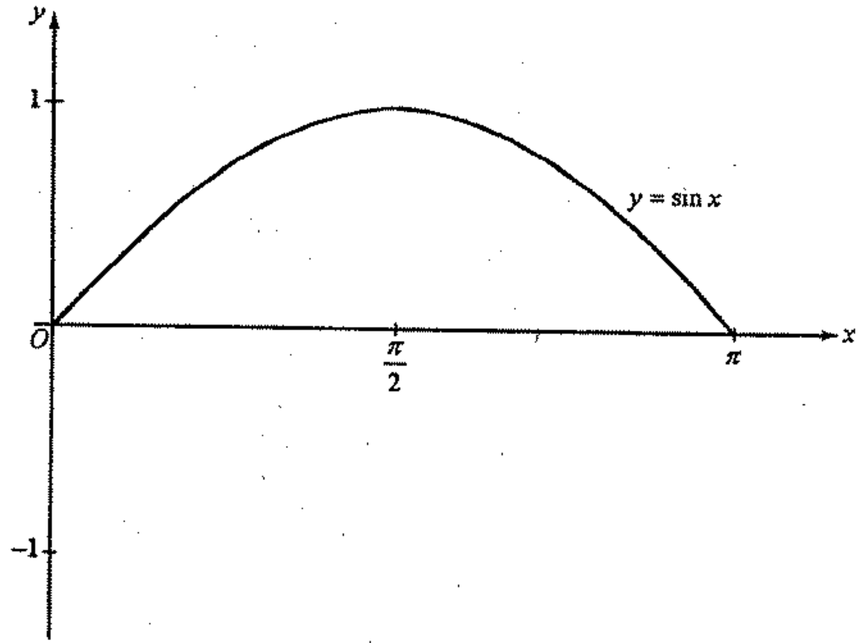

Figure 2(b)

Go on to the next page

This is a blank page.

11. (a) Using the substitution $u = \cot\theta$, find

$$\int \cot^n\theta \csc^2\theta \, d\theta,$$

where n is a non-negative integer.

(3 marks)

(b) By writing $\cot^{n+2}\theta$ as $\cot^n\theta\cot^2\theta$, show that

$$\int \cot^{n+2}\theta \, d\theta = -\frac{\cot^{n+1}\theta}{n+1} - \int \cot^n\theta \, d\theta,$$

where n is a non-negative integer.

(4 marks)

(c) Using (b), or otherwise, show that

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cot^2\theta \, d\theta = 1 - \frac{\sqrt{3}}{3} - \frac{\pi}{12}.$$

(3 marks)

(d) Using the substitution $x = \sec\theta$, evaluate

$$\int_{\sqrt{2}}^{2} \frac{dx}{x\sqrt{(x^2-1)^5}}.$$

(6 marks)

12.

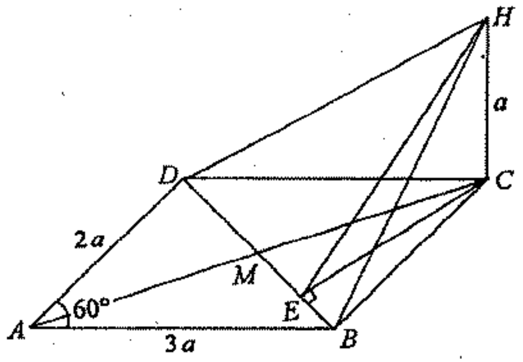

Figure 3(a)

In Figure 3(a), $ABCD$ is a parallelogram on a horizontal plane with $AB = 3a$, $AD = 2a$ and $\angle BAD = 60°$. H is a point vertically above C and $HC = a$.

(a) (i) Find AC in terms of a.

(ii) If M is the mid-point of AC, find the angle of elevation of H from M to the nearest degree.

(4 marks)

(b) E is a point on BD such that CE is perpendicular to BD.

(i) Find BD and CE in terms of a.

(ii) Using Pythagoras' theorem and its converse, show that HE is perpendicular to BD.

Hence find the angle between the planes HBD and $ABCD$ to the nearest degree.

(9 marks)

(c)

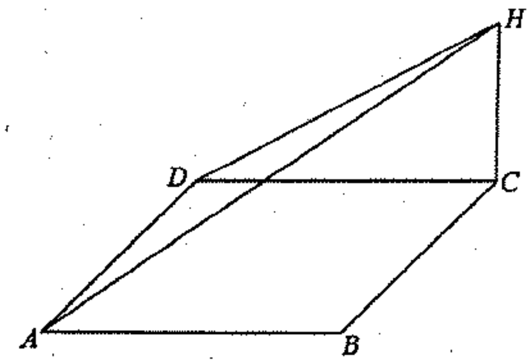

Figure 3(b)

Figure 3(b) shows the planes HAD and $ABCD$. X is a point lying on both planes such that the angle between the two planes is $\angle HXC$. Find AX in terms of a.

(3 marks)

Go on to the next page

13. Given a family of circles $F: x^2 + y^2 - 6x - 2y + k(2x - 4y + 3) = 0$, where k is real. All circles in F pass through two fixed points A and B.

(a) Find, in terms of k, the centre of a circle in F and show that the radius of the circle is $\sqrt{5(k^2 - k + 2)}$.

(4 marks)

(b) By considering the radius of the smallest circle in F, or otherwise, find the length of AB.

(4 marks)

(c) Given a straight line $L: 4x + 2y - 9 = 0$.

(i) Show that the distance from the centre of a circle in F to the line L is a constant.

State the geometrical relationship between the locus of the centres of the circles in F and the line L.

(ii)

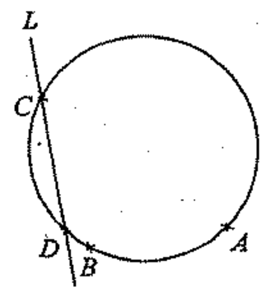

Figure 4

A circle in F cuts the line L at two points C and D such that the chords CD and AB are equal. (See Figure 4.) Find the equations of the two possible circles satisfying this condition.

(8 marks)

END OF PAPER