
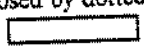


1996 I

GENERAL INSTRUCTIONS TO MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method is specified in the question.
2. In the marking scheme, marks are classified as follows :
 - 'M' marks – awarded for knowing a correct method of solution and attempting to apply it;
 - 'A' marks – awarded for the accuracy of the answer;
 - Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answer should NOT be awarded. Unless otherwise specified, no marks in the marking scheme are subdivisible.
3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.
4. The symbol pp-1 should be used to denote marks deducted for poor presentation (p.p.). Marks entered in the Page Total Box should be the net total score on that page. Note the following points :
 - (a) At most deduct 1 mark for p.p. in each question, up to a maximum of 3 marks for the whole paper.
 - (b) For similar p.p., deduct only 1 mark for the first time that it occurs, i.e. do not penalise candidates twice in the whole paper for the same p.p.
 - (c) In any case, do not deduct any marks for p.p. in those parts where candidates could not score any marks.
 - (d) Some cases in which marks should be deducted for p.p. are specified in the marking scheme. However the lists are by no means exhaustive. Markers should exercise their professional judgement to give p.p.s in other situations.
5. In the Marking Scheme, steps which can be omitted are enclosed by dotted rectangles , whereas alternative answers are enclosed by solid rectangles .
6. Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.
7. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

Solution	Marks	Remarks
1. $f'(x) = 3\sin^2 x \cos x$ $f''(x) = 6\sin x \cos^2 x - 3\sin^3 x$	1A $\frac{1M+1A}{3}$	Accept equivalent forms
2. $\frac{d}{dx}(x^2) = \lim_{\Delta x \rightarrow 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x}$ $= \lim_{\Delta x \rightarrow 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$ $= \lim_{\Delta x \rightarrow 0} (2x + \Delta x)$ $= 2x$	1A 1A 1A $\frac{1A}{4}$	Withhold this mark if $\lim_{\Delta x \rightarrow 0}$ is omitted. } For simplification
3. $\frac{2x-3}{x+1} \leq 1$ $\frac{2x-3}{x+1} - 1 \leq 0$ $\frac{x-4}{x+1} \leq 0$ $-1 < x \leq 4$	IM 1A $\frac{2A}{4}$	1A only for $-1 \leq x \leq 4$
<p><u>Alternative Solution (1)</u></p> <p>Consider the following cases (i) $x > -1$, (ii) $x < -1$.</p> <p>Case 1: $x > -1$</p> $2x - 3 \leq x + 1$ $x \leq 4$ <p>Since $x > -1$, $\therefore -1 < x \leq 4$.</p> <p>Case 2: $x < -1$</p> $2x - 3 \geq x + 1$ $x \geq 4$ <p>Since $x < -1$, \therefore there is no solution.</p> <p>Combining the 2 cases, $-1 < x \leq 4$.</p>	1M → 1A 2A	Awarded even if equality sign is included 1A only for $-1 \leq x \leq 4$

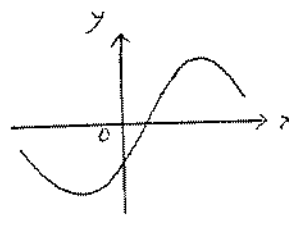
Solution	Marks	Remarks
<p><u>Alternative Solution (2)</u></p> $\frac{2x-3}{x+1} \leq 1$ $(2x-3)(x+1) \leq (x+1)^2 \quad \text{and } x \neq -1$ $x^2 - 3x - 4 \leq 0 \quad \text{and } x \neq -1$ $(x+1)(x-4) \leq 0 \quad \text{and } x \neq -1$ $-1 < x \leq 4$	<p>1M</p> <p>1A</p> <p>2A</p>	<p>1A only for $-1 \leq x \leq 4$</p>
<p>4. (a) $x^2 - 6x + 11 \equiv (x^2 - 6x + 9) + 2$</p> $= (x-3)^2 + 2$ <p>$\therefore a = -3, b = 2$</p>	<p>1M</p> <p>1A</p>	<p>For using method of completing square</p>
<p><u>Alternative Solution (1)</u></p> $x^2 - 6x + 11 \equiv (x+a)^2 + b$ $= x^2 + 2ax + a^2 + b$ <p>Comparing coefficients,</p> $\begin{cases} 2a = -6 \\ a^2 + b = 11 \end{cases}$ <p>$\therefore a = -3, b = 2$</p>	<p>1M</p> <p>1A</p>	<p>For comparing coefficients</p>
<p><u>Alternative Solution (2)</u></p> $x^2 - 6x + 11 \equiv (x+a)^2 + b$ <p>Put $x = 0, a^2 + b = 11$</p> $x = 1, (a+1)^2 + b = 6$ <p>Solving the 2 equations, $a = -3, b = 2$.</p>	<p>1M</p> <p>1A</p>	<p>For substituting any two real values of x</p>
<p>The least value of $x^2 - 6x + 11$ is 2.</p>	<p>1A</p>	

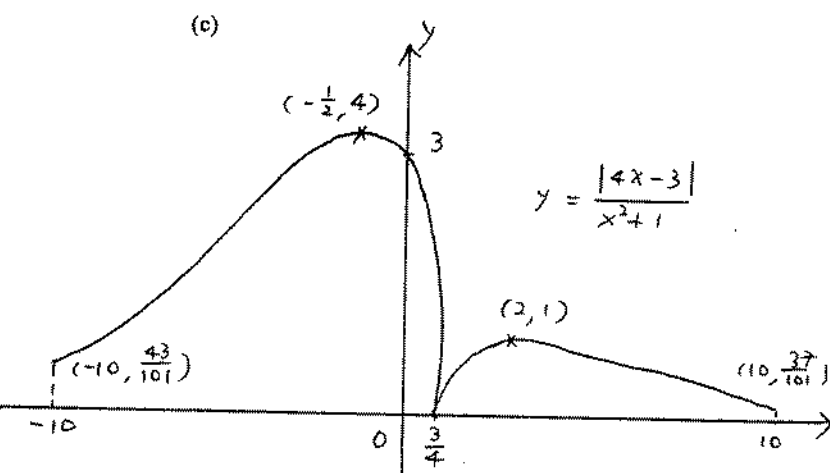
Solution	Marks	Remarks
<p>(b) The range of possible values of $\frac{1}{x^2 - 6x + 11}$ is</p> $0 < \frac{1}{x^2 - 6x + 11} \leq \frac{1}{2}$	<p><u>1M+1A</u></p> <p><u>5</u></p>	<p>1M for $\leq \frac{1}{\text{least value found in (a)}}$</p> <p>1A if all correct</p>
<p><u>Alternative solution</u></p> <p>Let $\frac{1}{x^2 - 6x + 11} = r$</p> $rx^2 - 6rx + (11r - 1) = 0$ $\Delta = 36r^2 - 4r(11r - 1) \geq 0$ $2r^2 - r \leq 0$ $r(2r - 1) \leq 0$ $0 < r \leq \frac{1}{2} \quad \text{Since } r \neq 0$	<p>1M</p> <p>1A</p>	
<p>5. (a) $\frac{2+4i}{1-i} = \frac{2+4i}{1-i} \cdot \frac{1+i}{1+i}$</p> $= \frac{2+2i+4i-4}{2}$ $= -1+3i$ <p>(b) $p+qi = \frac{2+4i}{1-i}(q+i)$</p> $p+qi = (-1+3i)(q+i)$ $= -q-3+(3q-1)i$ <p>Comparing coefficients,</p> $\begin{cases} p = -q - 3 \\ q = 3q - 1 \end{cases}$	<p>1M</p> <p>1A</p> <p>1M</p> <p>1M</p>	<p>For simplifying to the form $a+bi=c+di$</p>
<p><u>Alternative solution</u></p> $p+qi = \frac{2+4i}{1-i}(q+i)$ $(p+qi)(1-i) = (2+4i)(q+i)$ $(p+q)+(-p+q)i = (2q-4)+(2+4q)i$ <p>Comparing coefficients,</p> $\begin{cases} p+q = 2q-4 \\ -p+q = 2+4q \end{cases}$	<p>1M</p> <p>1M</p>	<p>For simplifying to the form $a+bi=c+di$</p>
<p>$\therefore q = \frac{1}{2}$</p> $p = \frac{-7}{2}$	<p>1A</p> <p><u>1A</u></p> <p><u>6</u></p>	

Solution	Marks	Remarks
<p>6. $\frac{dy}{dx} = \frac{-6}{(x+1)^2}$</p> $\frac{-6}{(x+1)^2} = -\frac{1}{6}$ $(x+1)^2 = 36$ $x = 5 \text{ or } -7$ <p>The points of contact are (5, 1) and (-7, -1).</p> <p>Equations of tangents are</p> $\frac{y-1}{x-5} = -\frac{1}{6} \quad \text{and} \quad \frac{y+1}{x+7} = -\frac{1}{6}$ $x+6y-11=0 \quad \text{and} \quad x+6y+13=0.$	<p>1A</p> <p>1M+1A</p> <p>1A+1A</p> <p>1M</p> <p>1A</p>	<p>1A for slope of line = $-\frac{1}{6}$</p> <p>For point-slope form</p>
<p><u>Alternative Solution</u></p> <p>Let the equation of the tangent be $x+6y+c=0$</p> <p>Put $x=-(6y+c)$ into C.</p> $y = \frac{6}{-6y+(1-c)}$ $6y^2+(c-1)y+6=0$ $\Delta = (c-1)^2 - 144 = 0$ $c = 13 \text{ or } -11$ <p>\therefore The equations of the tangents are $x+6y+13=0$ and $x+6y-11=0$.</p>	<p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A+1A</p> <p>1A</p>	
<hr/> <p>7</p> <hr/>		

<p>7. (a) Unit vector = $\frac{1}{\sqrt{4^2+3^2}}(4\vec{i}+3\vec{j})$</p>	1M	Accept $\frac{\vec{OA}}{ \vec{OA} }$
$= \frac{4}{5}\vec{i} + \frac{3}{5}\vec{j}$	1A	
$\vec{OC} = \frac{16}{5}\left(\frac{4}{5}\vec{i} + \frac{3}{5}\vec{j}\right)$ $= \frac{64}{25}\vec{i} + \frac{48}{25}\vec{j}$	1A	OR $\frac{16}{25}(4\vec{i}+3\vec{j})$

Solution	Marks	Remarks
<p>(b) $\vec{BC} = \vec{OC} - \vec{OB}$</p> $= \left(\frac{64}{25}\vec{i} + \frac{48}{25}\vec{j}\right) - (\vec{i} + 4\vec{j})$ $= \frac{39}{25}\vec{i} - \frac{52}{25}\vec{j}$ <p>$\vec{BC} \cdot \vec{OA} = \left(\frac{39}{25}\vec{i} - \frac{52}{25}\vec{j}\right) \cdot (4\vec{i} + 3\vec{j})$</p> $= \frac{39}{25}(4) - \frac{52}{25}(3)$ $= 0$ <p>$\therefore BC$ is perpendicular to OA.</p>	<p>1M</p> <p>1M</p> <p>1</p> <hr/> <p>6</p>	<p>For $\vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = 1$ and $\vec{i} \cdot \vec{j} = 0$</p> <p>Omitting vector sign generally (pp-1) Omitting dot sign generally (pp-1) (pp-1) for writing \vec{i}^2 or $\frac{\vec{a}}{b} = \frac{1}{2}$ etc.</p>
<p><u>Alternative solution</u></p> <p>(b) $\vec{OB} = \sqrt{1^2 + 4^2} = \sqrt{17}$</p> $ \vec{OC} = \sqrt{\left(\frac{64}{25}\right)^2 + \left(\frac{48}{25}\right)^2} = \frac{16}{5}$ <p>$\vec{BC} = \vec{OC} - \vec{OB}$</p> $= \left(\frac{64}{25}\vec{i} + \frac{48}{25}\vec{j}\right) - (\vec{i} + 4\vec{j})$ $= \frac{39}{25}\vec{i} - \frac{52}{25}\vec{j}$ <p>$\vec{BC} = \sqrt{\left(\frac{39}{25}\right)^2 + \left(\frac{-52}{25}\right)^2} = \frac{13}{5}$</p> $ \vec{OC} ^2 + \vec{BC} ^2 = \left(\frac{16}{5}\right)^2 + \left(\frac{13}{5}\right)^2$ $= \left(\frac{16}{5}\right)^2 + \left(\frac{13}{5}\right)^2$ $= 17 = OB^2$ <p>$\therefore BC$ is perpendicular to OA.</p>	<p>1M</p> <p>1M</p> <p>1</p>	<p>For using Pythagoras Theorem</p>
<p>8. (a) $(k-2)^2 - 4(k+1) > 0$</p> $k^2 - 8k > 0$ $k(k-8) > 0$ $k > 8 \text{ or } k < 0$ <p>(b) $\alpha + \beta = k - 2$</p> $ k - 2 < 5$ $-5 < k - 2 < 5$ $-3 < k < 7$ <p>Combining with (a), $-3 < k < 0$.</p>	<p>1M</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1M</p> <p>1A</p> <hr/> <p>1A</p> <hr/> <p>7</p>	<p>No mark for ≥ 0</p> <p>For simplifying L.H.S.</p> <p>For $-5 < \alpha + \beta < 5$ OR $(\alpha + \beta)^2 < 25$</p>

Solution	Marks	Remarks
9. (a) (i) The x-intercept is $\frac{3}{4}$. The y-intercept is -3 .	1A 1A	Accept $(\frac{3}{4}, 0)$ Accept $(0, -3)$
(ii) $\frac{dy}{dx} = \frac{4(x^2+1)-2x(4x-3)}{(x^2+1)^2}$	1M+1A	1M for quotient rule. Awarded even if included in (iii)
$\frac{4(x^2+1)-2x(4x-3)}{(x^2+1)^2} \leq 0$ $-4x^2+6x+4 \leq 0$	1M	Accept < 0
$(2x+1)(x-2) \geq 0$ $x \geq 2 \text{ or } x \leq -\frac{1}{2}$	1A	Accept $x > 2$ or $x < -\frac{1}{2}$
(iii) $\frac{dy}{dx} = \frac{4(x^2+1)-2x(4x-3)}{(x^2+1)^2}$	1M	
$\frac{dy}{dx} = 0$ at $x = 2$ or $-\frac{1}{2}$	1M	
The turning points are $(2, 1)$ and $(-\frac{1}{2}, -4)$.		
$(2, 1)$ is a maximum point.	1A	Awarded independent from the answer in (ii).
$(-\frac{1}{2}, -4)$ is a minimum point.	<hr/> 1A <hr/> 9	
(b)	1A <hr/> 3	Shape Accept  Labelling the end-points Labelling the intercepts and turning points

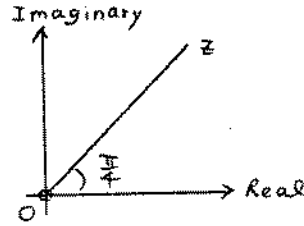
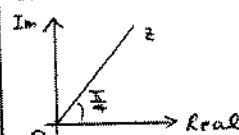
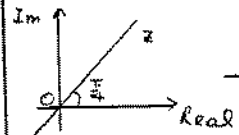
Solution	Marks	Remarks
<p>(c)</p>  <p style="text-align: center;">$y = \frac{ 4x-3 }{x^2+1}$</p> <p>The greatest value is 4.</p> <p>The least value is 0.</p>	<p>2M</p> <p>1A</p> <p><u>1A</u></p> <p><u>4</u></p>	<p>For reflection (Accept no labelling)</p>

Solution	Marks	Remarks
10. (a) (i) $\vec{AE} = \frac{2\vec{a} + t\vec{b}}{1+t}$	1A	
(ii) $\vec{AE} = \frac{7\vec{a} + \vec{AF}}{8}$	2A	
<u>Alternative Solution</u> $\vec{AE} = \vec{AD} + \vec{DE}$ $= \vec{a} + \frac{1}{8}(\vec{DF})$ $= \vec{a} + \frac{1}{8}(\vec{AF} - \vec{a})$ $= \frac{7}{8}\vec{a} + \frac{1}{8}\vec{AF}$	1A 1A	
$\frac{2\vec{a} + t\vec{b}}{1+t} = \frac{7\vec{a} + \vec{AF}}{8}$ $\vec{AF} = 8\left(\frac{2\vec{a} + t\vec{b}}{1+t}\right) - 7\vec{a}$ $= \frac{9-7t}{1+t}\vec{a} + \frac{8t}{1+t}\vec{b}$	1M 1	
<u>Alternative Solution</u> $\vec{AF} = \vec{AD} + \vec{DF}$ $= \vec{AD} + 8\vec{DE}$ $= \vec{a} + 8(\vec{AE} - \vec{AD})$ $= \vec{a} + 8\left(\frac{2\vec{a} + t\vec{b}}{1+t} - \vec{a}\right)$ $= \frac{9-7t}{1+t}\vec{a} + \frac{8t}{1+t}\vec{b}$	1M 1	
	<hr/> <u>5</u>	
(b) (i) Since A, B, F are collinear,		
$\frac{9-7t}{1+t} = 0$	1M	
$t = \frac{9}{7}$	1A	
(ii) (1) $\vec{a} \cdot \vec{b} = \vec{a} \vec{b} \cos \angle BAC$		
$= 3(2)\left(\frac{1}{3}\right)$		
$= 2$	1A	
(2) $\vec{AB} \cdot \vec{BC} = \vec{b} \cdot (\vec{AC} - \vec{AB})$	1M	For $\vec{BC} = \vec{AC} - \vec{AB}$
$= \vec{b} \cdot (2\vec{a} - \vec{b})$		
$= 2\vec{b} \cdot \vec{a} - \vec{b} \cdot \vec{b}$	1M	For distributive law
$= 2(2) - 4$	1A	For $\vec{b} \cdot \vec{b} = 4$
$= 0$	1A	

Solution	Marks	Remarks
$\vec{AD} \cdot \vec{DE} = \vec{AD} \cdot (\vec{AE} - \vec{AD})$ $= \vec{a} \cdot \left(\frac{2\vec{a} + \frac{9}{7}\vec{b}}{1 + \frac{9}{7}} - \vec{a} \right)$ $= \vec{a} \cdot \left(\frac{-2\vec{a} + 9\vec{b}}{16} \right)$ $= -\frac{2}{16}(\vec{a} \cdot \vec{a}) + \frac{9}{16}(\vec{b} \cdot \vec{a})$ $= -\frac{2}{16}(3)^2 + \frac{9}{16}(2)$ $= 0$ <p>(3) Since $\vec{AB} \cdot \vec{BC} = \vec{AD} \cdot \vec{DE} = 0$,</p> $\therefore \angle CBF = \angle CDF = \frac{\pi}{2}$ <p>\therefore The points E, C, D and F are concyclic. Converse of \angles in the same segment.</p> <p>\therefore The circle also passes through F.</p>	<p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <hr/> <p>11</p>	<p>For substituting t to find \vec{DE}</p> <p>Omitting vector sign generally (pp-1) Omitting dot sign generally (pp-1) (pp-1) for writing \vec{i}^2 or $\frac{\vec{a}}{b} = \frac{1}{2}$ etc.</p>

Solution	Marks	Remarks
<p>11. (a) $V = \left(\frac{1}{2}x^2 \sin 60^\circ\right) \ell$ $= \frac{\sqrt{3}}{4}x^2 \ell$</p> <p>Since $V = 24$, $\frac{\sqrt{3}}{4}x^2 \ell = 24$ $x^2 \ell = 32\sqrt{3}$</p> <p>$S = 2\left(\frac{1}{2}x^2 \sin 60^\circ\right) + 2x\ell$ $= \frac{\sqrt{3}}{2}x^2 + 2x\left(\frac{32\sqrt{3}}{x^2}\right)$</p> <p>$S = \frac{\sqrt{3}}{2}x^2 + \frac{64\sqrt{3}}{x}$</p>	<p>1A</p> <p>1M</p> <p>1A</p> <p>1</p> <p>4</p>	<p>For establishing a relation in x and ℓ</p>
<p>(b) $\frac{dS}{dx} = \sqrt{3}x - \frac{64\sqrt{3}}{x^2}$ $\frac{dS}{dx} = 0$ $\sqrt{3}x = \frac{64\sqrt{3}}{x^2}$ $x = 4$</p> <p>$\frac{d^2S}{dx^2} = \sqrt{3} + \frac{128\sqrt{3}}{x^3} > 0$ at $x = 4$</p> <p>$\therefore S$ is minimum at $x = 4$.</p> <p>When $x = 4$, $\ell = 2\sqrt{3}$.</p>	<p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>5</p>	<p>For checking</p> <p>Withhold this mark if checking is omitted</p>
<p>(c) (i) $V = \frac{1}{2}h(2h \tan 30^\circ)(2\sqrt{3})$ $= 2h^2$ $A = (2h \tan 30^\circ)(2\sqrt{3})$ $= 4h$</p> <p>$\frac{dV}{dt} = -\frac{1}{10}A$ $\frac{dV}{dh} \frac{dh}{dt} = -\frac{1}{10}A$ $(4h) \frac{dh}{dt} = -\frac{1}{10}(4h)$ $\frac{dh}{dt} = -\frac{1}{10}$</p>	<p>1</p> <p>1</p> <p>1M</p> <p>1A</p> <p>1A</p>	<p>For chain rule</p> <p>For $\frac{dV}{dh} = 4h$ only.</p>

Solution	Marks	Remarks
<p><u>Alternative Solution (1)</u></p> $V = 2h^2$ $\frac{dV}{dt} = 4h \frac{dh}{dt}$ $\frac{1}{10}(4h) = 4h \frac{dh}{dt}$ $\frac{dh}{dt} = -\frac{1}{10}$	<p>1A</p> <p>1M</p> <p>1A</p>	<p>For substitution in LHS</p>
<p><u>Alternative Solution (2)</u></p> $\frac{dV}{dt} = -\frac{1}{10}A$ $\frac{dV}{dh} \frac{dh}{dt} = -\frac{1}{10}A$ $A \frac{dh}{dt} = -\frac{1}{10}A \quad \therefore \frac{dV}{dh} = A$ $\frac{dh}{dt} = -\frac{1}{10}$	<p>1M</p> <p>1A</p> <p>1A</p>	<p>For chain rule</p> <p>For $\frac{dV}{dh} = A$</p>
<p>(ii) At $t = 0$, $h = 4 \cos 30^\circ = 2\sqrt{3}$</p> <p>$\therefore$ Time required = $\frac{2\sqrt{3}}{\frac{1}{10}}$</p> <p style="padding-left: 100px;">$= 20\sqrt{3}$</p>	<p>1M</p> <hr/> <p>1A</p> <hr/> <p>7</p>	

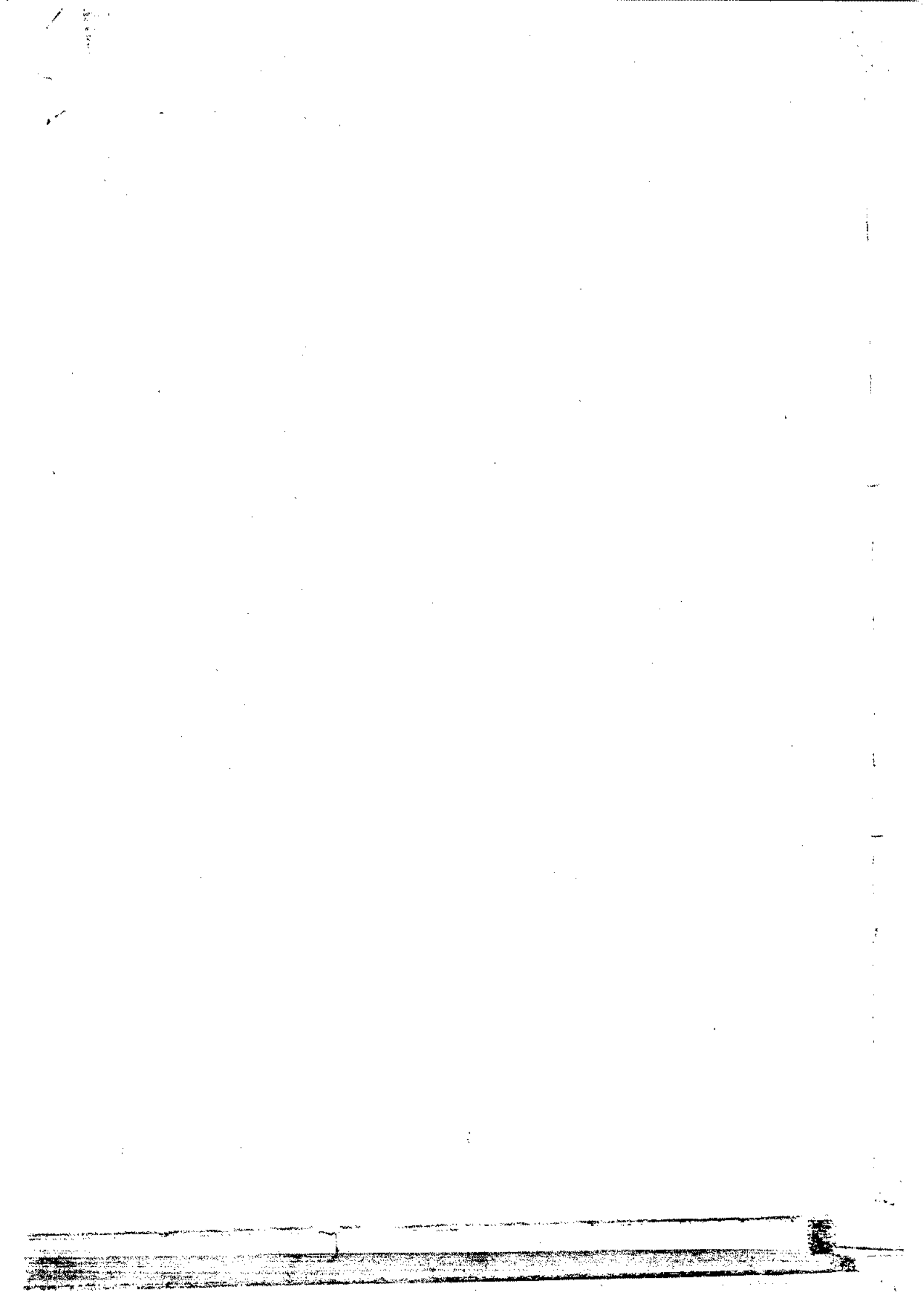
Solution	Marks	Remarks
<p>12. (a) $\arg(1-i) = -\frac{\pi}{4}$ OR - 45°</p> <p>$\arg\left(\frac{z}{1-i}\right) = \frac{\pi}{2}$</p> <p>$\arg z - \arg(1-i) = \frac{\pi}{2}$</p> <p>$\arg z - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$</p> <p>$\arg z = \frac{\pi}{4}$ OR 45°</p>	<p>1A</p> <p>2M</p> <p>1A</p>	<p>Accept $-\frac{\pi}{4} + 2k\pi$</p> <p>Accept $\frac{\pi}{4} + 2k\pi$</p>
<p><u>Alternative solution for evaluating $\arg(z)$</u></p> <p>Let $z = x + yi$</p> $\frac{z}{1-i} = \frac{x+yi}{1-i} \cdot \frac{1+i}{1+i}$ $= \frac{(x-y) + (x+y)i}{2}$ <p>Since $\arg\left(\frac{z}{1-i}\right) = \frac{\pi}{2}$, $\therefore \operatorname{Re}\left(\frac{z}{1-i}\right) = 0$ and $\operatorname{Im}\left(\frac{z}{1-i}\right) > 0$</p> <p style="margin-left: 150px;">i.e. $x-y=0$</p> <p style="margin-left: 150px;">$x=y$</p> <p style="margin-left: 150px;">$x+y > 0$</p> <p style="margin-left: 150px;">$x+y > 0$</p> <p>$\therefore \arg z = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}(1)$</p> <p style="margin-left: 150px;">$= \frac{\pi}{4}$</p>	<p>1M</p> <p>1M</p> <p>1A</p>	
<p>Imaginary</p> 	<p>1A+1A</p> <hr style="width: 50px; margin: 0 auto;"/> <p>6</p>	<p>1A for a straight line through O</p> <p>1A if argument is also correct</p> <p>(pp-1) if axes are not labelled</p>  <p style="text-align: right;">— 2A</p>  <p style="text-align: right;">— 1A only</p>
<p>(b) Let $w = r(\cos\theta + i\sin\theta)$</p> <p>$w^3 = (\bar{w})^3$</p> <p>$r^3(\cos 3\theta + i\sin 3\theta) = [r(\cos(-\theta) + i\sin(-\theta))]^3$</p> <p>$= r^3[\cos(-3\theta) + i\sin(-3\theta)]$</p>	<p>1A+1M</p>	<p>1A for LHS</p> <p>1M for $\bar{w} = r[\cos(-\theta) + i\sin(-\theta)]$ (can be on</p>

Solution	Marks	Remarks
$= r^3 (\cos 3\theta - i \sin 3\theta)$ $\therefore r^3 \sin 3\theta = 0$ $\sin 3\theta = 0 \quad \because r > 0$ $3\theta = n\pi$ $\theta = \frac{2\pi}{3} \quad \because \frac{\pi}{2} < \theta < \pi$ $\therefore \arg w = \frac{2\pi}{3}$	<p>1A</p> <p>1A</p> <p>1A</p>	<p>(can be omitted)</p> <p>Accept degrees</p>
<p><u>Alternative Solution (1)</u></p> <p>Let $w = r(\cos \theta + i \sin \theta)$</p> $w^3 = (\overline{w})^3$ $w^3 = \overline{w^3}$ <p>\therefore Imaginary part of $w^3 = 0$</p> <p>Since $w^3 = r^3(\cos 3\theta + i \sin 3\theta)$</p> $r^3 \sin 3\theta = 0$ $\sin 3\theta = 0$ $\theta = \frac{2\pi}{3}$	<p>1M</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p>	<p>(can be omitted)</p>
<p><u>Alternative Solution (2)</u></p> <p>Let $w = x + yi$</p> $w^3 = (\overline{w})^3$ $(x + yi)^3 = (x - yi)^3$ $x^3 + 3x^2(yi) + 3x(yi)^2 + (yi)^3 = x^3 - 3x^2(yi) + 3x(yi)^2 - (yi)^3$ $6x^2yi + 2(yi)^3 = 0$ $3x^2y - y^3 = 0$ $y(3x^2 - y^2) = 0$ $y = 0 \text{ or } \frac{y}{x} = \pm\sqrt{3}$ <p>Since $\frac{\pi}{2} < \arg w < \pi$, $\frac{y}{x} = -\sqrt{3}$.</p> <div style="border: 1px dashed black; padding: 5px; width: fit-content; margin: 10px auto;"> $y = 0 \text{ and } \frac{y}{x} = \sqrt{3} \text{ are rejected}$ </div> $\therefore \arg w = \tan^{-1} \left(\frac{y}{x} \right)$ $= \tan^{-1}(-\sqrt{3})$ $= \frac{2\pi}{3}$	<p>1M</p> <p>1A</p> <p>1A</p> <p>1A</p>	<p>1A</p>
<hr style="width: 50%; margin: 0 auto;"/> <p>5</p> <hr style="width: 50%; margin: 0 auto;"/>		

Solution	Marks	Remarks
<p>(c) $z = 2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$</p> <p>$w = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$</p> <p>The complex number represented by Q</p> <p>$= z + w$</p> <p>$= 2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) + 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$</p> <p>$= 2\sqrt{2}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) + 2\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$</p> <p>$= 1 + (2 + \sqrt{3})i$</p>	<p>} 1M</p> <p>2M</p> <p>1A</p> <hr/> <p>1A</p> <hr/> <p>5</p>	<p>Awarded if either one is correct</p> <p>(can be omitted)</p>

Solution	Marks	Remarks
<p>13. (a) $\alpha + \beta = \lambda$ and $\alpha\beta = 1$</p> $S_2 = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $= \lambda^2 - 2$ $S_3 = \alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha^2\beta - 3\alpha\beta^2$ $= (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ $= \lambda^3 - 3\lambda$ <p>(b) $\alpha^5 - \lambda\alpha^4 + \alpha^3$</p> $= \alpha^3(\alpha^2 - \lambda\alpha + 1)$ $= 0 \quad \because \alpha \text{ is a root of } x^2 - \lambda x + 1 = 0$	<p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <hr/> <p>5</p> <p>1A</p> <p>1A</p>	<p>(can be omitted)</p> <div style="border: 1px dashed black; padding: 5px; margin-top: 20px;"> <p>OR $= (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$</p> <p>$= (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$</p> </div>
<p><u>Alternative Solution</u></p> <p>Since α is a root of $x^2 - \lambda x + 1 = 0$</p> $\alpha^2 - \lambda\alpha + 1 = 0$ $\alpha^3(\alpha^2 - \lambda\alpha + 1) = 0$ $\alpha^5 - \lambda\alpha^4 + \alpha^3 = 0 \quad \text{--- (1)}$	<p>1A</p> <p>1A</p>	
$S_5 - \lambda S_4 + S_3 = (\alpha^5 + \beta^5) - \lambda(\alpha^4 + \beta^4) + (\alpha^3 + \beta^3)$ $= (\alpha^5 - \lambda\alpha^4 + \alpha^3) + (\beta^5 - \lambda\beta^4 + \beta^3)$ $= 0 + \beta^3(\beta^2 - \lambda\beta + 1)$ $= 0 + \beta^3(0)$ $= 0$	<p>1A</p> <hr/> <p>1</p> <hr/> <p>4</p>	<p>For $\beta^2 - \lambda\beta + 1 = 0$ or $\beta^5 - \lambda\beta^4 + \beta^3 = 0$</p>
<p>(c) Let $S_3 = 10k, S_4 = 7\lambda k, S_5 = 25k$, where $k \neq 0$.</p> $S_5 - \lambda S_4 + S_3 = 0$ $25k - \lambda(7\lambda k) + 10k = 0$ $7\lambda^2 = 35$ $\lambda = \sqrt{5} \quad \because \lambda \geq 2$	<p>1M</p> <p>1A</p>	

Solution	Marks	Remarks
<p><u>Alternative Solution</u></p> <p>Put $S_5 = \frac{25}{10}S_3, S_4 = \frac{7\lambda}{10}S_3$.</p> $S_5 - \lambda S_4 + S_3 = 0$ $\frac{25}{10}S_3 - \lambda\left(\frac{7\lambda}{10}\right)S_3 + S_3 = 0$ $\frac{25}{10} - \frac{7\lambda^2}{10} + 1 = 0$ $7\lambda^2 = 35$ $\lambda = \sqrt{5} \quad \because \lambda \geq 2$	<p>1M</p> <p>1A</p>	<p>OR Putting $S_3 = \frac{10}{25}S_5, S_4 = \frac{7\lambda}{25}S_5$;</p> $S_3 = \frac{10}{7\lambda}S_4, S_5 = \frac{25}{7\lambda}S_4 \text{ etc.}$
<p>(i) $S_3 = \lambda^3 - 3\lambda$</p> $= (\sqrt{5})^3 - 3\sqrt{5}$ $= 2\sqrt{5}$	<p>1M</p> <p>1A</p>	
<p>(ii) Put $\lambda = \sqrt{5}, \alpha, \beta = \frac{\sqrt{5} \pm 1}{2}$</p> $\therefore \left(\frac{\sqrt{5}+1}{2}\right)^5 + \left(\frac{\sqrt{5}-1}{2}\right)^5$ $= S_5 \quad \boxed{\text{or } = \alpha^5 + \beta^5}$ $= \frac{25}{10}S_3$ $= \frac{25}{10}(2\sqrt{5})$ $= 5\sqrt{5}$	<p>1M</p> <p>1M</p> <hr/> <p>1A</p> <hr/> <p>7</p>	<p>(pp-1) if explanation was not given</p>



1996 II

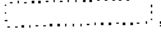

GENERAL INSTRUCTIONS TO MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method is specified in the question.
2. In the marking scheme, marks are classified as follows :

'M' marks – awarded for knowing a correct method of solution and attempting to apply it;

'A' marks – awarded for the accuracy of the answer;

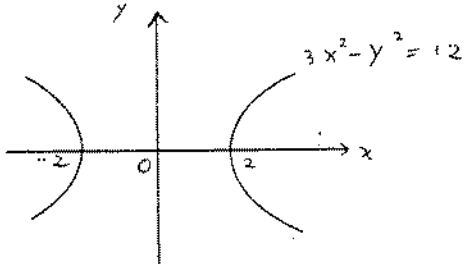
Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answer should NOT be awarded. Unless otherwise specified, no marks in the marking scheme are subdivisible.
3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.
4. The symbol pp-1 should be used to denote marks deducted for poor presentation (p.p.). Marks entered in the Page Total Box should be the net total score on that page. Note the following points :
 - (a) At most deduct 1 mark for p.p. in each question, up to a maximum of 3 marks for the whole paper.
 - (b) For similar p.p., deduct only 1 mark for the first time that it occurs, i.e. do not penalise candidates twice in the whole paper for the same p.p.
 - (c) In any case, do not deduct any marks for p.p. in those parts where candidates could not score any marks.
 - (d) Some cases in which marks should be deducted for p.p. are specified in the marking scheme. However the lists are by no means exhaustive. Markers should exercise their professional judgement to give p.p.s in other situations.
5. In the Marking Scheme, steps which can be omitted are enclosed by dotted rectangles , whereas alternative answers are enclosed by solid rectangles .
6. Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.
7. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

Solution	Marks	Remarks
<p>1. $\sin 5\theta + \sin 3\theta = \cos \theta$</p> <p>$2 \sin 4\theta \cos \theta = \cos \theta$</p> <p>$\cos \theta = 0$ or $\sin 4\theta = \frac{1}{2}$</p> <p>$4\theta = n\pi + (-1)^n \left(\frac{\pi}{6}\right)$ n is an integer</p> <p>$\theta = 2n\pi \pm \frac{\pi}{2}$ or $\theta = \frac{n\pi}{4} + (-1)^n \left(\frac{\pi}{24}\right)$</p> <div style="border: 1px solid black; padding: 5px; display: inline-block;"> <p>or $360n^\circ \pm 90^\circ$ or $45n^\circ + (-1)^n 7.5^\circ$</p> </div>	<p>1M</p> <p>1A+1A</p> <p>1A+1A</p> <hr/> <p>5</p>	<p>For using</p> $\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$ <p>$2n\pi \pm 90^\circ$ etc (pp-1)</p>
<p>2. (a) $(1+x+ax^2)^6 = [1+x(1+ax)]^6$</p> <p>$= 1 + 6x(1+ax) + 15x^2(1+ax)^2 + 20x^3(1+ax)^3 + \dots$</p> <p>$= 1 + 6x + (6a+15)x^2 + (30a+20)x^3 + \dots$</p> <p>$\therefore k_1 = 6a + 15$</p> <p>$k_2 = 30a + 20$</p> <p>(b) $6 + k_2 = 2k_1$</p> <p>$6 + (30a + 20) = 2(6a + 15)$</p> <p>$a = \frac{2}{9}$</p>	<p>1M</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1M</p> <p>1A</p> <hr/> <p>6</p>	<p>Accept $[(1+x) + ax^2]^6$ etc.</p> <p>or $(1+x)^6 + 6(1+x)^5 ax^2 + \dots$</p> <p>Accept ${}_6C_r$ notations</p> <p>(pp-1) for omitting dots</p> <p>or $k_2 - k_1 = k_1 - 6$</p>
<p>3. (a) $\begin{cases} \frac{x^2}{2} + \frac{y^2}{7} = 1 \\ y = mx + c \end{cases}$</p> <p>$7x^2 + 2(mx+c)^2 = 14$</p> <p>$(7+2m^2)x^2 + 4mcx + (2c^2-14) = 0$</p> <p>Since the line is a tangent to E,</p> <p>$(4mc)^2 - 4(7+2m^2)(2c^2-14) = 0$</p> <p>$16m^2c^2 - 4(14c^2 - 98 + 4m^2c^2 - 28m^2) = 0$</p> <p>$14c^2 = 98 + 28m^2$</p> <p>$c^2 = 2m^2 + 7$</p>	<p>1M</p> <p>1A</p> <p>1M</p> <p>1</p>	<p>For substitution</p>

Solution	Marks	Remarks
<p><u>Alternative solution</u> Let (x_0, y_0) be the point of contact of the tangent with E.</p> <p>The equation of the tangent is $7(xx_0) + 2(yy_0) = 14$</p> <p>Compare with $y = mx + c$,</p> $\frac{7x_0}{m} = \frac{2y_0}{-1} = \frac{-14}{c}$ $\therefore x_0 = \frac{-2m}{c}, y_0 = \frac{7}{c}$ <p>Substitute into E,</p> $7\left(\frac{-2m}{c}\right)^2 + 2\left(\frac{7}{c}\right)^2 = 14$ $2m^2 + 7 = c^2$	<p>1A</p> <p>1M</p> <p>1M</p> <p>1</p>	
<p>(b) Substitute $(0, 5)$ into $y = mx + c, c = 5$</p> <p>Put $c = 5, 25 = 2m^2 + 7$</p> <p>$m = \pm 3$.</p> <p>\therefore The equations of the two tangents are $y = 3x + 5$ and $y = -3x + 5$.</p>	<p>1A</p> <p>1M</p> <p><u>1A</u> <u>7</u></p>	<p><u>OR</u> Substituting $y = mx + 5$ into E and set $\Delta = 0$</p>

Solution	Marks	Remarks
<p>4. For $n = 1$, $2n^3 + n = 3$ which is divisible by 3.</p> <p>\therefore the statement is true for $n = 1$.</p> <p>Assume $2k^3 + k$ is divisible by 3 for some +ve integer k.</p> <p>i.e. $2k^3 + k = 3m$, where m is an integer</p> <p>Then $2(k+1)^3 + (k+1) = 2(k^3 + 3k^2 + 3k + 1) + (k+1)$</p> $= (2k^3 + k) + (6k^2 + 6k + 3)$ $= 3m + 3(2k^2 + 2k + 1)$ <p>Since both terms are divisible by 3,</p> <p>$\therefore 2(k+1)^3 + (k+1)$ is also divisible by 3.</p>	<p>1</p> <p>1</p> <p>1</p> <p>2</p>	
<p><u>Alternative solution (1)</u></p> <p>Put $2k^3 = 3m - k$</p> <p>$2(k+1)^3 + (k+1) = 2(k^3 + 3k^2 + 3k + 1) + (k+1)$</p> $= 3m - k + 6k^2 + 6k + 2 + (k+1)$ $= 3m + 3(2k^2 + 2k + 1)$	<p>1</p> <p>2</p>	
<p><u>Alternative solution (2)</u></p> <p>Let $f(k) = 2k^3 + k$</p> <p>$f(k+1) - f(k) = 2(k^3 + 3k^2 + 3k + 1) + (k+1) - (2k^3 + k)$</p> $= 3(2k^2 + 2k + 1)$ <p>$f(k+1) = f(k) + 3(2k^2 + 2k + 1)$</p> <p>Since $f(k)$ is divisible by 3, $\therefore f(k+1)$ is also divisible by 3.</p>	<p>1</p> <p>2</p>	
<p>\therefore The statement is also true for $n = k + 1$ if it is true for $n = k$.</p> <p>By the principle of mathematical induction,</p> <p>the statement is true for all positive integers n.</p>	<p>1</p> <hr/> <p>6</p>	
<p>5. (a) Area bounded by C and $OA = \int_0^4 (4x - x^2) dx$</p> $= [2x^2 - \frac{x^3}{3}]_0^4$ $= \frac{32}{3}$ <p>(b) Area of $\triangle OPA = \frac{1}{2} (4)(3)$</p> $= 6$ <p>\therefore Area of shaded region $= \frac{32}{3} - 6$</p> $= \frac{14}{3}$	<p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1M</p> <p>1A</p> <hr/> <p>6</p>	<p>(pp-1) for omitting dx</p> <p>or $= \int_0^1 3x dx + \int_1^4 (-x+4) dx$</p> $= \frac{3}{2} + \frac{9}{2} = 6$

Solution	Marks	Remarks
<p>6. $y = \int \tan^3 x \sec x \, dx$</p> <p>Let $u = \sec x$</p> $du = \sec x \tan x \, dx$ $y = \int (u^2 - 1) du$ $= \frac{u^3}{3} - u + c$ $= \frac{1}{3} \sec^3 x - \sec x + c \quad \text{where } c \text{ is a constant}$	<p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p>	<p>Withhold this mark if "y=" is omitted</p> <p>Omit 'du' (pp-1)</p> <p>no mark if c is omitted</p>
<p><u>Alternative solution</u></p> $y = \int \tan^3 x \sec x \, dx$ $= \int \tan^2 x \, d \sec x$ $= \int (\sec^2 x - 1) \, d \sec x$ $= \frac{1}{3} \sec^3 x - \sec x + c \quad \text{where } c \text{ is a constant}$	<p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p>	<p>Omit 'dx' (pp-1)</p>
<p>Since the curve passes through O,</p> $0 = \frac{1}{3} - 1 + c$ $c = \frac{2}{3}$ <p>\therefore The equation of the curve is $y = \frac{1}{3} \sec^3 x - \sec x + \frac{2}{3}$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $\text{or } y = \frac{1}{3 \cos^3 x} - \frac{1}{\cos x} + \frac{2}{3}$ </div>	<p>1M</p> <p>1A</p> <hr/> <p>6</p>	
<p>7. (a)</p> $\sqrt{(x-4)^2 + y^2} = 2 x-1 \quad \text{or } (x-4)^2 + y^2 = 4(x-1)^2$ $x^2 - 8x + 16 + y^2 = 4(x^2 - 2x + 1)$ $3x^2 - y^2 = 12 \quad \text{or } \frac{x^2}{4} - \frac{y^2}{12} = 1, 3x^2 - y^2 - 12 = 0 \text{ etc.}$ <p>The locus is a hyperbola.</p> <p>(b)</p> 	<p>1A+1A</p> <p>1A</p> <p>1A</p> <hr/> <p>1M+1A</p> <hr/> <p>6</p>	<p>1A for LHS, 1A for RHS (pp-1) for omitting absolute sign</p> <p>(pp-1) for not labelling the axes.</p>

Solution	Marks	Remarks
<p>8. (a) The equation of F is</p> $2x - y - 4 + k(x - 2y + 4) = 0 \quad \text{where } k \text{ is real}$ <p>Substitute $(0, 0)$ into the equation,</p> $-4 + 4k = 0$ $k = 1$ <p>\therefore The equation of L is $2x - y - 4 + (x - 2y + 4) = 0$</p> $\text{i.e. } y - x = 0.$	<p>1A</p> <p>1M</p> <p>1A</p> <p>1A</p>	<p>or $x - 2y + 4 + k(2x - y - 4) = 0$</p>
<p><u>Alternative solution</u></p> $\begin{cases} x - 2y + 4 = 0 \\ 2x - y - 4 = 0 \end{cases}$ <p>Solving the two equations, $x = 4, y = 4$.</p> <p>\therefore The coordinates of the point of intersection of L_1 and L_2 are $(4, 4)$.</p> <p>Let m be the slope of a line passing through $(4, 4)$.</p> <p>The equation of F is</p> $\frac{y - 4}{x - 4} = m$ $y = mx - 4m + 4.$ <p>Substitute $(0, 0)$ into the equation,</p> $m = 1$ <p>\therefore The equation of L is $y = x$.</p>	<p>1A</p> <p>1M</p> <p>1A</p> <p>1A</p>	
<p>(b) (i) $m_{L_1} = 2, m_{L_2} = \frac{1}{2}, m_L = 1$</p> $\tan \angle CAB = \frac{m_{L_1} - m_L}{1 + m_{L_1} m_L}$ $= \frac{2 - 1}{1 + 2}$ $= \frac{1}{3}$ $\tan \angle CAD = \frac{m_L - m_{L_2}}{1 + m_{L_2} m_L}$ $= \frac{1 - \frac{1}{2}}{1 + \frac{1}{2}}$ $= \frac{1}{3}$	<p style="text-align: center;">4</p> <p>1A</p> <p>1M</p> <p>1</p> <p>1</p>	<p>Awarded if at least two are correct.</p> <p>Accept $\left \frac{m_L - m_{L_1}}{1 + m_{L_1} m_L} \right$</p>

Solution	Marks	Remarks
<p>(ii) (1) Let $BC = x$</p> $\tan \angle CAB = \frac{BC}{AB} = \frac{1}{3}$ $\therefore AB = 3x$ $\text{Area of } \triangle CBA = \frac{1}{2}(BC)(AB)$ $= \frac{3x^2}{2}$ <p>Since $\triangle CBA$ and $\triangle CDA$ are congruent,</p> $\frac{3x^2}{2} = \frac{240}{2}$ $x = 4\sqrt{5} \quad \boxed{\text{or } = \sqrt{80}}$	<p>1A</p> <p>1A</p> <p>1M</p> <p>1A</p>	<div style="border: 1px solid black; padding: 5px;"> <p>or Let $CD = x$</p> $AD = 3x$ $\text{Area} = \frac{1}{2}(CD)(AD)$ </div> <p>(can be omitted)</p>
<p>(2) Let the coordinates of C be (h, h).</p> $BC = \frac{ 2h - h - 4 }{\sqrt{5}}$ <p>Since $BC = 4\sqrt{5}$, $\frac{ 2h - h - 4 }{\sqrt{5}} = 4\sqrt{5}$</p> $ h - 4 = 20$ <p>$h = 24$ (rejected) or -16</p> <p>$h = -16$</p> <p>\therefore The coordinates of C are $(-16, -16)$.</p>	<p>1A</p> <p>1M</p> <p>1A</p> <p>1A</p>	<p>For distance formula</p> <p>Accept $\frac{h-4}{\sqrt{5}} = -4\sqrt{5}$</p> <p>(pp-1) for $\frac{h-4}{\sqrt{5}} = 4\sqrt{5}$</p>
<div style="border: 1px solid black; padding: 5px;"> <p>or $CD = \frac{ h - 2h + 4 }{\sqrt{5}}$</p> <p>Since $CD = 4\sqrt{5}$, $\frac{ h - 2h + 4 }{\sqrt{5}} = 4\sqrt{5}$</p> $4 - h = 20$ <p>$h = 24$ (rejected) or -16</p> <p>$h = -16$</p> <p>\therefore The coordinates of C are $(-16, -16)$.</p> </div>	<p>1M</p> <p>1A</p> <p>1A</p>	<p>For distance formula</p> <p>Accept $\frac{4-h}{\sqrt{5}} = 4\sqrt{5}$</p> <p>(pp-1) for $\frac{h-4}{\sqrt{5}} = 4\sqrt{5}$</p>

Solution	Marks	Remarks
<p><u>Alternative Solution for (2)</u></p> $AC^2 = BC^2 + AB^2$ $= (4\sqrt{5})^2 + (12\sqrt{5})^2$ $AC = \sqrt{800}$ <p>Let the coordinates of C be (h, h).</p> <p>Coordinates of A are (4, 4).</p> $\sqrt{(h-4)^2 + (h-4)^2} = \sqrt{800}$ $(h-4)^2 = 400$ $h = 24 \text{ (rejected) or } -16$ $\therefore h = -16$ <p>\therefore The coordinates of C are (-16, -16).</p>	<p>1A</p> <p>1M+1A</p> <p>1A</p>	<p>or $= CD^2 + DA^2$</p>
<p><u>Alternative Solution for (ii)</u></p> <p>(1)&(2) Let the coordinates of C be (h, h).</p> $BC = \frac{ 2h-h-4 }{\sqrt{5}} \quad \text{or} \quad = -\frac{2h-h-4}{\sqrt{5}}$ $= \frac{ h-4 }{\sqrt{5}}$ <p>Since $\tan \angle CAB = \frac{BC}{AB} = \frac{1}{3}$</p> $AB = 3 \frac{ h-4 }{\sqrt{5}} \quad \text{or} \quad = 3\left(\frac{4-h}{\sqrt{5}}\right)$ <p>Area of $\triangle CBA = \frac{1}{2}(BC \times AB)$</p> $= \frac{3}{10}(h-4)^2$ <hr/> <p>or $CD = \frac{ h-2h+4 }{\sqrt{5}} \quad \text{or} \quad = \frac{h-2h+4}{\sqrt{5}}$</p> $= \frac{ 4-h }{\sqrt{5}}$ <p>Since $\tan \angle CAD = \frac{CD}{AD} = \frac{1}{3}$</p> $\therefore AD = 3 \frac{ 4-h }{\sqrt{5}} \quad \text{or} \quad = 3\left(\frac{4-h}{\sqrt{5}}\right)$ <p>Area of $\triangle CDA = \frac{1}{2}(CD \times AD)$</p> $= \frac{3}{10}(4-h)^2$	<p>1A</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1A</p>	<p>For distance formula (pp-1) for $\frac{2h-h-4}{\sqrt{5}}$</p> <p>For distance formula (pp-1) for $-\frac{h-2h+4}{\sqrt{5}}$</p>

Solution	Marks	Remarks
<p>Since $\triangle CBA$ and $\triangle CDA$ are congruent,</p> $\frac{3}{10}(h-4)^2 = \frac{240}{2}$ $(h-4)^2 = 400$ $h = 24 \text{ (rejected) or } -16$ $\therefore h = -16$ $\therefore \text{The coordinates of } C \text{ are } (-16, -16)$ <p>Put $h = -16$</p> $BC = \frac{ -16-4 }{\sqrt{5}}$ $= 4\sqrt{5}.$	<p>1M+1A</p> <p>1A</p> <p>1A</p> <hr/> <p>12</p>	

Solution	Marks	Remarks
9. (a) Let $t = \cos x$, $dt = -\sin x dx$ $\int_0^{\pi} \sin^5 x dx = \int_1^{-1} -(1-t^2)^2 dt$ $= \int_{-1}^1 (1-2t^2+t^4) dt$ $= \left[t - \frac{2t^3}{3} + \frac{t^5}{5} \right]_{-1}^1$ $= \frac{16}{15}$	1A+1A 1A 1A	1A for integrand, 1A for limits For the primitive function
<p><u>Alternative solution</u></p> $\int_0^{\pi} \sin^5 x dx = \int_0^{\pi} -\sin^4 x d \cos x$ $= \int_0^{\pi} -(1-\cos^2 x)^2 d \cos x$ $= \int_0^{\pi} -(1-2\cos^2 x + \cos^4 x) d \cos x$ $= -\left[\cos x - \frac{2\cos^3 x}{3} + \frac{\cos^5 x}{5} \right]_0^{\pi}$ $= \frac{16}{15}$	2A 1A 1A	For the primitive function
<hr/> 4 <hr/>		
(b) $u = \pi - x$, $du = -dx$ $\int_0^{\pi} x \sin^5 x dx = \int_{\pi}^0 (\pi-u) \sin^5(\pi-u) (-du)$ $= \int_0^{\pi} (\pi-u) \sin^5 u du$ $= \pi \int_0^{\pi} \sin^5 u du - \int_0^{\pi} u \sin^5 u du$	1A+1A 1M 1A 1A 5	1A for integrand, 1A for limits (can be omitted)
$\therefore \int_0^{\pi} x \sin^5 x dx = \int_0^{\pi} u \sin^5 u du$ $\therefore 2 \int_0^{\pi} x \sin^5 x dx = \pi \int_0^{\pi} \sin^5 u du$ $\int_0^{\pi} x \sin^5 x dx = \frac{\pi}{2} \left(\frac{16}{15} \right)$ $= \frac{8\pi}{15}$		

Solution	Marks	Remarks
<p>(c) $y = x^2 \sin^5 x$</p> $\frac{dy}{dx} = 2x \sin^5 x + 5x^2 \sin^4 x \cos x$ <p>Integrating with respect to x from 0 to π,</p> $[x^2 \sin^5 x]_0^\pi = \int_0^\pi (2x \sin^5 x + 5x^2 \sin^4 x \cos x) dx$ $0 = 2 \int_0^\pi x \sin^5 x dx + 5 \int_0^\pi x^2 \sin^4 x \cos x dx$ $\int_0^\pi x^2 \sin^4 x \cos x dx = -\frac{2}{5} \int_0^\pi x \sin^5 x dx$ $= -\frac{2}{5} \left(\frac{8\pi}{15} \right)$ $= -\frac{16\pi}{75}$	<p>1M+1A</p> <p>1M</p> <p>1A</p> <hr/> <p>1A</p> <hr/> <p>5</p> <hr/> <p>2A</p> <hr/> <p>2</p>	<p>1M for product rule</p> <p>(pp-1) for omitting the limits in the L.H.S.</p> <p>For LHS = 0</p>
<p>(d) I_2 is equal to I_1 because</p> $ x = x \quad \text{for } 0 < x < \pi.$ <div style="border: 1px solid black; padding: 5px; display: inline-block;"> <p>or $\cos x = \cos x \quad \text{for all values of } x.$</p> </div>		

Solution	Marks	Remarks
<p>10. (a) (i) $x^2 + y^2 - 8ky - 6ky + 25(k^2 - 1) = 0$</p> <p>The centre is $(4k, 3k)$.</p> <p>Since $3x - 4y = 3(4k) - 4(3k) = 0$,</p> <p>$\therefore$ the centre always lies on the line $3x - 4y = 0$.</p> <p>(ii) $(x - 4k)^2 + (y - 3k)^2 = -25(k^2 - 1) + 25k^2$ $= 25$</p> <p>\therefore All circles in F have the same radius 5.</p>	<p>1A</p> <p>1</p> <hr/> <p>$\frac{1}{3}$</p>	<p>OR $r = \sqrt{\left(\frac{-8k}{2}\right)^2 + \left(\frac{-6k}{2}\right)^2 - 25(k^2 - 1)}$ $= 5$</p>
<p>(b) Slopes of the common tangents $= \frac{3}{4}$.</p> <p>Let the equation of the tangents be $y = \frac{3}{4}x + c$.</p> <p>Distance between centre $(4k, 3k)$ and the tangents $= 5$.</p> $\left \frac{\frac{3}{4}(4k) - (3k) + c}{\sqrt{\left(\frac{3}{4}\right)^2 + (-1)^2}} \right = 5$ <p>$c = \pm \frac{25}{4}$</p> <p>\therefore The equations of the two tangents are $y = \frac{3}{4}x + \frac{25}{4}$ and $y = \frac{3}{4}x - \frac{25}{4}$</p>	<p>1A</p> <p>1M</p> <p>2M</p> <p>1A+1A</p>	<p>OR $3x - 4y + c = 0$</p> <p>OR $(4, 3)$ or any point on the line $3x - 4y = 0$</p> <p>(pp-1) for omitting absolute sign</p> <p>$3x - 4y + 25 = 0$ and $3x - 4y - 25 = 0$</p>
<p><u>Alternative Solution (1)</u></p> <p>Slope of the common tangents $= \frac{3}{4}$.</p> <p>Let the equation of the tangent be $y = \frac{3}{4}x + c$.</p> <p>Substitute into F,</p> $x^2 + \left(\frac{3}{4}x + c\right)^2 - 8kx - 6k\left(\frac{3}{4}x + c\right) + 25(k^2 - 1) = 0$ $\frac{25}{16}x^2 + \left(\frac{3c}{2} - \frac{25k}{2}\right)x + (c^2 - 6kc + 25k^2 - 25) = 0$ <p>Discriminant $= \left(\frac{3c}{2} - \frac{25k}{2}\right)^2 - 4\left(\frac{25}{16}\right)(c^2 - 6kc + 25k^2 - 25) = 0$</p> $\frac{9c^2}{4} - \frac{625k^2}{4} - \frac{75}{2}kc - \frac{25}{4}c^2 + \frac{75}{2}kc - \frac{625}{4}k^2 + \frac{625}{4} = 0$ $16c^2 = 625$ <p>$c = \pm \frac{25}{4}$</p> <p>\therefore The equations for the tangents are $y = \frac{3}{4}x + \frac{25}{4}$ and $y = \frac{3}{4}x - \frac{25}{4}$</p>	<p>1A</p> <p>1M</p> <p>1M</p> <p>1M</p> <p>1A+1A</p>	

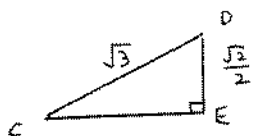
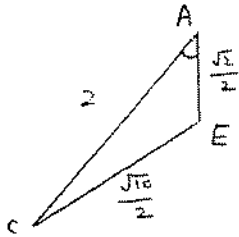
Solution	Marks	Remarks
<p><u>Alternative Solution (2)</u></p> <p>Slope of the common tangents = $\frac{3}{4}$.</p> <p>Let the equation of the tangent be $y = \frac{3}{4}x + c$.</p> <p>Substitute into the circle $x^2 + y^2 - 25 = 0$,</p> $x^2 + \left(\frac{3}{4}x + c\right)^2 - 25 = 0$ $25x^2 + 24cx + 16(c^2 - 25) = 0$ <p>Discriminant = $(24c)^2 - 4(25)(16)(c^2 - 25) = 0$</p> $16c^2 = 625$ $c = \pm \frac{25}{4}$ <p>\therefore The equations for the tangents are</p> $y = \frac{3}{4}x + \frac{25}{4} \text{ and } y = \frac{3}{4}x - \frac{25}{4}$	<p>1A</p> <p>1M</p> <p>1M</p> <p>1M</p> <p>1A+1A</p>	<p>or using other circles in F</p>
<p><u>Alternative Solution (3)</u></p> <p>Slope of the common tangents = $\frac{3}{4}$.</p> <p>Equation of the line through O and perpendicular to the common tangents is $y = -\frac{4}{3}x$.</p> <p>Putting $k = 0$, equation of circle in F with O as centre is $x^2 + y^2 = 25$.</p> $\begin{cases} x^2 + y^2 = 25 \\ y = -\frac{4}{3}x \end{cases}$ $x^2 + \left(-\frac{4}{3}x\right)^2 = 25$ $x = \pm 3$ <p>When $x = 3$, $y = -4$</p> <p>When $x = -3$, $y = 4$</p> <p>\therefore Equations of the common tangents are</p> $\frac{y+4}{x-3} = \frac{3}{4} \quad \text{and} \quad \frac{y-4}{x+3} = \frac{3}{4}$ $3x - 4y - 25 = 0 \quad \text{and} \quad 3x - 4y + 25 = 0$	<p>1A</p> <p>1M</p> <p>1M</p> <p>1M</p> <p>1A+1A</p>	

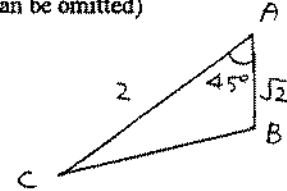
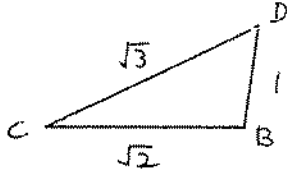
Solution	Marks	Remarks
<p>(c) Since the coordinates of the centre are $(4k, 3k)$, distance from centre to x-axis</p> $= 3 k $ <div style="border: 1px solid black; padding: 5px; display: inline-block;"> $\text{or} = \begin{cases} 3k & \text{if } k \geq 0 \\ -3k & k < 0 \end{cases}$ </div> <p>Let M be the mid-point of AB and C be the centre.</p> <p>Since $AB = 8$, $AM = 4$</p> $AC = 5$ $\therefore CM = \sqrt{5^2 - 4^2} = 3$ $\therefore 3 k = 3$ $k = \pm 1$ <p>The equations of the two circles are</p> $x^2 + y^2 - 8x - 6y = 0 \quad \text{and} \quad x^2 + y^2 + 8x + 6y = 0.$	<p>2A</p> <p>1A</p> <p>1A</p> <p>1M</p> <p>1A+1A</p>	<p>Withhold 1 mark if absolute sign was omitted.</p> <p>(can be omitted)</p> <p>(can be omitted)</p> <p>$(x-4)^2 + (y-3)^2 = 25,$ $(x+4)^2 + (y+3)^2 = 25$</p>
<p><u>Alternative Solution (1)</u></p> <p>Since $AB = 8$, $AM = 4$.</p> <p>Coordinates of A are $(4k-4, 0)$.</p> <p>Substitute $A(4k-4, 0)$ into F,</p> $(4k-4)^2 - 8k(4k-4) + 25k^2 - 25 = 0$ $9k^2 - 9 = 0$ $k = \pm 1$ <p>\therefore The equations of the two circles are</p> $x^2 + y^2 - 8x - 6y = 0 \quad \text{and} \quad x^2 + y^2 + 8x + 6y = 0.$	<p>1A</p> <p>1A</p> <p>1M</p> <p>1A+1A</p>	<p>(can be omitted)</p> <p><u>or</u> $B(4k+4, 0)$</p>
<p><u>Alternative solution (2)</u></p> <p>Since $AB = 8$, $AM = 4$.</p> <p>Coordinates of A are $(4k-4, 0)$.</p> <p>Distance between A (or B) and the centre is 5.</p> $\therefore \sqrt{(4k-4-4k)^2 + (0-3k)^2} = 5$ $9k^2 + 16 = 25$ $k = \pm 1$ <p>\therefore The equation of the circles are</p> $x^2 + y^2 - 8x - 6y = 0 \quad \text{and} \quad x^2 + y^2 + 8x + 6y = 0.$	<p>1A</p> <p>1A</p> <p>1M</p> <p>1A+1A</p>	<p>(can be omitted)</p> <p><u>or</u> $B(4k+4, 0)$</p>

Solution	Marks	Remarks
<p><u>Alternative Solution (3)</u></p> <p>Put $y=0$, $x^2 - 8kx + 25(k^2 - 1) = 0$</p> $x = \frac{8k \pm \sqrt{64k^2 - 100(k^2 - 1)}}{2}$ $= 4k \pm \sqrt{25 - 9k^2}$ <p>$AB = (4k + \sqrt{25 - 9k^2}) - (4k - \sqrt{25 - 9k^2})$</p> $= 2\sqrt{25 - 9k^2}$ <p>$\therefore 2\sqrt{25 - 9k^2} = 8$</p> $25 - 9k^2 = 16$ $k = \pm 1$ <p>\therefore The equations of the two circles are $x^2 + y^2 - 8x - 6y = 0$ and $x^2 + y^2 + 8x + 6y = 0$.</p>	<p>1A</p> <p>1A</p> <p>1M</p> <p>1A+1A</p> <hr/> <p>7</p>	

Solution	Marks	Remarks
11. (a) The coordinates of B are $(3, 4)$.	1A 1	
(b) (i) $y = 4 - (x - 3)^2$ $(x - 3)^2 = 4 - y$ $x - 3 = -\sqrt{4 - y}$ $\therefore x < 3$ $x = 3 - \sqrt{4 - y}$	1	
(ii) The equation of BC is $x = 3 + \sqrt{4 - y}$.	1A 2	
(c) (i) Volume $= \pi \int_0^h [(3 + \sqrt{4 - y})^2 - (3 - \sqrt{4 - y})^2] dy$ $= 12\pi \int_0^h \sqrt{4 - y} dy$ $= 12\pi \left[-\frac{2}{3}(4 - y)^{\frac{3}{2}} \right]_0^h$ $= 8\pi [8 - (4 - h)^{\frac{3}{2}}]$	1M+1A+1A 1A 1	1M for $V = \pi \int_a^b x^2 dy$. 1A for integrand, 1A for limits For the primitive function ↑ or $= -12\pi \left[\frac{2}{3} u^{\frac{3}{2}} \right]_4^{4-h}$ ($u = 4 - y$)
<u>Alternative Solution</u> Volume formed by revolving lower part of curve BC $V_1 = \pi \int_0^h (3 + \sqrt{4 - y})^2 dy$ $= \pi \int_0^h (13 - y + 6\sqrt{4 - y}) dy$ $= \pi \left[13y - \frac{y^2}{2} - 4(4 - y)^{\frac{3}{2}} \right]_0^h$ $= \pi \left[13h - \frac{1}{2}h^2 - 4(4 - h)^{\frac{3}{2}} + 32 \right]$ Volume formed by revolving lower part of curve AB $V_2 = \pi \int_0^h (3 - \sqrt{4 - y})^2 dy$ $= \pi \int_0^h (13 - y - 6\sqrt{4 - y}) dy$ $= \pi \left[13y - \frac{1}{2}y^2 + 4(4 - y)^{\frac{3}{2}} \right]_0^h$ $= \pi \left[13h - \frac{1}{2}h^2 + 4(4 - h)^{\frac{3}{2}} - 32 \right]$ Volume of lower layer of jelly ring $= V_1 - V_2$ $= 64\pi - 8\pi(4 - h)^{\frac{3}{2}}$ $= 8\pi [8 - (4 - h)^{\frac{3}{2}}]$	1M+1A 1A 1	1M for $V = \pi \int_a^b x^2 dy$.
(ii) Volume of jelly ring $= 8\pi [8 - (4 - 4)^{\frac{3}{2}}]$ $= 64\pi$	1M 1A	For putting $h = 4$

Solution	Marks	Remarks
<p>If the two layers have equal volumes,</p> $8\pi[8 - (4 - h)^{\frac{3}{2}}] = \frac{1}{2}(64\pi)$ $(4 - h)^{\frac{3}{2}} = 4$ <p>$h = 1.48$ (correct to 3 sig. figures)</p>	<p>1M</p> <hr/> <p>1A</p> <hr/> <p>9</p>	
<p><u>Alternative solution for (ii)</u></p> <p>Volume of upper layer</p> $= \pi \int_h^4 [(3 + \sqrt{4 - y})^2 - (3 - \sqrt{4 - y})^2] dy$ $= 12\pi \int_h^4 \sqrt{4 - y} dy$ $= 12\pi \left[-\frac{2}{3}(4 - y)^{\frac{3}{2}} \right]_h^4$ $= 8\pi(4 - h)^{\frac{3}{2}}$ <p>If the two layers have equal volumes,</p> $= 8\pi(4 - h)^{\frac{3}{2}} = 8\pi[8 - (4 - h)^{\frac{3}{2}}]$ $(4 - h)^{\frac{3}{2}} = 4$ <p>$h = 1.48$ (correct to 3 sig. figures)</p>	<p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p>	
<p>(d) Volume of milk</p> $= \pi \int_0^4 (3 - \sqrt{4 - y})^2 dy$ $= \pi \int_0^4 (13 - y - 6\sqrt{4 - y}) dy$ $= \pi \left[13y - \frac{1}{2}y^2 + 4(4 - y)^{\frac{3}{2}} \right]_0^4$ $= \pi[52 - 8 - 32] = 12\pi$	<p>1A+1M</p> <p>1A</p> <p>1A</p>	<p>1A for integrand, 1M for limits</p> <p>For the primitive function</p>
<p><u>Alternative Solution (1)</u></p> <p>Substitute $h = 4$ in v_2 in the alternative solution (c) (i),</p> $\text{Volume of milk} = \pi \left[13(4) - \frac{1}{2}(4)^2 + 4(4 - 4)^{\frac{3}{2}} - 32 \right]$ $= \pi(52 - 8 - 32)$ $= 12\pi$	<p>2M+1A</p> <p>1A</p>	

Solution	Marks	Remarks
<p>12. (a) By Sine Law,</p> $\frac{AB}{\sin 30^\circ} = \frac{2}{\sin 45^\circ}$ $AB = \sqrt{2}$ $BD = \sqrt{2} \cos 45^\circ = 1$ $DC = 2 \cos 30^\circ = \sqrt{3}$	<p>1A 1A <hr/>1A <hr/>3</p>	
<p>(b) (i) θ is $\angle DCE$.</p> $DE = \sin 45^\circ = \frac{\sqrt{2}}{2}$ <p>In $\triangle CDE$, $\sin \theta = \frac{DE}{CD}$</p> $= \frac{\sqrt{2}/2}{\sqrt{3}}$ $\sin \theta = \frac{\sqrt{6}}{6}$	<p>1A 1A 1M 1</p>	<p>(can be omitted)</p> 
<p>(ii) $CE = \sqrt{(CD)^2 - (DE)^2}$</p> $= \sqrt{(\sqrt{3})^2 - \left(\frac{\sqrt{2}}{2}\right)^2}$ $= \frac{\sqrt{10}}{2} \text{ (or } \sqrt{\frac{5}{2}})$	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> <p>OR $CE = CD \cos \theta$</p> $= \sqrt{3} \left(\frac{\sqrt{30}}{6}\right)$ $= \frac{\sqrt{10}}{2}$ </div> <p>1M 1A</p>	
<p>$AE = \frac{1}{2} AB = \frac{\sqrt{2}}{2}$</p> <p>In $\triangle EAC$, by Cosine Law</p> $\cos \angle EAC = \frac{(AE)^2 + (AC)^2 - (CE)^2}{2(AE)(AC)}$ $= \frac{\left(\frac{\sqrt{2}}{2}\right)^2 + 2^2 - \left(\frac{\sqrt{10}}{2}\right)^2}{2\left(\frac{\sqrt{2}}{2}\right)(2)}$ $= \frac{1}{\sqrt{2}}$ <p>$\therefore \angle EAC = 45^\circ$</p>	<p>1A 1M 1</p>	

Solution	Marks	Remarks
<p>(iii) $\angle BDC$ represents the angle between the 2 planes.</p> $BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos 45^\circ$ $= (\sqrt{2})^2 + 2^2 - 2(\sqrt{2})(2)\cos 45^\circ$ $= 2$ <p>$\therefore BC = \sqrt{2}$</p>	<p>1A</p>	<p>(can be omitted)</p> 
<p>In $\triangle BDC$, by Cosine Law,</p>	<p>1A</p>	
$\cos \angle BDC = \frac{(BD)^2 + (CD)^2 - (BC)^2}{2(BD)(CD)}$ $= \frac{1^2 + (\sqrt{3})^2 - (\sqrt{2})^2}{2(1)(\sqrt{3})} = \frac{1}{\sqrt{3}}$	<p>1M</p>	
<p>$\therefore \angle BDC = 55^\circ$ (correct to the nearest degree)</p>	<p>1A</p>	<p>(Do not accept radian)</p>
<p>or Since $BC^2 + BD^2 = CD^2$, $\therefore \angle CBD = 90^\circ$</p> $\sin \angle BDC = \frac{BC}{CD}$ $= \frac{\sqrt{2}}{\sqrt{3}}$ <p>$\angle BDC = 55^\circ$ (correct to the nearest degree)</p>	<p>1M 1A</p>	
	<p style="text-align: center;">13</p>	