Section A (42 marks)

Answer ALL questions in this section.

1. Let $f(x) = \sin^3 x$.

   Find $f'(x)$ and $f''(x)$.

   (3 marks)

2. Find $\frac{d}{dx}(x^2)$ from first principles.

   (4 marks)

3. Solve the inequality $\frac{2x-3}{x+1} \le 1$.

   (4 marks)

4. Given $x^2 - 6x + 11 \equiv (x+a)^2 + b$, where x is real.

   (a) Find the values of a and b.

   Hence write down the least value of $x^2 - 6x + 11$.

   (b) Using (a), or otherwise, write down the range of possible values of $\frac{1}{x^2 - 6x + 11}$.

   (5 marks)

5. (a) Express the complex number $\frac{2+4i}{1-i}$ in standard form.

   (b) If $p + qi = \frac{2+4i}{1-i}(q+i)$, where p and q are real constants, find the values of p and q.

   (6 marks)

6. Find the equations of the two tangents to the curve $C : y = \frac{6}{x+1}$ which are parallel to the line $x + 6y + 10 = 0$.

   (7 marks)

7. Given $\overrightarrow{OA} = 4\mathbf{i} + 3\mathbf{j}$ and C is a point on OA such that $|\overrightarrow{OC}| = \frac{16}{5}$.

   (a) Find the unit vector in the direction of $\overrightarrow{OA}$.

   Hence find $\overrightarrow{OC}$.

   (b) If $\overrightarrow{OB} = \mathbf{i} + 4\mathbf{j}$, show that BC is perpendicular to OA.

   (6 marks)

8. The graph of $y = x^2 - (k-2)x + k + 1$ intersects the x-axis at two distinct points $(\alpha, 0)$ and $(\beta, 0)$, where k is real.

   (a) Find the range of possible values of k.

   (b) Furthermore, if $|\alpha + \beta| < 5$, find the range of possible values of k.

   (7 marks)

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Section B (48 marks)

Answer any THREE questions in this section.
Each question carries 16 marks.

9. C_1 is the curve $y = \dfrac{4x-3}{x^2+1}$.

(a) Find

(i) the x- and y- intercepts of the curve C_1 ;

(ii) the range of values of x for which $\dfrac{4x-3}{x^2+1}$ is decreasing;

(iii) the turning point(s) of C_1, stating whether each point is a maximum or a minimum point. (Testing for maximum/minimum is not required.)

(9 marks)

(b) In Figure 1(a), sketch the curve C_1 for $-10 \le x \le 10$.

(3 marks)

(c) C_2 is the curve $y = \dfrac{|4x-3|}{x^2+1}$.

Using the result of (b), sketch the curve C_2 for $-10 \le x \le 10$ in Figure 1(b).

Hence write down the greatest and least values of $\dfrac{|4x-3|}{x^2+1}$ for $-10 \le x \le 10$.

(4 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page

If you attempt Question 9, fill in the details in the first three boxes above and tie this sheet into your answer book.

9. (b) **(continued)**

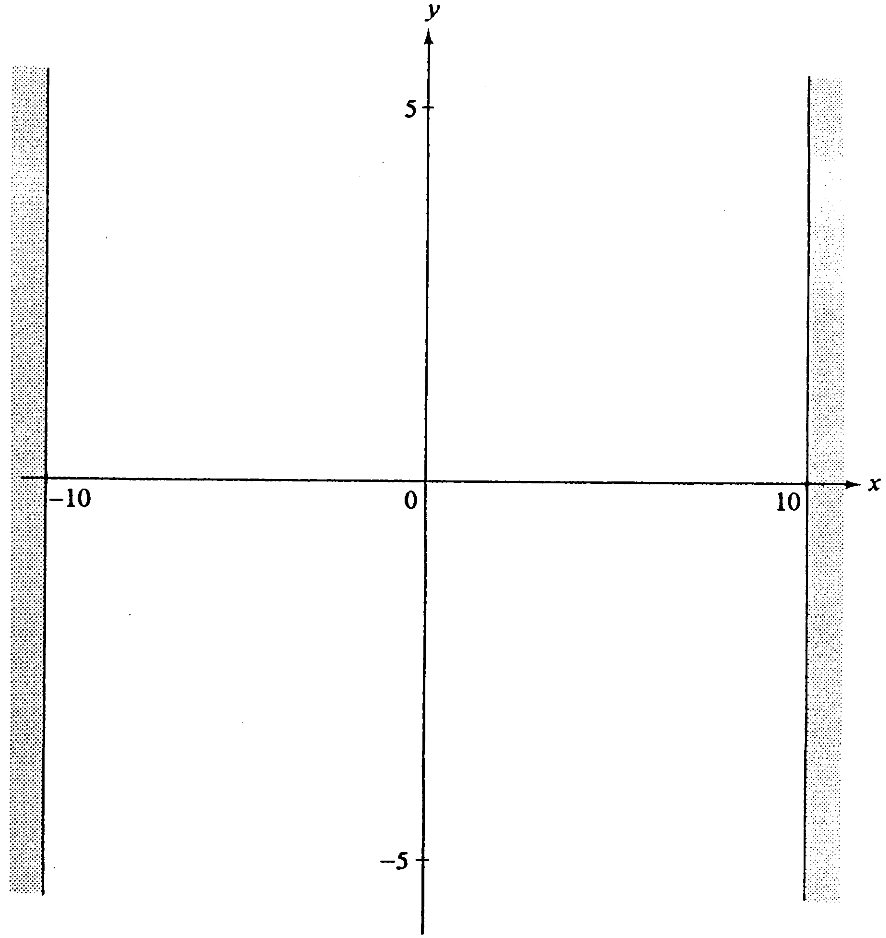

Figure 1(a)

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Total Marks on this page

9. (c) **(continued)**

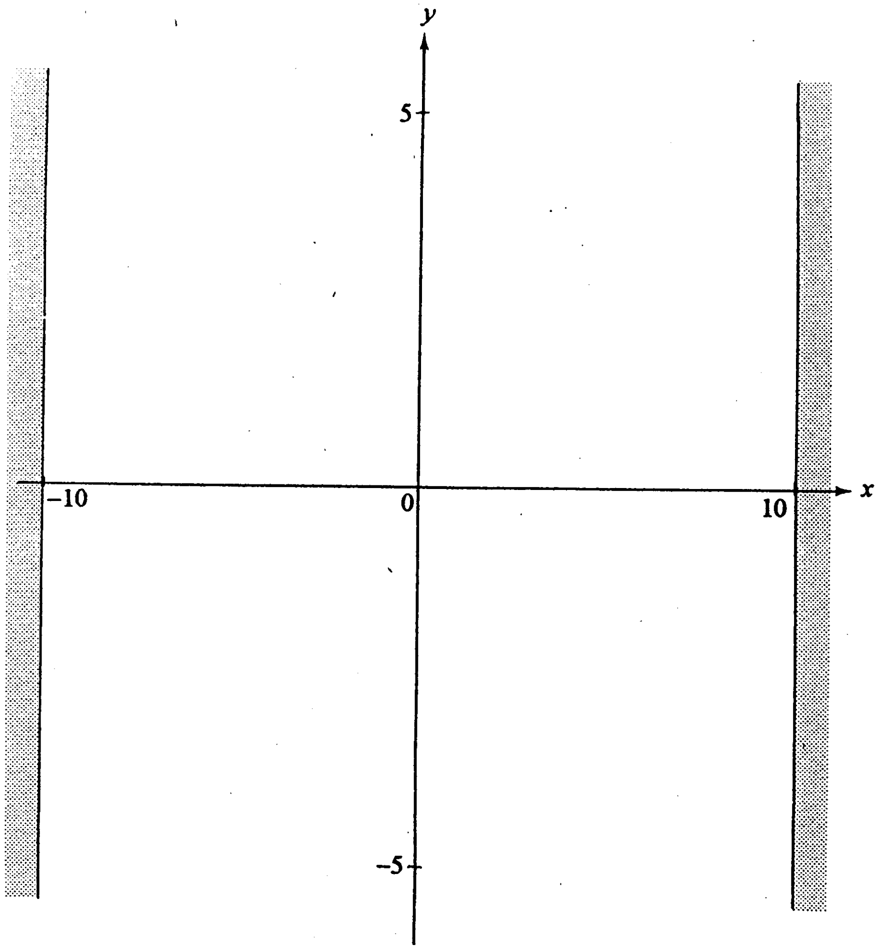

Figure 1(b)

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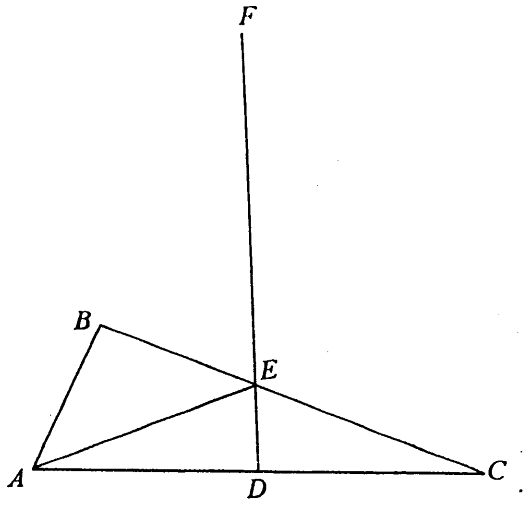

Figure 2

In Figure 2, D is the mid-point of AC and E is a point on BC such that $BE : EC = 1 : t$, where $t > 0$. DE is produced to a point F such that $DE : EF = 1 : 7$. Let $\overrightarrow{AD} = \mathbf{a}$ and $\overrightarrow{AB} = \mathbf{b}$.

(a) (i) Express $\overrightarrow{AE}$ in terms of t, $\mathbf{a}$ and $\mathbf{b}$.

(ii) Express $\overrightarrow{AE}$ in terms of $\mathbf{a}$ and $\overrightarrow{AF}$.

Hence, or otherwise, show that $\overrightarrow{AF} = \frac{9-7t}{1+t}\mathbf{a} + \frac{8t}{1+t}\mathbf{b}$.

(5 marks)

(b) Suppose that A, B and F are collinear.

(i) Find the value of t.

(ii) It is given that $|\mathbf{a}| = 3$, $|\mathbf{b}| = 2$ and $\cos \angle BAC = \frac{1}{3}$.

(1) Find $\mathbf{a} \cdot \mathbf{b}$.

(2) Find $\overrightarrow{AB} \cdot \overrightarrow{BC}$ and $\overrightarrow{AD} \cdot \overrightarrow{DE}$.

(3) Does the circle passing through points B, C and D also pass through point F? Explain your answer.

(11 marks)

11.

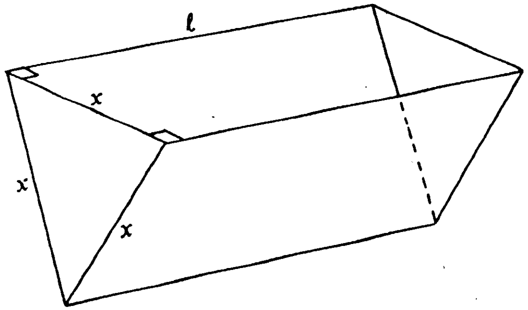

Figure 3(a)

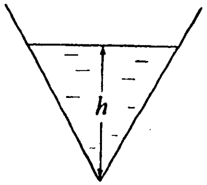

Figure 3(b)

Figure 3(a) shows a vessel with a capacity of 24 cubic units. The length of the vessel is ℓ and its vertical cross-section is an equilateral triangle of side x. The vessel is made of thin metal plates and has no lid. Let S be the total area of metal plates used to make the vessel.

(a) Show that $S = \frac{\sqrt{3}}{2}x^2 + \frac{64\sqrt{3}}{x}$. (4 marks)

(b) Find the values of x and ℓ such that the area of metal plates used to make the vessel is minimum.

(5 marks)

(c) At time $t = 0$, the vessel described in part (b) is completely filled with water. Suppose the water evaporates at a rate proportional to the area of water surface at that instant such that

$$\frac{dV}{dt} = -\frac{1}{10}A\ ,$$ where V and A are respectively the volume of water and the area of water surface at time t.

(i) Let h be the depth of water in the vessel at time t. (See Figure 3(b).) Show that $A = 4h$ and $V = 2h^2$.

Hence, or otherwise, find $\frac{dh}{dt}$.

(ii) Find the time required for the water in the vessel to evaporate completely.

(7 marks)

12. (a) P is a point in an Argand diagram representing a non-zero complex number z such that

$$\arg\left(\frac{z}{1-i}\right) = \frac{\pi}{2}.$$

Find $\arg(1-i)$ and $\arg z$.

Hence, or otherwise, sketch the locus of P in an Argand diagram.

(6 marks)

(b) R is a point in an Argand diagram representing a non-zero complex number w such that

$$w^3 = (\bar{w})^3 \text{ and } \frac{\pi}{2} < \arg w < \pi.$$

Find $\arg w$.

[Hint : You may let $w = r(\cos\theta + i\sin\theta)$.]

(5 marks)

(c) It is given that in (a), $|z| = 2\sqrt{2}$ and in (b), $|w| = 2$. Furthermore, $OPQR$ is a parallelogram in an Argand diagram, where O represents the complex number 0.

Find the complex number represented by the point Q, giving your answer in standard form.

(5 marks)

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13. Let α, β be the real roots of the quadratic equation

$$x^2 - \lambda x + 1 = 0, \text{ where } \lambda \geq 2.$$

Let $S_n = \alpha^n + \beta^n$, where n is a positive integer.

(a) Express S_2 and S_3 in terms of λ.

(5 marks)

(b) Find the value of $\alpha^5 - \lambda\alpha^4 + \alpha^3$.

Hence show that $S_5 - \lambda S_4 + S_3 = 0 \cdots\cdots (*)$.

(4 marks)

(c) It is known that S_3, S_4 and S_5 are non-zero.
Suppose $S_3 : S_4 : S_5 = 10 : 7\lambda : 25$.

Using (*) in (b), find the value of λ.

Hence (i) find the value of S_3,

(ii) evaluate $\left(\frac{\sqrt{5}+1}{2}\right)^5 + \left(\frac{\sqrt{5}-1}{2}\right)^5$ without using a binomial expansion.

(7 marks)

END OF PAPER

Section A (42 marks)

Answer ALL questions in this Section.

1. Find the general solution of the equation

$$\sin 5\theta + \sin 3\theta = \cos\theta .$$

(5 marks)

2. It is given that

$$(1+x+ax^2)^6 = 1+6x+k_1x^2+k_2x^3 + \text{ terms involving higher powers of } x .$$

(a) Express k_1 and k_2 in terms of a .

(b) If 6 , k_1 and k_2 are in A.P. , find the value of a .

(6 marks)

3. E is the ellipse $\frac{x^2}{2}+\frac{y^2}{7}=1$.

(a) Let the line $y=mx+c$ be a tangent to E . Show that $c^2 = 2m^2+7$.

(b) Using (a) , find the equations of the two tangents from the point (0, 5) to E .

(7 marks)

4. Prove by mathematical induction, that for all positive integers n ,

$(2n^3 + n)$ is divisible by 3 .

(6 marks)

5.

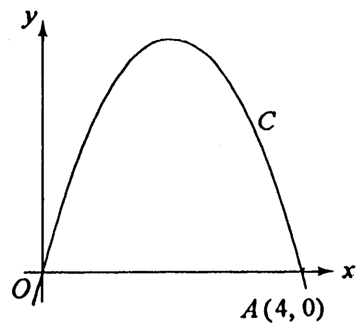

Figure 1(a)

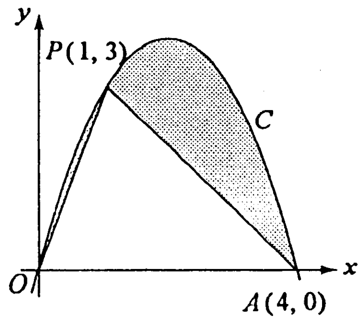

Figure 1(b)

The curve $C: y = 4x - x^2$ cuts the x-axis at the origin O and the point $A(4, 0)$ as shown in Figure 1(a).

(a) Find the area of the region bounded by C and the line segment OA.

(b) In Figure 1(b), the shaded region is enclosed by the curve C and the line segements OP and PA , where P is the point (1, 3). Using (a), find the total area of the shaded region.

(6 marks)

6. The slope at any point (x , y) of a curve is given by $\frac{dy}{dx} = \tan^3 x \sec x$.

If the curve passes through the origin, find its equation.

[Hint : Let $u = \sec x$.]

(6 marks)

7. $P(x , y)$ is a variable point such that the distance between P and the point (4, 0) is always equal to twice the distance from P to the line $x - 1 = 0$.

(a) Find the equation of the locus of P . State whether the locus is a circle, an ellipse, a hyperbola or a parabola.

(b) Sketch the locus of P .

(6 marks)

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Section B (48 marks)

Answer any THREE questions in this Section.
Each question carries 16 marks.

8. Given two straight lines $L_1 : 2x - y - 4 = 0$ and $L_2 : x - 2y + 4 = 0$. Let F be the family of straight lines passing through the point of intersection of L_1 and L_2.

(a) Write down the equation of F.

Hence find the equation of the line L in F which passes through the origin.

(4 marks)

(b)

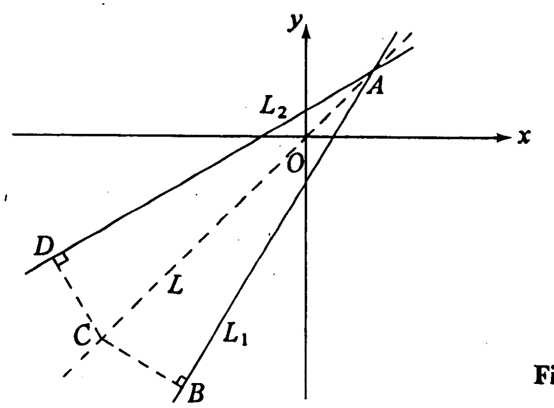

Figure 2

Let A denote the point of intersection of L_1 and L_2. C is a point in the third quadrant lying on the line L found in (a). Points B and D are the feet of perpendicular from C to L_1 and L_2 respectively. (See Figure 2.)

(i) Show that both $\tan\angle CAB$ and $\tan\angle CAD$ are equal to $\frac{1}{3}$.

(ii) If the area of quadrilateral $ABCD$ is 240, find

(1) the length of BC,

(2) the coordinates of point C.

(12 marks)

9. (a) Evaluate $\int_0^{\pi} \sin^5 x \, dx$.

[Hint : Let $t = \cos x$.]

(4 marks)

(b) Using the substitution $u = \pi - x$ and the result of (a), evaluate

$$\int_0^{\pi} x \sin^5 x \, dx.$$

(5 marks)

(c) By differentiating $y = x^2 \sin^5 x$ with respect to x and using the result of (b), evaluate

$$I_1 = \int_0^{\pi} x^2 \sin^4 x \cos x \, dx.$$

(5 marks)

(d) Let $I_2 = \int_0^{\pi} x^2 \sin^4 x \cos |x| \, dx$.

State, with a reason, whether I_2 is smaller than, equal to or larger than I_1 in (c).

(2 marks)

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10. The equation of a family of circles F is given by

$$x^2+y^2-8kx-6ky+25(k^2-1)=0,$$

where k is real.

(a) (i) Find the centre of a circle in F in terms of k.

Hence show that the centres of all circles in F lie on the line $3x-4y=0$.

(ii) Show that all circles in F have the same radius 5.

(3 marks)

(b)

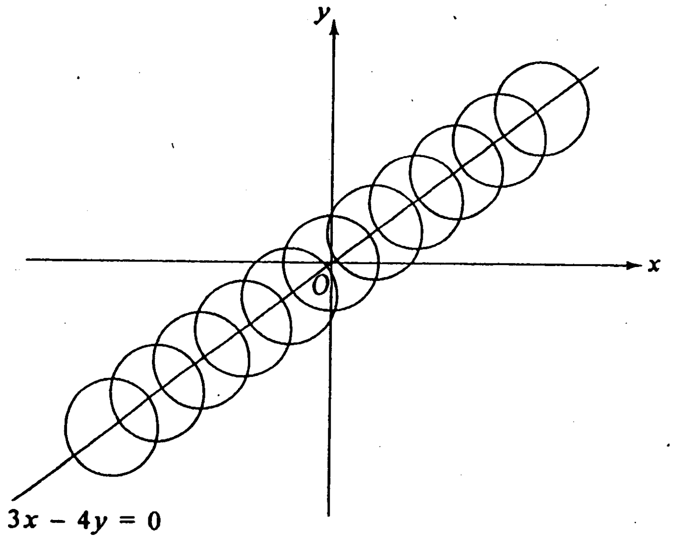

Figure 3

Figure 3 shows some circles in F. It is given that there are two parallel lines, both of which are common tangents to all circles in F.

Write down the slope of these two common tangents.

Hence find the equations of these two common tangents.

(6 marks)

(c) A circle in F cuts the x-axis at two points A and B.

Using (a) (i), write down the distance from the centre of the circle to the x-axis in terms of k.

Hence, or otherwise, find the equations of the two possible circles in F satisfying the condition $AB=8$.

(7 marks)

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11.

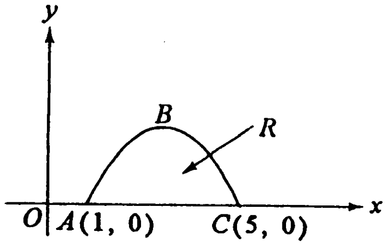

Figure 4 (a)

In Figure 4(a), the region R is enclosed by the parabola $y = 4 - (x-3)^2$ and the line segment AC, where A and C are the points (1, 0) and (5, 0) respectively. B is the vertex of the parabola.

(a) Write down the coordinates of B.

(1 mark)

(b) (i) Show that the equation of the curve AB is

$$x = 3 - \sqrt{4-y}.$$

(ii) Write down the equation of the curve BC.

(2 marks)

(c)

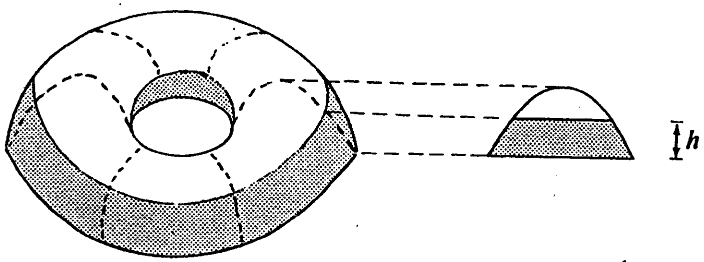

Figure 4(b)

(c) **(Continued)**

A jelly ring is in the shape of the solid of revolution of the region R in Figure 4(a) about the y-axis. Furthermore, the jelly ring contains two layers. Let h be the height of the lower layer. (See Figure 4(b) .)

(i) Show that the volume of the lower layer of the jelly ring is

$$8\pi [8 - (4-h)^{\frac{3}{2}}].$$

(ii) If the two layers have equal volumes, find the value of h correct to 3 significant figures.

(9 marks)

(d)

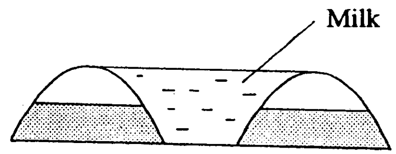

Figure 4(c)

If milk is poured into the centre of the jelly ring in (c) until it is completely filled (see Figure 4(c)), find the volume of milk required.

(4 marks)

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12.

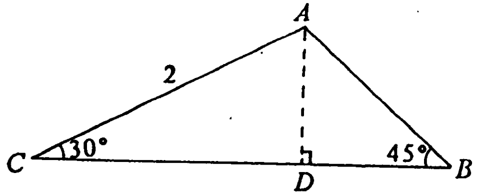

Figure 5(a)

In Figure 5(a), ABC is a triangular piece of paper such that $\angle B = 45°$, $\angle C = 30°$ and $AC = 2$. D is the foot of perpendicular from A to BC.

(a) Find AB, BD and DC. (3 marks)

(b)

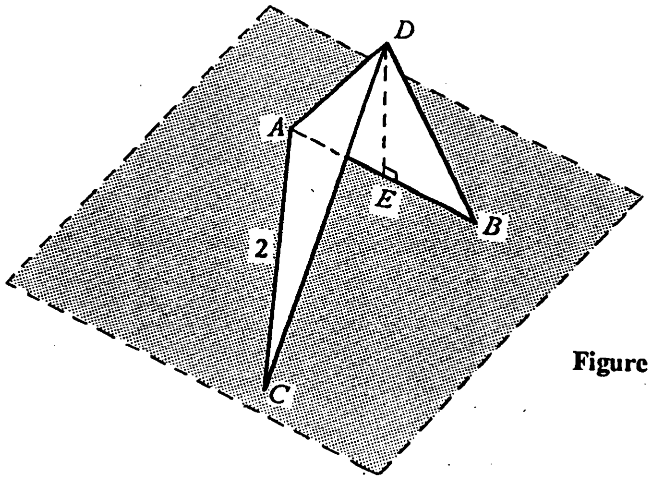

Figure 5(b)

The paper is folded along AD. It is then placed on a horizontal table such that the edges AB and AC lie on the table and the plane DAB is vertical. (See Figure 5(b).) E is the foot of perpendicular from D to AB.

(i) If θ is the angle between DC and the horizontal, show that $\sin\theta = \frac{\sqrt{6}}{6}$.

(ii) Find CE. Hence show that $\angle EAC = 45°$.

(iii) Find the angle between the two planes DAB and DAC to the nearest degree.

[Hint : You may use a ruler to tear off Figure 5(c) to help you answer part (b).] (13 marks)

Note : You need not hand in Figure 5(c).

12. (b) (Continued)

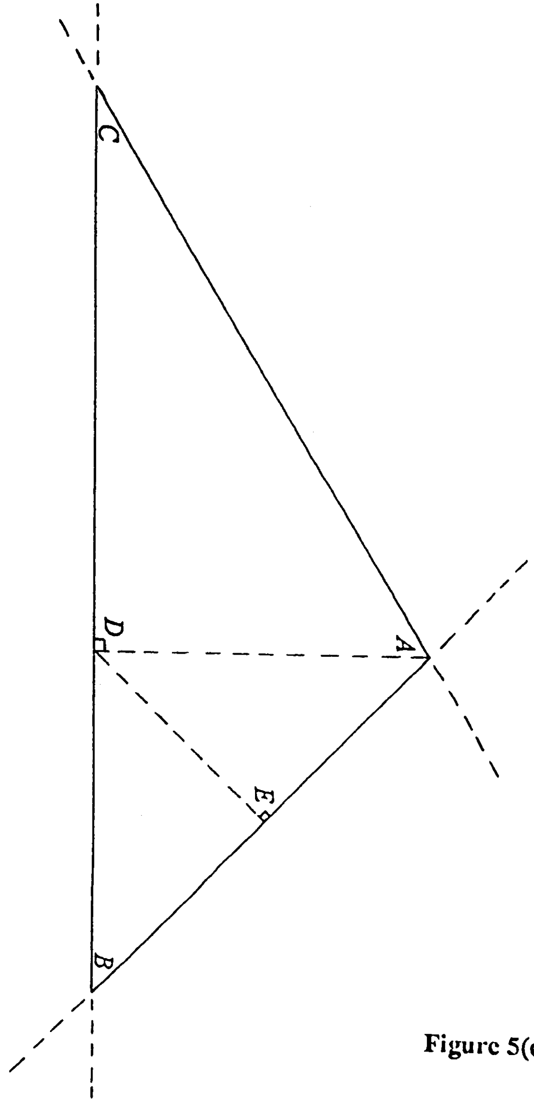

Figure 5(c)

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