2 JUN 1994

I

香港考試局
HONG KONG EXAMINATIONS AUTHORITY

一九九四年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1994

附加數學卷一
ADDITIONAL MATHEMATICS PAPER I

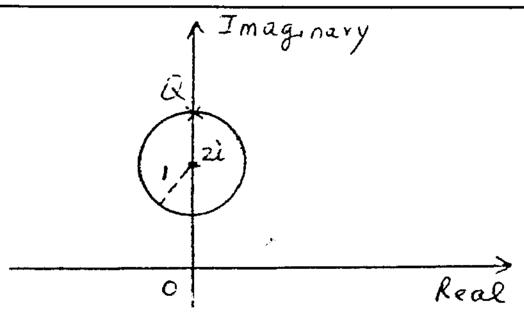
評卷參考
MARKING SCHEME

It is highly undesirable that this marking scheme should fall into the hands of students. They are likely to regard it as a set of model answers, which it certainly is not.

Markers should therefore resist pleas from their students to have access to this document. Making it available to students would constitute misconduct on the part of the marker.

本評卷參考並非標準答案，故極不宜落於學生手中，以免引起誤會。

遇有學生求取此文件時，閱卷員應嚴予拒絕。閱卷員如向學生披露本評卷參考內容，即違背閱卷員守則。

Solution	Marks	Remarks
1. $\dfrac{2(x+1)}{x-2} \ge 1$		
$\dfrac{2(x+1)}{x-2} - 1 \ge 0$	1M	
$\dfrac{x+4}{x-2} \ge 0$	1A	
$x > 2$ or $x \le -4$	2A	1A for $\underline{x \ge 2}$ or $x \le -4$
	4	
Alternative solution (1)		
$\dfrac{2(x+1)}{x-2} \ge 1$		
Consider the following 2 cases (i) $x > 2$, (ii) $x < 2$:	1M	Awarded even if equality sign is included.
Case 1 : $x > 2$		
$2(x+1) \ge x - 2$		
$x \ge -4$		
Since $x > 2$, $\therefore x > 2$	1A	
Case 2 : $x < 2$		
$2(x+1) \le x - 2$		
$x \le -4$		
Since $x < 2$, $\therefore$ $x \le -4$		
Combining the 2 cases, $x > 2$ or $x \le -4$	2A	1A for $\underline{x \ge 2}$ or $x \le -4$
Alternative solution (2)		
$\dfrac{2(x+1)}{(x-2)} \ge 1$		
$2(x+1)(x-2) \ge (x-2)^2$ (and $x \ne 2$)	1M	
$x^2 + 2x - 8 \ge 0$ (and $x \ne 2$)		
$(x-2)(x+4) \ge 0$ (and $x \ne 2$)	1A	
$x > 2$ or $x \le -4$	2A	1A for $\underline{x \ge 2}$ or $x \le -4$
2. 	1A	For circle
	1A	For centred at $z = 2i$
	1A	For radius = 1
Position of Q	1M+1A	1M for being farthest away from O
		Axes not labelled – (pp-1)
	5	

Solution	Marks	Remarks
3. (a) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$	1M	Omit vector sign (pp-1)
$= 2\vec{i} - \vec{j}$	1A	
$\lvert\overrightarrow{PQ}\rvert = \sqrt{2^2 + (-1)^2} = \sqrt{5}$	1A	
(b) Let $\angle QPR = \theta$		
$\overrightarrow{PQ} \cdot \overrightarrow{PR} = (2\vec{i} - \vec{j})\cdot(-3\vec{i} - 2\vec{j}) = -4$	1A	Omit dot sign (pp-1)
$\cos\theta = \dfrac{\overrightarrow{PQ}\cdot\overrightarrow{PR}}{\lvert\overrightarrow{PQ}\rvert\lvert\overrightarrow{PR}\rvert}$	1M	
$= \dfrac{-4}{\sqrt{65}}$	1A 6	
Alternative solution		
(b) $\lvert\overrightarrow{PR}\rvert = \sqrt{13}$		
$\overrightarrow{RQ} = 5\vec{i} + \vec{j}$		
$\lvert\overrightarrow{RQ}\rvert = \sqrt{26}$	1A	
$\cos\angle QPR = \dfrac{\lvert\overrightarrow{PQ}\rvert^2 + \lvert\overrightarrow{PR}\rvert^2 - \lvert\overrightarrow{QR}\rvert^2}{2\lvert\overrightarrow{PQ}\rvert\lvert\overrightarrow{PR}\rvert}$	1M	
$= \dfrac{5 + 13 - 26}{2\sqrt{5}\sqrt{13}}$		
$= \dfrac{-4}{\sqrt{65}}$	1A	
4. $y = \tan\left(\dfrac{1}{x}\right)$		
$\dfrac{dy}{dx} = -\dfrac{1}{x^2}\sec^2\left(\dfrac{1}{x}\right)$	1M+1A	1M for $\dfrac{d}{dx}(\tan x) = \sec^2 x$
$x^2\dfrac{dy}{dx} + (y^2+1) = -\sec^2\left(\dfrac{1}{x}\right) + \tan^2\left(\dfrac{1}{x}\right) + 1$	1M	or $= -(1+y^2) + y^2 + 1$
$= 0$	1	
Differentiating $x^2\dfrac{dy}{dx} + (y^2+1) = 0$ with respect to x		
$2x\dfrac{dy}{dx} + x^2\dfrac{d^2y}{dx^2} + 2y\dfrac{dy}{dx} = 0$	1A	
$x^2\dfrac{d^2y}{dx^2} + 2(x+y)\dfrac{dy}{dx} = 0$		
$\dfrac{d^2y}{dx^2} + \dfrac{2(x+y)}{x^2}\dfrac{dy}{dx} = 0$	1 6	

Solution	Marks	Remarks
5. $z^2 - \sqrt{2}z + 1 = 0$		
$z = \frac{\sqrt{2}}{2} \pm \frac{\sqrt{2}}{2}i$	1A	Accept $\frac{\sqrt{2}}{2} \pm \frac{\sqrt{-2}}{2}$
$= \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$ or $\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4})$	1A+1A	Do not accept degrees Do not accept $\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}$ (Note : Mark the rest of the Q if z is correct but not in the specified format.)
$w^4 - \sqrt{2}w^2 + 1 = 0$		
$w^2 = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$ or $w^2 = \cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4})$	1M	
$w = \cos(\frac{2k\pi + \frac{\pi}{4}}{2}) + i\sin(\frac{2k\pi + \frac{\pi}{4}}{2})$ $= \cos(k\pi + \frac{\pi}{8}) + i\sin(k\pi + \frac{\pi}{8})$ or $w = \cos(\frac{2k\pi - \frac{\pi}{4}}{2}) + i\sin(\frac{2k\pi - \frac{\pi}{4}}{2})$, $= \cos(k\pi - \frac{\pi}{8}) + i\sin(k\pi - \frac{\pi}{8})$	1M+1A +1A	1M for De Moivre's Theorem
where k = 0, 1 (or any 2 consecutive integers)		
	7	
or $w = \cos\theta + i\sin\theta$, where $\theta = \frac{\pi}{8}$(or $-15\frac{\pi}{8}$ etc.), $-\frac{\pi}{8}$(or $\frac{15\pi}{8}$), $\frac{7\pi}{8}$(or $-\frac{9\pi}{8}$), $-\frac{7\pi}{8}$(or $\frac{9\pi}{8}$)	1A	Accept other equivalent values. Accept degrees

Solution	Marks	Remarks
6. (a) $x^2 + y\cos x - y^2 = 0$		
$2x + \cos x\frac{dy}{dx} - y\sin x - 2y\frac{dy}{dx} = 0$	1A+1A	1A for $\frac{d}{dx}(y\cos x)$
		1A for other terms
$\frac{dy}{dx} = \frac{y\sin x - 2x}{\cos x - 2y}$	1A	
(b) At P, $\frac{dy}{dx} = \frac{-\frac{\pi}{2}\sin\frac{\pi}{2} - \pi}{\cos\frac{\pi}{2} - (-\pi)}$	1M	
$= -\frac{3}{2}$	1A	
Equation of tangent is		
$\frac{y + \frac{\pi}{2}}{x - \frac{\pi}{2}} = -\frac{3}{2}$	1M	
$6x + 4y - \pi = 0$	1A	or $y = -\frac{3}{2}x + \frac{\pi}{4}$
	7	

Solution	Marks	Remarks
7. $(x-3)^2 - \|x-3\| - 12 = 0$		
Solution (1) :		
$(x-3)^2 = \|x-3\|^2$	2M	
$\|x-3\|^2 - \|x-3\| - 12 = 0$		
$(\|x-3\|+3)(\|x-3\|-4) = 0$	1A	
$\|x-3\| = 4$ or $\|x-3\| = -3$		
$\therefore \|x-3\| = 4$		
$\therefore x = 7$ or -1	1A+1A+2A	2A for rejecting $\|x-3\| = -3$
	7	
Solution (2) :		
Consider 2 cases : (1) $x \geq 3$ (2) $x < 3$	1M	Accept omitting equality sign
Case (1) : $x \geq 3$		
$(x-3)^2 - (x-3) - 12 = 0$	1A	
$[(x-3)+3][(x-3)-4] = 0$		
$x^2 - 7x = 0$		
$x = 0$ or 7	1A	
Rejecting $x = 0$, $\therefore x = 7$	1A	
Case (2) : $x < 3$		
$(x-3)^2 + (x-3) - 12 = 0$	1A	
$[(x-3)-3][(x-3)+4]$		
$x^2 - 5x - 6 = 0$		
$x = 6$ or -1	1A	
Rejecting $x = 6$, $\therefore x = -1$	1A	
Combining the 2 cases, $x = -1$ or 7		
Solution (3) :		
$(x-3)^2 - \|x-3\| - 12 = 0$		
Let $x - 3 = u$		
$u^2 - 12 = \|u\|$		
$u^4 - 24u^2 + 144 = u^2$	2M	
$u^4 - 25u^2 + 144 = 0$		
$(u^2-9)(u^2-16) = 0$	1A	
$u = \pm 3$ or $u = \pm 4$		
$x = 0$ or 6 or $x = -1$ or 7	1A+1A	
Rejecting $x = 0$ and 6, $\therefore x = -1$ or 7	1A+1A	

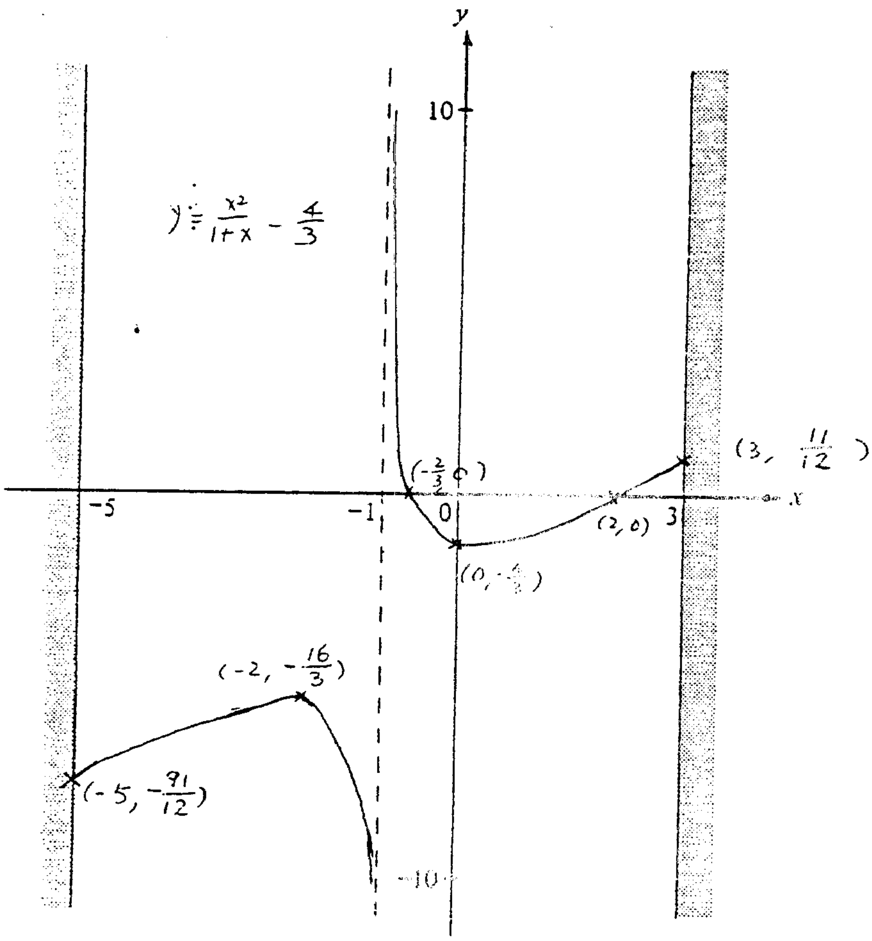
Solution	Marks	Remarks
8. (a) $x = 0$	1A	
	1	
(b) $x^2 + kx + (2k - 3) = 0$ has no real root.	1M	Can be omitted
$\Delta = k^2 - 4(2k - 3) < 0$	1M	
$k^2 - 8k + 12 < 0$		
$(k - 2)(k - 6) < 0$	1A	
$2 < k < 6$	1	
	4	
(c) (i) $f'(x) = 3x^2 + 2kx + (2k - 3)$	1A	
$\begin{cases} \alpha + \beta = \dfrac{-2k}{3} \\ \alpha\beta = \dfrac{2k - 3}{3} \end{cases}$	1M	
$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$	1A	Can be omitted
$= (\frac{-2k}{3})^2 - 4(\frac{2k - 3}{3})$		
$= \frac{4}{9}(k^2 - 6k + 9)$	1A	
$= \frac{4}{9}(k - 3)^2$	1A	$\Delta = 4k^2 - 12(2k - 3)$
Since $\alpha \neq \beta$, $\therefore k \neq 3$	1	$= 4(k - 3)^2$ .... 1A Since $\Delta > 0$, $\therefore k \neq 3$ .. 1
1		
(ii) $\lvert \frac{2}{3}(k - 3) \rvert \leq \frac{2}{3}$	1M	
$\lvert k - 3 \rvert \leq 1$	1A	or $-1 \leq k - 3 \leq 1$
$2 \leq k \leq 4$	1A	
Combining with $2 < k < 6$, $k \neq 3$ and k is an integer,		
$k = 4$	2A	
	11	

Alternative solution

Solution	Marks
(c) (ii) $\frac{4}{9}(k^2 - 6k + 9) \leq \frac{4}{9}$	1M
$k^2 - 6k + 8 \leq 0$	1A
$(k - 2)(k - 4) \leq 0$	
$2 \leq k \leq 4$	1A
$\therefore k = 4$	2A

Solution	Marks	Remarks
9. (a) Put $x = 0$, $y = -\frac{4}{3}$ $\therefore y$-intercept is $-\frac{4}{3}$	1A	
Put $y = 0$, $\frac{x^2}{1+x} - \frac{4}{3} = 0$		
$3x^2 - 4x - 4 = 0$		
$x = 2, -\frac{2}{3}$		
$\therefore x$-intercepts are 2 and $-\frac{2}{3}$.	1A	
	2	
(b) $\frac{dy}{dx} = \frac{2x(1+x) - x^2}{(1+x)^2}$	1M	For quotient rule
$= \frac{2x + x^2}{(1+x)^2}$	1A	
$\frac{d^2y}{dx^2} = \frac{(2+2x)(1+x)^2 - 2(1+x)(2x+x^2)}{(1+x)^4}$		
$= \frac{2}{(1+x)^3}$	1	
	3	
(c) $\frac{2x + x^2}{(1+x)^2} = 0$	1M	
$x = 0$ or -2	1A+1A	
When $x = 0$, $y = -\frac{4}{3}$		
$\frac{d^2y}{dx^2} = 2$ (or > 0)	1M	
$\therefore (0, -\frac{4}{3})$ is a minimum point	1A	No mark if checking is omitted
When $x = -2$, $y = -\frac{16}{3}$		
$\frac{d^2y}{dx^2} = -2$ (or < 0)		
$\therefore (-2, -\frac{16}{3})$ is a maximum point	1A	No mark if checking is omitted
	6	

Solution	Marks	Remarks
(d)		



Solution	Marks	Remarks
(i) For $-5 \le x < -1$	1A	Shape
	$\frac{1}{2}$A	For $(-5, -\frac{91}{12})$. Accept $-\frac{91}{12} \approx -7.6$
	$\frac{1}{2}$A	For $(-2, -\frac{16}{3})$ as a maximum point
(ii) For $-1 < x \le 3$	1A	Shape
	$\frac{1}{2}$A	For $(-\frac{2}{3}, 0)$
	$\frac{1}{2}$A	For (2, 0)
	$\frac{1}{2}$A	For $(3, \frac{11}{12})$. Accept $\frac{11}{12} \approx 0.92$
	$\frac{1}{2}$A	For $(0, -\frac{4}{3})$ as a minimum point
		Note : Round up to the nearest mark
	5	

Solution	Marks	Remarks
10. (a) $\overrightarrow{OC} = \frac{1}{3}\vec{a} + \frac{2}{3}\vec{b}$	1A	Omit vector sign (pp-1)
$\overrightarrow{DA} = \overrightarrow{OA} - \overrightarrow{OD}$	1M	Can be omitted
$= \vec{a} - \frac{1}{2}\vec{b}$	1A 3	
(b) $\overrightarrow{BE} = \overrightarrow{OE} - \overrightarrow{OB}$		
$= (k+1)\overrightarrow{OC} - \overrightarrow{OB}$	1A	For $\overrightarrow{OE} = (k+1)\overrightarrow{OC}$
$= \frac{k+1}{3}\vec{a} + \frac{2k-1}{3}\vec{b}$	1 2	$\frac{\overrightarrow{OC}}{\overrightarrow{CE}} = \frac{1}{k}$ (pp-1)
Alternative solution $\overrightarrow{BE} = \overrightarrow{BC} + \overrightarrow{CE}$		
$= \frac{1}{3}\overrightarrow{BA} + k\overrightarrow{OC}$	1A	
$= \frac{1}{3}(\vec{a} - \vec{b}) + k(\frac{\vec{a} + 2\vec{b}}{3})$		
$= \frac{k+1}{3}\vec{a} + \frac{2k-1}{3}\vec{b}$	1	
(c) $\vec{a} - \frac{1}{2}\vec{b} = \lambda(\frac{k+1}{3}\vec{a} + \frac{2k-1}{3}\vec{b})$	1M	No mark if λ is omitted
$\begin{cases} 1 = \lambda(\frac{k+1}{3}) \\ -\frac{1}{2} = \lambda(\frac{2k-1}{3}) \end{cases}$	1M	No mark if λ is omitted
$k = \frac{1}{5}$	1A	
Alternative solution $\frac{\frac{k+1}{3}}{1} = \frac{\frac{2k-1}{3}}{-\frac{1}{2}}$	2M	(pp-1) for considering the slope of the vectors
$k = \frac{1}{5}$	1A	
	3	
(d) (i) $\vec{a} \cdot \vec{b} = \lvert\vec{a}\rvert \lvert\vec{b}\rvert \cos\frac{\pi}{3}$	1M	Omit dot sign (pp-1)
$= (1)(2) \cos\frac{\pi}{3} = 1$	1A	

Solution	Marks	Remarks
(ii) $\overrightarrow{BE}\cdot\overrightarrow{OE}=0$	1M	
$\left(\frac{k+1}{3}\vec{a}+\frac{2k-1}{3}\vec{b}\right)\cdot\frac{k+1}{3}(\vec{a}+2\vec{b})=0$		
$\frac{k+1}{9}[(k+1)\vec{a}\cdot\vec{a}+(2k+2+2k-1)$		
$\vec{a}\cdot\vec{b}+2(2k-1)\vec{b}\cdot\vec{b}]=0$	1M	For distribution
$k+1+4k+1+8(2k-1)=0\quad(k\neq-1)$		
$k=\frac{2}{7}$	1A	
For $k=\frac{2}{7}$, $\overrightarrow{BE}=\frac{3}{7}\vec{a}-\frac{1}{7}\vec{b}$	1M	Trying to find $\overrightarrow{BE}$
$\lvert\overrightarrow{BE}\rvert^2=\overrightarrow{BE}\cdot\overrightarrow{BE}$	1M	Trying to find $\lvert\overrightarrow{BE}\rvert$
$=\left(\frac{3}{7}\vec{a}-\frac{1}{7}\vec{b}\right)\cdot\left(\frac{3}{7}\vec{a}-\frac{1}{7}\vec{b}\right)$		
$=\frac{9}{49}\vec{a}\cdot\vec{a}-\frac{6}{49}\vec{a}\cdot\vec{b}+\frac{1}{49}\vec{b}\cdot\vec{b}$		
$=\frac{9}{49}-\frac{6}{49}+\frac{4}{49}=\frac{1}{7}$		
$\therefore$ Distance of B from $OC=\frac{\sqrt{7}}{7}$	1A	
	8	

Solution	Marks	Remarks
11. (a) (i) $\dfrac{z^2}{\bar{z}} = \dfrac{r^2(\cos 2\theta + i\sin 2\theta)}{r(\cos\theta - i\sin\theta)}$	1A+1A	1A for denominator 1A for numerator
$= \dfrac{r^2(\cos 2\theta + i\sin 2\theta)}{r(\cos(-\theta) + i\sin(-\theta))}$	1A	For denominator
$= r(\cos 3\theta + i\sin 3\theta)$	1	
(ii) $z^2 = i\bar{z}$		
$r(\cos 3\theta + i\sin 3\theta) = i$	1A	
$= \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$	1A	Or other equivalent polar form (can be omitted)
$r = 1$	1A	
$3\theta = 2n\pi + \frac{\pi}{2}$		
$\theta = -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$	1A+1A	Any one correct - 1A
(or $\theta = \frac{2n\pi}{3} + \frac{\pi}{6}$, where $k = -1, 0, 1$)		All correct - 2A
<u>Alternative solution</u>		
(ii) $r(\cos 3\theta + i\sin 3\theta) = i$	1A	
$\begin{cases} r\cos 3\theta = 0 \\ r\sin 3\theta = 1 \end{cases}$	1A	or $\begin{cases} \cos 3\theta = 0 \\ \sin 3\theta = 1 \end{cases}$ (can be omitted)
$r^2(\sin^2 3\theta + \cos^2 3\theta) = 1$		
$r = 1$	1A	
$3\theta = 2n\pi + \frac{\pi}{2}$		
$\theta = -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$	1A+1A	Any one correct - 1A All correct - 2A
	9	

Solution	Marks	Remarks
(b) (i) $w - i = \cos\alpha + i\sin\alpha$	1A	
$\lvert w - i \rvert = (\sqrt{\cos^2\alpha + \sin^2\alpha}) = 1$	1A	
(ii) Since $(w - i)^2 = = i\overline{w} - 1$		
$= i(\overline{w - i})$,	1A	
$\therefore w - i$ satisfies the equation $z^2 = i\overline{z}$		
Using the result of (a),		
$w - i = \cos\theta + i\sin\theta$, where $\theta = -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$	1M	
$w - i = -i, \frac{\sqrt{3}}{2} + \frac{1}{2}i, \frac{-\sqrt{3}}{2} + \frac{1}{2}i$		
$w = 0, \frac{\sqrt{3}}{2} + \frac{3}{2}i, \frac{-\sqrt{3}}{2} + \frac{3}{2}i$	1A+1A+1A	(pp-1) for $w = \cos\alpha + i(1 + \sin\alpha)$
	7	where $\alpha = -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

RESTRICTED 内部文件

Solution	Marks	Remarks
12. (a) $x = 4\sin\theta$	1A	
$\frac{dx}{dt} = 4\cos\theta \frac{d\theta}{dt}$	1M	For differentiating *wrt t*
Put $\frac{dx}{dt} = \frac{1}{2}$		
$\frac{d\theta}{dt} = \frac{1}{8\cos\theta}$	1A	
	3	
(b) $y = 4\cos\theta$	1A	
$\frac{dy}{dt} = -4\sin\theta \frac{d\theta}{dt}$		
$= -\frac{\tan\theta}{2}$	1A	
$z = \sqrt{25 - 16\sin^2\theta}$	1M	or $\sqrt{25 - x^2}$
$\frac{dz}{dt} = \frac{-32\sin\theta\cos\theta}{2\sqrt{25 - 16\sin^2\theta}} \frac{d\theta}{dt}$		
$= \frac{-2\sin\theta}{\sqrt{25 - 16\sin^2\theta}}$	1A	
Rate of change $= \frac{dy}{dt} + \frac{dz}{dt}$	1M	
At $\theta = \frac{\pi}{6}$, Rate $= -\frac{\tan\frac{\pi}{6}}{2} - \frac{2\sin\frac{\pi}{6}}{\sqrt{25 - 16\sin^2\frac{\pi}{6}}}$		
$= -0.507$ (m s^{-1})	1A	
	6	
(c) Let A be the area of ΔOPR		
$A = \frac{1}{2}xy$		
$= 8\sin\theta\cos\theta$	1A	
$= 4\sin 2\theta$		
A is maximum when $\sin 2\theta = 1$	2M	
$2\theta = \frac{\pi}{2}$		
$\therefore \theta = \frac{\pi}{4}$	1A	

Solution	Marks	Remarks
Alternative solution (1)		
$A = 8\sin\theta\cos\theta$	1A	
$\frac{dA}{d\theta} = 8(\cos^2\theta - \sin^2\theta)$		
$\frac{dA}{d\theta} = 0$, $\cos^2\theta - \sin^2\theta = 0$	1M	
$\tan^2\theta = 1$		
$\theta = \frac{\pi}{4}$	1A	
$\frac{d^2A}{d\theta^2} = -32\sin\theta\cos\theta$		
$\frac{d^2A}{d\theta^2} < 0$ at $\theta = \frac{\pi}{4}$		
$\therefore$ A is maximum when $\theta = \frac{\pi}{4}$	1M	For checking
Alternative solution (2)		
$A = 4\sin 2\theta$	1A	
$\frac{dA}{d\theta} = 8\cos 2\theta$		
$\frac{dA}{d\theta} = 0$, $\cos 2\theta = 0$	1M	
$\theta = \frac{\pi}{4}$	1A	
$\frac{d^2A}{d\theta^2} = -16\sin 2\theta$		
$\frac{d^2A}{d\theta^2} < 0$ at $\theta = \frac{\pi}{4}$		
$\therefore$ A is a maximum when $\theta = \frac{\pi}{4}$	1M	For checking
Similarly, area of ΔORQ is maximum when $\angle OQR = \frac{\pi}{4}$	1M	
By Sine Law,		
$\frac{\sin\frac{\pi}{4}}{4} = \frac{\sin\theta}{5}$	1M	
$\theta = 1.08$	1A	
	7	

II

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Solution	Marks	Remarks
1. $\int(\sin x - \cos x)^2 dx$		
$= \int(\sin^2 x - 2\sin x\cos x + \cos^2 x)\,dx$	1A	
$= \int(1 - 2\sin x\cos x)\,dx$		
$= x - \sin^2 x + c$ (where c is a constant)	1A+2A	1A for $\int dx = x$ 2A for $\int 2\sin x\cos x\,dx = \sin^2 x$ (pp-1) for omitting c (pp-1) for omitting dx
Alternative solution $x + \cos^2 x + c$, $x + \frac{1}{2}\cos 2x + c$		
	4	
2. $\cos(x - 7°) = 2\cos(x + 7°)$		
$\cos x\cos 7° + \sin x\sin 7° = 2\cos x\cos 7° - 2\sin x\sin 7°$	1A+1A	1A for L.H.S., 1A for R.H.S.
$3\sin x\sin 7° = \cos x\cos 7°$		
$\tan x = \frac{\cos 7°}{3\sin 7°}$		
$x = 180n° + 69.8°$	1M+1A	1M for $180n° + \alpha$
	4	
3. (a) $(1-2x)^3 = 1 - 6x + 12x^2 - 8x^3$	1A	
$(1 + \frac{1}{x})^5 = 1 + \frac{5}{x} + \frac{10}{x^2} + \frac{10}{x^3} + \frac{5}{x^4} + \frac{1}{x^5}$	1A	
(b) $(1-2x)^3(1 + \frac{1}{x})^5 = (1 - 6x + 12x^2 - 8x^3)(1 + \frac{5}{x} + \frac{10}{x^2} + \frac{10}{x^3} + \frac{5}{x^4} + \frac{1}{x^5})$		
	1M	For collecting terms
(i) Constant term $= 1 - 6(5) + 12(10) - 8(10)$		
$= 11$	1A	
(ii) Coefficient of $x = -6 + 12(5) - 8(10)$		
$= -26$	1A	
	5	

Solution	Marks	Remarks
4. (a) Area of $\Delta PQR = \frac{1}{2}\left\|4x + 0 + 2y - 6x - 8\right\|$		
$= \left\|-x + y - 4\right\|$	1A+1A	1A for $(-x + y - 4)$ or $(x - y + 4)$
(b) $\left\|-x + y - 4\right\| = 4$	1M	Accept no absolute value
$-x + y - 4 = \pm 4$		
$x - y = 0$ or $x - y + 8 = 0$	1A+1A	
$\therefore$ the equation of the curve is $x - y = 0$ or $x - y + 8 = 0$		
Alternative solution (b) $(-x + y - 4)^2 = 4^2$	1M	
$x^2 - 2xy + y^2 + 8x - 8y = 0$		
$\therefore$ the equation of the curve is $x^2 - 2xy + y^2 + 8x - 8y = 0$	2A	
	5	
5. For n = 1, L.H.S. $= \frac{1}{2}$		
R.H.S. $= 3 - \frac{2+3}{2} = \frac{1}{2}$	1	
$\therefore$ the statement is true for n = 1		
Assume $\frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \ldots + \frac{2k-1}{2^k} = 3 - \frac{2k+3}{2^k}$	1	
(for some positive integer k)		
Then $\frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \ldots + \frac{2k-1}{2^k} + \frac{2(k+1)-1}{2^{k+1}}$		
$= 3 - \frac{2k+3}{2^k} + \frac{2k+1}{2^{k+1}}$	1	
$= 3 - \frac{2(2k+3)-(2k+1)}{2^{k+1}}$		
$= 3 - \frac{2k+5}{2^{k+1}}$ $\left(= 3 - \frac{2(k+1)+3}{2^{k+1}}\right)$	1	
($\therefore$ the statement is also true for n = k + 1 if it is true for n = k)		
(By the Principle of mathematical induction) $\therefore$ the statement is true for all +ve integers n.	1	
	5	

Solution	Marks	Remarks
6. (a) Slope of $L_1 = 2$, slope of $L_2 = 3$	1A	
$\tan\theta = \dfrac{3-2}{1+3(2)}$	1M	Accept $\dfrac{2-3}{1+2(3)}$
$= \dfrac{1}{7}$	1A	1M for using inclination
(b) Let m be the slope of the line		
$\dfrac{2-m}{1+2m} = \dfrac{1}{7}$	1M	Accept $\dfrac{m-2}{1+2m} = \dfrac{1}{7}$
$m = \dfrac{13}{9}$	1A	
$\therefore$ the equation of the line is $y = \dfrac{13}{9}x$	1A	or $13x - 9y = 0$
	6	
7. (a) $x^3 = x^3 - 6x^2 + 12x$	1A	
$6x^2 - 12x = 0$		
$x = 0$ or 2		
The coordinates of A are (2, 8)	1A	
(b) Area $= \int_0^2 [(x^3 - 6x^2 + 12x) - x^3]\,dx$	1M+1A	1M for $\int_\alpha^\beta (y_2 - y_1)\,dx$
$= \int_0^2 (-6x^2 + 12x)\,dx$		
$= \left[-2x^3 + 6x^2\right]_0^2$	1A	For primitive function only
$= 8$	1A	
	6	
8. (a) $\dfrac{dy}{dx} = 8 - 10x$		
$y = 8x - 5x^2 + k$	1A	
Put $x = 1$, $y = 13$, $k = 10$	1M	For substituting (1, 13) and finding k
$\therefore$ The equation of C is $y = 8x - 5x^2 + 10$	1A	
(b) At $x = 0$,		
$\dfrac{dy}{dx} = 8$	1A	
$\therefore$ slope of normal $= -\dfrac{1}{8}$	1M	
$y = 10$	1M	
The equation of the normal is		
$\dfrac{y-10}{x-0} = -\dfrac{1}{8}$		
$y = -\dfrac{1}{8}x + 10$	1A	or $x + 8y - 80 = 0$
	7	

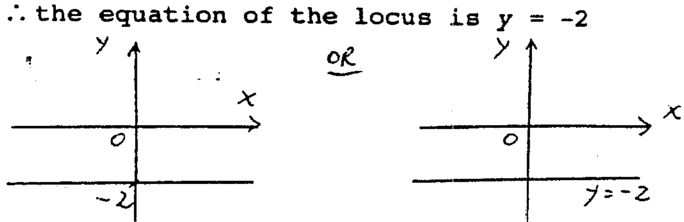
Solution	Marks	Remarks
9. (a) A, B are equidistant from the centre		
$(h-5)^2 + (k-5)^2 = (h-7)^2 + (k-1)^2$	2A	
$h^2 - 10h + 25 + k^2 - 10k + 25 = h^2 - 14h + 49 + k^2 - 2k + 1$		
$h = 2k$	1A	
<u>Alternative solution</u>		
Mid point of $AB = (6, 3)$, slope of $AB = -2$	1A	
Equation of perpendicular bisection of AB		
$\frac{y-3}{x-6} = \frac{1}{2}$	1M	
$x = 2y$		
Since (h, k) lies on the perpendicular bisector,		
$\therefore h = 2k$	1A	
Equation of C is		
$(x-h)^2 + (y-k)^2 = (h-7)^2 + (k-1)^2$	1M	or $= (h-5)^2 + (k-5)^2$
$(x-2k)^2 + (y-k)^2 = (2k-7)^2 + (k-1)^2$		
$x^2 + y^2 - 4kx - 2ky + 30k - 50 = 0$	1 / 5	
(b) Slope of line joining centre $(2k, k)$ and $B(7, 1)$		
$= \frac{k-1}{h-7}$	1M	Accept $\frac{k-1}{h-k}$
Slope of tangent at $B = \frac{7-2k}{k-1}$	1M	or $\frac{7-h}{k-1}$
Since slope of tangent at $B = \frac{1}{2}$		
$\frac{7-2k}{k-1} = \frac{1}{2}$	1M	In one unknown
$k = 3$	← 1A	
$\therefore$ Equation of C is $x^2 + y^2 - 12x - 6y + 40 = 0$	1A / 5	or $(x-6)^2 + (y-3)^2 = 5$

Solution	Marks	Remarks
<u>Alternative solution</u> :		
<u>Method (2)</u> :		
$x^2 + y^2 - 4ky - 2ky + 30k - 50 = 0$		
$2x + 2y\frac{dy}{dx} - 4k - 2k\frac{dy}{dx} = 0$	1M	
$\frac{dy}{dx} = \frac{4k - 2x}{2y - 2k}$		
At B (7, 1), $\frac{dy}{dx} = \frac{4k - 14}{2 - 2k} = \frac{1}{2}$	1M+1M	1M for substituting (7, 1) 1M for equating $\frac{1}{2}$
$k = 3$	1A	
$\therefore$ equation of C is $x^2 + y^2 - 12x - 6y + 40 = 0$	1A	
<u>Method (3)</u> :		
The equation of tangent at B is		
$y - 1 = \frac{1}{2}(x - 7)$	1A	
$x - 2y - 5 = 0$		
Distance from centre $(2k, k)$ of circle to the line		
$= \left\vert\frac{2k - 2k - 5}{\sqrt{5}}\right\vert$	1M	
$= \sqrt{5}$		
$\sqrt{5} = \sqrt{5k^2 - 30k + 50}$	1M	
$k^2 - 6k + 9 = 0$		
$k = 3$	1A	
$\therefore$ equation of C is $x^2 + y^2 - 12x - 6y + 40 = 0$	1A	
<u>Method (4)</u> :		
The equation of tangent at B is		
$y - 1 = \frac{1}{2}(x - 7)$	1A	
$y = \frac{1}{2}(x - 5)$ (or $x = 2y + 5$)		
Substitute into C		
$x^2 + \frac{1}{4}(x - 5)^2 - 4kx - 2k\frac{1}{2}(x - 5) + 30k - 50 = 0$	1M	or substitute $x = 2y + 5$
$5x^2 - 10(2k + 1)x + 140k - 175 = 0$		$5y^2 + (20 - 10k)y + (10k - 25) = 0$
Dis. $= 100(2k + 1)^2 - 20(140k - 175) = 0$	1M	$(20 - 10k)^2 - 20(10k - 25) = 0$
$k^2 - 6k + 9 = 0$		
$k = 3$	1A	
$\therefore$ equation of C is $x^2 + y^2 - 12x - 6y + 40 = 0$	1A	

Solution	Marks	Remarks
(c) Radius of $C = \sqrt{(h-7)^2+(k-1)^2}$	1M	or $\sqrt{(h-5)^2+(k-5)^2}$
$= \sqrt{5k^2-30k+50}$		
Distance from $(2k, k)$ to the line $y = 3x$ $= \left\lvert\frac{3(2k)-k}{\sqrt{10}}\right\rvert = \left\lvert\frac{5k}{\sqrt{10}}\right\rvert$	1M	Accept missing absoulte signs
If the circle touches the line, $\left\lvert\frac{5k}{\sqrt{10}}\right\rvert = \sqrt{5k^2-30k+50}$	1M	For equating and expressing in one unknown
$k^2 - 12k + 20 = 0$		
$k = 2$ or 10	1A	
∴ The equations of the circles are $x^2+y^2-8x-4y+10=0$	1A	or $(x-4)^2+(y-2)^2=10$
and $x^2+y^2-40x-20y+250=0$	1A 6	or $(x-20)^2+(y-10)^2=250$
Alternative solution		
Substitute $y = 3x$ into C		
$x^2+(3x)^2-4kx-2k(3x)+30k-50=0$	1M	
$10x^2-10kx+30k-50=0$		
Discriminant $= 100k^2-40(30k-50)=0$	1M+1A	
$k^2-12k+20=0$		
$k = 2$ or 10	1A	
∴ The equations of the circles are $x^2+y^2-8x-4y+10=0$	1A	
and $x^2+y^2-40x-20y+250=0$	1A	

Solution	Marks	Remarks
10. (a) $\int_0^1 \frac{dx}{1+x^2} = \int_0^{\frac{\pi}{4}} \frac{\sec^2\theta d\theta}{1+\tan^2\theta}$	1A+1A	1A for integrand 1A for limits
$= \int_0^{\frac{\pi}{4}} d\theta$	1A	
$= \frac{\pi}{4}$	1A 4	
(b) $3 + 2\sin x + \cos x$		
$= 3 + 2(\frac{2t}{1+t^2}) + \frac{1-t^2}{1+t^2}$	1A+1A	1A for sinx 1A for cosx
$= \frac{2(2+2t+t^2)}{1+t^2}$	1	
$t = \tan\frac{x}{2}$		
$dt = \frac{1}{2}\sec^2\frac{x}{2}dx$		
$dx = \frac{2dt}{1+t^2}$		
$\int \frac{dx}{3+2\sin x+\cos x} = \int \frac{1+t^2}{2(2+2t+t^2)} \cdot \frac{2dt}{1+t^2}$	1A	
$= \int \frac{dt}{2+2t+t^2}$		
$= \int \frac{dt}{1+(1+t)^2}$	1 5	
(c) Put $t = \tan\frac{x}{2}$		
$\int_{-\frac{\pi}{2}}^{0} \frac{dx}{3+2\sin x+\cos x} = \int_{-1}^{0} \frac{dt}{1+(1+t)^2}$	1A	
Put $u = 1 + t$	1A	
$\int_{-1}^{0} \frac{dt}{1+(1+t)^2} = \int_0^1 \frac{du}{1+u^2}$	1A	
$= \frac{\pi}{4}$ (using the result of (a))	1A 4	

Solution	Marks	Remarks
(d) $\int_{-\frac{\pi}{2}}^{0} \frac{(2\sin x + \cos x)dx}{3 + 2\sin x + \cos x}$		
$= \int_{-\frac{\pi}{2}}^{0} (1 - \frac{3}{3 + 2\sin x + \cos x})dx$	1M+1A	1M for $k_1 + \frac{k_2}{3 + 2\sin x + \cos x}$
$= \int_{-\frac{\pi}{2}}^{0} dx - 3\int_{-\frac{\pi}{2}}^{0} \frac{dx}{3 + 2\sin x + \cos x}$		
$= \frac{\pi}{2} - 3(\frac{\pi}{4})$		
$= -\frac{\pi}{4}$	1A	
	3	

Solution	Marks	Remarks
11. (a) Substitute $y = m_1x + c_1$ into $x^2 = 8y$		
$x^2 = 8(m_1x + c_1)$	1A	or $(\frac{y - c_1}{m_1})^2 = 8y$
$x^2 - 8m_1x - 8c_1 = 0$		
Discriminant $= 64m_1^2 + 32c_1 = 0$	1M	
$c_1 = -2m_1^2$	1A	
	3	
(b) Equation of L_2 is $y = m_2x - 2m_2^2$	1A	
$y = m_1x - 2m_1^2$ $y = m_2x - 2m_2^2$	1M	For solving the 2 eqns.
$0 = (m_1 - m_2)x - 2(m_1^2 - m_2^2)$		
$x = 2(m_1 + m_2)$	1	
$y = m_1x - 2m_1^2$		
$= m_1[2(m_1 + m_2)] - 2m_1^2$		
$= 2m_1m_2$	1	
	4	
(c) Let (x, y) be a point on the locus.		
$\begin{cases} x = 2(m_1 + m_2) \\ y = 2m_1m_2 \end{cases}$		
$\left\lvert \frac{m_1 - m_2}{1 + m_1m_2} \right\rvert = \tan\frac{\pi}{4}$	1A	Accept no absolute sign
$(\frac{m_1 - m_2}{1 + m_1m_2})^2 = 1$	1M	For squaring both sides
$(m_1 + m_2)^2 - 4m_1m_2 = (1 + m_1m_2)^2$	1A	
$(\frac{x}{2})^2 - 2y = (1 + \frac{y}{2})^2$	1M	For substituting $m_1 + m_2 = \frac{x}{2}$, $m_1m_2 = \frac{y}{2}$
$x^2 - y^2 - 12y - 4 = 0$	1A	
	5	
(d) Let (x, y) be a point on the locus		
$\begin{cases} x = 2(m_1 + m_2) \\ y = 2m_1m_2 \end{cases}$		
Since $L_1 \perp L_2$, $m_1m_2 = -1$	1A	
$y = 2m_1m_2 = -2$		
$\therefore$ the equation of the locus is $y = -2$	1A	
	1A	For a line below and parallel the x-axis
	1A	For labelling the axes and the line
	4	

Solution	Marks	Remarks
12. (a) By Sine law,		
$\frac{AC}{\sin\beta} = \frac{100}{\sin(\pi - \alpha - \beta)}$	1M+1A	1M for $\frac{AC}{\sin B} = \frac{AB}{\sin C}$
$AC = \frac{100\sin\beta}{\sin(\alpha+\beta)}$ (km)	1	
$PC = AC\tan\theta$	1A	
$= \frac{100\sin\beta\tan\theta}{\sin(\alpha+\beta)}$ (km)	1 5	
(b) (i) $AC = \frac{100\sin 30°}{\sin(45°+30°)}$		
$= 51.76$ (km)	1A	
$AC' = \frac{100\sin 43°}{\sin(37°+43°)}$		
$= 69.25$ (km)	1A	
(ii) $\angle CAC' = 45° - 37° = 8°$	1A	
By Cosine law,		
$CC'^2 = AC^2 + AC'^2 - 2(AC)(AC')\cos\angle CAC'$		
$= (51.76)^2 + (69.25)^2 - 2(51.76)(69.25)\cos 8°$	1M	
$CC' = 19.38$ (km)	1A	
(iii) Increase in height		
$= P'C' - PC$		
$= \frac{100\sin 43°\tan 17°}{\sin(43°+37°)} - \frac{100\sin 30°\tan 20°}{\sin(30°+45°)}$	1M+1A	or $AC'\tan 17° - AC\tan 20°$ – 1M
$= 2.33$ (km)	1A	
(iv) Let the angle of elevation be γ		
$\tan\gamma = \frac{P'C' - PC}{CC'}$	2M	
$\gamma = 6.86°$	1A 11	

Solution	Marks	Remarks
13. (a) (i) $V = \int_{-a}^{-a+h} \pi x^2 dy$ (Accept $\int_{a-h}^{a} \pi x^2 dy$)	1M+1A	1M for $\pi\int_{\alpha}^{\beta} x^2 dy$
$= \int_{-a}^{-a+h} \pi(a^2 - y^2) dy$	1M	For integrand only
$= \pi[a^2y - \frac{1}{3}y^3]_{-a}^{-a+h}$	1A	For primitive function only
$= \pi[a^2(h - a) - \frac{1}{3}(h-a)^3 + a^3 - \frac{1}{3}a^3]$		
$= \pi[a^2h - a^3 - \frac{1}{3}(h^3 - 3h^2a + 3ha^2 - a^3) + a^3 - \frac{1}{3}a^3]$		
$= \pi h^2(a - \frac{1}{3}h)$	1	
(ii) Put $y = -a + h$ into $x^2 + y^2 = a^2$		
$x^2 = a^2 - (-a + h)^2$	1M	For finding the radius
$= 2ah - h^2$		
$\therefore A = \pi h(2a - h)$	1 / 7	
(b) (i) $\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$	1M	For chain rule
$\frac{dV}{dh} = \pi h(2a - h)$ $(= A)$	1A	Accept $\frac{dV}{dh} = A$
$-kA = A\frac{dh}{dt}$		
$\frac{dh}{dt} = -k$	1A	
$\therefore \frac{dh}{dt}$ is a constant.		
Alternative solution		
$\frac{dV}{dt} = -kA$		
$\frac{dV}{dh}\frac{dh}{dt} = -kA$	1M	
$\pi(2ah - h^2)\frac{dh}{dt} = -k\pi h(2a - h)$	1A	For finding $\frac{dV}{dh}$
$\frac{dh}{dt} = -k$	1A	
$\therefore \frac{dh}{dt}$ is a constant.		

Solution	Marks	Remarks
(ii) (1) $\frac{dh}{dt} = -k$		
$h = -kt + c$ (where c is a constant)	1M	Accept $h = -kt$
At $t = 0$, $h = \frac{3}{4}a$ $\therefore c = \frac{3}{4}a$	1M	
At $t = 30$, $h = 0$ $\therefore k = \frac{1}{40}a$	1M	
$\therefore h = \frac{3}{4}a - \frac{1}{40}at = \frac{a}{40}(30 - t)$	1	
(2) At $t = 10$		
$h = \frac{1}{40}a(30 - 10) = \frac{1}{2}a$	1A	
$V = \pi h^2(a - \frac{h}{3})$		
$= \pi(\frac{a}{2})^2(a - \frac{1}{6}a)$		
$= \frac{5}{24}\pi a^3$	1A 9	