Solution — 1993 Add. maths (Paper 1)	Marks	Remarks
1. (a) $(\sqrt{2(x+\Delta x)} - \sqrt{2x})(\sqrt{2(x+\Delta x)} + \sqrt{2x})$		
$= 2(x+\Delta x) - 2x$		
$= 2\Delta x$	1A	
(b) $\frac{d}{dx}\sqrt{2x} = \lim_{\Delta x \to 0} \frac{1}{\Delta x}(\sqrt{2(x+\Delta x)} - \sqrt{2x})$	1A	
$= \lim_{\Delta x \to 0} \frac{1}{\Delta x}(\sqrt{2(x+\Delta x)} - \sqrt{2x}) \cdot \frac{\sqrt{2(x+\Delta x)} + \sqrt{2x}}{\sqrt{2(x+\Delta x)} + \sqrt{2x}}$	1M	
$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \cdot \frac{2\Delta x}{\sqrt{2(x+\Delta x)} + \sqrt{2x}}$		
$= \lim_{\Delta x \to 0} \frac{2}{\sqrt{2(x+\Delta x)} + \sqrt{2x}}$	1A	
$= \frac{1}{\sqrt{2x}}$	1A	
	5	
2. (a) $\frac{50}{4+3i} = \frac{50}{4+3i}\left(\frac{4-3i}{4-3i}\right)$	1M	
$= 8 - 6i$	1A	
(b) $5z + 3\bar{z} = \frac{50}{4+3i}$		
$5(a+bi) + 3(a-bi) = 8 - 6i$	1A	For $\bar{z} = a - bi$
$\begin{cases} 5a + 3a = 8 \\ 5b - 3b = -6 \end{cases}$	1M	
$\therefore\ a = 1,\ b = -3$	1A	
$z = 1 - 3i$		
	5	
3. $\alpha + \beta = -p,\ \alpha\beta = q$	1A	
$-q = (\alpha+3) + (\beta+3)$	1M	
$= (\alpha+\beta) + 6$		
$-q = -p + 6$	1A	
$p = (\alpha+3)(\beta+3)$	1M	
$= \alpha\beta + 3(\alpha+\beta) + 9$		
$p = q - 3p + 9$	1A	
Solving the equations, $p = 1,\ q = -5$	1A	
	6	

Solution	Marks	Remarks 1983
Alternative solution		
α, β are the roots of $(x+3)^2 + q(x+3) + p = 0$	1M	
$x^2 + (q+6)x + (p+3q+9) = 0$	1A	
Comparing coefficient with $x^2 + px + q = 0$	1M	
$\begin{cases} p = q + 6 \\ q = p + 3q + 9 \end{cases}$	1A+1A	
Solving the equations, $p = 1$, $q = -5$.	1A	
i. $\sin\frac{2\pi}{3} + i\cos\frac{2\pi}{3}$		
$= \frac{\sqrt{3}}{2} - \frac{1}{2}i$	1A	
$= \cos\frac{\pi}{6} - i\sin\frac{\pi}{6}$	1A	(can be omitted)
$= \cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6})$	1A	or $\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}$ etc. Accept degree measures
Alternative solution		
$\sin\frac{2\pi}{3} + i\cos\frac{2\pi}{3} = \cos(\frac{\pi}{2} - \frac{2\pi}{3}) + i\sin(\frac{\pi}{2} - \frac{2\pi}{3})$	1A	
$= \cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6})$	2A	
$\sin\frac{2\pi}{3} + i\cos\frac{2\pi}{3} = \sin(\frac{\pi}{2} + \frac{\pi}{6}) + i\cos(\frac{\pi}{2} + \frac{\pi}{6})$	1A	
$= \cos\frac{\pi}{6} - i\sin\frac{\pi}{6}$	1A	(can be omitted)
$= \cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6})$	1A	
$(\sin\frac{2\pi}{3} + i\cos\frac{2\pi}{3})^{\frac{1}{3}} = [\cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6})]^{\frac{1}{3}}$		
$= \cos\frac{2k\pi - \frac{\pi}{6}}{3} + i\sin\frac{2k\pi - \frac{\pi}{6}}{3}$,	1M+1A	1M for De Moivre's Theorem
where $k = -1, 0, 1$,	1A	1A if others correct or $k = 0, 1, 2$ or etc.
OR $(\sin\frac{2\pi}{3} + i\cos\frac{2\pi}{3})^{\frac{1}{3}} = \cos(-\frac{\pi}{18}) + i\sin(-\frac{\pi}{18})$,	1A	
$\cos\frac{11\pi}{18} + i\sin\frac{11\pi}{18}$,	1A	
$\cos(-\frac{13\pi}{18}) + i\sin(-\frac{13\pi}{18})$	1A	
(or $\cos\frac{23\pi}{18} + i\sin\frac{23\pi}{18}$)		
	6	

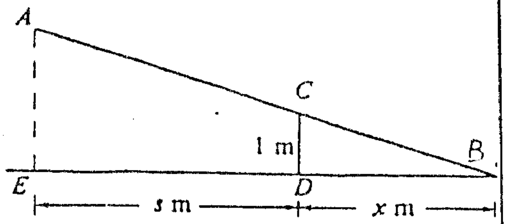
Solution	Marks	Remarks 1993
5. $\lvert -x^2 + 2x + 3 \rvert \geq 5$		
$-x^2 + 2x + 3 \geq 5$ or $-x^2 + 2x + 3 \leq -5$	2A	use 'and' or ' ,,, and' (no mark)
$x^2 - 2x + 2 \leq 0$ or $x^2 - 2x - 8 \geq 0$		
$(x - 1)^2 + 1 \leq 0$ or $(x + 2)(x - 4) \geq 0$	1A	For factorisation
No solution or $x \geq 4$ or $x \leq -2$	1A+1A	
$\therefore x \leq -2$ or $x \geq 4$	1A	cannot omit 'or'
<u>Alternative solution (1)</u>		
$(-x^2 + 2x + 3)^2 \geq 5^2$	1A	
$(-x^2 + 2x + 3 + 5)(-x^2 + 2x + 3 - 5) \geq 0$	1A	
$(-x^2 + 2x + 8)(-x^2 + 2x - 2) \geq 0$		
$(x^2 - 2x + 2)(x^2 - 2x - 8) \geq 0$		
$\because x^2 - 2x + 2 = (x - 1)^2 + 1 > 0$	1M	
$(x^2 - 2x - 8) \geq 0$	1A	
$(x - 4)(x + 2) \geq 0$	1A	
$x \geq 4$ or $x \leq -2$	1A	
<u>Alternative solution (2)</u>		
$\lvert (x + 1)(x - 3) \rvert \geq 5$	1M	
Consider the following cases :		
case 1 : $x \geq 3$		
$(x + 1)(x - 3) \geq 5$		
$(x - 4)(x + 2) \geq 0$		
$x \geq 4$ or $x \leq -2$		
since $x \geq 3$, $\therefore x \geq 4$	1A	
case 2 : $-1 < x < 3$		
$-(x + 1)(x - 3) \geq 5$		
$x^2 - 2x + 2 \leq 0$		
$(x - 1)^2 + 1 \leq 0$		
no solution	1A	
case 3 : $x \leq -1$		
$(x + 1)(x - 3) \geq 5$		
$x \geq 4$ or $x \leq -2$		
since $x \leq -1$, $\therefore x \leq -2$	1A	
Combining the 3 cases,		
$x \leq -2$ or $x \geq 4$	2A	
	6	

Solution	Marks	Remarks
6. (a) $\vec{AB} = \vec{OB} - \vec{OA}$	1A	Omit vector sign (pp-1)
$= -2\vec{i} + 3\vec{j}$	1A	
(b) $\vec{AB} \cdot \vec{AB} = (-2)^2 + 3^2$	1M	Omit dot sign (pp-1)
$= 13$	1A	
or $\vec{AB} \cdot \vec{AB} = (-2i + 3j) \cdot (-2i + 3j)$	1M	
$= 4 + 9 = 13$	1A	
Since $AB \perp BC$, $\vec{AB} \cdot \vec{BC} = 0$	1A	
$\vec{AB} \cdot \vec{AC} = \vec{AB} \cdot (\vec{AB} + \vec{BC})$	1M	For $\vec{AC} = \vec{AB} + \vec{BC}$
$= \vec{AB} \cdot \vec{AB} + \vec{AB} \cdot \vec{BC}$		
$= 13$	1A	
	7	
7. (a) $2x - 2y^2 - 4xy\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$	1A+1A	1A for $\frac{d}{dx}(-2xy^2)$ 1A for other terms
$\frac{dy}{dx} = \frac{2x - 2y^2}{4xy - 3y^2}$	1A	
(b) At $(2, -1)$, $\frac{dy}{dx} = \frac{2(2) - 2(-1)^2}{4(2)(-1) - 3(-1)^2}$	1M	
$= -\frac{2}{11}$	1A	
Equation of tangent is		
$\frac{y+1}{x-2} = -\frac{2}{11}$	1M	
$2x + 11y + 7 = 0$	1A	or $y = -\frac{2}{11}x - \frac{7}{11}$
	7	

Solution	Marks	Remarks
8. (a) $\vec{OP} = \dfrac{\vec{a} + r\vec{b}}{1 + r}$	1A	Omit vector sign (pp-1)
$\vec{OQ} = \dfrac{\vec{OP} + r\vec{OB}}{1 + r}$	1A	
$= \dfrac{\frac{1}{1+r}(\vec{a} + r\vec{b}) + r\vec{b}}{1 + r}$		
$= \dfrac{\vec{a} + (r^2 + 2r)\vec{b}}{(1 + r)^2}$	1A	
<u>Alternative solutions for $\vec{OQ}$</u>		
$\vec{PQ} = \dfrac{r}{r^2 + 2r + 1}\vec{AB}$	1A	
$\vec{OQ} = \vec{OP} + \vec{PQ}$		
$= \dfrac{\vec{a} + r\vec{b}}{1 + r} + \dfrac{r}{(1 + r)^2}(\vec{b} - \vec{a})$		
$= \dfrac{\vec{a} + (r^2 + 2r)\vec{b}}{(1 + r)^2}$	1A	
$AQ : QB = (r^2 + 2r) : 1$	1A	
$\vec{OQ} = \dfrac{\vec{a} + (r^2 + 2r)\vec{b}}{(1 + r)^2}$	1A	
	3	
(b) $\vec{OT} = \dfrac{1}{1 + r}\vec{b}$	1A	
$\vec{TQ} = \vec{OQ} - \vec{OT}$		
$= \dfrac{\vec{a} + (r^2 + 2r)\vec{b}}{(1 + r)^2} - \dfrac{1}{(1 + r)}\vec{b}$	1M	
$= \dfrac{\vec{a} + (r^2 + r - 1)\vec{b}}{(1 + r)^2}$	1	
	3	
(c) Since $\vec{OA} \parallel \vec{TQ}$,		
$r^2 + r - 1 = 0$	2M	or $\dfrac{r^2 + r - 1}{(1 + r)^2} = 0$
$r = \dfrac{-1 \pm \sqrt{5}}{2}$		
Since $r > 0$, $\therefore r = \dfrac{-1 + \sqrt{5}}{2}$	1A	
	3	

Solution	Marks	Remarks
Alternative solution		
$AQ : QB = (r^2 + 2r) : 1$		
$OT : TB = 1 : r$		
Since $\overrightarrow{OA} \parallel \overrightarrow{TB}$,		
$\frac{r^2 + 2r}{1} = \frac{1}{r}$	1M+1A	
$r^3 + 2r^2 - 1 = 0$		
$(r + 1)(r^2 + r - 1) = 0$		
$r = -1, \frac{-1 \pm \sqrt{5}}{2}$		
Since $r > 0$, $\therefore r = \frac{-1 + \sqrt{5}}{2}$	1A	
(d) (i) $\vec{a} \cdot \vec{a} = 4$	1A	Omit dot sign (pp-1)
$\vec{a} \cdot \vec{b} = 2(16)(\cos\frac{\pi}{3})$	1M	
$= 16$	1A	
(ii) $\overrightarrow{OA} \cdot \overrightarrow{TQ} = 0$	1M	
$\vec{a} \cdot \left[\frac{\vec{a} + (r^2 + r - 1)\vec{b}}{(1 + r)^2}\right] = 0$		
$\frac{1}{(1 + r)^2}[\vec{a} \cdot \vec{a} + (r^2 + r - 1)\vec{a} \cdot \vec{b}] = 0$		
$\frac{1}{(1 + r)^2}[4 + (r^2 + r - 1)16] = 0$	1M	
$16r^2 + 16r - 12 = 0$	1A	
$r = \frac{1}{2}$ or $-\frac{3}{2}$ (rejected)		
$\therefore r = \frac{1}{2}$	1A	
	7	

Solution	Marks	Remarks 1993
9. (a) $\Delta BCD \sim \Delta BAE$		
$\frac{BC}{BA} = \frac{BD}{BE}$	1M	
$\frac{\sqrt{x^2+1}}{8} = \frac{x}{x+s}$	1A	
$s = \frac{8x}{\sqrt{1+x^2}} - x$	1	
	3	
(b) $\frac{ds}{dx} = \frac{8\sqrt{1+x^2} - \frac{8x^2}{\sqrt{1+x^2}}}{1+x^2} - 1$	1M+1A	1M for quotient rule
$= \frac{8}{(1+x^2)^{3/2}} - 1$		
$\frac{ds}{dx} = 0$	1M	
$(1+x^2)^{3/2} = 8$		
$x = \pm\sqrt{3}$		
Since $x > 0$, $\therefore x = \sqrt{3}$	1A	
$\frac{d^2s}{dx^2} = \frac{-24x}{(1+x^2)^{5/2}}$	1A	When $0 < x < \sqrt{3}$, $\frac{ds}{dx} > 0$; When $\sqrt{3} < x < 3\sqrt{7}$, $\frac{ds}{dx} < 0$ } 1A
At $s = \sqrt{3}$, $\frac{d^2s}{dx^2}$ $(= -\frac{3\sqrt{3}}{4}) < 0$ $\therefore s$ is a maximum	1M	$\therefore s$ is a maximum 1M
$s_{max} = \frac{8\sqrt{3}}{\sqrt{1+3}} - \sqrt{3} = 3\sqrt{3}$	1A	Awarded if checking is omitted
	7	
(c) (i) P = Area of ΔABE − area of ΔCBD		
$= \frac{1}{2}(s+x)(8)\sin\angle CBD - \frac{x}{2}$	1M	
$= \frac{1}{2}(s+x)(8)\cdot\frac{1}{\sqrt{1+x^2}} - \frac{x}{2}$	1A	
$= \frac{1}{2}\left(\frac{8x}{\sqrt{1+x^2}} - x + x\right)\cdot\frac{8}{\sqrt{1+x^2}} - \frac{x}{2}$		
$= \frac{32x}{1+x^2} - \frac{x}{2}$	1	



Solution	Marks	Remarks 1993
Alternative solution		
Area of ΔABE : Area of ΔCBD		
$= (s + x)^2 : x^2$	1M	
$= \dfrac{64x^2}{1 + x^2} : x^2$		
Area of $\Delta ABE = \dfrac{32x}{1 + x^2}$	1A	
Area of $\Delta CBD = \dfrac{x}{2}$		
$\therefore P = \dfrac{32x}{1 + x^2} - \dfrac{x}{2}$	1	
$P = \frac{1}{2}(1 + AE)s$	~~1M~~	
$= \frac{1}{2}(1 + \frac{s + x}{x})s$	1M	
$= \frac{1}{2}(1 + \frac{8}{\sqrt{1 + x^2}})(\frac{8x}{\sqrt{1 + x^2}} - x)$	1A	
$= \frac{x}{2}(1 + \frac{8}{\sqrt{1 + x^2}})(\frac{8}{\sqrt{1 + x^2}} - 1)$		
$= \dfrac{32x}{1 + x^2} - \dfrac{x}{2}$	1	
(ii) $\dfrac{dp}{dx} = \dfrac{32(1 + x^2) - 32x(2x)}{(1 + x^2)^2} - \dfrac{1}{2}$	1M	
$= \dfrac{32(1 - x^2)}{(1 + x^2)^2} - \dfrac{1}{2}$		$x^4 + 66x^2 - 63 = 0$
From (b), s attains its maximum at $x = \sqrt{3}$		
At $x = \sqrt{3}$, $\dfrac{dp}{dx} = -\dfrac{9}{2}$	1A	
Since $\dfrac{dp}{dx} \neq 0$ at $x = \sqrt{3}$, P does not attain a maximum when s attains its maximum	1	
	6	

1993

Solution	Marks	Remarks
10. (a) Let α, β be the roots of the equation		
$\frac{1}{k+1}[2x^2 + (k+7)x + 4] = 0$		
$\begin{cases} \alpha + \beta = -\frac{k+7}{2} \\ \alpha\beta = 2 \end{cases}$	1A	
$PQ = \lvert \alpha - \beta \rvert = 1$	1M	
$(\alpha - \beta)^2 = 1$		
$(\alpha + \beta)^2 - 4\alpha\beta = 1$	1M	
$(-\frac{k+7}{2})^2 - 8 = 1$	1A	
Alternative solution		
$\frac{1}{k+1}[2x^2 + (k+7)x + 4] = 0$		
$x = \frac{-(k+7) \pm \sqrt{(k+7)^2 - 32}}{4}$	1A	
$PQ = \frac{-(k+7) + \sqrt{(k+7)^2 - 32}}{4} - \frac{-(k+7) - \sqrt{(k+7)^2 - 32}}{4}$	2M	
$1 = \frac{\sqrt{(k+7)^2 - 32}}{2}$	1A	
$k^2 + 14k + 13 = 0$	1A	
$k = -1$ or -13		(can be omitted)
$\because k \neq -1$, $\therefore k = -13$	1A 6	
(b) Discriminant $= \frac{(k+7)^2 - 32}{(k+1)^2} < 0$	1M+1A	Accept $\frac{(k+7)^2}{4} - 8 < 0$, $(k+7)^2 - 32 < 0$ $k^2 + 14k + 17 < 0$
$-7 - 4\sqrt{2} < k < -7 + 4\sqrt{2}$	2A 4	

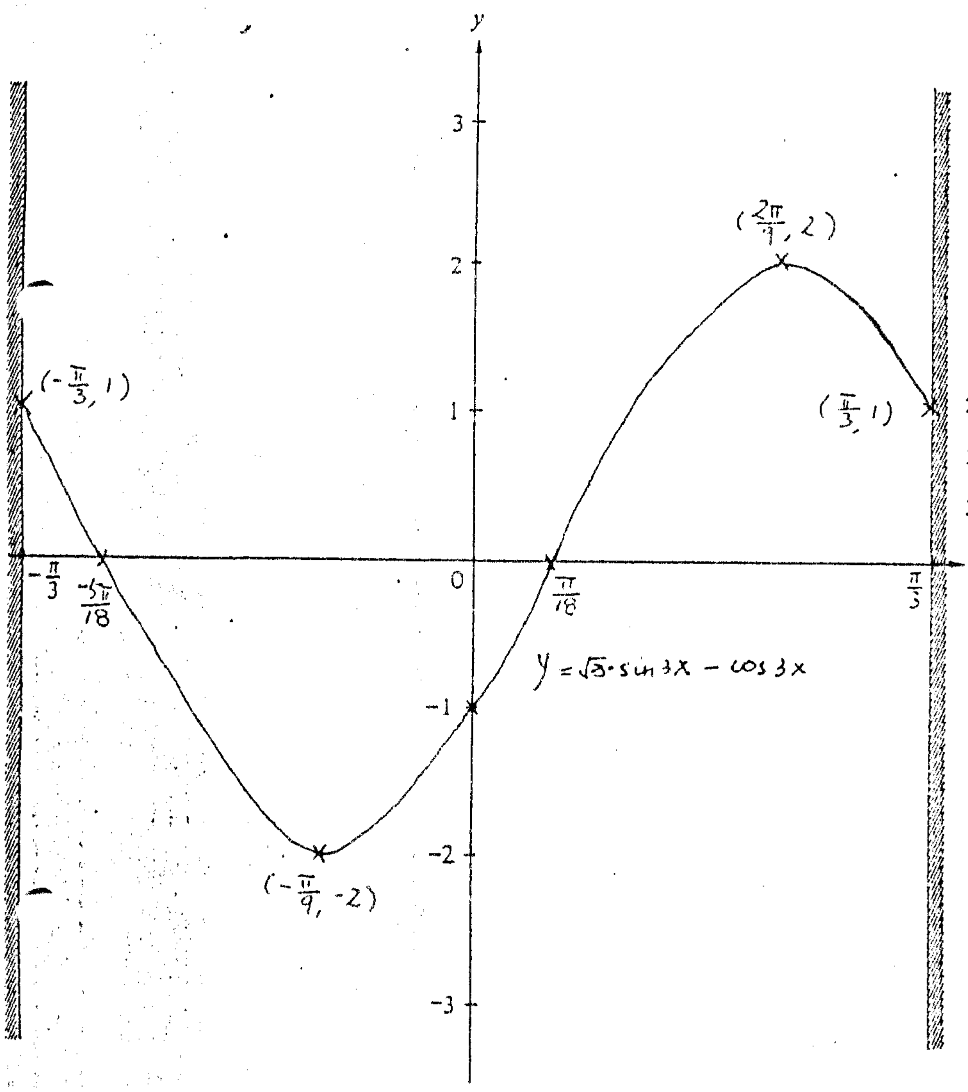
Solution	Marks	Remarks
(c) Put $k = 0$, C becomes $y = 2x^2 + 7x + 4$ .... (1)		
$k = 1$, C becomes $y = x^2 + 4x + 2$ .... (2)	2M	or any other values
Solving (1) and (2), the points of intersection are		
$(-1, -1)$ and $(-2, -2)$	1A+1A	
Put $x = -1$, $y = -1$ into C		
LHS $= y = -1$		
RHS $= \frac{1}{k+1}[2 + (k+7)(-1) + 4] = -1 =$ LHS $\forall k \neq -1$	1	
Put $x = -2$, $y = -2$ into C		
LHS $= -2$		
RHS $= \frac{1}{k+1}[8 + (k+7)(-2) + 4] = -2 =$ LHS $\forall k \neq -1$	1	
$\therefore C$ always passes through $(-1, -1)$ and $(-2, -2)$	6	

<u>Alternative solution</u>

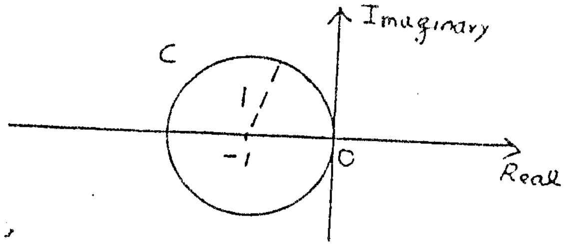
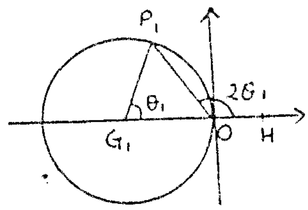
Solution	Marks	Remarks
(c) $y = \frac{1}{k+1}[2x^2 + (k+7)x + 4]$		
$(k+1)y = 2x^2 + (k+7)x + 4$		
$(y - 2x^2 - 7x - 4) + k(y - x) = 0$	1M+1A	
C always passes through the intersection points of the 2 curves $y - 2x^2 - 7x - 4 = 0$ and $y - x = 0$	2M	
$\begin{cases} y - 2x^2 - 7x - 4 = 0 \\ y - x = 0 \end{cases}$		
Solving, the 2 points are $(-1, -1)$ and $(-2, -2)$	1A+1A	
(c) Let k_1, k_2 be two distinct values of k		
$\begin{cases} (k_1 + 1)y = 2x^2 + (k_1 + 7)x + 4 \quad \ldots (1) \\ (k_2 + 1)y = 2x^2 + (k_2 + 7)x + 4 \quad \ldots (2) \end{cases}$	1M	
(1) − (2) : $(k_1 - k_2)y = (k_1 - k_2)x$	1M	
$y = x$ (since $k_1 \neq k_2$)	1A	
Subs. into (1) :		
$(k_1 + 1)x = 2x^2 + (k_1 + 7)x + 4$	1M	
$2x^2 + 6x + 4 = 0$		
$x = -1$ or -2		
when $x = -1$, $y = -1$	1A	
$y = -2$, $y = -2$	1A	
$\therefore C$ always passes through 2 fixed points whose coordinates are $(-1, -1)$ and $(-2, -2)$		

Solution	Marks	Remarks
11. (a) Put $x = 0$, $f(x) = -1$	1A	
$\therefore$ The y-intercept is -1.		
Put $f(x) = 0$, $\sqrt{3}\sin 3x - \cos 3x = 0$		
$\tan 3x = \frac{1}{\sqrt{3}}$	1A	
$x = \frac{\pi}{18}$ or $\frac{-5\pi}{18}$	1A+1A	No mark for degrees
$\therefore$ The x-intercepts are $\frac{\pi}{18}$ and $\frac{-5\pi}{18}$		
	4	
(b) $f'(x) = 3\sqrt{3}\cos 3x + 3\sin 3x$	1A	
$f''(x) = -9\sqrt{3}\sin 3x + 9\cos 3x$	1A	
	2	
(c) $f'(x) = 3\sqrt{3}\cos 3x + 3\sin 3x = 0$	1M	
$\tan 3x = -\sqrt{3}$		
$x = \frac{2\pi}{9}$ or $-\frac{\pi}{9}$	1A+1A	
$f''(-\frac{\pi}{9})$ (= 18) > 0 $\therefore$ it is a minimum	1M	
The minimum point is $(-\frac{\pi}{9}, -2)$	1A	
$f''(\frac{2\pi}{9})$ (= -18) < 0 $\therefore$ it is a maximum		
The maximum point is $(\frac{2\pi}{9}, 2)$	1A	
	6	

Solution	Marks	Remarks
Alternative solution		
(a) $f(x) = 2\sin(3x - \frac{\pi}{6})$		or $f(x) = -2\cos(3x + \frac{\pi}{3})$
$f(0) = -1$	1A	
$\therefore$ The y-intercept is -1		
Put $f(x) = 0$, $\sin(3x - \frac{\pi}{6}) = 0$	1A	or $\cos(3x + \frac{\pi}{3}) = 0$
$x = \frac{\pi}{18}$ or $\frac{-5\pi}{18}$		
$\therefore$ The x-intercepts are $\frac{\pi}{18}$ or $\frac{-5\pi}{18}$	1A+1A	
	4	
(b) $f'(x) = 6\cos(3x - \frac{\pi}{6})$	1A	
$f''(x) = -18\sin(3x - \frac{\pi}{6})$	1A	
	2	
(c) $f'(x) = 6\cos(3x - \frac{\pi}{6}) = 0$	1M	
$x = \frac{2\pi}{9}$ or $-\frac{\pi}{9}$	1A+1A	
$f''(-\frac{\pi}{9})$ $(= 18) > 0$ $\therefore$ it is a minimum	1M	
The minimum point is $(-\frac{\pi}{9}, -2)$	1A	
$f''(\frac{2\pi}{9})$ $(= -18) < 0$ $\therefore$ it is a maximum		
The maximum point is $(\frac{2\pi}{9}, 2)$	1A	
OR		
$f(x) = 2\sin(3x - \frac{\pi}{6})$		
$f(x)$ is maximum when $\sin(3x - \frac{\pi}{6}) = 1$	1M	
$x = \frac{2\pi}{9}$	1A	
$\therefore$ The maximum point is $(\frac{2\pi}{9}, 2)$	1A	
$f(x)$ is minimum when $\sin(3x - \frac{\pi}{6}) = -1$	1M	
$x = -\frac{\pi}{9}$	1A	
$\therefore$ The minimum point is $(-\frac{\pi}{9}, -2)$	1A	
	6	

Solution	Marks	Remarks 1993
(d)		
	2A	Shape
	1A	Labelled end points
	1A	Label the turning points and intercepts
	4	

1993

Solution	Marks	Remarks
12.(a) $z + 1 = \cos\theta + i\sin\theta$	1A	
$\lvert z + 1 \rvert \ (= \sqrt{\cos^2\theta + \sin^2\theta}) = 1$	1	
	1A	For circle
	1A	For centre at $z = -1$
	1A	For radius = 1
	5	PP-1 if not properly label the axes
(b) $\tan 2\theta_1 = \dfrac{\sin\theta_1}{\cos\theta_1 - 1}$	1A	
$\dfrac{2\sin\theta_1\cos\theta_1}{2\cos^2\theta - 1} = \dfrac{\sin\theta_1}{\cos\theta_1 - 1}$	1M	
$\sin\theta_1(2\cos\theta_1 - 1) = 0$		
$\cos\theta_1 = \frac{1}{2}$ or $\sin\theta = 0$ (rejected $\because 0 < \theta < \frac{\pi}{2}$)	1A+1A	
$\theta_1 = \frac{\pi}{3}$	1A	
Alternative solution		
Let G be the centre of C and H be a point on the positive real axis 		
$\angle OGP_1 = \theta_1$	1A	
$\angle HOP_1 = 2\theta_1$	1A	
Since $GP_1 = GO$, ΔGOP_1 is isosceles.		
$\angle GOP_1 = \dfrac{\pi - \theta_1}{2}$	1A	$\angle OP_1G = \pi - 2\theta_1$
$\dfrac{\pi - \theta_1}{2} + 2\theta_1 = \pi$	1A	$(\pi - 2\theta_1) \times 2 + \theta_1 = \pi$
$\theta_1 = \frac{\pi}{3}$	1A	
$z_1 = \cos\frac{\pi}{3} - 1 + i\sin\frac{\pi}{3}$	1M	
$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i$	1A	
	7	

(1993)

Solution	Marks	Remarks
(c) $z_2 = \cos(\frac{\pi}{3} + \pi) - 1 + i\sin(\frac{\pi}{3} + \pi)$	1M+1M+1A	1M for using C 1M for π
$= -\frac{3}{2} - \frac{\sqrt{3}}{2}i$	1A	
<u>Alternative solutions</u>		
P_1P_2 is a diameter of the circle C.	1A	
Let P_2 represent the complex no. $x + yi$		
$\frac{x - \frac{1}{2}}{2} = -1$, $\frac{y + \frac{\sqrt{3}}{2}}{2} = 0$	2M	
$x = -\frac{3}{2}$ $y = -\frac{\sqrt{3}}{2}$		
$\therefore P_2$ represents $-\frac{3}{2} - \frac{\sqrt{3}}{2}i$	1A	
$\lvert z_2 \rvert = \sqrt{3}$	1A	
$\angle GOP_2 = \frac{\pi}{6}$	1M	
$\operatorname{Arg} z_2 = \frac{\pi}{6} - \pi$	1M	Accept $\pi + \frac{\pi}{6}$
$= \frac{-5\pi}{6}$		$\frac{7\pi}{6}$
$\therefore z_2 = \sqrt{3}(\cos\frac{-5\pi}{6} + i\sin\frac{-5\pi}{6})$		
$= -\frac{3}{2} - \frac{\sqrt{3}}{2}i$	1A	
	4	

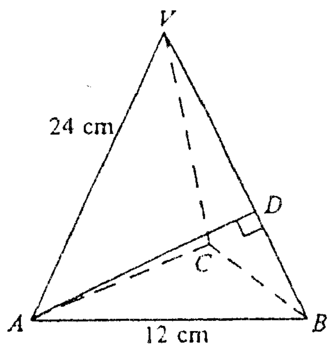
Solution 1993 (paper 2)	Marks	Remarks 1993
1. For $n = 1$, L.H.S. $= 2$		
R.H.S. $= \frac{1}{12} \times 2 \times 3 \times 4 = 2$		
$\therefore$ The statement is true for $n = 1$	1	
Assume $1^2 \times 2 + 2^2 \times 3 + \ldots + k^2(k+1) = \frac{k(k+1)(k+2)(3k+1)}{12}$	1	
(for some positive integer k)		
Then $1^2 \times 2 + 2^2 \times 3 + \ldots + k^2(k+1) + (k+1)^2(k+2)$		
$= \frac{k(k+1)(k+2)(3k+1)}{12} + (k+1)^2(k+2)$	1	
$= \frac{(k+1)(k+2)}{12}[k(3k+1) + 12(k+1)]$		
$= \frac{(k+1)(k+2)(3k^2+13k+12)}{12}$		
$= \frac{(k+1)(k+2)(k+3)[3(k+1)+1]}{12}$	1	
$\therefore$ The statement is also true for $n = k+1$ (if it is true for $n = k$)		
(By the principle of mathematical induction)		
$\therefore$ the statement is true for all +ve integers n.	1	
	5	
2. (a) $\sqrt{3}\cos x - \sin x$		OR
$= 2(\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x)$		$r\cos\alpha = \sqrt{3}$ $r\sin\alpha = 1$
$= 2\cos(x + \frac{\pi}{6})$	1A+1A	$r = 2$, $\alpha = \frac{\pi}{6}$ or $30°$
$2\cos(x + \frac{\pi}{6}) = 1$		
$x + \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{3}$	1M+1A	1M for $(2n\pi \pm \alpha)$ 1A for $\frac{\pi}{3}$
$x = 2n\pi + \frac{\pi}{6}$ or $2n\pi - \frac{\pi}{2}$	1A	no mark if degree is used
	5	

Solution	Marks	Remarks 1983
3. (a) $(1+4x+x^2)^n$		
$= [1 + x(4+x)]^n$	1M	For separating into 2 terms
$= 1 + n\,x(4+x) + \frac{n(n-1)}{2}x^2(4+x)^2 + \ldots\ldots$	1A	Accept ${}_nC_r$ notation (pp − 1) for omitting dots
$\therefore a = 4n$	1A	
$b = n + 8n(n-1)$		
$= 8n^2 - 7n$	1A	
(b) $n = 5$	1A	
$b = 165$	1A	
	6	
4. Let slope of the line be m		
$\frac{m - \frac{1}{3}}{1 + \frac{m}{3}} = \pm 1$	1A+1A	$\left\lvert\frac{m - \frac{1}{3}}{1 + \frac{m}{3}}\right\rvert = 1$ (2A)
$m = 2$ or $-\frac{1}{2}$	1A+1A	
Equation of lines are		
$\frac{y-3}{x-4} = 2$ i.e. $y = 2x - 5$		$2x - y - 5 = 0$
$\frac{y-3}{x-4} = -\frac{1}{2}$ $y = -\frac{x}{2} + 5$	1A+1A	$x + 2y - 10 = 0$
	6	
Alternative solution		
Let the angle of inclination of $y = \frac{1}{3}x$ be θ		
$\tan\theta = \frac{1}{3}$		
Angles of inclination of the two lines $= \theta \pm \frac{\pi}{4}$	1M+1M	
Slope $m = \tan(\theta + \frac{\pi}{4}) = \frac{\tan\theta + \tan\frac{\pi}{4}}{1 - \tan\theta\tan\frac{\pi}{4}} = 2$	1A+1A	
or $m = \tan(\theta - \frac{\pi}{4}) = \frac{\tan\theta - \tan\frac{\pi}{4}}{1 + \tan\theta\tan\frac{\pi}{4}} = -\frac{1}{2}$		
Equation of the lines are $y = 2x - 5$ and $y = \frac{-x}{2} + 5$	1A+1A	

1993

Solution	Marks	Remarks
$\left\|1+\frac{m}{3}\right\| = \left\|m-\frac{1}{3}\right\|$	2A	
$1+\frac{2}{3}m+\frac{m^2}{9} = m^2-\frac{2}{3}m+\frac{1}{9}$		
$2m^2-3m-2=0$		
$m = 2$ or $-\frac{1}{2}$	2A	
Equation of lines are		
$y = 2x-5$ and $y = \frac{-x}{2}+5$	1A+1A	
5. (a) $\sin x = \cos x$		
$\tan x = 1$		
$x = \frac{\pi}{4}, \frac{5\pi}{4}$	1A	
∴ The coordinates of A and B are		
$(\frac{\pi}{4}, \frac{\sqrt{2}}{2})$ and $(\frac{5\pi}{4}, \frac{-\sqrt{2}}{2})$ respectively	1A	Do not accept degrees
(b) Area $= \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)dx$	1M+1A	1A for $\int(\sin x - \cos x)dx$ 1M for limits
$= [-\cos x - \sin x]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$	1A	
$= 2\sqrt{2}$	1A 6	

Solution	Marks	Remarks 1983
6. (a) $\frac{dy}{dx} = 3x^2 - 6x - 1$	1A	$y = \int (3x^2 - 6x - 1)dx$ omit (PP-1)
$y = x^3 - 3x^2 - x + k$	1A	
Put $x = 1$, $y = 0$. $k = 3$	1M+1A	withhold 1A for giving $x^3 - 3x^2 - x + 3 = 0$
$\therefore y = x^3 - 3x^2 - x + 3$		
(b) At $x = 0$, $y = 3$	1A	
$\frac{dy}{dx} = -1$	1A	marked independently
$\therefore$ Equation of tangent is $y = -x + 3$	1A	
	7	
7. (a) $\cos \angle VBA = \frac{6}{24} = \frac{1}{4}$	1M	
$\angle VBA = 75.5°$ (75.52° No mark)	1A	or 1.32 radian
$AD = 12 \sin \angle VBA$		
$= 11.6$ cm (11.619 No mark)	1A	
(b) The angle between the two planes is $\angle ADC$	1A	
By symmetry, $CD = AD$	1A	
$\sin \frac{\angle ADC}{2} = \frac{\frac{1}{2}AC}{AD}$	1M	
$= \frac{6}{11.619}$		
$\angle ADC = 62.2°$ (62.3° No mark)	1A	or 1.09 radian



Alternative solution

Solution	Marks
(b) The angle between the two planes is $\angle ADC$	1A
By symmetry, $CD = AD$	1A
$\cos \angle ADC = \frac{AD^2 + CD^2 - AC^2}{2(AD)(CD)}$	1M
$= \frac{(11.619)^2 + (11.619)^2 - 12^2}{2(11.619)(11.619)}$	
$\angle ADC = 62.2°$	1A
	7

Solution	Marks	Remarks 1993
8. (a) $\cos x = \dfrac{1-t^2}{1+t^2}$	1A	
$\sin x = \dfrac{2t}{1+t^2}$	1A	
$a\cos x + b\sin x = c$		
$a\left(\dfrac{1-t^2}{1+t^2}\right) + b\left(\dfrac{2t}{1+t^2}\right) = c$	1M	
$a(1-t^2) + 2bt = c(1+t^2)$		
$(a+c)t^2 - 2bt + (c-a) = 0$ .....(*)	1	
If E has solutions in x, (*) has solutions in t		
$(2b)^2 - 4(a+c)(c-a) \geq 0$	1M	
$b^2 - (c^2 - a^2) \geq 0$		
$a^2 + b^2 \geq c^2$	1 / 6	
(b) (i) Put $a = 5$, $b = 6$, $c = 7$ into (*)	1M	
$12t^2 - 12t + 2 = 0$	1A	
The roots are $\tan\frac{x_1}{2}$, $\tan\frac{x_2}{2}$		
$\therefore \tan\frac{x_1}{2} + \tan\frac{x_2}{2} = 1$	1M	or $\tan\frac{x_1}{2} = \frac{3+\sqrt{3}}{6}$
$\tan\frac{x_1}{2}\tan\frac{x_2}{2} = \frac{1}{6}$	1M	$\tan\frac{x_2}{2} = \frac{3-\sqrt{3}}{6}$
$\tan\left(\frac{x_1+x_2}{2}\right)$		
$= \dfrac{\tan\frac{x_1}{2} + \tan\frac{x_2}{2}}{1 - \tan\frac{x_1}{2}\tan\frac{x_2}{2}}$	1A	
$= \dfrac{1}{1-\frac{1}{6}} = \dfrac{6}{5}$	1A	
(ii) $\tan x_1 \tan x_2$		
$= \dfrac{2\tan\frac{x_1}{2}}{1-\tan^2\frac{x_1}{2}} \cdot \dfrac{2\tan\frac{x_2}{2}}{1-\tan^2\frac{x_2}{2}}$	1A	
$= \dfrac{4\tan\frac{x_1}{2}\tan\frac{x_2}{2}}{1-(\tan\frac{x_1}{2}+\tan\frac{x_2}{2})^2 + 2\tan\frac{x_1}{2}\tan\frac{x_2}{2} + (\tan\frac{x_1}{2}\tan\frac{x_2}{2})^2}$	1M	For expressing the denominator in terms of sum and product.
$= \dfrac{4(\frac{1}{6})}{1-1+2(\frac{1}{6})+(\frac{1}{6})^2}$		
$= \dfrac{24}{13}$	2A / 10	

Solution	Marks	Remarks 1983
9. (a) $\frac{d}{dx}(\sin^{m-1}x\cos^{n+1}x)$		
$= (m-1)\sin^{m-2}x\cos^{n+2}x - (n+1)\sin^m x\cos^n x$	1A+1A	
	2	
(b) Integrating with respect to x,		
$[\sin^{m-1}x\cos^{n+1}x]_0^{\pi/2} = (m-1)\int_0^{\pi/2}\sin^{m-2}x\cos^{n+2}x\,dx$ $- (n+1)\int_0^{\pi/2}\sin^m x\cos^n x\,dx$	1M+1A	(pp − 1) for omitting limits
$0 = (m-1)\int_0^{\pi/2}\sin^{m-2}x\cos^{n+2}x\,dx$ $- (n+1)\int_0^{\pi/2}\sin^m x\cos^n x\,dx$	1A	For L.H.S. = 0
$(n+1)\int_0^{\pi/2}\sin^m x\cos^n x\,dx =$ $(m-1)\int_0^{\pi/2}\sin^{m-2}x\cos^n x(1-\sin^2 x)\,dx$	1M	For rewriting $\cos^{n+2}x = \cos^n x(1-\sin^2 x)$
$(n+1+m-1)\int_0^{\pi/2}\sin^m x\cos^n x\,dx =$ $(m-1)\int_0^{\pi/2}\sin^{m-2}x\cos^n x\,dx$		
$\int_0^{\pi/2}\sin^m x\cos^n x\,dx = \frac{m-1}{m+n}\int_0^{\pi/2}\sin^{m-2}x\cos^n x\,dx$	1	
	5	
(c) Put $x = \frac{\pi}{2} - y$, $dx = -dy$		
$\int_0^{\pi/2}\sin^n x\cos^m x\,dx = \int_{\pi/2}^{0}\sin^n(\frac{\pi}{2}-y)\cos^m(\frac{\pi}{2}-y)(-dy)$	1A	
$= \int_0^{\pi/2}\cos^n y\sin^m y\,dy$		
$= \int_0^{\pi/2}\sin^m x\cos^n x\,dx$	1A	
$= \frac{m-1}{m+n}\int_{\pi/2}^{0}\sin^{m-2}(\frac{\pi}{2}-y)\cos^n(\frac{\pi}{2}-y)(-dy)$	1A	
$= \frac{m-1}{m+n}\int_0^{\pi/2}\cos^{m-2}y\sin^n y\,dy$		
$= \frac{m-1}{m+n}\int_0^{\pi/2}\sin^n x\cos^{m-2}x\,dx$	1	

1993

Solution	Marks	Remarks
Alternative solution		
Put $x = \frac{\pi}{2} - y$, $dx = -dy$		
The identity in (b) becomes		
$\int_{\pi/2}^{0} \sin^m(\frac{\pi}{2} - y)\cos^n(\frac{\pi}{2} - y)(-dy)$		
$= \frac{m-1}{m+n} \int_{\pi/2}^{0} \sin^{m-2}(\frac{\pi}{2} - y)\cos^n(\frac{\pi}{2} - y)(-dy)$	1A+1A	
$\int_0^{\pi/2} \cos^m y \sin^n y\, dy = \frac{m-1}{m+n} \int_0^{\pi/2} \cos^{m-2} y \sin^n y\, dy$	1A	
$\int_0^{\pi/2} \sin^n x \cos^m x\, dx = \frac{m-1}{m+n} \int_0^{\pi/2} \sin^n x \cos^{m-2} x\, dx$	1	
	4	
(d) $\int_0^{\pi/2} \sin^4 x \cos^6 x\, dx$		
$= \frac{4-1}{4+6} \int_0^{\frac{\pi}{2}} \sin^2 x \cos^6 x\, dx$ (using (b))	1M	For using (b)
$= \frac{3}{10} \cdot \frac{2-1}{2+6} \int_0^{\frac{\pi}{2}} \sin^0 x \cos^6 x\, dx$ (using (b))		
$= \frac{3}{10} \cdot \frac{1}{8} \cdot \frac{6-1}{6} \int_0^{\frac{\pi}{2}} \cos^4 x\, dx$ (using (c))	1M	For using (c)
$= \frac{3}{10} \cdot \frac{1}{8} \cdot \frac{5}{6} \cdot \frac{4-1}{4} \int_0^{\frac{\pi}{2}} \cos^2 x\, dx$ (using (c))		
$= \frac{3}{10} \cdot \frac{1}{8} \cdot \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{2-1}{2} \int_0^{\frac{\pi}{2}} \cos^0 x\, dx$ (using (c))	1M	For evaluating the last integral, accept stopping at $\int \cos^2 x\, dx$, $\int \sin^2 x\, dx$ $\int \sin^0 x\, dx$ or $\int \cos^2 x \sin^2 x\, dx$
$= \frac{3}{10} \cdot \frac{1}{8} \cdot \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}$		
$= \frac{3\pi}{512}$	2A	
	5	

1983

Solution	Marks	Remarks
10. (a) $\frac{y - s^2}{x - 2s} = \frac{t^2 - s^2}{2t - 2s}$	1A	
$2y - 2s^2 = (t + s)x - 2s(t + s)$		
$y = \frac{s + t}{2}x - st$	1A	$(s + t)x - 2y - 2st = 0$
	2	
(b) Put $t = s$	1A	
Equation of tangent is $y = sx - s^2$	1A	
Alternative solutions		
Using the formula $\frac{1}{2}(y + y_1) = \frac{1}{4}xx_1$		
Equation of tangent is $\frac{1}{2}(y + s^2) = \frac{1}{4}x(2s)$	1A	
$y = sx - s^2$	1A	
$\frac{dy}{dx} = \frac{1}{2}x$		
At $(2s, s^2)$, $\frac{dy}{dx} = s$	1A	
Equation of tangent is		
$\frac{y - s^2}{x - 2s} = s$		
$y = sx - s^2$	1A	
	2	
(c) (i) Substitute (0, 1) into $y = \frac{s + t}{2}x - st$	1M	
$1 = \frac{s + t}{2}(0) - st$		
$st = -1$	1	
(ii) slope of $PS = s$ slope of $PT = t$	1A	
From (i), $st = -1$	1A	
$\therefore PS$ and PT are $\perp$, angle btn them $= \frac{\pi}{2}$	1A	Accept $PS \perp PT$
Alternative solution		
Let θ be the angle between PS and PT		
$\tan\theta = \frac{m_{PS} - m_{PT}}{1 + m_{PS}m_{PT}} = \frac{s - t}{1 + st}$	1A	
$\because st = -1$	1A	
$\therefore \theta = \frac{\pi}{2}$	1A	Accept $PS \perp PT$

Solution	Marks	Remarks
(iii) $\begin{cases} y = sx - s^2 \\ y = tx - t^2 \end{cases}$	1A	Equation of *PT*
Eliminating x,		
$\frac{y + s^2}{s} = \frac{y + t^2}{t}$	1M	For solving y
$ty + s^2t = sy + st^2$		
$(t - s)y = st(t - s)$		
$y = st = -1 \quad (\because s \neq t)$		
$\therefore P$ lies on the line $y + 1 = 0$	1	
<u>Alternative solution</u>		
$\begin{cases} y = sx - s^2 \ldots\ldots (1) \\ y = tx - t^2 \ldots\ldots (2) \end{cases}$	1A	Equation of *PT*
Since $st = -1$, (2) becomes $y = \frac{-1}{s}x - \frac{1}{s^2}$		
$s^2y = -sx - 1 \ldots\ldots (3)$		
(1) + (3) : $(1 + s^2)y = -(1 + s^2)$	1M	For solving y
$y = -1$		
$\therefore P$ lies on the line $y + 1 = 0$	1	
(iv) Let (x, y) be a point on the locus.		
$\begin{cases} x = s + t \\ y = \frac{1}{2}(s^2 + t^2) \end{cases}$	1A	
$2y = s^2 + t^2$		
$= (s + t)^2 - 2st$	1M	1M for completing square
$2y = x^2 + 2$	1M+1A	1M for using $st = -1$
$\therefore$ The equation of the locus is $2y = x^2 + 2$		
<u>Alternative solution</u>		
$\begin{cases} x = s + t \\ y = \frac{1}{2}(s^2 + t^2) \end{cases}$	1A	
Since $st = -1$		
$\begin{cases} x = s - \frac{1}{s} \\ y = \frac{1}{2}(s^2 + \frac{1}{s^2}) \end{cases}$	1M	For using $st = -1$
$x^2 = (s - \frac{1}{s})^2$	1M	For completing square
$= (s^2 + \frac{1}{s^2}) - 2$		
$x^2 = 2y - 2$	1A	
$\therefore$ The equation of the locus is $x^2 = 2y - 2$		
	12	

Solution	Marks	Remarks
11. (a) $AB = \sqrt{(0-3)^2 + (2-\frac{3}{4})^2}$		
$= \frac{13}{4}$	1A	
Radius of $C_2 = \frac{3}{4}$	1A	
Radius of C_1 − radius of C_2	1M	
$= 4 - \frac{3}{4} = \frac{13}{4} = AB$		
$\therefore C_1$ and C_2 touch each other.	1 / 4	
(b) $PA = \sqrt{s^2 + (t-2)^2}$	1A	
If the circle touches the x-axis and C_1,		
$\sqrt{s^2 + (t-2)^2} = 4 - t$	1M	no mark for $t - 4$
$s^2 + (t-2)^2 = (4-t)^2$		
$4t = 12 - s^2$	1 / 3	
(c) $PB = \sqrt{(s-3)^2 + (t-\frac{3}{4})^2}$	1A	
If the circle touches the x-axis and C_2,		
$\sqrt{(s-3)^2 + (t-\frac{3}{4})^2} = t + \frac{3}{4}$	1M	
$(s-3)^2 + (t-\frac{3}{4})^2 = (t+\frac{3}{4})^2$		
$3t = (s-3)^2$	1 / 3	
(d) $\begin{cases} 4t = 12 - s^2 \\ 3t = (s-3)^2 \end{cases}$		
Eliminating t,		
$\frac{12 - s^2}{4} = \frac{(s-3)^2}{3}$	1M	or eliminating s
$36 - 3s^2 = 4s^2 - 24s + 36$		
$7s^2 - 24s = 0$	1A	
$s = 0$, $t = 3$	1A	
or $s = \frac{24}{7}$, $t = \frac{3}{49}$	1A	
$\therefore$ The equations of the 2 circles are		
$x^2 + (y-3)^2 = 3^2$	1A	
and $(x - \frac{24}{7})^2 + (y - \frac{3}{49})^2 = (\frac{3}{49})^2$	1A / 6	

Solution	Marks	Remarks
12. (a) Capacity $= \int_0^{\frac{\pi}{2}} \pi x^2 dy$	1A+1A	1A for $\int \pi x^2 dy$ 1A if others correct
$= \int_0^{\frac{\pi}{2}} \pi k^2 \sin^2 y dy$	1M	Substituting $x = k\sin y$
$= \pi k^2 \int_0^{\frac{\pi}{2}} \frac{1}{2}(1 - \cos 2y) dy$	1M	For $\sin^2 y = \frac{1}{2}(1 - \cos 2y)$
$= \pi k^2 [\frac{1}{2}y - \frac{1}{4}\sin 2y]_0^{\frac{\pi}{2}}$	1A	
$= \frac{1}{4}k^2\pi^2$	1 6	
(b) (i) Put $x = 4$, $y = \frac{\pi}{2}$ in $x = k\sin y$		
$k = 4$	1A	
$\therefore$ Volume of water $= \frac{1}{4}(4)^2\pi^2 = 4\pi^2$	1A	
(ii) Let V be the volume of water remaining after t minutes		
$\frac{dV}{dt} = -(\pi + 2t)$	1A	
$V = -(\pi t + t^2) + c$	1A	
After $t = 0$, $V = 4\pi^2$, $\therefore c = 4\pi^2$	1M+1A	
$\therefore V = 4\pi^2 - (\pi t + t^2)$		
Alternative solution		
Volume remaining, $V = 4\pi^2 - \int_0^t (\pi + 2t) dt$	1M+1A	
$= 4\pi^2 - [\pi t + t^2]_0^t$	1A	
$= 4\pi^2 - (\pi t + t^2)$	1A	
Let V be the volume of water pumped away		
$\frac{dV}{dt} = \pi + 2t$	1A	
$V = \pi t + t^2 + c$	1A	
At $t = 0$, $V = 0$ $\therefore c = 0$	1M+1A	
$\therefore V = \pi t + t^2$		
Volume pumped away $= \int_0^t (\pi + 2t) dt$	1M+1A	
$= [\pi t + t^2]_0^t$	1A	
$= \pi t + t^2$	1A	

Solution	Marks	Remarks
Put $V = 2\pi^2$	1M	
$t^2 + \pi t - 2\pi^2 = 0$		
$t = \pi$ [or -2π (rejected)]	1A	
∴ Time required to pump out half of the water $= \pi$ (minutes)		
Put $V = 0$,		
$t^2 + \pi t - 4\pi^2 = 0$		
$t = \frac{-\pi + \sqrt{17}\pi}{2}$ [or $\frac{-\pi - \sqrt{17}\pi}{2}$ (rejected)]	1A	
∴ Time required to pump out the remaining water		
$= (\frac{\sqrt{17} - 1}{2})\pi - \pi$		
$= (\frac{\sqrt{17} - 3}{2})\pi$ (minutes)	1A	
	10	