93-CE
A MATHS
PAPER I

HONG KONG EXAMINATIONS AUTHORITY
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1993

# ADDITIONAL MATHEMATICS PAPER I

8.30 am-10.30 am (2 hours)
This paper must be answered in English

Answer **ALL** questions in Section A and any **THREE** questions in Section B.

All working must be clearly shown.

Unless otherwise specified in a question, the **exact values** of numerical answers must be given.

In this paper, vectors may be represented by bold-type letters such as **u**, but candidates are expected to use appropriate symbols such as $\vec{u}$ in their working.

**Section A** (42 marks)

Answer **ALL** questions in this section.

1. (a) Simplify $(\sqrt{2(x+\Delta x)} - \sqrt{2x})(\sqrt{2(x+\Delta x)} + \sqrt{2x})$.

   (b) Find $\frac{d}{dx}(\sqrt{2x})$ from first principles.

(5 marks)

2. (a) Express $\frac{50}{4+3i}$ in standard form.

   (b) By putting $z = a + bi$, where $a$, $b$ are real numbers, solve the equation $5z + 3\bar{z} = \frac{50}{4+3i}$.

(5 marks)

3. $\alpha$, $\beta$ are the roots of the equation $x^2 + px + q = 0$ and $\alpha + 3$, $\beta + 3$ are the roots of the equation $x^2 + qx + p = 0$. Find the values of $p$ and $q$.

(6 marks)

4. Express $\sin\frac{2\pi}{3} + i\cos\frac{2\pi}{3}$ in polar form.

   Hence find the three cube roots of $\sin\frac{2\pi}{3} + i\cos\frac{2\pi}{3}$, giving your answers in polar form.

(6 marks)



5. Solve $\left| -x^2 + 2x + 3 \right| \geq 5$ for real values of $x$.

(6 marks)

6. Given $\vec{OA} = 3\mathbf{i} - 2\mathbf{j}$, $\vec{OB} = \mathbf{i} + \mathbf{j}$. $C$ is a point such that $\angle ABC$ is a right angle.

   (a) Find $\vec{AB}$.

   (b) Find $\vec{AB} \cdot \vec{AB}$ and $\vec{AB} \cdot \vec{BC}$.

   Hence find $\vec{AB} \cdot \vec{AC}$.

(7 marks)

7. Given the curve $C : x^2 - 2xy^2 + y^3 + 1 = 0$.

   (a) Find $\frac{dy}{dx}$.

   (b) Find the equation of the tangent to $C$ at the point (2, −1).

(7 marks)

**Section B** (48 marks)

Answer any **THREE** questions in this section.
Each question carries 16 marks.

8.

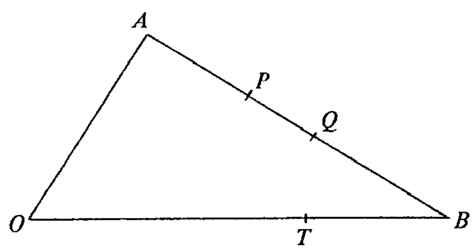


Figure 1

In Figure 1, $OAB$ is a triangle. $P$, $Q$ are two points on $AB$ such that $AP : PB = PQ : QB = r : 1$, where $r > 0$. $T$ is a point on $OB$ such that $OT : TB = 1 : r$. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Express $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ in terms of $r$, $\mathbf{a}$ and $\mathbf{b}$.
(3 marks)

(b) Express $\overrightarrow{OT}$ in terms of $r$ and $\mathbf{b}$.

Hence show that $\overrightarrow{TQ} = \dfrac{\mathbf{a} + (r^2 + r - 1)\mathbf{b}}{(r+1)^2}$.
(3 marks)

(c) Find the value(s) of $r$ such that $\overrightarrow{OA}$ is parallel to $\overrightarrow{TQ}$.
(3 marks)

(d) Suppose $OA = 2$, $OB = 16$ and $\angle AOB = \frac{\pi}{3}$.

(i) Find $\mathbf{a} \cdot \mathbf{a}$ and $\mathbf{a} \cdot \mathbf{b}$.

(ii) Find the value(s) of $r$ such that $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{TQ}$.
(7 marks)



9.

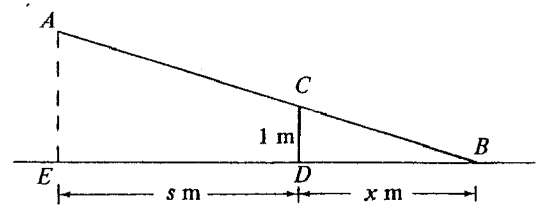


Figure 2

Figure 2 shows a straight rod $AB$ of length 8 m resting on a vertical wall $CD$ of height 1 m. The end $B$ is free to slide along a horizontal rail such that $AB$ is vertically above the rail. Let $E$ be the projection of $A$ on the rail, $DE = s$ m and $BD = x$ m, where $0 < x < 3\sqrt{7}$.

(a) Show that $s = \dfrac{8x}{\sqrt{1+x^2}} - x$.
(3 marks)

(b) Find the maximum value of $s$.
(7 marks)

(c) Let $P$ m$^2$ be the area of the trapezium $CAED$.

(i) Show that $P = \dfrac{32x}{1+x^2} - \dfrac{x}{2}$.

(ii) Does $P$ attain a maximum when $s$ attains its maximum? Explain your answer.
(6 marks)

10. $C$ is the curve $y = \frac{1}{k+1}[2x^2 + (k+7)x + 4]$, where $k$ is a real number not equal to $-1$.

(a) If $C$ cuts the $x$-axis at two points $P$ and $Q$ and $PQ = 1$, find the value(s) of $k$.
(6 marks)

(b) Find the range of values of $k$ such that $C$ does not cut the $x$-axis.
(4 marks)

(c) Show that $C$ always passes through two fixed points for all values of $k$ not equal to $-1$. What are the coordinates of the two points ?
(6 marks)

11. Let $f(x) = \sqrt{3}\sin 3x - \cos 3x$, where $-\frac{\pi}{3} \le x \le \frac{\pi}{3}$.

(a) Find the $x$- and $y$-intercepts of the curve $y = f(x)$.
(4 marks)

(b) Find $f'(x)$ and $f''(x)$.
(2 marks)

(c) Find the turning point(s) of the curve $y = f(x)$.
For each point, test whether it is a maximum or a minimum point.
(6 marks)

(d) In Figure 3, sketch the curve $y = f(x)$.
(4 marks)



| Candidate Number | Centre Number | Seat Number | Total Marks on this page |
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**If you attempt Question 11, fill in the details in the first three boxes above and tie this sheet into your answer book.**

11. (d) **(Continued)**

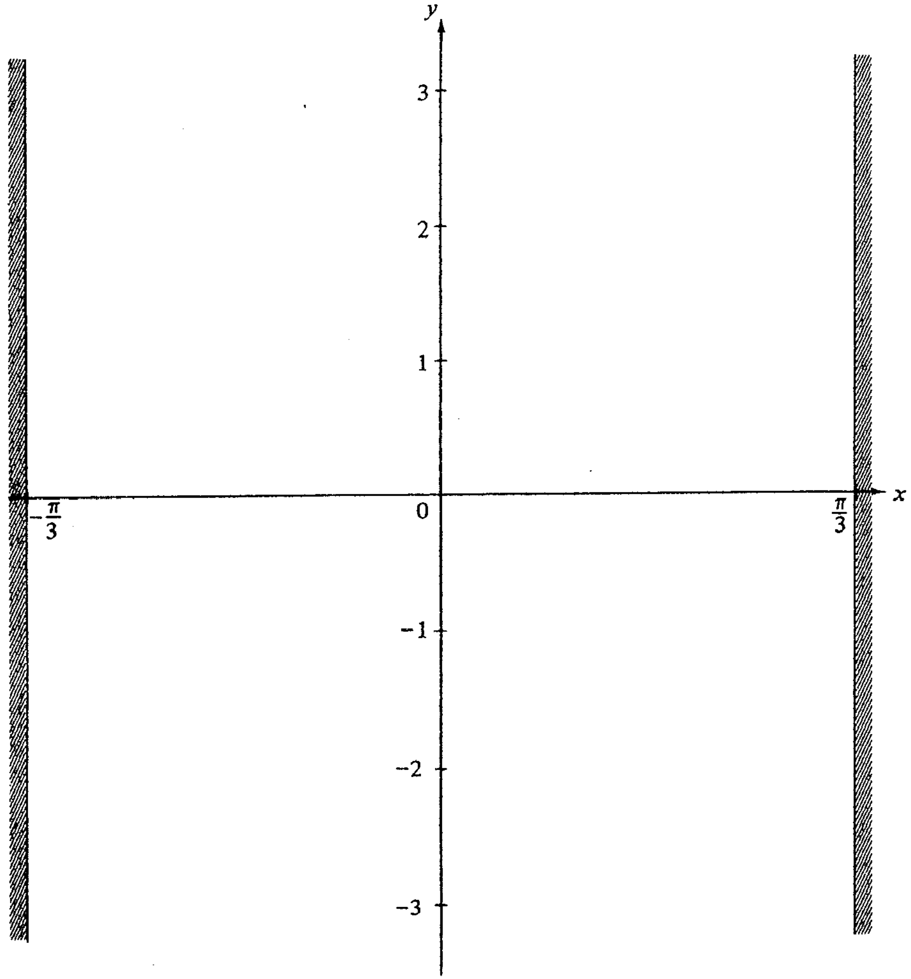


**Figure 3**

12. Let $C$ be the locus of the point in an Argand diagram representing the complex number $z = (\cos\theta - 1) + i\sin\theta$, where $0 \le \theta < 2\pi$.

(a) Show that $|z + 1| = 1$.
Hence sketch $C$ in an Argand diagram.
(5 marks)

(b) Let $P_1$ be the point on $C$ representing the complex number $z_1 = (\cos\theta_1 - 1) + i\sin\theta_1$, such that $\arg z_1 = 2\theta_1$, and $0 < \theta_1 < \frac{\pi}{2}$.

Find the value of $\theta_1$ and express $z_1$ in standard form.
(7 marks)

(c) Let $P_2$ be the point on $C$ which is farthest away from the point $P_1$ in (b). Find the complex number represented by $P_2$ in standard form.
(4 marks)

**END OF PAPER**



93-CE
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PAPER II

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# ADDITIONAL MATHEMATICS PAPER II

11.15 am-1.15 pm (2 hours)
This paper must be answered in English

Answer **ALL** questions in Section A and any **THREE** questions in Section B.

All working must be clearly shown.

Unless otherwise specified in a question, the **exact values** of numerical answers must be given.

## Section A (42 marks)

Answer **ALL** questions in this section.

1. Prove that

$$1^2 \times 2 + 2^2 \times 3 + \ldots + n^2(n+1) = \frac{n(n+1)(n+2)(3n+1)}{12}$$

for any positive integer $n$ .

(5 marks)

2. By expressing $\sqrt{3}\cos x - \sin x$ in the form $r\cos(x+\alpha)$, find the general solution of the equation

$$\sqrt{3}\cos x - \sin x = 1 ,$$

giving your answer in terms of $\pi$ .

(5 marks)

3. Given $(1 + 4x + x^2)^n = 1 + ax + bx^2 +$ other terms involving higher powers of $x$ , where $n$ is a positive integer.

(a) Express $a$ and $b$ in terms of $n$ .

(b) If $a = 20$ , find $n$ and $b$.

(6 marks)



4. Two lines pass through (4, 3) and each line makes an angle $\frac{\pi}{4}$ with the line $y = \frac{1}{3}x$. Find the equations of the two lines.

(6 marks)

5.

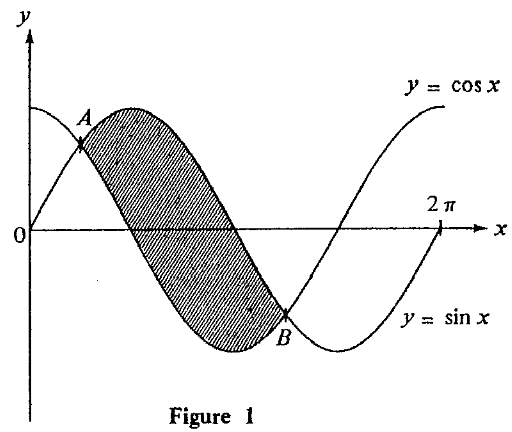


Figure 1

Figure 1 shows the curves of $y = \sin x$ and $y = \cos x$ , where $0 \le x \le 2\pi$ , intersecting at points $A$ and $B$.

(a) Find the coordinates of $A$ and $B$ .

(b) Find the area of the shaded region as shown in Figure 1.

(6 marks)

6. The slope of a curve $C$ at any point $(x, y)$ on $C$ is $3x^2 - 6x - 1$. $C$ passes through the point (1, 0).

(a) Find the equation of $C$.

(b) Find the equation of the tangent to $C$ at the point where $C$ cuts the $y$-axis.

(7 marks)

7.

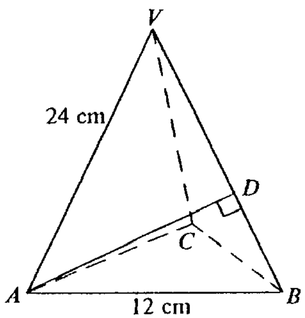


**Figure 2**

In Figure 2, $VABC$ is a right pyramid whose base $ABC$ is an equilateral triangle. $AB$ = 12 cm and $VA$ = 24 cm. $D$ is a point on $VB$ such that $AD$ is perpendicular to $VB$. Find, correct to 3 significant figures,

(a) $\angle VBA$ and $AD$,

(b) the angle between the faces $VAB$ and $VBC$.

(7 marks)



## Section B (48 marks)

Answer any **THREE** questions in this section.
Each question carries 16 marks.

8. Given $-\pi < x < \pi$ and $t = \tan\frac{x}{2}$.

(a) By expressing $\sin x$ and $\cos x$ in terms of $t$, show that the equation in $x$

$$E : a\cos x + b\sin x = c$$

can be expressed as $(a + c)t^2 - 2bt + (c - a) = 0$.

Hence show that if $E$ has solutions, then

$$a^2 + b^2 \geq c^2.$$

(6 marks)

(b) Let $x_1$, $x_2$ be the two values of $x$ satisfying the equation

$$5\cos x + 6\sin x = 7.$$

Without evaluating $x_1$ and $x_2$, find

(i) $\tan(\frac{x_1 + x_2}{2})$.

(ii) $\tan x_1 \tan x_2$.

(10 marks)

9. Let $m, n$ be integers such that $m > 1$ and $n \geq 0$.

(a) Find $\frac{d}{dx}(\sin^{m-1}x\cos^{n+1}x)$.

(2 marks)

(b) Using the result of (a), show that

$$\int_0^{\frac{\pi}{2}} \sin^m x\cos^n x\,dx = \frac{m-1}{m+n}\int_0^{\frac{\pi}{2}} \sin^{m-2}x\cos^n x\,dx .$$

(5 marks)

(c) Using the result of (b) and the substitution $x = \frac{\pi}{2} - y$, show that

$$\int_0^{\frac{\pi}{2}} \sin^n x\cos^m x\,dx = \frac{m-1}{m+n}\int_0^{\frac{\pi}{2}} \sin^n x\cos^{m-2}x\,dx .$$

(4 marks)

(d) Using the results of (b) and (c), evaluate

$$\int_0^{\frac{\pi}{2}} \sin^4 x\cos^6 x\,dx .$$

(5 marks)



10. $S(2s, s^2)$, $T(2t, t^2)$ are two distinct points on the parabola $y = \frac{1}{4}x^2$.

(a) Find the equation of the chord $ST$.

(2 marks)

(b) Find the equation of the tangent to the parabola at $S$.

(2 marks)

(c) Suppose $ST$ passes through the point $F(0, 1)$ and the tangents to the parabola at $S$ and $T$ meet at a point $P$.

(i) Show that $st = -1$.

(ii) Find the angle between $PS$ and $PT$.

(iii) Show that $P$ lies on the line $y + 1 = 0$.

(iv) Find the equation of the locus of the mid-point of $ST$ as $S$ and $T$ move along the parabola.

(12 marks)

11.

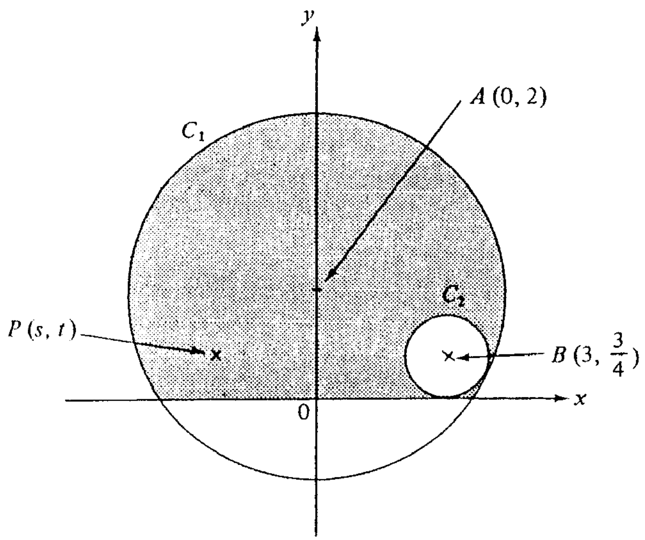


Figure 3

$A\ (0, 2)$ is the centre of circle $C_1$ with radius 4. $B(3, \frac{3}{4})$ is the centre of circle $C_2$ which touches the $x$-axis. $P(s, t)$ is any point in the shaded region as shown in Figure 3.

(a) Find $AB$ and the radius of $C_2$.

Hence show that $C_1$ and $C_2$ touch each other.

(4 marks)

(b) If $P$ is the centre of a circle which touches the $x$-axis and $C_1$, show that $4t = 12 - s^2$ .

(3 marks)



(c) If $P$ is the centre of a circle which touches the $x$-axis and $C_2$, show that $3t = (s - 3)^2$ .

(3 marks)

(d) Given that there are two circles in the shaded region, each of which touches the $x$-axis, $C_1$ and $C_2$. Using (b) and (c), find the equations of the two circles, giving your answers in the form $(x - h)^2 + (y - k)^2 = r^2$.

(6 marks)

12.

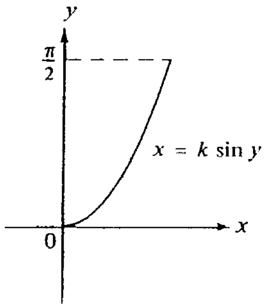


Figure 4 (a)

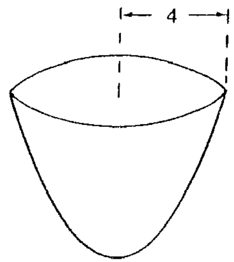


Figure 4 (b)

Figure 4 (a) shows the curve $x = k\sin y$, where $k > 0$ and $0 \le y \le \frac{\pi}{2}$. A bowl is formed by revolving the curve about the $y$-axis.

(a) Show that the capacity of the bowl is $\frac{1}{4}k^2\pi^2$ cubic units.

(6 marks)

(b) Given that the radius of the rim of the bowl is 4 units. (See Figure 4 (b).) The bowl is full of water.

(i) Find the volume of water.

(ii) The water is now pumped out of the bowl at a rate of $(\pi + 2t)$ cubic units per minute, where $t$ is the time in minutes after pumping starts.

Find the time taken to pump out half of the water and the time taken to pump out the remaining water in the bowl. Give both answers in terms of $\pi$.

(10 marks)

**END OF PAPER**