Solution	Marks	Remarks
1. (a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$	1M	
$= (-3\vec{i} + 5\vec{j}) - (5\vec{i} - \vec{j})$		
$= -8\vec{i} + 6\vec{j}$	1A	Omit vector sign(pp-1)
$\lvert\overrightarrow{AB}\rvert = \sqrt{(-8)^2 + 6^2}$		
$= 10$	1A	
(b) $\overrightarrow{AP} = \frac{4}{10}\overrightarrow{AB}$	1M	$\overrightarrow{OP} = \frac{4\overrightarrow{OB} + 6\overrightarrow{OA}}{10}$
$= -\frac{16}{5}\vec{i} + \frac{12}{5}\vec{j}$	1A 5	$-3.2\vec{i} + 2.4\vec{j}$
2. (a) $\frac{\sqrt{3}+i}{\sqrt{3}-i} = \frac{2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})}{2(\cos\frac{-\pi}{6} + i\sin\frac{-\pi}{6})}$ (or $\frac{2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})}{2(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6})}$)	1A+1A	$\frac{2(\cos 30° + i\sin 30°)}{2(\cos(-30°) + i\sin(-30°))}$
$= \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$	1A	$\cos 60° + i\sin 60°$
Alternative solution $\frac{\sqrt{3}+i}{\sqrt{3}-i} = \frac{(\sqrt{3}+i)(\sqrt{3}+i)}{(\sqrt{3}-i)(\sqrt{3}+i)}$	1A	
$= \frac{1}{2} + \frac{\sqrt{3}}{2}i$	1A	
$= \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$	1A	
(b) $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^{92} = \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{92}$		
$= \cos\frac{92\pi}{3} + i\sin\frac{92\pi}{3}$	1M	$\cos 5520° + i\sin 5520°$
$= \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$	1A	$\cos 120° + i\sin 120°$ (can be omitted)
$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i$	1A 6	(pp-1) for omitting degree sign.

Solution	Marks	Remarks
3. $\lvert x(x+5)\rvert > 6$		
$x(x+5) > 6$ or $x(x+5) < -6$	2A	Use "and" or ",,, and " (no mark)
$(x+6)(x-1) > 0$ or $(x+2)(x+3) < 0$	1A+1A	
$x > 1$ or $x < -6$ or $-3 < x < -2$	2A	"or" cannot be omitted.
$\therefore x < -6$ or $-3 < x < -2$ or $x > 1$		
	6	
Alternative solution		
(1) $x^2(x+5)^2 > 36$	1A	
$[x(x+5)-6][x(x+5)+6] > 0$	1M	
$(x+6)(x-1)(x+2)(x+3) > 0$	1A+1A	
$x < -6$ or $-3 < x < -2$ or $x > 1$	2A	
(2) Case 1 : $x \geq 0$		
$x(x+5) > 6$		
$(x+6)(x-1) > 0$		
$x > 1$ or $x < -6$		
Since $x \geq 0$, $\therefore x > 1$	1A	
Case 2 : $-5 < x < 0$		
$-x(x+5) > 6$		
$(x+2)(x+3) < 0$		
$-3 < x < -2$		
Since $-5 < x < 0$, $\therefore -3 < x < -2$	1A	
Case 3 : $x \leq -5$		
$x(x+5) > 6$		
$x > 1$ or $x < -6$		
Since $x \leq -5$, $\therefore x < -6$	1A	
	1M	For consider the 3 cases. (pp-1 for omitting some equality signs)
Combining the 3 cases,		
$x < -6$ or $-3 < x < -2$ or $x > 1$	2A	

Solution	Marks	Remarks
4. (a) $z_1 = 2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$	1A	Accept degree measures (can be omitted)
$= \sqrt{3} + i$	1A	
$z_3 = \cos(\frac{\pi}{2} + \frac{\pi}{6}) + i\sin(\frac{\pi}{2} + \frac{\pi}{6})$ OR $z_3 = \frac{1}{2}iz_1$	1A	$z_3 = -\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$ (can be omitted)
$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i$	1A	
(b) $z_2 = z_1 + z_3$	1M	
$= (\sqrt{3} - \frac{1}{2}) + (\frac{\sqrt{3}}{2} + 1)i$	1A	
	6	
5. (a) Put $x = 0$,	1M	
$y = \pm 1$	1A	
∴ The points are (0, 1) and (0, -1)		
(b) Differentiate with respect to x,		$y^2 = \frac{2 + 3x}{2 - x}$
$(y^2 + 3) + (x - 2)(2y\frac{dy}{dx}) = 0$ (For product rule)	1M	$2y\frac{dy}{dx} = \frac{3(2 - x) + (2 + 3x)}{(2 - x)^2}$
$\frac{dy}{dx} = \frac{y^2 + 3}{2y(2 - x)}$		$2y\frac{dy}{dx} = \frac{8}{(2 - x)^2}$
$\frac{dy}{dx}\Big\vert_{(0,1)} = 1$	1M+1A	Subs. (0, 1), $\frac{dy}{dx} = 1$
$\frac{dy}{dx}\Big\vert_{(0,-1)} = -1$	1A	Subs. (0, -1), $\frac{dy}{dx} = 1$
	6	

Solution	Marks	Remarks
6. Consider 2 cases : (1) $\alpha = \beta$ (2) $\alpha = -\beta$	1M	
(1) $\alpha = \beta$		
Discriminant $= (2 - k)^2 + 4(k - 1) = 0$	1M+1A	1M for $\Delta = 0$
$k = 0$	1A	
(2) $\alpha = -\beta$		
Sum of roots $= -(k - 2) = 0$	1M	
$k = 2$	1A 6	
Alternative solutions		
$x^2 + (k - 2)x - (k - 1) = 0$		
$(x - 1)(x - (1 - k)) = 0$	1A	For factorisation
$x = 1$ or $1 - k$	1A+1A	
Since $\lvert\alpha\rvert = \lvert\beta\rvert$		
$\lvert 1 - k\rvert = 1$	1M	
$k = 0$ or 2	1A+1A	
$\alpha^2 = \beta^2$		
$(\alpha + \beta)(\alpha - \beta) = 0$	1M	
(1) $\alpha + \beta = 0$		
$-(k - 2) = 0$	1M	
$k = 2$	1A	
(2) $\alpha - \beta = 0$		
$(\alpha - \beta)^2 = 0$		
$(\alpha + \beta)^2 - 4\alpha\beta = 0$	1M	
$(2 - k)^2 + 4(k - 1) = 0$	1A	
$k = 0$	1A	

Solution	Marks	Remarks
7. (a) Let r cm be the radius of water surface when the depth of water is h cm.		
$\tan 30° = \frac{r}{h}$		
$r = \frac{h}{\sqrt{3}}$	1A	
$V = \frac{1}{3}\pi(\frac{h}{\sqrt{3}})^2 h$	1M	
$= \frac{\pi}{9}h^3$	1A	
(b) $\frac{dV}{dt} = \frac{\pi}{3}h^2\frac{dh}{dt}$	1M+1A	1M for chain rule
Put $\frac{dV}{dt} = -\pi$	1A	
At $h = 4$, $-\pi = \frac{\pi}{3}(4)^2\frac{dh}{dt}$		
$\frac{dh}{dt} = \frac{-3}{16}$	1A	
∴ The water is falling at a rate of $\frac{3}{16}$ cms^{-1}		
	7	

RESTRICTED 内部文件

Solution	Marks	Remarks
8. (a) $\vec{a}\cdot\vec{a} = \lvert\vec{a}\rvert^2$		Omit dot sign (pp-1)
$= 4$	1A	Omit vector sign (pp-1)
$\vec{a}\cdot\vec{b} = 2(3)\cos\frac{\pi}{3}$	1M	
$= 3$	1A	
	3	
(b) $OD = 2\cos\frac{\pi}{3} = 1$	1A	
$\vec{OD} = \frac{1}{3}\vec{b}$	1A	
	2	
(c) (i) $\vec{OH} = \dfrac{k\vec{a} + \frac{1}{3}\vec{b}}{k+1}$	1M+1A	
$\vec{OH}\cdot\vec{AB} = 0$	1M	
$\left(\dfrac{k\vec{a} + \frac{1}{3}\vec{b}}{k+1}\right)\cdot(\vec{b} - \vec{a}) = 0$		
$\dfrac{1}{k+1}(k\vec{a}\cdot\vec{b} - k\vec{a}\cdot\vec{a} + \frac{1}{3}\vec{b}\cdot\vec{b} - \frac{1}{3}\vec{b}\cdot\vec{a}) = 0$	1M	
$3k - 4k + \frac{1}{3}(9) - 1 = 0$		
$k = 2$	1A	
(ii) (1) $\vec{OC} = \dfrac{m\vec{a} + \vec{b}}{m+1}$	1A	
(2) $\vec{OC} = (n+1)\left(\dfrac{2\vec{a} + \frac{1}{3}\vec{b}}{3}\right)$	1M+1A	
(3) $\begin{cases} \dfrac{m}{m+1} = \dfrac{2(n+1)}{3} \\ \dfrac{1}{m+1} = \dfrac{n+1}{9} \end{cases}$	1M	
Solving, $m = 6$	1A	
$n = \frac{2}{7}$	1A	
	11	

Solution	Marks	Remarks
9. (a) Discriminant $\Delta = (p+1)^2 - 4(p-1)$	1A	
$= p^2 - 2p + 5$		
$= (p-1)^2 + 4 > 0$	1M+1	1M for knowing $\Delta > 0$ 1 for a correct proof.
$\therefore \alpha, \beta$ are real and distinct.		
	3	[$\Delta' = (-2)^2 - 4(1)(5) = -16 < 0$ ∴ real and distinct.]
(b) $\begin{cases} \alpha + \beta = -(p+1) \\ \alpha\beta = (p-1) \end{cases}$	1A	
$(\alpha - 2)(\beta - 2) = \alpha\beta - 2(\alpha + \beta) + 4$		
$= (p-1) + 2(p+1) + 4$	1M	For complete substitution
$= 3p + 5$	1A 3	
(c) (i) Since $\beta < 2 < \alpha$,		
$\alpha - 2 > 0$, $\beta - 2 < 0$	1M	
From (b) $(\alpha - 2)(\beta - 2) = 3p + 5 < 0$		
$\therefore p < -\frac{5}{3}$	1	
(ii) $(\alpha - \beta)^2 < 24$		
$(\alpha + \beta)^2 - 4\alpha\beta < 24$	1A	
$(p+1)^2 - 4(p-1) < 24$		
$p^2 - 2p - 19 < 0$	1A	
$(p-1)^2 < 20$		or $(p-1+\sqrt{20})(p-1-\sqrt{20})<0$,
$1 - 2\sqrt{5} < p < 1 + 2\sqrt{5}$	1M+2A	or $1 - \sqrt{20} < p < 1 + \sqrt{20}$ ($-3.47 < P < 5.47$ - 1M only)
Combining with (i)		
$1 - 2\sqrt{5} < p < -\frac{5}{3}$	1A	or $1 - \sqrt{20} < p < -\frac{5}{3}$
The possible integral values of p are -2 or -3 .	1M+1A 10	

Alternative solution

Solution	Marks	Remarks
(c) (ii) $x^2 + (p+1)x + (p-1) = 0$		
$x = \frac{-(p+1) \pm \sqrt{p^2 - 2p + 5}}{2}$	1A	
$(\alpha - \beta)^2 < 24$		
$\left[\frac{-(p+1) + \sqrt{p^2 - 2p + 5}}{2} - \frac{-(p+1) - \sqrt{p^2 - 2p + 5}}{2}\right]^2 < 24$		
$p^2 - 2p - 19 < 0$	1A	

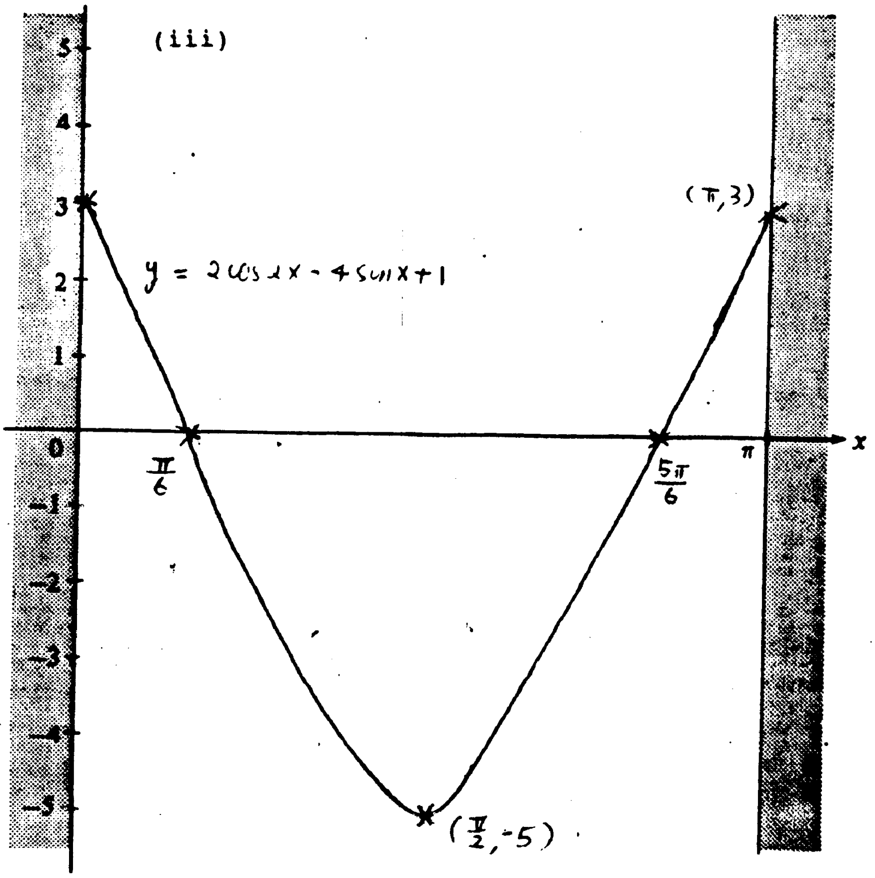
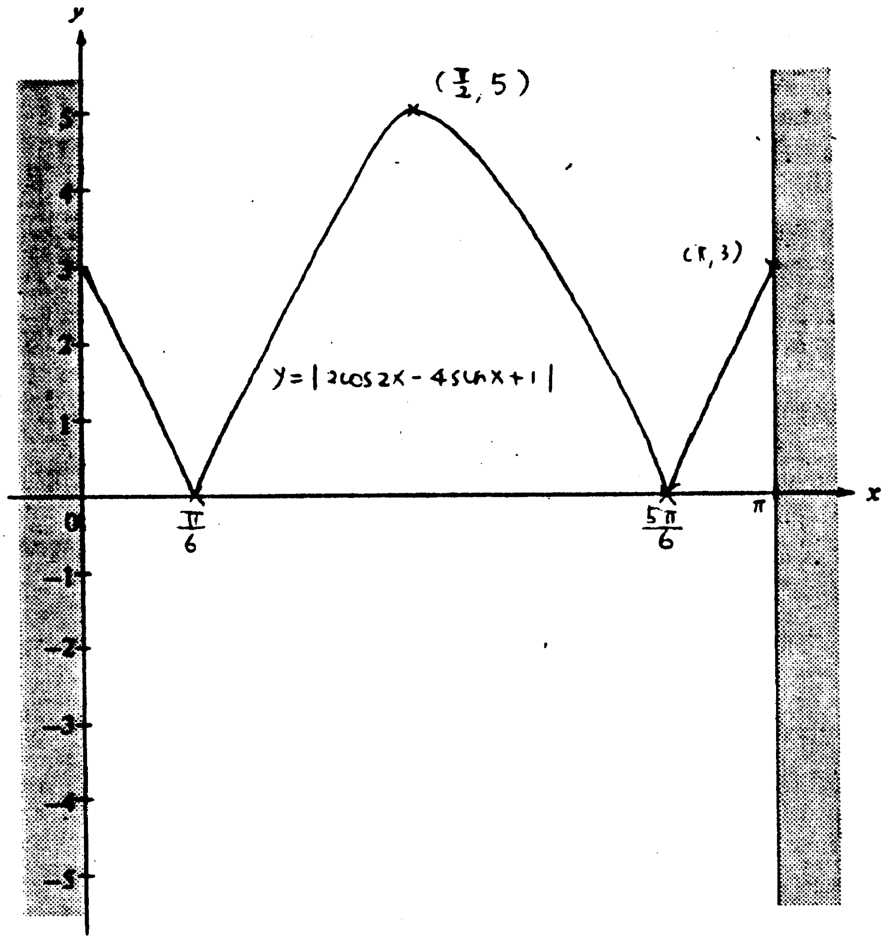
Solution	Marks	Remarks
10. (a) $(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta$	1A	
$(\cos\theta + i\sin\theta)^3 = \cos^3\theta + 3\cos^2\theta(i\sin\theta) + 3\cos\theta(i\sin\theta)^2 + (i\sin\theta)^3$	1A	
Equating imaginary parts,		
$\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$		
$= 3\sin\theta(1 - \sin^2\theta) - \sin^3\theta$		
$= 3\sin\theta - 4\sin^3\theta$	1 / 3	
(b) $16\sin^3\theta\cos^2\theta = 16[\frac{1}{2i}(z - \frac{1}{z})]^3 [\frac{1}{2}(z + \frac{1}{z})]^2$	1A+1A	
$= \frac{-1}{2i}(z^2 - \frac{1}{z^2})^2(z - \frac{1}{z})$		
$= \frac{-1}{2i}(z^4 - 2 + \frac{1}{z^4})(z - \frac{1}{z})$		
$= \frac{-1}{2i}(z^5 - 2z + \frac{1}{z^3} - z^3 + \frac{2}{z} - \frac{1}{z^5})$	1A	or $\frac{i}{2}$ (......)
$= \frac{-1}{2i}[(z^5 - \frac{1}{z^5}) - 2(z - \frac{1}{z}) - (z^3 - \frac{1}{z^3})]$	1M	For collecting terms
$= -\frac{1}{2i}(2i\sin 5\theta - 4i\sin\theta - 2i\sin 3\theta)$		
$= 2\sin\theta + \sin 3\theta - \sin 5\theta$	1 / 5	
(c) $\sin 5\theta + 9\sin 3\theta$		
$= (2\sin\theta + \sin 3\theta - 16\sin^3\theta\cos^2\theta) + 9\sin 3\theta$	1M	For using (b)
$= 2\sin\theta + 10(3\sin\theta - 4\sin^3\theta) - 16\sin^3\theta(1 - \sin^2\theta)$	1M	For using (a)
$= 16\sin^5\theta - 56\sin^3\theta + 32\sin\theta$	1A	
$\sin 5\theta + 9\sin 3\theta - 8\sin\theta = 0$		
$16\sin^5\theta - 56\sin^3\theta + 24\sin\theta = 0$	1A	
$8\sin\theta(2\sin^4\theta - 7\sin^2\theta + 3) = 0$		
$\sin\theta = 0$ or $\sin^2\theta = \frac{1}{2}$ or $\sin^2\theta = 3$	1A+1A	1A for $\sin\theta = 0$, 1A for others
$\sin\theta = 0$ or $\sin\theta = \pm\frac{\sqrt{2}}{2}$		
$\theta = 0, \pi$ or $\frac{\pi}{4}, \frac{3\pi}{4}$	1A+1A / 8	1A for $0, \pi$ 1A for $\frac{\pi}{4}, \frac{3\pi}{4}$ no mark for degrees for extra −1 for each.

Solution	Marks	Remarks
<u>Alternative solution</u>		
$\sin5\theta + 9\sin3\theta - 8\sin\theta = 0$		
$(\sin5\theta + \sin3\theta) + 8(\sin3\theta - \sin\theta) = 0$		
$2\sin4\theta\cos\theta + 16\sin\theta\cos2\theta = 0$	1M	For sum to product
$8\sin\theta\cos^2\theta\cos2\theta + 16\sin\theta\cos2\theta = 0$		
$8\sin\theta\cos2\theta(\cos^2\theta + 2) = 0$		
$\sin\theta = 0$ or $\cos2\theta = 0$	1A+1A	
$\theta = 0, \pi$ or $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$	1A+1A	

Solution	Marks	Remarks
11. (a) Diagonal of base $= \sqrt{x^2 + x^2}$	1M	
$= \sqrt{2}x$ (cm)		
Height of pyramid $= \sqrt{(\frac{\sqrt{6}x}{2})^2 - (\frac{\sqrt{2}x}{2})^2}$		
$= x$ (cm)	1A	
$\therefore h = (10 - 2x) + x$		
$= 10 - x$	1 / 3	
(b) (i) $V = \frac{1}{3}x^2(x) + x^2(10 - 2x)$		
$= 10x^2 - \frac{5}{3}x^3$	1	
(ii) $\frac{dV}{dx} = 20x - 5x^2$	1A	
$\frac{dV}{dx} \geq 0$	1M	or $\frac{dV}{dx} > 0$
$20x - 5x^2 \geq 0$		
$5x(4 - x) \geq 0$		
$0 \leq x \leq 4$		
Since $0 < x < 5$, $\therefore 0 < x \leq 4$	1A	or $0 < x < 4$
The range of values of x for which V is decreasing is $4 \leq x < 5$.	1M+1A / 6	or $4 < x < 5$
(c) (i) Base side length $x \leq 3.5$		
$h = 10 - x \leq 7$	1A	
$\therefore 3 \leq x \leq 3.5$	1	
(ii) From (b) (ii), V is increasing on this interval		
$\therefore V$ is greatest when $x = 3.5$	1M	
Greatest volume $= 10(3.5)^2 - \frac{5}{3}(3.5)^3$		
$= 51.0$ (cm^3)	1A / 4	
(d) $x \leq 4.7$ and $10 - x \leq 5.5$		
$\therefore 4.5 \leq x \leq 4.7$	1A	
Since V is decreasing on this interval,		
$\therefore V$ is greatest when $x = 4.5$	1M	
Greatest volume $= 10(4.5)^2 - \frac{5}{3}(4.5)^3$		
$= 50.6$ (cm^3)	1A / 3	

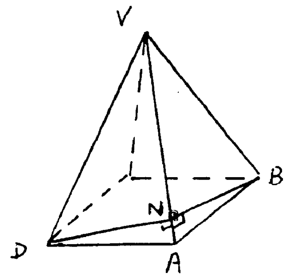
Solution	Marks	Remarks
12. (a) (i) When $x = 0$, $y = 3$	1A	Accept (0, 3)
$\therefore$ The y-intercept is 3.		
When $y = 0$, $2\cos 2x - 4\sin x + 1 = 0$		
$2(1 - 2\sin^2 x) - 4\sin x + 1 = 0$	1A	
$4\sin^2 x + 4\sin x - 3 = 0$		
$\sin x = \frac{1}{2}$ or $\sin x = \frac{-3}{2}$ (rejected)	1A	
$x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$	1A	Accept $(\frac{\pi}{6}, 0)$, $(\frac{5\pi}{6}, 0)$, no mark for degrees, (pp-1) if other correct roots are included.
$\therefore$ The x-intercepts are $\frac{\pi}{6}$ and $\frac{5\pi}{6}$		
(ii) $\frac{dy}{dx} = -4\sin 2x - 4\cos x$	1A	
$-4\sin 2x - 4\cos x = 0$	1M	
$\cos x(2\sin x + 1) = 0$		
$\cos x = 0$ or $\sin x = -\frac{1}{2}$ (rejected)		
$x = \frac{\pi}{2}$	1A	(pp-1) if other correct roots are included
$\frac{d^2y}{dx^2} = -8\cos 2x + 4\sin x$		
$\left.\frac{d^2y}{dx^2}\right\|_{x = \frac{\pi}{2}} > 0$	1M	
$\therefore (\frac{\pi}{2}, -5)$ is a minimum point.	1A	Accept turning point

RESTRICTED 内部文件

Solution	Marks	Remarks
(iii)		
	1A	For V-shape
	1A	labelled end points
	1A	For labelling $(\frac{\pi}{6},0)$ $(\frac{5\pi}{6},0)$ and $(\frac{\pi}{2},-5)$
	<u>12</u>	
(b) (i)		
	2M	For reflection
		Accept no labelling
(ii) Greatest value = 5	1A	
Least value = 0	<u>1A</u>	
	<u>4</u>	

Solution	Marks	Remarks
1. For $n = 1$, L.H.S. $= 1 \times 2 = 2$		$n=1$, $2=2$ ✓
R.H.S. $= 1^2(1 + 1) = 2$	1	
∴ the statement is true for $n = 1$		
Assume $1 \times 2 + 2 \times 5 + \ldots + k(3k - 1) = k^2 (k + 1)$ (for some positive integer k)	1	
Then $1 \times 2 + 2 \times 5 + \ldots + k(3k - 1) + (k + 1)[3(k + 1) - 1]$		
L.H.S. $= k^2(k + 1) + (k + 1) [3(k + 1) - 1]$	1	
$= (k + 1) (k^2 + 3k + 2)$		Assume $n=k$ is true
$= (k + 1)^2 (k + 2)$	1	
∴ the statement is also true for $n = k + 1$		the statement is true for all real number ✗
(if it is true for $n = k$)		
(By the principle of mathematical induction)		
∴ the statement is true for all +ve integers n.	1 / 5	
2. (a) Coefficient of $x = n + 6$	1A	
$n + 6 = 10$		
$n = 4$	1A	
(b) Coefficient of $x^2 = \frac{n(n-1)}{2} + 6n + 9$	2A	Accept nCr notation
$= \frac{4(4-1)}{2} + 6(4) + 9$		
$= 39$	1A / 5	
3. (a) $\frac{y-7}{x-4} = m$	1A	
$mx - y + (7 - 4m) = 0$		
(b) $\left\lvert \frac{7 - 4m}{\sqrt{m^2 + 1}} \right\rvert = 1$	1M+1A	Omit absolute sign (PP-1)
$(7 - 4m)^2 = m^2 + 1$		
$15m^2 - 56m + 48 = 0$	1A	
$m = \frac{4}{3}$ or $\frac{12}{5}$	2A / 6	1A + 1A

Solution	Marks	Remarks
4. (a) $\frac{dy}{dx} = x^2 - 2$	1A	or $y = \int (x^2 - 2)dx$
$y = \frac{1}{3}x^3 - 2x + c$	1A	
Put $x = 3$, $y = 4$	1M	
$c = 1$	1A	
$\therefore y = \frac{1}{3}x^3 - 2x + 1$		
(b) $x^2 - 2 = -2$	1A	
$x = 0$, $y = 1$	1A	
$\therefore$ The coordinates of the point is (0, 1)	6	
5. $\sin 2\theta(4\cos^2\theta - 3) - \sin\theta = 0$		
$2\sin\theta\cos\theta(4\cos^2\theta - 3) - \sin\theta = 0$	1A	For $\sin 2\theta = 2\sin\theta\cos\theta$
$2\sin\theta\cos 3\theta - \sin\theta = 0$	1M	For using the identity
$\sin\theta(2\cos 3\theta - 1) = 0$		
$\sin\theta = 0$ or $\cos 3\theta = \frac{1}{2}$	1A+1A	
$\theta = n\pi$ or $\theta = \frac{2n\pi}{3} \pm \frac{\pi}{9}$ (n is an integer)	1A+1A	or $180n°$, $120n° \pm 20°$
	6	① P.P-1 for mixed units ② 120n P.P-1
6. (a) $x^3 - x^2 - 2x = 0$	1A	
$x = 0, -1, 2$		
$\therefore a = -1$, $b = 2$	1A	
(b) Area $= \int_{-1}^{0}(x^3 - x^2 - 2x)dx - \int_{0}^{2}(x^3 - x^2 - 2x)dx$	1M+1M	1M for $\int y dx$ 試用
		1M for $\int_a^0 - \int_0^b$ or $\int_0^a + \int_0^b$ or $\lvert\int_0^a\rvert + \lvert\int_0^b\rvert$
$= [\frac{x^4}{4} - \frac{x^3}{3} - x^2]_{-1}^{0} - [\frac{x^4}{4} - \frac{x^3}{3} - x^2]_{0}^{2}$	1A	For correct integration
$= \frac{5}{12} + \frac{8}{3}$		
$= \frac{37}{12}$	1A	
	6	

Solution	Marks	Remarks
7. (a) $BD = \sqrt{6^2 + 6^2}$	1A	
$= \sqrt{72}$		
$\cos\angle VBD = \dfrac{\frac{1}{2}BD}{VB}$	1M	
$= \sqrt{18}/9$		
$\angle VBD = 61.9°$	1A	
(b) Let N be the point on VA such that $DN \perp VA$, $BN \perp VA$. The angle between the 2 planes is $\angle BND$.	1M	$\angle VAB = 70.5°$ $\angle AVB = 38.9°$
$\cos\angle VAB = \dfrac{1}{3}$		
$BN = 6\sin\angle VAB$ (or $9\sin\angle AVB$)	1M	
$= 4\sqrt{2}$	1A	Accept 5.7
$\sin\dfrac{\angle BND}{2} = \dfrac{\frac{1}{2}BD}{BN}$	1M	
$= \dfrac{\sqrt{18}}{4\sqrt{2}} = \dfrac{3}{4}$		
$\angle BND = 97.2°$	1A 8	
Alternative solution		
(b) Let N be the point on VA such that $DN \perp VA$, $BN \perp VA$. The angle between the 2 planes is $\angle BND$.	1M	
$BN = DN = 4\sqrt{2}$	1M+1A	Accept 5.7
$\cos\angle BND = \dfrac{BN^2 + DN^2 - BD^2}{2BN.DN}$	1M	
$= \dfrac{(4\sqrt{2})^2 + (4\sqrt{2})^2 - (\sqrt{72})^2}{2(4\sqrt{2})(4\sqrt{2})}$		
$= -0.125$		
$\angle BND = 97.2°$	1A	

Solution	Marks	Remarks
8. (a) $\frac{dy}{dx} = \frac{(2+\cos x)\cos x + \sin^2 x}{(2+\cos x)^2}$	1M+1A	1M for quotient rule or product rule
$= \frac{2\cos x + 1}{(2+\cos x)^2}$	1A	
$= \frac{(2\cos x + 4) - 3}{(2+\cos x)^2}$		
$= \frac{2}{2+\cos x} - \frac{3}{(2+\cos x)^2}$	1	
	4	
(b) $dt = \sqrt{3}\sec^2\theta\, d\theta$	1A	
$\int_0^1 \frac{dt}{t^2+3} = \int_0^{\frac{\pi}{6}} \frac{\sqrt{3}}{3}\, d\theta$	1A+1A	1A for limits 1A for integrand
$= \frac{\sqrt{3}\pi}{18}$	1A	Accept $\frac{\pi}{6\sqrt{3}}$
	4	
(c) $dx = \frac{2dt}{1+t^2}$	1A	or $dt = \frac{1}{2}\sec^2\frac{x}{2}dx$
Since $\cos x = \frac{1-t^2}{1+t^2}$,	1A	
$\therefore \int_0^{\frac{\pi}{2}} \frac{dx}{2+\cos x} = \int_0^1 \frac{2}{t^2+3}dt$	1A	
$= \frac{\sqrt{3}\pi}{9}$	1A	Accept $\frac{\pi}{3\sqrt{3}}$
	4	
(d) $\frac{dy}{dx} = \frac{2}{2+\cos x} - \frac{3}{(2+\cos x)^2}$ Integrating with respect to x,		
$\int_0^{\frac{\pi}{2}} [\frac{2}{2+\cos x} - \frac{3}{(2+\cos x)^2}]dx = [\frac{\sin x}{2+\cos x}]_0^{\frac{\pi}{2}}$	1M+1A	1M for integrating both sides, (pp-1) for omitting limits
$= \frac{1}{2}$	1A	
$2\int_0^{\frac{\pi}{2}} \frac{dx}{2+\cos x} - 3\int_0^{\frac{\pi}{2}} \frac{dx}{(2+\cos x)^2} = \frac{1}{2}$		
$\int_0^{\frac{\pi}{2}} \frac{dx}{(2+\cos x)^2} = \frac{2\sqrt{3}\pi}{27} - \frac{1}{6}$	1A	Accept $\frac{2\pi}{9\sqrt{3}} - \frac{1}{6}$
	4	

Solution	Marks	Remarks
9. (a) Substitute $y = mx + c$ into E,		
$16x^2 + 25(mx + c)^2 = 400$	1M	
$(25m^2 + 16)x^2 + 50mcx + 25c^2 - 400 = 0$	1A	
Since L is a tangent to E,		
$(50\,mc)^2 - 4(25m^2 + 16)\,(25c^2 - 400) = 0$	1M	
$(50mc)^2 - 4[(25mc)^2 - 400(25m^2) + 400c^2 - 400(16)] = 0$		
$c^2 = 25m^2 + 16$	1	
	4	
(b) Substitute (h, k) into L		
$c = k - mh$	1A	
Substitute into (a),		
$(k - mh)^2 = 25m^2 + 16$	1M	
$(h^2 - 25)m^2 - 2hkm + (k^2 - 16) = 0$	1	
	3	
(c) Put $h = 7$, $k = 4$		
$24m^2 - 56m + 0 = 0$	1M	
$m = 0$ or $\frac{7}{3}$	1A+1A	
$m = 0$: The equation of tangent is $y = 4$	1A	
$m = \frac{7}{3}$: The equation of tangent is $\frac{y - 4}{x - 7} = \frac{7}{3}$		
$7x - 3y - 37 = 0$	1A	$y = \frac{7}{3}x - \frac{37}{3}$
	5	
(d) Let $p(h, k)$ be a point on the locus		
$\frac{k^2 - 16}{h^2 - 25} = -1$	1M+2A	1M for $m_1m_2 = -1$
$h^2 + k^2 - 41 = 0$	1A	
$\therefore$ The equation of the locus is $x^2 + y^2 - 41 = 0$.		
	4	

Alternative solution

Solution	Marks
(d) $(x^2 - 25)m^2 - 2xym + (y^2 - 16) = 0$	
$m = \frac{2xy \pm \sqrt{4(x^2 - 25)(y^2 - 16)}}{2(x^2 - 25)}$	1A
$m_1m_2 = -1$	1M
$\frac{2xy + \sqrt{4(x^2 - 25)(y^2 - 16)}}{2(x^2 - 25)} \cdot \frac{2xy - \sqrt{4(x^2 - 25)(y^2 - 16)}}{2(x^2 - 25)} = -1$	
$(x^2 - 25)(x^2 + y^2 - 41) = 0$	
$x^2 - 25 = 0$ (rejected), $x^2 + y^2 - 41 = 0$	2A

Solution	Marks	Remarks
10. (a) $x^2 + y^2 - 2y - 4 = 0$		
$x^2 + (y - 1)^2 = 5$		
C_1 is centred at (0, 1) with radius $\sqrt{5}$	1A	
Distance between centres		
$= \sqrt{(8 - 0)^2 + (5 - 1)^2} = 4\sqrt{5}$		
Radius of circle $= 4\sqrt{5} \pm \sqrt{5}$	1M+1M	1M for +, 1M for -
$= 3\sqrt{5}$ or $5\sqrt{5}$	1A+1A	
$\therefore$ Equations of circles are		
$(x - 8)^2 + (y - 5)^2 = 45$	1A	$x^2 + y^2 - 16x - 10y + 44 = 0$
$(x - 8)^2 + (y - 5)^2 = 125$	1A	$x^2 + y^2 - 16x - 10y - 36 = 0$
	7	
__Alternative solution__		
Let the equation of circle be		
$x^2 + y^2 - 16x - 10y + k = 0$	1A	
$C_1 : x^2 + y^2 - 2y - 4 = 0$		
$\therefore 16x + 8y - 4 - k = 0$	1A	
$\left(\frac{k + 4 - 8y}{16}\right)^2 + y^2 - 2y - 4 = 0$	1M	
$320y^2 - 16(36 + k)y + (k^2 + 8k - 1008) = 0$		
$256(36 + k)^2 - 4(320)(k^2 + 8k - 1008) = 0$	1M	
$k^2 - 8k - 1584 = 0$	1A	
$k = 44$ or -36	1A+1A	
Equations of circle are		
$x^2 + y^2 - 16x - 10y + 44 = 0$		
$x^2 + y^2 - 16x - 10y - 36 = 0$		
(b) $2x + 2y\frac{dy}{dx} - 2\frac{dy}{dx} = 0$		
At $(-1, 3)$, $\frac{dy}{dx} = \frac{-2x}{(2y - 2)} = \frac{1}{2}$	1A	
The equation is		
$\frac{y - 3}{x + 1} = \frac{1}{2}$		
$x - 2y + 7 = 0$	1A	
	2	
__Alternative solutions__		
(1) By using the formula $xx_1 + yy_1 - (y + y_1) - 4 = 0$		
The equation is $x(-1) + y(3) - (y + 3) - 4 = 0$	1A	
$x - 2y + 7 = 0$	1A	
(2) Slope of $L_1 = {}^{-1}\big/\frac{3 - 1}{-1 - 0} = \frac{1}{2}$	1A	
The equation is $x - 2y + 7 = 0$	1A	

Solution	Marks	Remarks
(c) (i) $x^2 + y^2 - 2y - 4 + k(x - 2) = 0$ (k is a constant)	2A	Accept $(x - 2) + k(x^2 + \ldots) = 0$
(ii) Substitute L_1 into F,		
$(2y - 7)^2 + y^2 - 2y - 4 + k(2y - 7) - 2k = 0$	1M	
$5y^2 + (2k - 30)y + (45 - 9k) = 0$	1A	
$(2k - 30)^2 - 20(45 - 9k) = 0$	1M	
$4k^2 + 60k = 0$		
$k = -15$ ($k = 0$ (rejected))	1A	
$\therefore$ Equation of C_2 is $x^2 + y^2 - 15x - 2y + 26 = 0$	1A	
	7	
Alternative solutions for (ii)		
(1) Substitute $y = \frac{1}{2}(x + 7)$	1M	
$5x^2 + (4k + 10)x + (5 - 8k) = 0$	1A	
$(4k + 10)^2 - 20(5 - 8k) = 0$	1M	
$4k^2 + 60k = 0$		
(2) $x^2 + y^2 + kx - 2y = 2k + 4$		
$(x + \frac{k}{2})^2 + (y - 1)^2 = \frac{k^2}{4} + 2k + 5$		
centre is $(-\frac{k}{2}, 1)$, radius $= \sqrt{\frac{k^2}{4} + 2k + 5}$	1M	
If L_1 is tangent to a circle in F,		
$\left\lvert \frac{-\frac{k}{2} - 2 + 7}{\sqrt{5}} \right\rvert = \sqrt{\frac{k^2}{4} + 2k + 5}$	1M+1A	
$(5 - \frac{k}{2})^2 = 5(\frac{k^2}{4} + 2k + 5)$		
$4k^2 + 60k = 0$		

Solution	Marks	Remarks
11. (a) Volume $= \int_{-b}^{-\frac{b}{2}} \pi x^2 dy$	1A+1A	1A for $\int \pi x^2 dy$, 1A if others correct
$= \int_{-b}^{-\frac{b}{2}} \pi a^2 (1 - \frac{y^2}{b^2}) dy$	1M	
$= \pi a^2 [y - \frac{y^3}{3b^2}]_{-b}^{-\frac{b}{2}}$	1A	
$= \pi a^2 [\frac{-b}{2} + \frac{b}{24} + b - \frac{b}{3}]$		
$= \frac{5\pi a^2 b}{24}$	1	
	5	
(b) (i) Equation of ellipse is $\frac{x^2}{100} + \frac{y^2}{36} = 1$	1A	Accept $a = 10$, $b = 6$
Put $y = -3$	1A	
$x^2 = 75$		
$\therefore$ surface area $= \pi x^2$		
$= 75\pi$	1A	
(ii) Put $a = 10$, $b = 6$ into (a)	1A	
Volume $= \frac{5\pi(10)^2(6)}{24}$		
$= 125\pi$	1A	
(iii) (1) Let V be the volume of water remaining in the bowl.		
$\frac{dV}{dt} = -\frac{\pi}{100}(25 + 2t)$	1A	
$V = \frac{-\pi}{100}(25t + t^2) + c$	1A	
At $t = 0$, $V = 125\pi$ $\therefore c = 125\pi$	1M + 1A	
$\therefore V = 125\pi - \frac{\pi}{100}(25t + t^2)$	[redacted]	

Solution	Marks	Remarks
Alternative solutions		
$V = 125\pi - \int_0^t \frac{\pi}{100}(25 + 2t)dt$	1M+1A	1M
$= 125\pi - \frac{\pi}{100}[25t + t^2]_0^t$	1A	
$= 125\pi - \frac{\pi}{100}(25t + t^2)$	1A	
Let V be the volume of water lost.		
$\frac{dV}{dt} = \frac{\pi}{100}(25 + 2t)$	1A	
$V = \frac{\pi}{100}(25t + t^2) + c$	1A	
At $t = 0$, $V = 0$ $\therefore c = 0$ Volume remaining	1M	
$= 125\pi - \frac{\pi}{100}(25t + t^2)$	1A	
(2) $125\pi - \frac{\pi}{100}(25t + t^2) = 0$	1M	
$t^2 + 25t - 12500 = 0$		
$t = 100$ or -125 (rejected)		
$\therefore$ $t = 100$ (seconds)	1A	
	11	

Solution	Marks	Remarks
12. (a) $2[\cos\theta + \cos(\theta + 2\alpha) + \dots + \cos(\theta + 8\alpha)]\sin\alpha$		
$= 2\cos\theta\sin\alpha + 2\cos(\theta + 2\alpha)\sin\alpha + \dots + 2\cos(\theta + 8\alpha)\sin\alpha$	1A	
$= \sin(\theta + \alpha) - \sin(\theta - \alpha) + \sin(\theta + 3\alpha) - \sin(\theta + \alpha) + \dots + \sin(\theta + 9\alpha) - \sin(\theta + 7\alpha)$	1M	For using the identity.
$= \sin(\theta + 9\alpha) - \sin(\theta - \alpha)$	1	
Put $\alpha = \frac{\pi}{5}$	1A	
$2[\cos\theta + \cos(\theta + \frac{2\pi}{5}) + \dots + \cos(\theta + \frac{8\pi}{5})]\sin\frac{\pi}{5}$		
$= \sin(\theta + \frac{9\pi}{5}) - \sin(\theta - \frac{\pi}{5})$	1A	
$= 2\sin\pi\cos(\theta + \frac{4\pi}{5})$		OR $= \sin(\theta + \frac{9\pi}{5}) - \sin(\theta + \frac{9\pi}{5})$
$= 0$	1	OR $= \sin(\theta - \frac{\pi}{5}) - \sin(\theta - \frac{\pi}{5})$
$\therefore \cos\theta + \cos(\theta + \frac{2\pi}{5}) + \dots + \cos(\theta + \frac{8\pi}{5}) = 0$	1	OR $\because (\theta + \frac{9\pi}{5}) - (\theta - \frac{\pi}{5}) = 2\pi$
	7	
(b) (i) $PD^2 = r^2 + r^2 - 2r^2\cos(\frac{4\pi}{5} - \theta)$	1A	
$= 2r^2 - 2r^2\cos(\theta + \frac{6\pi}{5})$	1	
OR $PD^2 = [2r\sin\frac{1}{2}(\frac{4\pi}{5} - \theta)]^2$	1A	
$= 2r^2[1 - \cos(\frac{4\pi}{5} - \theta)]$		
$= 2r^2 - 2r^2\cos(\theta + \frac{6\pi}{5})$	1	

Solution	Marks	Remarks
(ii) $PA^2 = 2r^2 - 2r^2\cos\theta$	1A	Any one of PA^2, PB^2 or P
$PB^2 = 2r^2 - 2r^2\cos(\theta + \frac{2\pi}{5})$		
$PC^2 = 2r^2 - 2r^2\cos(\theta + \frac{4\pi}{5})$		
$PE^2 = 2r^2 - 2r^2\cos(\theta + \frac{8\pi}{5})$	1A	or $2r^2 - 2r^2\cos(\frac{2\pi}{5} - \theta)$
$\therefore PA^2 + PB^2 + PC^2 + PD^2 + PE^2$		
$= 10r^2 - 2r^2[\cos\theta + \cos(\theta + \frac{2\pi}{5})$		
$+ \ldots + \cos(\theta + \frac{8\pi}{5})]$	1A	
$= 10r^2$	1	
(iii) $QA^2 = QP^2 + PA^2 \quad (\because \angle QPA = \frac{\pi}{2})$	1A	
$= 4r^2 + PA^2$		
Similarly, $QB^2 = 4r^2 + PB^2, \ldots, QE^2 = 4r^2 + PE^2$		
$\therefore QA^2 + QB^2 + QC^2 + QD^2 + QE^2$		
$= 20r^2 + (PA^2 + PB^2 + PC^2 + PD^2 + PE^2)$	1A	
$= 20r^2 + 10r^2$		
$= 30r^2$	1A	
	9	