92-CE
A MATHS
PAPER I

HONG KONG EXAMINATIONS AUTHORITY
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1992

ADDITIONAL MATHEMATICS PAPER I

8.30 am-10.30 am (2 hours)
This paper must be answered in English

Answer **ALL** questions in Section A and any **THREE** questions in Section B.

All working must be clearly shown.

Unless otherwise specified in a question, numerical answers must be given in **exact value**.

Section A (42 marks)

Answer **ALL** questions in this section.

1. Given $\overrightarrow{OA} = 5\mathbf{i} - \mathbf{j}$, $\overrightarrow{OB} = -3\mathbf{i} + 5\mathbf{j}$ and APB is a straight line.

   (a) Find $\overrightarrow{AB}$ and $|\overrightarrow{AB}|$.

   (b) If $|\overrightarrow{AP}| = 4$, find $\overrightarrow{AP}$.

(5 marks)

2. (a) Express the complex number $\frac{\sqrt{3}+i}{\sqrt{3}-i}$ in polar form.

   (b) Using the result of (a), express $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^{92}$ in the form $a + bi$.

(6 marks)

3. Solve $|x(x+5)| > 6$ for real values of x .

(6 marks)

4.

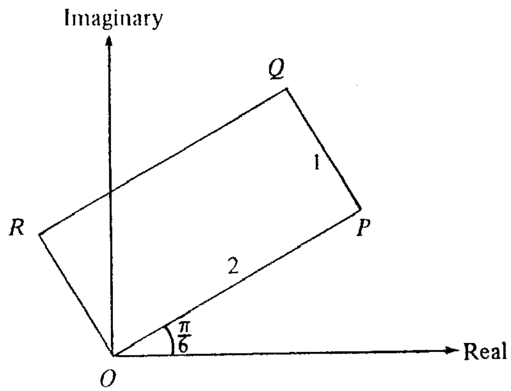

Figure 1

Figure 1 shows an Argand diagram in which $OPQR$ is a rectangle. $OP = 2$, $PQ = 1$ and OP makes an angle $\frac{\pi}{6}$ with the positive real axis. Let z_1, z_2 and z_3 be the complex numbers represented by vertices P, Q and R respectively.

(a) Express z_1 and z_3 in the form $a + bi$.

(b) Using the result of (a), find z_2.

(6 marks)

5. The curve $(x - 2)(y^2 + 3) = -8$ cuts the y-axis at two points. Find

(a) the coordinates of the two points;

(b) the slope of the tangent to the curve at each of the two points.

(6 marks)

6. α, β are the real roots of the equation

$$x^2 + (k - 2)x - (k - 1) = 0.$$

If $|\alpha| = |\beta|$, find k.

(6 marks)

7.

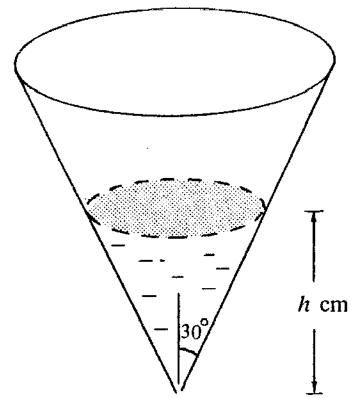

Figure 2

Figure 2 shows a vessel in the shape of a right circular cone with semi-vertical angle 30°. Water is flowing out of the cone through its apex at a constant rate of π cm^{3}s^{-1}.

(a) Let V cm^3 be the volume of water in the vessel when the depth of water is h cm. Express V in terms of h.

(b) How fast is the water level falling when the depth of water is 4 cm ?

(7 marks)

Section B (48 marks)

Answer any **THREE** questions in this section.

Each question carries 16 marks.

8.

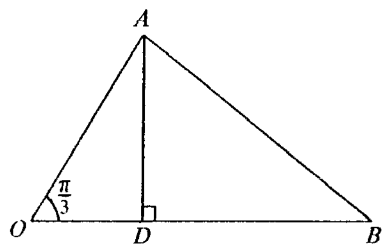

Figure 3

In Figure 3, $OA = 2$, $OB = 3$ and $\angle AOB = \frac{\pi}{3}$. D is a point on OB such that AD is perpendicular to OB. Let $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$.

(a) Find $\mathbf{a} \cdot \mathbf{a}$ and $\mathbf{a} \cdot \mathbf{b}$. (3 marks)

(b) Find the length of OD.
Hence express $\overrightarrow{OD}$ in terms of $\mathbf{b}$. (2 marks)

(c) Let H be a point on AD such that $AH : HD = 1 : k$ and $\overrightarrow{OH}$ is perpendicular to $\overrightarrow{AB}$.

(i) Express $\overrightarrow{OH}$ in terms of k, $\mathbf{a}$ and $\mathbf{b}$.
Hence find the value of k.

(ii) OH produced meets AB at a point C. Let $AC : CB = 1 : m$ and $OH : HC = 1 : n$.

(1) Express $\overrightarrow{OC}$ in terms of m, $\mathbf{a}$, $\mathbf{b}$.

(2) Express $\overrightarrow{OC}$ in terms of n, $\mathbf{a}$, $\mathbf{b}$.

(3) Hence find m and n. (11 marks)

9. α, β are the roots of the quadratic equation

$$x^2 + (p + 1)x + (p - 1) = 0,$$

where p is a real number.

(a) Show that α, β are real and distinct. (3 marks)

(b) Express $(\alpha - 2)(\beta - 2)$ in terms of p. (3 marks)

(c) Given $\beta < 2 < \alpha$.

(i) Using the result of (b), show that $p < -\frac{5}{3}$.

(ii) If $(\alpha - \beta)^2 < 24$, find the range of possible values of p.

Hence write down the possible integral value(s) of p.

(10 marks)

10. (a) Using De Moivre's Theorem, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$

(3 marks)

(b) Let the complex number $z = \cos\theta + i\sin\theta$.

Using the fact that

$$z^n + \frac{1}{z^n} = 2\cos n\theta \quad \text{and} \quad z^n - \frac{1}{z^n} = 2i\sin n\theta$$

for any positive integer n, show that

$$16\sin^3\theta\cos^2\theta = 2\sin\theta + \sin 3\theta - \sin 5\theta.$$

(5 marks)

(c) Using (a) and (b), express $\sin 5\theta + 9\sin 3\theta$ as a polynomial in $\sin\theta$.

Hence, or otherwise, solve

$$\sin 5\theta + 9\sin 3\theta - 8\sin\theta = 0$$

for $0 \le \theta \le \pi$.

(8 marks)

11.

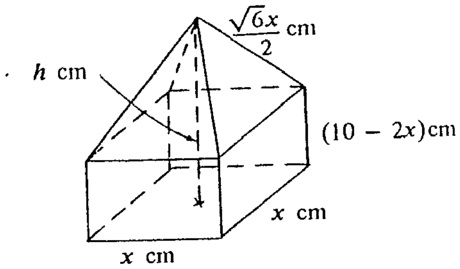

Figure 4 (a)

Figure 4 (a) shows a solid consisting of a right pyramid and a cuboid with a common face which is a square of side x cm. The slant edge of the pyramid is $\frac{\sqrt{6}x}{2}$ cm and the height of the cuboid is $(10 - 2x)$cm, where $0 < x < 5$.

(a) Let h cm be the height of the solid. Show that $h = 10 - x$.

(3 marks)

(b) Let V cm^3 be the volume of the solid.

(i) Show that $V = 10x^2 - \frac{5}{3}x^3$.

(ii) Find the range of values of x for which V is increasing.

Hence write down the range of values of x for which V is decreasing.

(6 marks)

(c)

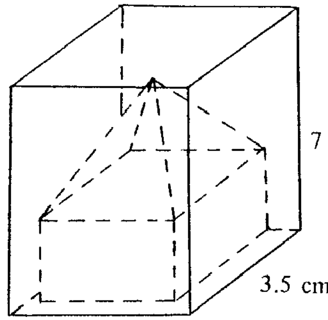

Figure 4 (b)

The solid is placed COMPLETELY inside a rectangular box as shown in Figure 4 (b). The base of the box is a square of side 3.5 cm and the height of the box is 7 cm.

(i) Show that $3 \le x \le 3.5$.

(ii) Hence find, correct to one decimal place, the greatest volume of the solid.

(4 marks)

(d) The side of the square base of the box in (c) is now changed to 4.7 cm and the height 5.5 cm. Find, correct to one decimal place, the greatest volume of the solid that can be placed COMPLETELY inside the box.

(3 marks)

12. (a) C_1 is the curve $y = 2\cos 2x - 4\sin x + 1$, where $0 \le x \le \pi$.

(i) Find the x- and y- intercepts of C_1.

(ii) Find the turning point(s) of C_1.

(iii) Sketch the curve C_1 in Figure 5 (a).

(12 marks)

(b) C_2 is the curve $y = |2\cos 2x - 4\sin x + 1|$, where $0 \le x \le \pi$.

(i) Using the result of (a) (iii), sketch the curve C_2 in Figure 5 (b).

(ii) Hence write down the greatest and least values of $|2\cos 2x - 4\sin x + 1|$ for $0 \le x \le \pi$.

(4 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page	

12. **(Continued)**

If you attempt Question 12, fill in the details in the first three boxes above and tie this sheet into your answer book.

(a) (iii)

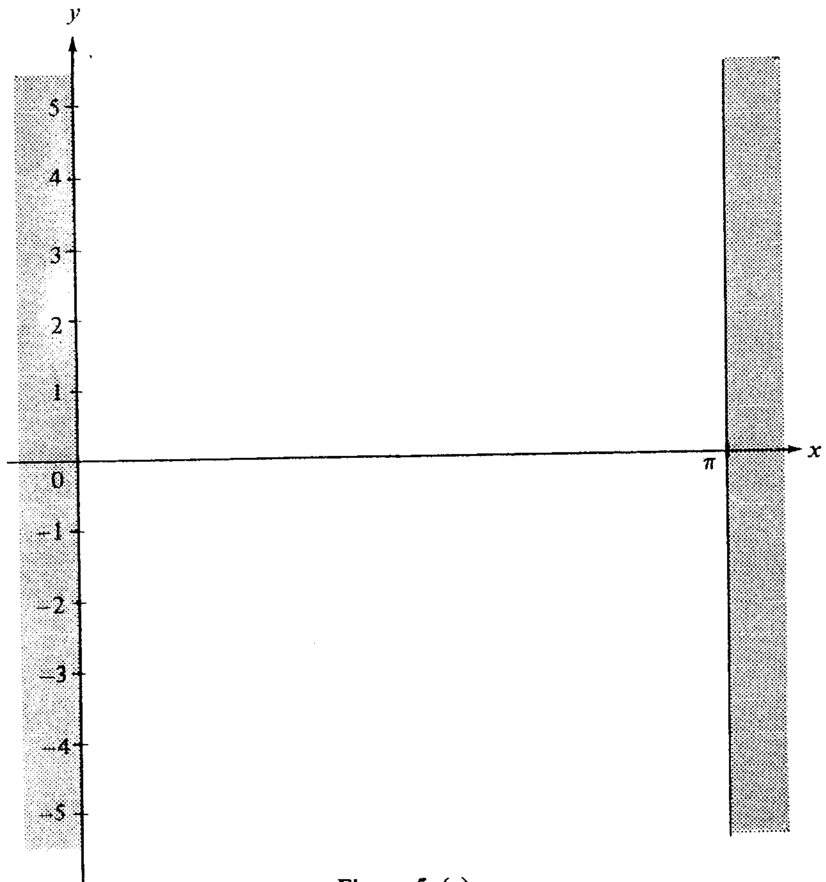

Figure 5 (a)

12. **(Continued)**

Total Marks on this page

(b) (i)

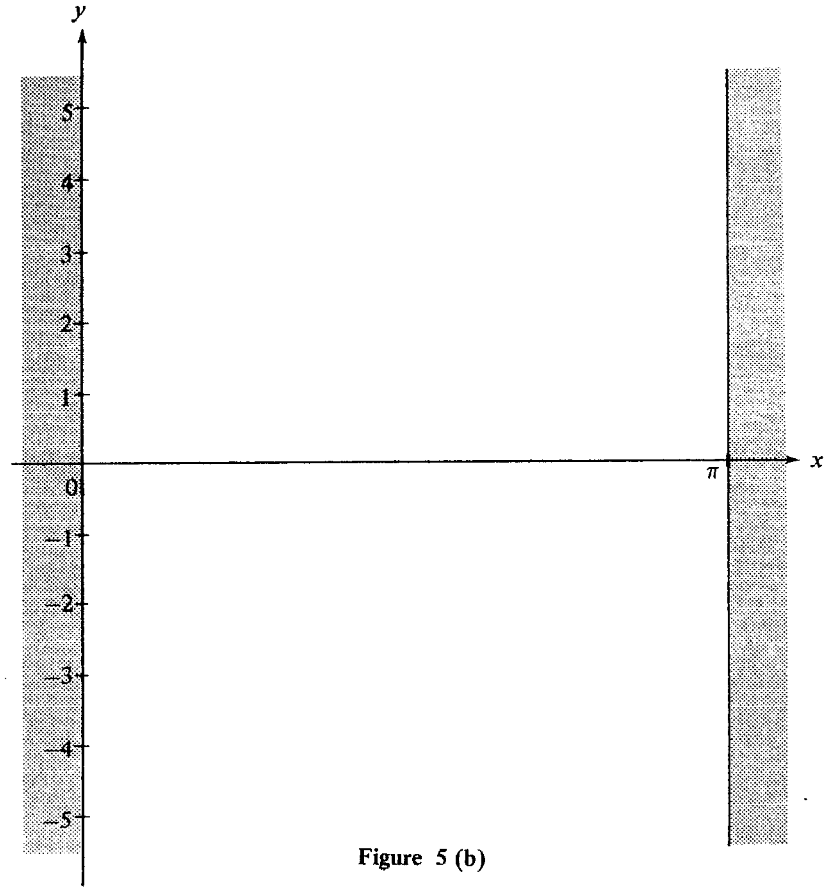

Figure 5 (b)

END OF PAPER

92-CE
A MATHS
PAPER II

HONG KONG EXAMINATIONS AUTHORITY
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1992

ADDITIONAL MATHEMATICS PAPER II

11.15 am-1.15 pm (2 hours)
This paper must be answered in English

Answer **ALL** questions in Section A and any **THREE** questions in Section B.

All working must be clearly shown.

Unless otherwise specified in a question, numerical answers must be given in exact value.

Section A (42 marks)

Answer **ALL** questions in this section.

1. Prove, by mathematical induction, that

$$1 \times 2 + 2 \times 5 + 3 \times 8 + \ldots + n(3n - 1) = n^2(n + 1)$$

for all positive integers n .

(5 marks)

2. In the expansion of $(1 + 3x)^2(1 + x)^n$, where n is a positive integer, the coefficient of x is 10.

(a) Find the value of n .

(b) Find the coefficient of x^2 .

(5 marks)

3. A straight line with slope m passes through the point (4, 7).

(a) Write down the equation of the line.

(b) If the distance from the origin to the line is 1, find the two possible values of m .

(6 marks)

4. The slope of the tangent to a curve C at any point (x, y) on C is $x^2 - 2$. C passes through the point (3, 4).

(a) Find an equation of C .

(b) Find the coordinates of the point on C at which the slope of the tangent is -2.

(6 marks)

5. By using the identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$, find the general solution of the equation

$$\sin 2\theta\,(4\cos^2\theta - 3) - \sin\theta = 0.$$

(6 marks)

6.

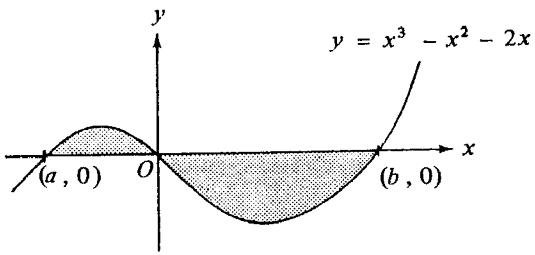

Figure 1

The curve $y = x^3 - x^2 - 2x$ cuts the x-axis at the origin and the points $(a, 0)$ and $(b, 0)$, as shown in Figure 1.

(a) Find the values of a and b.

(b) Find the total area of the shaded parts.

(6 marks)

7.

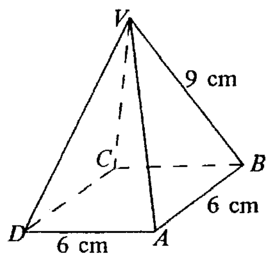

Figure 2

In Figure 2, $VABCD$ is a right pyramid with a square base of side 6 cm. VB = 9 cm. Find, correct to the nearest 0.1 degree,

(a) the angle between edge VB and the base $ABCD$,

(b) the angle between the planes VAB and VAD.

(8 marks)

Section B (48 marks)

Answer any **THREE** questions in this section.
Each question carries 16 marks.

8. (a) Let $y = \dfrac{\sin x}{2 + \cos x}$.

Show that $\dfrac{dy}{dx} = \dfrac{2}{2 + \cos x} - \dfrac{3}{(2 + \cos x)^2}$.

(4 marks)

(b) Using the substitution $t = \sqrt{3}\tan\theta$, evaluate

$$\int_0^1 \frac{dt}{t^2 + 3}.$$

(4 marks)

(c) Using the substitution $t = \tan\frac{x}{2}$ and the result of (b), evaluate

$$\int_0^{\frac{\pi}{2}} \frac{dx}{2 + \cos x}.$$

(4 marks)

(d) Using the results of (a) and (c), evaluate

$$\int_0^{\frac{\pi}{2}} \frac{dx}{(2 + \cos x)^2}.$$

(4 marks)

9. Given an ellipse $E : \frac{x^2}{25} + \frac{y^2}{16} = 1$.

Let the line $L : y = mx + c$ be a tangent to E.

(a) Show that $c^2 = 25m^2 + 16$.

(4 marks)

(b) Suppose L passes through the point (h, k). Using the result of (a), show that

$$(h^2 - 25)m^2 - 2hkm + (k^2 - 16) = 0.$$

(3 marks)

(c) Find equations of the two tangents from the point (7, 4) to E.

(5 marks)

(d) P is a variable point outside E and the two tangents from P to E are at right angles. Find an equation of the locus of P.

(4 marks)

10. Given a circle $C_1 : x^2 + y^2 - 2y - 4 = 0$.

(a) Find equations of the two circles centred at the point (8, 5) and touching C_1.

(7 marks)

(b) Find an equation of the line L_1 which touches C_1 at the point (−1, 3).

(2 marks)

(c) F is the family of circles passing through the points of intersection of C_1 and the line $x - 2 = 0$.

(i) Write down an equation of F.

(ii) If L_1 in (b) also touches another circle C_2 of F, find an equation of C_2.

(7 marks)

11. (a)

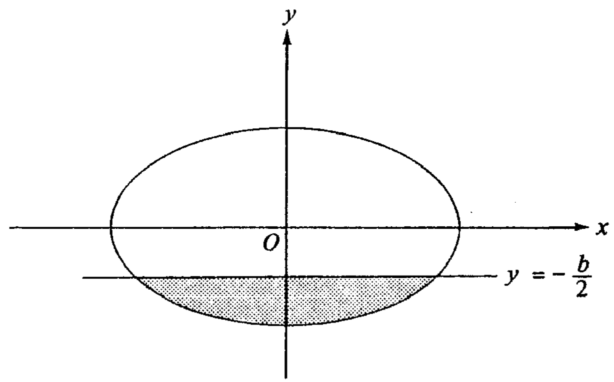

Figure 3 (a)

The shaded region enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $y = -\frac{b}{2}$, as shown in Figure 3 (a), is revolved about the y-axis. Show that the volume of the solid of revolution is $\frac{5\pi a^2 b}{24}$.

(5 marks)

(b)

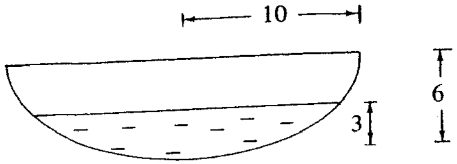

Figure 3 (b)

A bowl is generated by revolving the lower half of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the y-axis. The depth of the bowl is 6 units and the radius of its rim is 10 units. The bowl contains water to a depth of 3 units. (See Figure 3 (b).)

(i) Find the area of the water surface.

(ii) Using the result of (a), find the volume of water.

(iii) The water in the bowl is heated. At time t seconds after heating, the volume of water decreases at a rate of $\frac{\pi}{100}(25 + 2t)$ cubic units per second.

(1) Find the volume of water remaining in the bowl after t seconds.

(2) Calculate the time required to dry up the water in the bowl.

(11 marks)

12. (a) Using the identity $2\cos x \sin y = \sin(x + y) - \sin(x - y)$, show that

$$2[\cos\theta + \cos(\theta + 2\alpha) + \cos(\theta + 4\alpha) + \cos(\theta + 6\alpha) + \cos(\theta + 8\alpha)]\sin\alpha$$
$$= \sin(\theta + 9\alpha) - \sin(\theta - \alpha).$$

Hence show that

$$\cos\theta + \cos(\theta + \frac{2\pi}{5}) + \cos(\theta + \frac{4\pi}{5}) + \cos(\theta + \frac{6\pi}{5}) + \cos(\theta + \frac{8\pi}{5}) = 0.$$

(7 marks)

(b)

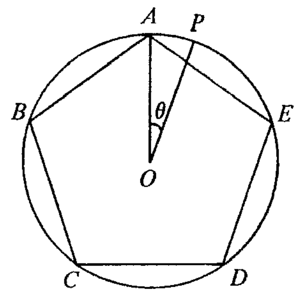

Figure 4

A, B, C, D and E are the vertices of a regular pentagon inscribed in a circle of radius r and centred at O. P is a point on the circumference of the circle such that $\angle POA = \theta$, as shown in Figure 4.

(i) By considering ΔOPD, show that $PD^2 = 2r^2 - 2r^2\cos(\theta + \frac{6\pi}{5})$.

(ii) Show that $PA^2 + PB^2 + PC^2 + PD^2 + PE^2 = 10r^2$.

(iii) QP is a line perpendicular to the plane of the circle such that $QP = 2r$.
Find $QA^2 + QB^2 + QC^2 + QD^2 + QE^2$.

(9 marks)

END OF PAPER

1993

Additional Mathematics I

1. (a) $2\Delta x$

   (b) $\frac{1}{\sqrt{2x}}$

2. (a) $8 - 6i$

   (b) $1 - 3i$

3. 1, −5

4. $\cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6})$

   $\cos(\frac{2k\pi}{3} - \frac{\pi}{18}) + i\sin(\frac{2k\pi}{3} - \frac{\pi}{18})$, $k = -1, 0, 1$

5. $x \le -2$ or $x \ge 4$

6. (a) $-2\mathbf{i} + 3\mathbf{j}$

   (b) 13, 0, 13

7. (a) $\frac{2x - 2y^2}{4xy - 3y^2}$

   (b) $2x + 11y + 7 = 0$

8. (a) $\frac{\overline{a} + r\overline{b}}{1 + r}$, $\frac{\overline{a} + (r^2 + 2r)\overline{b}}{(1 + r)^2}$

   (b) $\frac{1}{1 + r}\overline{b}$

   (c) $\frac{-1 + \sqrt{5}}{2}$

   (d) (i) 4, 16

   (ii) $\frac{1}{2}$

9. (b) $3\sqrt{3}$

10. (a) −13

    (b) $-7 - 4\sqrt{2} < k < -7 + 4\sqrt{2}$

    (c) (−1, −1), (−2, −2)

11. (a) $\frac{\pi}{18}$, $\frac{-5\pi}{18}$; −1

    (b) $3\sqrt{3}\cos 3x + 3\sin 3x$, $-9\sqrt{3}\sin 3x + 9\cos 3x$

    (c) $(-\frac{\pi}{9}, -2)$ is a minimum point.

    $(\frac{2\pi}{9}, 2)$ is a maximum point.

12. (b) $\frac{\pi}{3}$, $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$

    (c) $-\frac{3}{2} - \frac{\sqrt{3}}{2}i$

1993

Additional Mathematics II

2. $2\cos(x + \frac{\pi}{6})$

   $2n\pi + \frac{\pi}{6}$, $2n\pi - \frac{\pi}{2}$

3. (a) $4n$, $8n^2 - 7n$

   (b) 5, 165

4. $y = 2x - 5$, $y = -\frac{x}{2} + 5$

5. (a) $(\frac{\pi}{4}, \frac{\sqrt{2}}{2})$, $(\frac{5\pi}{4}, \frac{-\sqrt{2}}{2})$

   (b) $2\sqrt{2}$

6. (a) $y = x^3 - 3x^2 - x + 3$

   (b) $y = -x + 3$

7. (a) 75.5°, 11.6 cm

   (b) 62.2°

8. (b) (i) $\frac{6}{5}$

   (ii) $\frac{24}{13}$

9. (a) $(m - 1)\sin^{m-2}x\cos^{n+2}x - (n + 1)\sin^m x\cos^n x$

   (d) $\frac{3\pi}{512}$

10. (a) $y = \frac{s + t}{2}x - st$

    (b) $y = sx - s^2$

    (c) (ii) $\frac{\pi}{2}$

    (iv) $2y = x^2 + 2$

11. (a) $\frac{13}{4}$, $\frac{3}{4}$

    (d) $x^2 + (y - 3)^2 = 3^2$, $(x - \frac{24}{7})^2 + (y - \frac{3}{49})^2 = (\frac{3}{49})^2$

12. (b) (i) $4\pi^2$

    (ii) π minutes, $(\frac{\sqrt{17} - 3}{2})\pi$ minutes

1992

Additional Mathematics I

1. (a) $-8\mathbf{i} + 6\mathbf{j}$, 10

   (b) $-\frac{16}{5}\mathbf{i} + \frac{12}{5}\mathbf{j}$

2. (a) $\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$

   (b) $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$

3. $x < -6$ or $-3 < x < -2$ or $x > 1$

4. (a) $\sqrt{3} + i$, $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$

   (b) $(\sqrt{3} - \frac{1}{2}) + (\frac{\sqrt{3}}{2} + 1)i$

5. (a) (0, 1), (0, -1)

   (b) 1, -1

6. 0, 2

7. (a) $V = \frac{\pi}{9}h^3$

   (b) $\frac{3}{16}$ cm s^{-1}

8. (a) 4, 3

   (b) 1, $\frac{1}{3}\mathbf{b}$

   (c) (i) $\frac{k\mathbf{a} + \frac{1}{3}\mathbf{b}}{k + 1}$, 2

   (ii) (1) $\frac{m\mathbf{a} + \mathbf{b}}{m + 1}$

   (2) $(n + 1)(\frac{2}{3}\mathbf{a} + \frac{1}{9}\mathbf{b})$

   (3) 6, $\frac{2}{7}$

9. (b) $3p + 5$

   (c) (ii) $1 - 2\sqrt{5} < p < -\frac{5}{3}$

   -2, -3

10. (c) $16\sin^5\theta - 56\sin^3\theta + 32\sin\theta$

    0, $\frac{\pi}{4}$, $\frac{3\pi}{4}$, π

11. (b) (ii) $0 < x \leq 4$, $4 \leq x < 5$

    (c) (ii) 51.0 cm^3

    (d) 50.6 cm^3

12. (a) (i) $\frac{\pi}{6}$, $\frac{5\pi}{6}$

    3

    (ii) $(\frac{\pi}{2}, -5)$

    (b) (ii) 5, 0

1992

Additional Mathematics II

2. (a) 4

   (b) 39

3. (a) $mx - y + (7 - 4m) = 0$

   (b) $\frac{4}{3}$, $\frac{12}{5}$

4. (a) $y = \frac{1}{3}x^3 - 2x + 1$

   (b) (0, 1)

5. $n\pi$, $\frac{2n\pi}{3} \pm \frac{\pi}{9}$

6. (a) −1, 2

   (b) $\frac{37}{12}$

7. (a) 61.9°

   (b) 97.2°

8. (b) $\frac{\sqrt{3}\pi}{18}$

   (c) $\frac{\sqrt{3}\pi}{9}$

   (d) $\frac{2\sqrt{3}\pi}{27} - \frac{1}{6}$

9. (c) $y = 4$, $7x - 3y - 37 = 0$

   (d) $x^2 + y^2 - 41 = 0$

10. (a) $(x - 8)^2 + (y - 5)^2 = 45$, $(x - 8)^2 + (y - 5)^2 = 125$

    (b) $x - 2y + 7 = 0$

    (c) (i) $x^2 + y^2 - 2y - 4 + k(x - 2) = 0$

    (ii) $x^2 + y^2 - 15x - 2y + 26 = 0$

11. (b) (i) 75π

    (ii) 125π

    (iii) (1) $V = 125\pi - \frac{\pi}{100}(25t + t^2)$

    (2) 100 s

12. (b) (iii) $30r^2$