RESTRICTED 内部文件

香港考試局
HONG KONG EXAMINATIONS AUTHORITY

一九九一年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1991

附加數學卷一
ADDITIONAL MATHEMATICS PAPER I

評卷參考
MARKING SCHEME

It is highly undesirable that this marking scheme should fall into the hands of students. They are likely to regard it as a set of model answers, which it certainly is not.

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本評卷參考並非標準答案，故極不宜落於學生手中，以免引起誤會。

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GENERAL INSTRUCTIONS TO MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method. <u>In general, a correct answer merits all the marks allocated to that part, provided that the method used is sound</u>.

2. In a question consisting of several parts each depending on the previous parts, marks should be awarded to steps or methods correctly deduced from previous erroneous answers. However, marks for the corresponding answer should <u>NOT</u> be awarded. In the marking scheme, 'M' marks are awarded for showing correct method use, and 'A' marks are awarded for the accuracy of the answers.

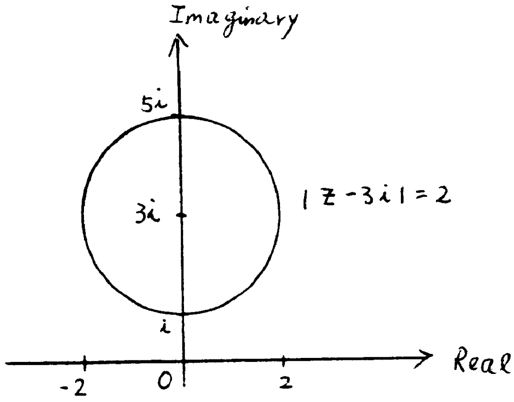
3. The symbol (pp-1) should be used to denote marks deducted for poor presentation (p.p.). Marks entered in the box should be the <u>net</u> total scored on that page. Note the following points :

   (a) At most deduct 1 mark for p.p. in each question, up to a maximum of 3 marks for the whole paper.

   (b) For similar p.p., deduct only 1 mark for the first time that it occurs, i.e. do not penalise candidates twice in the whole paper for the same p.p.

4. Numerical answers should be given in exact value unless otherwise specified in the question. However answers not in exact values would be accepted this year provided that they are correct to at least 3 significant figures.

1.



Solution	Marks	Remarks
circle	1A	Axes not labelled (pp-1)
correct centre	1A	
correct radius	1A	
$2+3i$	1A	(2,3) not accepted
	4	

2.

$$\frac{f(x+h)-f(x)}{h} = \frac{1}{h}\left(\frac{1}{1+x+h}-\frac{1}{1+x}\right) \quad \text{1A}$$

$$= \frac{1}{h}\,\frac{-h}{(1+x+h)(1+x)} \quad \text{1A}$$

$$= \frac{-1}{(1+x+h)(1+x)} \quad \text{1A}$$

$$f'(x) = \lim_{h\to 0}\frac{-1}{(1+x+h)(1+x)} \quad \text{1M}$$

$$= \frac{-1}{(1+x)^2} \quad \text{1A}$$

5

3.

$$|x-2| = |x^2-4|$$

$$|x-2| = |x-2|\,|x+2| \quad \text{2A}$$

$x = 2$ or $|x+2| = 1$

$x + 2 = \pm 1$

$x = -1$ or -3

$\therefore x = -1, -3$ or 2 1A+1A+1A

5

Alternative solutions

(1) $x - 2 = x^2 - 4$ or $x - 2 = -(x^2 - 4)$ 2A

$x - 2 = (x - 2)(x + 2)$ $x - 2 = -(x - 2)(x + 2)$

$x = 2$ or $x + 2 = 1$ $x = 2$ or $x + 2 = -1$

$x = -1$ $x = -3$

$\therefore x = -1, -3$ or 2 1A+1A+1A

(2)	$(x - 2)^2 = (x^2 - 4)^2$ $(x - 2)^2 [(x + 2)^2 - 1] = 0$ $(x - 2)^2 (x + 3)(x + 1) = 0$ $x = 2, -1$ or -3	2A 1A+1A+1A	
(3)	3 cases : $x \geq 2$, $-2 < x < 2$, $x \leq -2$ Case 1 : $x \geq 2$ $x - 2 = x^2 - 4$ $x = 2$ or -1 (rejected) $\therefore x = 2$ Case 2 : $-2 < x < 2$ $-(x - 2) = -(x^2 - 4)$ $x = -1$ or 2 (rejected) $\therefore x = -1$ Case 3 : $x \leq -2$ $-(x - 2) = x^2 - 4$ $x = -3$ or 2 (rejected) $\therefore x = -3$ $\therefore x = -1, 2$ or -3	1A 1A 1A+1A+1A	 Awarded only if the 3 equations are all correct

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4. (a) $\frac{dy}{dx} = 1 + 2\cos 2x$	1A	
$\frac{d^2y}{dx^2} = -4\sin 2x$	1A	
(b) $1 + 2\cos 2x = 0$	1M	
$\cos 2x = -\frac{1}{2}$		
$2x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$ $(0 \le x \le \pi)$		
$x = \frac{\pi}{3}$ or $\frac{2\pi}{3}$	1A	Do not accept degrees, but carry forward
$\left.\frac{d^2y}{dx^2}\right\|_{x=\frac{\pi}{3}} = -2\sqrt{3} < 0 \quad \therefore$ max	1M	Accept $\left.\frac{d^2y}{dx^2}\right\|_{x=\frac{\pi}{3}} < 0 \quad \therefore$ max
$y_{max} = \frac{\pi}{3} + \sin\frac{2\pi}{3}$		
$= \frac{\pi}{3} + \frac{\sqrt{3}}{2}$	1A	Accept 1.91 (awarded only if max. is checked)
$\left.\frac{d^2y}{dx^2}\right\|_{x=\frac{2\pi}{3}} = 2\sqrt{3} > 0 \quad \therefore$ min		
$y_{min} = \frac{2\pi}{3} + \sin\frac{4\pi}{3}$		
$= \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$	1A	Accept 1.23 (awarded only if min. is checked)
	7	

5. (a) $\vec{OC} = \dfrac{\vec{a} + 3\vec{b}}{4}$	1A	$\frac{1}{4}\vec{a} + \frac{3}{4}\vec{b}$ Omit vector sign (pp-1)
(b) (i) $\vec{OE} = \dfrac{k+1}{k}\vec{OC}$	1A	
$= \dfrac{k+1}{4k}(\vec{a} + 3\vec{b})$	1A	
$= \dfrac{k+1}{4k}\vec{a} + \dfrac{3(k+1)}{4k}\vec{b}$		
(ii) $\vec{OD} = 2\vec{b}$	1A	
$\vec{OE} = \dfrac{\vec{a} + 2m\vec{b}}{1+m}$	1A	
$= \dfrac{1}{1+m}\vec{a} + \dfrac{2m}{1+m}\vec{b}$		
$\therefore \begin{cases} \dfrac{k+1}{4k} = \dfrac{1}{1+m} \\ \dfrac{3(k+1)}{4k} = \dfrac{2m}{1+m} \end{cases}$	1M	
Solving, $m = \dfrac{3}{2}$, $k = \dfrac{5}{3}$	1A	
	7	

			Marks
6.	(a)	$y' = -\frac{1}{x^2} + 1$	1A
		$y'\|_{x=1} = 0$	1A
		Equation of tangent at P : $y = 2$	1A
		Equation of normal at P : $x = 1$	1A
	(b)	$y'\|_{x=\frac{1}{2}} = -3$	1A
		Equation of tangent at Q	
		$\dfrac{y - \frac{5}{2}}{x - \frac{1}{2}} = -3$	1A
		$y = -3x + 4$	
		Subs. $x = 0$, $y = 4$	
		$\therefore$ The tangent to C at Q passes through A.	1
			7

Alternative solution for (b)

	Marks
$y'\|_{x=\frac{1}{2}} = -3$	1A
Slope of $AQ = \dfrac{4 - \frac{5}{2}}{0 - \frac{1}{2}}$	
$= -3$	1A
$=$ slope of tangent at Q	
$\therefore$ The tangent to C at Q passes through A.	1

			Marks	Remarks
7.	(a)	$pq = 1 - k(p + q)$	1A	can be omitted
		$= 1 - k(2 - k)$	1A	
		$= 1 - 2k + k^2$		
	(b)	The equation is		
		$x^2 - (2 - k)x + (1 - 2k + k^2) = 0$	1A+1M	$x^2 + (k - 2)x + (k - 1)^2 = 0$
		$[-(2 - k)]^2 - 4(1 - 2k + k^2) \geq 0$	1M	do not accept > 0
		$4k - 3k^2 \geq 0$		
		$k(4 - 3k) \geq 0$	1A	or $k(3k - 4) \leq 0$
		$0 \leq k \leq \frac{4}{3}$	1A	
			7	

Solution	Marks	Remarks
8. (a) $\vec{CA} = \vec{OA} - \vec{OC}$	1M	Omit vector sign (pp-1)
$= (3 - x)\vec{i} - (y+1)\vec{j}$	1A	
$\vec{OB} = \vec{OC} - \vec{BC}$		
$= (x - 7)\vec{i} + (y - 1)\vec{j}$	1A	
$\vec{AB} = \vec{OB} - \vec{OA}$		
$= (x - 10)\vec{i} + y\vec{j}$	1A	
	4	
(b) (i) $\vec{AB} \cdot \vec{BC} = 4\vec{BC} \cdot \vec{CA}$		
$7(x - 10) + y = 4[7(3 - x) - (y + 1)]$	1M	
$5y = -35x + 150$		
$y = 30 - 7x$ ----- (1)	1	
(ii) (1) $\lvert\vec{BC}\rvert = \sqrt{5}\lvert\vec{CA}\rvert$		
$\sqrt{7^2 + 1^2} = \sqrt{5}\sqrt{(3-x)^2 + (y+1)^2}$	1M	
$(3 - x)^2 + (y + 1)^2 = 10$		
$x^2 + y^2 - 6x + 2y = 0$ ----- (2)	~~1A~~	
Subs. (1) int (2)		
$x^2 + (30 - 7x)^2 - 6x + 2(30 - 7x) = 0$	1M	For substitution
$50x^2 - 440x + 960 = 0$	1A	
$x = 4$ or $\frac{24}{5}$	1A	
$x = 4,\ y = 2$		
$x = \frac{24}{5},\ y = \frac{-18}{5}$ rejected $\because y > 0$		
$\therefore x = 4,\ y = 2$	1A	
	7	

(2)	$\vec{CA} = -\vec{i} - 3\vec{j}$		
	$\vec{AB} = -6\vec{i} + 2\vec{j}$		
	$\vec{CA} \cdot \vec{AB} = -(-6) - 3(2) = 0$	1A	Omit dot sign (pp-1)
	$\therefore CA \perp AB$	1	

Alternative solution for (b) (ii) (2)

Slope of $CA = 3$		
Slope of $AB = -\frac{1}{3}$		
Slope of CA . Slope of $AB = -1$	1A	
$\therefore CA \perp AB$	1	

(3)	$\vec{OA} = 3\vec{i} - \vec{j}$	OR $\vec{OA} = 3\vec{i} - \vec{j}$		
	$\vec{OB} = -3\vec{i} + \vec{j}$	$\vec{AB} = -6\vec{i} + 2\vec{j}$	1A	
	$\vec{OA} = -\vec{OB}$	$\vec{OA} = -\frac{1}{2}\vec{AB}$	1A	
	$\therefore$ O lies on AB		1	
			12	

Alternative solution for (b) (ii) (3)

(1)	$\vec{OB} = -3\vec{i} + \vec{j}$	1A	
	Slope of $OB = -\frac{1}{3}$ = slope of OA	1A	
	$\therefore$ O lies on AB	1	
(2)	Equation of AB : $x + 3y = 0$	1A	
	(0, 0) satisfy $x + 3y = 0$	1A	
	$\therefore$ O lies on AB.	1	

$\vec{a} : \vec{b} = k : 1$
pp-1

GA 29

9. (a) $g(x) = -2(x+3)^2 - 5$	1A+1A	1A for $-2(x+3)^2$ 1A for -5
$\because -2(x+3)^2 - 5 \le -5$ for all x		or $(x+3)^2 \ge 0$
$\therefore g(x) < 0$ for all x.	1	
	3	
(b) (i) $(x^2 + 2x - 2) + k(-2x^2 - 12x - 23) = 0$		
$(1 - 2k)x^2 + (2 - 12k)x - (2 + 23k) = 0$		
For equal roots		
$(2 - 12k)^2 + 4(1 - 2k)(2 + 23k) = 0$	1M	
$-40k^2 + 28k + 12 = 0$	1A	
$10k^2 - 7k - 3 = 0$		
$(10k + 3)(k - 1) = 0$		
$k = 1$ or $-\frac{3}{10}$		
$k_1 = 1,\ k_2 = -\frac{3}{10}$	1A+1A	Awarded only if the equation is correct
(ii) $f(x) + k_1 g(x)$		
$= (x^2 + 2x - 2) - (2x^2 + 12x + 23)$		
$= -x^2 - 10x - 25$		
$= -(x+5)^2$	1A	
$\therefore f(x) + g(x) \le 0$ for all x	1	
$f(x) + k_2 g(x)$		
$= (x^2 + 2x - 2) + \frac{3}{10}(2x^2 + 12x + 23)$		
$= \frac{8}{5}(x^2 + \frac{7}{2}x + \frac{49}{16})$		
$= \frac{8}{5}(x + \frac{7}{4})^2$	1A	$\frac{1}{10}(4x+7)^2$
$\therefore f(x) - \frac{3}{10}g(x) \ge 0$ for all x	1	
	8	

(c) $f(x) + g(x) \le 0$		
$f(x) \le -g(x)$	1A	
$\frac{f(x)}{g(x)} \ge -1$ for all x ($\because g(x) < 0$)	1M	accept omitting $g(x) < 0$
(and the equality holds when $x = -5$)		
$\therefore$ Least value $= -1$	1A	
$f(x) - \frac{3}{10} g(x) \ge 0$		
$f(x) \ge \frac{3}{10} g(x)$	1A	
$\frac{f(x)}{g(x)} \le \frac{3}{10}$ for all x ($\because g(x) < 0$)		accept omitting $g(x) < 0$
(and the equality holds when $x = -\frac{7}{4}$)		
$\therefore$ Greatest value $= \frac{3}{10}$	1A	(pp-1 if not specify the greatest and least values)
	5	

10. (a) (i) $t^2 + t + 1 = 0$		
$t = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$	1A	
$= \cos(\pm\frac{2\pi}{3}) + i\sin(\pm\frac{2\pi}{3})$	1A+1A	
(ii) Put $z^3 = t$	1M	
$z^3 = \cos(\pm\frac{2\pi}{3}) + i\sin(\pm\frac{2\pi}{3})$		
$z = \cos\frac{1}{3}(2k\pi \pm \frac{2\pi}{3}) + i\sin\frac{1}{3}(2k\pi \pm \frac{2\pi}{3})$	1A	
$K = -1, 0, 1$	2A	$k = 0,1,2$ (1A only)
OR $z = \cos\theta + i\sin\theta$ where $\theta = \pm\frac{2\pi}{9}, \pm\frac{4\pi}{9}, \pm\frac{8\pi}{9}$	1A+1A+1A	1 mark for every 2 answers.
	7	
(b) (i) $[z - \cos\theta - i\sin\theta][z - \cos(-\theta) - i\sin(-\theta)]$		
$= (z - \cos\theta - i\sin\theta)(z - \cos\theta + i\sin\theta)$	1A	For $\cos(-\theta) = \cos\theta$, $\sin(-\theta) = -\sin\theta$
$= (z - \cos\theta)^2 + \sin^2\theta$	1A	For an expression not involving i
$= z^2 - 2z\cos\theta + 1$	1	
(ii) $z^6 + z^3 + 1$		
$= [z - \cos\frac{2\pi}{9} - i\sin\frac{2\pi}{9}][z - \cos(-\frac{2\pi}{9}) - i\sin(-\frac{2\pi}{9})]$ $[z - \cos\frac{4\pi}{9} - i\sin\frac{4\pi}{9}][z - \cos(-\frac{4\pi}{9}) - i\sin(-\frac{4\pi}{9})]$ $[z - \cos\frac{8\pi}{9} - i\sin\frac{8\pi}{9}][z - \cos(-\frac{8\pi}{9}) - i\sin(-\frac{8\pi}{9})]$		1M
$= (z^2 - 2z\cos\frac{2\pi}{9} + 1)(z^2 - 2z\cos\frac{4\pi}{9} + 1)(z^2 - 2z\cos\frac{8\pi}{9} + 1)$ ...(*)	1	
	5	
(c) Put $z = i$ in (*)	1A	or $z = -i$
$-i = 8i\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}\cos\frac{8\pi}{9}$	1A+1A	1A for L.H.S. 1A for R.H.S.
$\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}\cos\frac{8\pi}{9} = -\frac{1}{8}$	1	
	4	

(ii) $\frac{dp}{dh} = 0$ when $h = 9$ — 1A

when $(6 <) h < 9$, $\frac{dp}{dh} < 0$ — $\frac{d^2p}{dh^2} = \frac{54}{h^{1/2}(h-6)^{5/2}}$

$h > 9$, $\frac{dp}{dh} > 0$ — $\frac{d^2p}{dh^2} > 0$ at $h = 9$

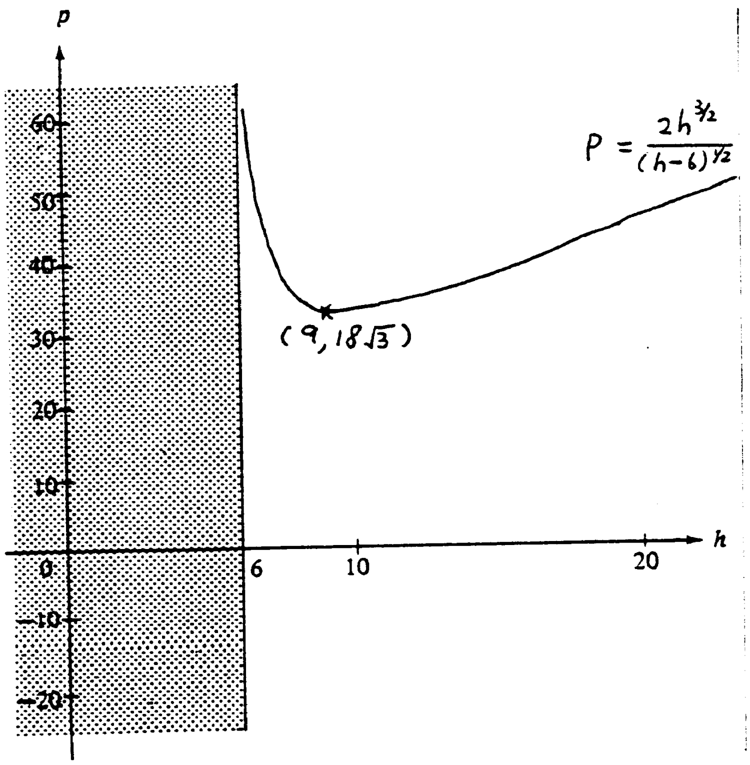
$\therefore p$ is minimum at $h = 9$ — 1M

$p_{min} = \frac{2 \cdot 9^{3/2}}{(9-6)^{1/2}}$

$= 18\sqrt{3}$ — 1A — Accept 31.2 (awarded even if min. is not checked)

6

(d) (i)

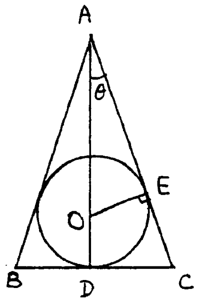


1A — Shape

1A — Labelled minimum point

(ii) From the graph, $p > 18\sqrt{3}$ — 2A

4



11. (a) $\Delta ACD \sim \Delta AOE$ (or $\frac{AE}{OE} = \frac{AD}{DC}$) — 1M

$\frac{\sqrt{(h-3)^2 - 3^2}}{3} = \frac{h}{t}$ — 1A

$t^2 = \frac{9h}{h-6}$ — 1

3

Alternative solution

$\Delta ACD \sim \Delta AOE$ (or $\frac{OE}{AO} = \frac{DC}{AC}$) — 1M

$\frac{t}{\sqrt{h^2 + t^2}} = \frac{3}{h-3}$ — 1A

$t^2(h^2 - 6h + 9) = 9(h^2 + t^2)$

$t^2 = \frac{9h}{h-6}$ — 1

(b) $p = 2t + 2\sqrt{h^2 + t^2}$ — 1A

$= 2\sqrt{\frac{9h}{h-6}} + 2\sqrt{h^2 + \frac{9h}{h-6}}$ — 1A

$= \frac{6h^{1/2} + 2h^{1/2}(h^2 - 6h + 9)^{1/2}}{\sqrt{h-6}}$

$= \frac{6h^{1/2} + 2(h-3)h^{1/2}}{(h-6)^{1/2}}$ ($\because h > 3$)

$= \frac{2h^{3/2}}{(h-6)^{1/2}}$ — 1

3

(c) (i) $\frac{dp}{dh} = \frac{(h-6)^{1/2} 3h^{1/2} - h^{3/2}(h-6)^{-1/2}}{(h-6)}$ — 1M+1A (1M for quotient rule)

$= \frac{3h^{1/2}(h-6) - h^{3/2}}{(h-6)^{3/2}}$

$= \frac{2h^{1/2}(h-9)}{(h-6)^{3/2}}$

$\frac{dp}{dh} > 0$

$\frac{2h^{1/2}(h-9)}{(h-6)^{3/2}} > 0$

$h - 9 > 0$ ($\because h > 6$)
$h > 9$ — 1A

12. (a) (i) $\angle OCP = \theta$		
$CP = \cos\theta$	1A	
Also $CP = 2\cos\phi$	1A	
$\therefore \cos\theta = 2\cos\phi$	1	
(ii) S = area of sector CAB − area of ΔCAB	1M	
$= \frac{1}{2}(2)^2(2\phi) - \frac{1}{2}(2)^2\sin 2\phi$		
$= 4\phi - 2\sin 2\phi$	1	
	5	
(b) (i) $\cos\theta = 2\cos\phi$		
$-\sin\theta = -2\sin\phi \frac{d\phi}{d\theta}$	1A	
$\frac{d\phi}{d\theta} = \frac{\sin\theta}{2\sin\phi}$	1A	
(ii) $\frac{dS}{d\theta} = \frac{dS}{d\phi}\frac{d\phi}{d\theta}$	1M	
$= (4 - 4\cos 2\phi)\frac{d\phi}{d\theta}$	1A	1A for $\frac{ds}{d\phi}$
$= (4 - 4\cos 2\phi)\frac{\sin\theta}{2\sin\phi}$	1A	
$= 4\sin\theta\sin\phi$		
$= 4\sin\theta\sqrt{1 - \frac{1}{4}\cos^2\theta}$	2A	$2\sin\theta\sqrt{4 - \cos^2\theta}$
	7	
(c) $\frac{d\theta}{dt} = -\frac{1}{30}$	1A	
$\frac{dS}{dt} = \frac{dS}{d\theta}\cdot\frac{d\theta}{dt}$	1M	
$= 2\sin\theta\sqrt{4 - \cos^2\theta}\frac{d\theta}{dt}$		
At $\theta = \frac{\pi}{3}$		
$\frac{ds}{dt} = 2\frac{\sqrt{3}}{2}\sqrt{4 - (\frac{1}{2})^2}(-\frac{1}{30})$	1M	for substitution
$= \frac{-\sqrt{5}}{20}$ (per second)	1A	Accept −0.112
	4	

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Solution	Marks	Remarks
1. (a) $(1+x+ax^2)^8 = [1+x(1+ax)]^8$ $= 1 + {}_8C_1x(1+ax) + {}_8C_2x^2(1+ax)^2 + {}_8C_3x^3(1+ax)^3 + \ldots$	1M	For grouping terms. (pp-1) for omitting dots in all expressions
$\therefore k_1 = 8a + 28$	1A	Accept ${}_8C_1a + {}_8C_2$
$k_2 = 56a + 56$	1A	$2\,{}_8C_2 + {}_8C_3$
(b) $k_1 = 8a + 28 = 4$ $a = -3$	1A	
$k_2 = 56(-3) + 56$ $= -112$	1A	
	5	
2. $\int_0^{\pi/2} (\sin x + \cos x)^2\,dx$		
$= \int_0^{\pi/2} (\sin^2 x + 2\sin x\cos x + \cos^2 x)\,dx$	1A	
$= \int_0^{\pi/2} (1 + 2\sin x\cos x)\,dx$ OR $= \int_0^{\pi/2} (1 + \sin 2x)\,dx$	1A	
$= [x + \sin^2 x]_0^{\pi/2}$ OR $= [x - \frac{1}{2}\cos 2x]_0^{\pi/2}$	1A+1A	
$= \frac{\pi}{2} + 1$	1A	Accept 2.57
	5	
3. $\cos 4\theta + \cos 2\theta = \cos\theta$ $2\cos 3\theta\cos\theta = \cos\theta$	1A	
$\cos\theta = 0$ or $\cos 3\theta = \frac{1}{2}$	1A+1A	
$3\theta = 2n\pi \pm \frac{\pi}{3}$		
$\theta = 2n\pi \pm \frac{\pi}{2}$ (or $(2n+1)\frac{\pi}{2}$) ; $\theta = \frac{2n\pi}{3} \pm \frac{\pi}{9}$ (n being any integer.)	1A+1A	$360n° \pm 90°$ (or $(2n+1)90°$), $120n° \pm 20°$
	5	
4. $\left\lvert\frac{4+3k}{\sqrt{(2-k)^2+(1+2k)^2}}\right\rvert = 1$ (or $\frac{4+3k}{\sqrt{(2-k)^2+(1+2k)^2}} = \pm 1$)	1A	Omit absolute sign (pp-1)
$(4+3k)^2 = (2-k)^2 + (1+2k)^2$ $4k^2 + 24k + 11 = 0$		
$k = -\frac{1}{2}$ or $\frac{-11}{2}$	1A+1A	
Equations of lines : $x = 1$	1A	
$3x - 4y + 5 = 0$	1A	$y = \frac{3}{4}x + \frac{5}{4}$
	5	

5. (a) $\frac{dy}{dx} = 4 - 2x$

$y = 4x - x^2 + c$ — 1A

Subs. (1, 0) — 1M

$c = -3$ — 1A

$\therefore y = -x^2 + 4x - 3$

(b) $y = 0$ at $x = 1$ or 3 — 1A

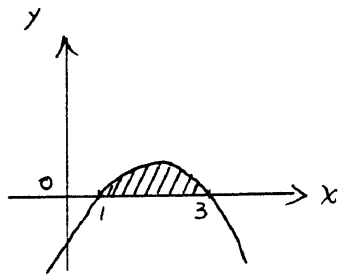
Area $= \int_1^3 (-x^2 + 4x - 3)\,dx$ — 1M

$= [\frac{-x^3}{3} + 2x^2 - 3x]_1^3$ — 1A

$= (-9 + 18 - 9) - (-\frac{1}{3} + 2 - 3)$

$= \frac{4}{3}$ — 1A

7



6. (a) Let M be the mid-point of AB
O be centre of $ABCD$

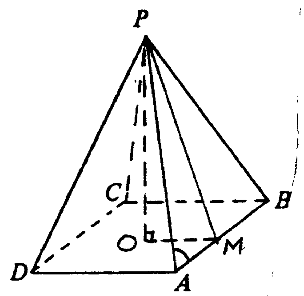
$PM = 2\tan 60°$ — 1A

$= 2\sqrt{3}$ — 1A

$\cos\angle PMO = \frac{OM}{PM}$

$= \frac{2}{2\sqrt{3}}$ — 1M

$\angle PMO = 54.7°$ — 1A



(b) Let X be the point on PA such that $DX \perp PA$, $BX \perp PA$

$BX = 4\sin 60°$

$= 2\sqrt{3}$ — 1A

$OB = \frac{1}{2}\sqrt{4^2 + 4^2}$

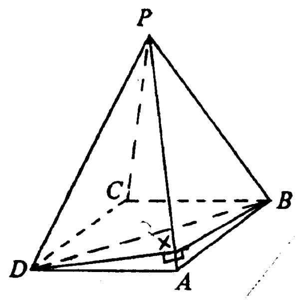
$= 2\sqrt{2}$ — 1A

$\sin\frac{\angle BXD}{2} = \frac{OB}{BX}$

$= \frac{2\sqrt{2}}{2\sqrt{3}}$ — 1M

$\angle BXD = 109.5°$ — 1A

7



Alternative solution for (b)

$BX = DX = 2\sqrt{3}$ — 1A

$BD = 4\sqrt{2}$ — 1A

$\cos\angle BXD = \frac{BX^2 + DX^2 - BD^2}{2BX.DX}$

$= \frac{(2\sqrt{3})^2 + (2\sqrt{3})^2 - (4\sqrt{2})^2}{2(2\sqrt{3})(2\sqrt{3})}$ — 1M — For cosine rule

$= -0.3333$

$\angle BXD = 109.5°$ — 1A

7. (a)	For $n = 1$, L.H.S. $= 1^2 = 1$		
	R.H.S. $= \frac{1}{6}(1)(2)(3) = 1$	1	
	$\therefore$ the statement is true for $n = 1$		
	Assume $1^2 + 2^2 + \ldots + k^2 = \frac{1}{6}k(k+1)(2k+1)$	1	
	(for some +ve integer k)		
	Then $1^2 + 2^2 + \ldots + k^2 + (k+1)^2$		
	$= \frac{1}{6}k(k+1)(2k+1) + (k+1)^2$	1	
	$= \frac{1}{6}(k+1)[k(2k+1) + 6(k+1)]$		
	$= \frac{1}{6}(k+1)(k+2)(2k+3)$	1	
	$\therefore$ the statement is also true for $n = k + 1$ (if it is true for $n = k$)		
	$\therefore$ (By the principle of mathematical induction) the statement is true for all +ve integers n	1	
(b)	$1\times2 + 2\times3 + \ldots + n(n+1)$		
	$= 1\times(1+1) + 2\times(2+1) + \ldots + n\times(n+1)$		
	$= (1^2 + 2^2 + \ldots + n^2) + (1 + 2 + \ldots + n)$.	1A	
	$= \frac{1}{6}n(n+1)(2n+1) + \frac{1}{2}n(n+1)$	1A	
	$= \frac{1}{3}n(n+1)(n+2)$	1A	$\frac{1}{3}(n^3 + 3n^2 + 2n)$
		8	

Solution	Marks	Remarks
8. (a) (i) $\tan x = k\tan y$		
$\sin x\cos y = k\cos x\sin y$	1A	
$\sin(x+y) = \sin x\cos y + \cos x\sin y$	1A	For addition formula
$= k\cos x\sin y + \cos x\sin y$		
$= (k+1)\cos x\sin y$	1	
(ii) $(k+1)\sin(x-y) = (k+1)(\sin x\cos y - \cos x\sin y)$	1A	For expanding $\sin(x-y)$
$= (k+1)(k\cos x\sin y - \cos x\sin y)$		
$= (k+1)(k-1)\cos x\sin y$		or $(k^2-1)\cos x\sin y$
$= (k-1)\sin(x+y)$	1M+1	1M for using result of (a) (i)
	6	
(b) (i) $\tan(\theta+10°) = k\tan(\theta-20°)$		
Using (a) (ii)		
$(k+1)\sin 30° = (k-1)\sin(2\theta-10°)$	1A	
$\sin(2\theta-10°) = \dfrac{k+1}{2(k-1)}$	1	
(ii) $\left\vert\dfrac{k+1}{2(k-1)}\right\vert \le 1$	2A	Do not accept < 1
$(k+1)^2 \le 4(k-1)^2$	1A	
$3k^2 - 10k + 3 \ge 0$		
$(3k-1)(k-3) \ge 0$	1A	
$k \ge 3$ or $k \le \frac{1}{3}$	1A	
	7	
Alternative solution		
$-1 \le \dfrac{k+1}{2(k-1)} \le 1$	1A+1A	
$\dfrac{k+1}{2(k-1)} \le 1$ and $\dfrac{k+1}{2(k-1)} \ge -1$		
$\dfrac{k+1}{2(k-1)} - 1 \le 0$; $\dfrac{k+1}{2(k-1)} + 1 \ge 0$		
$\dfrac{k-3}{k-1} \ge 0$; $\dfrac{3k-1}{k-1} \ge 0$		
$(k \ge 3$ or $k < 1)$ and $(k > 1$ or $k \le \frac{1}{3})$	1A+1A	Do not accept $k \le 1$
$\therefore k \ge 3$ or $k \le \frac{1}{3}$	1A	
(c) Subs. $k = -2$ into (b) (i)		
$\sin(2\theta-10°) = \frac{1}{6}$	1A	
$2\theta - 10° = 180n° + (-1)^n 9.6°$	1A	
$\theta = 90n° + 5° + (-1)^n 4.8°$	1A	
(n being any integer.)		
	3	

$\sin A\cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

		Marks	Remarks
9. (a)	$x = 1 + s\cos\theta$	1A	
	$y = 2 + s\sin\theta$	1A	
		2	
(b)	Subs. $x = 1 + s\cos\theta$, $y = 2 + s\sin\theta$ into C,		
	$(1 + s\cos\theta)^2 + (2 + s\sin\theta)^2 - 6(1 + s\cos\theta) - 10(2 + s\sin\theta) + 30 = 0$	1M	or Subs. $x = 1 + s_1\cos\theta$, $y = 2 + s_1\sin\theta$
	$s^2 - (4\cos\theta + 6\sin\theta)s + 9 = 0$	1	
	Since L and C intersects at H and K, so s_1 and s_2 are the roots of the above equation.	1	Similarly for subs. $x = 1 + s_2\cos\theta$, $y = 2 + s_2\sin\theta$
		3	
(c)	$HK^2 = (s_2 - s_1)^2$	1A	
	$= (s_1 + s_2)^2 - 4s_1s_2$	1A	
	$= (4\cos\theta + 6\sin\theta)^2 - 36$	1A	
	$= 16\cos^2\theta + 48\sin\theta\cos\theta + 36\sin^2\theta - 36$		
	$= 48\sin\theta\cos\theta - 20\cos^2\theta$	1	
		4	
(d)	$HK = 0$	1M	
	$48\sin\theta\cos\theta - 20\cos^2\theta = 0$		
	$\cos\theta = 0$ or $\tan\theta = \frac{5}{12}$	1A+1A	
	Equations of tangent :		
	$x = 1$	2A	
	and $\frac{y-2}{x-1} = \frac{5}{12}$	1M	
	$5x - 12y + 19 = 0$	1A	$y = \frac{5}{12}x + \frac{19}{12}$
		7	

10. (a)	$\frac{x}{8} - \frac{2y}{9}\frac{dy}{dx} = 0$ or $\frac{xx_1}{16} - \frac{yy_1}{9} = 1$	1A	~~For LHS~~ only
	$\frac{dy}{dx} = \frac{9x}{16y}$ or slope $= \frac{9x_1}{16y_1}$	1A	
	$= \frac{5}{4}$	1M	
	$y = \frac{9x}{20}$		
	$\frac{x^2}{16} - \frac{1}{9}(\frac{9x}{20})^2 = 1$	1M	For substitution
	$x = \pm 5$	1A	
	The points are $(5, \frac{9}{4})$ and $(-5, -\frac{9}{4})$.	1A+1A	
		7	

Alternative solution

(a)	$y = \frac{5}{4}x + c$	1A	
	$9x^2 - 16(\frac{5}{4}x + c)^2 = 144$	1M	For substitution
	$2x^2 + 5cx + 2(c^2 + 9) = 0$		
	$25c^2 - 16c^2 - 144 = 0$	1M	
	$c = \pm 4$	1A	
	$2x^2 \pm 20x + 50 = 0$		
	$x = \pm 5$	1A	
	The points are $(5, \frac{9}{4})$ and $(-5, \frac{-9}{4})$.	1A+1A	

(b) $\frac{x^2}{16} - \frac{1}{9}(\frac{5}{4}x + c)^2 = 1$ — ~~1M~~ — For substitution

$2x^2 + 5cx + 2(c^2 + 9) = 0$

$x = \frac{x_1 + x_2}{2}$ — 1M

$x = -\frac{5c}{4}$ — 1A

$y = \frac{-9c}{16}$ — 1A

4

<u>Alternative solution</u>

(b) $x = \frac{-5c \pm \sqrt{25c^2 - 16(c^2 + 9)}}{4}$ — ~~1M~~

$x = \frac{1}{2}(\frac{-5c + \sqrt{25c^2 - 16(c^2 + 9)}}{4} + \frac{-5c - \sqrt{25c^2 - 16(c^2 + 9)}}{4})$ — 1M — For $x = \frac{x_1 + x_2}{2}$

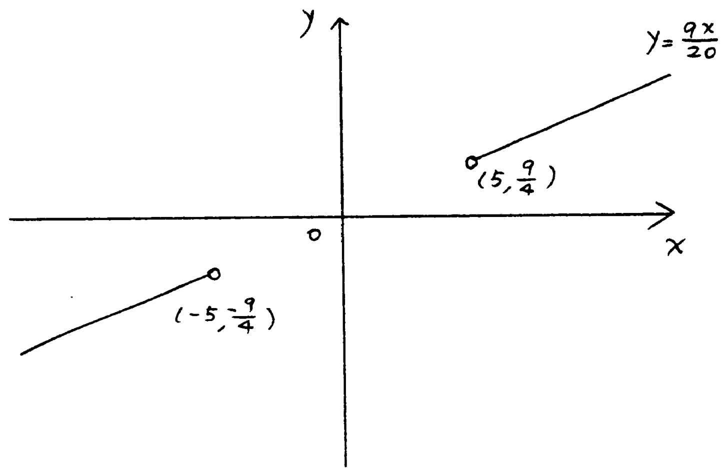
$= \frac{-5c}{4}$ — 1A

$y = \frac{-9c}{16}$ — 1A

(c) Eliminate c from $x = \frac{-5c}{4}$ and $y = \frac{-9c}{16}$, — 1M

Equation of locus : $y = \frac{9x}{20}$ $(x > 5 \text{ or } x < -5)$ — 1A — $(x > 5 \text{ or } x < -5)$ can be omitted.

(Note : The 2 limiting end-points can be included).



1A — straight line

2A — End points

5

11. (a) volume $= \int_0^4 \pi x^2 dy$	1A	
$= \int_0^4 \pi 4y dy$	1A	
$= [2\pi y^2]_0^4$		
$= 32\pi$	1A	
	3	
(b) (i) mass $= \int_0^4 \pi(16y - 3y^2) dy$	1A	
$= \pi[8y^2 - y^3]_0^4$		
$= 64\pi$	1A	
(ii) (1) $\int_0^h \pi x^2 dy = 16\pi$	1M	
$[2\pi y^2]_0^h = 16\pi$		
$2\pi h^2 = 16\pi$	1A	For $2\pi h^2$ only
$h = 2\sqrt{2}$	1A	Accept 2.83
(2) Mass of lower part $= \int_0^{2\sqrt{2}} \pi(16y - 3y^2) dy$	1M	
$= \pi[8y^2 - y^3]_0^{2\sqrt{2}}$		
$= (64 - 16\sqrt{2})\pi$	1A	Accept 41.4π, 130
Mass of upper part $= 64\pi - (64 - 16\sqrt{2})\pi$		
$= 16\sqrt{2}\pi$	1A	Accept 22.6π, 71.1
Ratio $= (64 - 16\sqrt{2})\pi : 16\sqrt{2}\pi$		
$= (2\sqrt{2} - 1) : 1$	1A	Accept 1.83 : 1, 1 : 0.546
	9	
(c) Volume of paint		
$= \pi\int_{-t}^4 4(y + t) dy - 32\pi$	1A+1M	1A for first term, 1M for difference
$= \pi[2(y + t)^2]_{-t}^4 - 32\pi$		
$= 2\pi(4 + t)^2 - 32\pi$		
$= 16\pi t + 2\pi t^2$	1A	
$\doteq 16\pi t$ ($\because$ t is small)	1	
	4	

12. (a) $y = (1+x)^{m+1}(1-x)^n$

$\frac{dy}{dx} = (m+1)(1+x)^m(1-x)^n - n(1+x)^{m+1}(1-x)^{n-1}$ — 1A+1A

$\therefore (m+1)\int(1+x)^m(1-x)^n dx$

$= (1+x)^{m+1}(1-x)^n + n\int(1+x)^{m+1}(1-x)^{n-1}dx$ — 1

3

(b) From (a),

$(m+1)\int_{-1}^{1}(1+x)^m(1-x)^n dx$

$= [(1+x)^{m+1}(1-x)^n]_{-1}^{1} + n\int_{-1}^{1}(1+x)^{m+1}(1-x)^{n-1}dx$ — 1A

$= n\int_{-1}^{1}(1+x)^{m+1}(1-x)^{n-1}dx$ — 1A

$\therefore \int_{-1}^{1}(1+x)^m(1-x)^n dx = \frac{n}{m+1}\int_{-1}^{1}(1+x)^{m+1}(1-x)^{n-1}dx$ — 1

3

(c) $\int_{-1}^{1}(1+x)^8 dx = [\frac{1}{9}(1+x)^9]_{-1}^{1}$ — 1A — Expansion not accepted

$= \frac{512}{9}$ — 1A — Accept $\frac{2^9}{9}$, 56.9

2

(d) $x = \tan\theta$

$\cos^2\theta = \frac{1}{1+x^2}$ — 1A — Accept $\cos\theta = \frac{1}{\sqrt{1+x^2}}$

$\cos 2\theta = \frac{1-x^2}{1+x^2}$ — 1A

$dx = \sec^2\theta d\theta$ — 1A

$d\theta = \frac{dx}{\sec^2\theta} = \frac{dx}{1+x^2}$ — 1A

$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{\cos^2 2\theta(1+\tan\theta)^4}{\cos^6\theta}d\theta$

$= \int_{-1}^{1}\frac{(\frac{1-x^2}{1+x^2})^2(1+x)^4}{(\frac{1}{1+x^2})^3}\frac{dx}{1+x^2}$ — ~~(pp-1) for not~~ changing the limits of integration

$= \int_{-1}^{1}(1-x^2)^2(1+x)^4 dx$

$= \int_{-1}^{1}(1+x)^6(1-x)^2 dx$ — 1

Alternative solution		
$x = \tan\theta$ $dx = \sec^2\theta d\theta$	1A	
$\int_{-1}^{1}(1+x)^6(1-x)^2dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}(1+\tan\theta)^6(1-\tan\theta)^2\sec^2\theta d\theta$	1A	(pp-1) for not changing the limits of integration
$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{(1+\tan\theta)^4}{\cos^2\theta}(1-\tan^2\theta)^2 d\theta$	1A	
$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{(1+\tan\theta)^4}{\cos^2\theta}\frac{(\cos^2\theta-\sin^2\theta)^2}{\cos^4\theta}d\theta$	1A	
$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{\cos^2 2\theta(1+\tan\theta)^4}{\cos^6\theta}d\theta$	1	

$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{\cos^2 2\theta(1+\tan\theta)^4}{\cos^6\theta}d\theta$

$= \int_{-1}^{1}(1+x)^6(1-x)^2 dx$

$= \frac{2}{7}\int_{-1}^{1}(1+x)^7(1-x)dx$ 1A

$= \frac{2}{7}\cdot\frac{1}{8}\int_{-1}^{1}(1+x)^8 dx$ 1A

$= \frac{2}{7}\cdot\frac{1}{8}\cdot\frac{512}{9}$

$= \frac{128}{63}$ 1A Accept $\frac{2^7}{63}$, 2.03

8