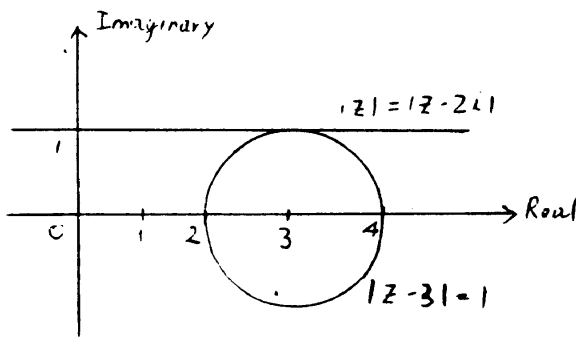


1990 HKCE Additional Mathematics I

Solutions	Marks	Remarks
<p>1. $f'(x) = \sqrt{x^2+k} \frac{d}{dx} \sin 2x + \sin 2x \frac{d}{dx} \sqrt{x^2+k}$</p> $= 2\sqrt{x^2+k} \cos 2x + \frac{x \sin 2x}{\sqrt{x^2+k}}$ <p>$f'(0) = 1$</p> <p>$2\sqrt{k} = 1$</p> <p>$k = \frac{1}{4}$</p>	<p>1M</p> <p>1A+1A</p> <p>1M</p> <p><u>1A</u></p> <p style="text-align: center;">5</p>	<p>For product rule.</p> <p>For substituting $x = 0$ in $f'(x)$</p>
<p>2. (a) $\vec{AB} = \vec{OB} - \vec{OA}$</p> $= -\hat{i} + 2\hat{j}$ <p>$\vec{OP} = \vec{OA} + t\vec{AB}$</p> $= \vec{OA} + t\vec{AB}$ $= -t\hat{i} + (2t+5)\hat{j}$	<p>1A</p> <p>1M</p> <p>1A</p>	<p>Omit vector sign (pp - 1)</p> <p>$\vec{OB} + \vec{BP}$ acceptable</p>
<p><u>Alt. Solution</u></p> <p>$\vec{AP} : \vec{PB} = t : 1 - t$</p> $\vec{OP} = \frac{t\vec{OB} + (1-t)\vec{OA}}{t + (1-t)}$ $= t(-\hat{i} + 7\hat{j}) + (1-t)(5\hat{j})$ $= -t\hat{i} + (2t+5)\hat{j}$	<p>1A</p> <p>1M</p> <p>1A</p>	<p>$\vec{AP} : \vec{PB} = t : 1 - t$ (pp - 1)</p>
<p>(b) (i) $\vec{OP} \cdot \vec{AB} = 0$</p> $-t(-1) + (2t+5)(2) = 0$ <p>$t = -2$</p> <p>(ii) $\vec{OP} = 2\hat{i} + \hat{j}$</p>	<p>1M</p> <p>1A</p> <p><u>1A</u></p> <p style="text-align: center;">6</p>	<p>Omit dot sign (pp - 1)</p>

Solutions	Marks	Remarks
4. (a) $\begin{cases} \alpha + \beta = k+2 \\ \alpha\beta = k \end{cases} (*)$ (b) $(\alpha + 1)(\beta + 2) = 4$ ----- (1) $\alpha\beta + (\alpha + \beta) + \alpha + 2 = 4$ $k + k + 2 + \alpha + 2 = 4$ $ = -2k$ Subs. into the equation $(-2k)^2 - (k + 2)(-2k) + k = 0$ $6k^2 + 5k = 0$ $k = 0 \text{ or } -5/6$	1A 1M 1 1M 1A+1A	For eliminating β .
<p><u>Alt. Solution 1</u></p> <p>Subs. $\alpha = -2k$ into (*)</p> $\begin{cases} -2k + \beta = k + 2 \\ -2k\beta = k \end{cases}$ <p>$k = 0$ or $\beta = -\frac{1}{2}$ $-2k - \frac{1}{2} = k + 2$ $k = \frac{-5}{6}$</p> <p><u>Alt. Solution 2</u></p> <p>Subs. $\alpha = -2k, \beta = 3k + 2$ into (1)</p> $(-2k + 1)(3k + 2 + 2) = 4$ $6k^2 + 5k = 0$ $k = 0 \text{ or } \frac{-5}{6}$	1M 1A 1A 1M 1A+1A	
<div style="border: 1px solid black; padding: 2px; display: inline-block;">6</div>		

5.  <p style="margin-top: 10px;">Circle and line touch at correct point. The intersection is the complex no. $3 + i$</p>	<p>Unit circle</p> <p>Correct centre</p> <p>Horizontal straight line</p> <p>Position correct</p>	1A 1A 1A 1A 1A 1A 6	<p>Axes or curves not labelled (pp - 1)</p> <p>Separate diagrams (pp - 1)</p> <p>Solve $\begin{cases} (x - 3)^2 + y^2 = 1 \\ y = 1 \end{cases}$ 1A</p> <p>Ans.: $3 + i$ 1A</p>
---	--	---	--

Solutions	Marks	Remarks
<p>6. $(x + 2)^2 - 8 x + 2 + 15 \geq 0$</p> <p>$x + 2 ^2 - 8 x + 2 + 15 \geq 0$</p> <p>$(x + 2 - 3)(x + 2 - 5) \geq 0$</p> <p>$x + 2 \geq 5 \text{ or } x + 2 \leq 3$</p> <p>$(x \geq 3 \text{ or } x \leq -7) \text{ or } -5 \leq x \leq 1$</p>	<p>2M</p> <p>1A</p> <p>1A</p> <p><u>1A+1A</u></p> <p style="text-align: center;">6</p>	<p>Omit 'or' (pp - 1) use 'and' (no mark) use ',' (pp - 1)</p>
<p><u>Alt. Solution</u></p> <p>Case (i) $x \geq -2$ (or $x > -2$)</p> <p>$(x + 2)^2 - 8(x + 2) + 15 \geq 0$</p> <p>$(x - 1)(x - 3) \geq 0$</p> <p>$x \geq 3 \text{ or } x \leq 1$</p> <p>Since $x \geq -2$ $x \geq 3 \text{ or } -2 \leq x \leq 1$</p> <p>Case (ii) $x < -2$ (or $x \leq -2$)</p> <p>$(x + 2)^2 + 8(x + 2) + 15 \geq 0$</p> <p>$(x + 5)(x + 7) \geq 0$</p> <p>$x \geq -5 \text{ or } x \leq -7$</p> <p>Since $x < -2$, $x \leq -7 \text{ or } -2 > x \geq -5$</p> <p>Combining the 2 cases,</p> <p>$x \geq 3 \text{ or } x \leq -7 \text{ or } -5 \leq x \leq 1$</p>	<p>1M</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p>	<p><u>Notes :</u></p> <p>(1) $x \geq -2$, $x \leq -2$ (deduct no mark)</p> <p>(2) Solve without stating range of x (no mark)</p>
<p>7. $2x + 4y + 4x \frac{dy}{dx} + 10y \frac{dy}{dx} = 0$</p> <p>$\frac{dy}{dx} = \frac{-(x + 2y)}{2x + 5y}$</p> <p>$\frac{-(x + 2y)}{2x + 5y} = \frac{-1}{2}$</p> <p>$y = 0$</p> <p>Subs. into the equation,</p> <p>$x = \pm 1$</p> <p>The equations are</p> <p>$y = \frac{-1}{2}x + \frac{1}{2}$ and $y = \frac{-1}{2}x - \frac{1}{2}$</p>	<p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1A</p> <p><u>1A+1A</u></p> <p style="text-align: center;">7</p>	<p>For implicit differentiation</p> <p>$x + 2y + 1 = 0$</p> <p>$x + 2y - 1 = 0$</p>

	Solutions	Marks	Remarks
8. (a) (i)	$\vec{x} \cdot \vec{z} = \vec{x} \vec{z} \cos\theta$ $= \vec{z} \cos\theta$	1A	Omit vector sign (pp - 1)
	$\vec{y} \cdot \vec{z} = \vec{y} \vec{z} \cos\theta$ $= \vec{z} \cos\theta$	1A	Omit dot sign (pp - 1)
	$\vec{x} \cdot \vec{z} = \vec{y} \cdot \vec{z}$	1	
(ii)	$\vec{x} \cdot \vec{z} = \vec{x} \cdot (m\vec{x} + n\vec{y})$ $= m\vec{x} \cdot \vec{x} + n\vec{x} \cdot \vec{y}$ $= m + n\cos 2\theta$	1A	
	$\vec{y} \cdot \vec{z} = \vec{y} \cdot (m\vec{x} + n\vec{y})$ $= m\cos 2\theta + n\vec{y} \cdot \vec{y}$ $= m\cos 2\theta + n$	1A	
	From (i), $m + n\cos 2\theta = m\cos 2\theta + n$	1M	
	$(m - n)(1 - \cos 2\theta) = 0$	1A	Accept $(m - n)$
	$\therefore m = n \quad (\because \cos 2\theta \neq 1)$	1	$(1 - \vec{x} \cdot \vec{y}) = 0$ Accept omitting $\cos 2\theta \neq 1$
		<u>8</u>	
(b) (i)	$\vec{OC} = \frac{\lambda(b\vec{v}) + (a\vec{u})}{1 + \lambda}$	1A	
(ii)	Using (a) (ii)		
	$\frac{a}{\lambda + 1} = \frac{b\lambda}{\lambda + 1}$	1M	
	$\lambda = \frac{a}{b}$	1	
(iii)	$ \vec{OA} = \sqrt{3^2 + 4^2} = 5$	1A	$\vec{OA} = 5$ (pp - 1)
	$\frac{AC}{CB} = \frac{5}{25/3}$	1M	
	$= \frac{3}{5}$	1A	
	$\vec{OC} = \frac{\frac{3}{5}(\frac{25}{3}\hat{i}) + (3\hat{i} + 4\hat{j})}{\frac{3}{5} + 1}$	1M	
	$= 5\hat{i} + \frac{5}{2}\hat{j}$	1A	
		<u>8</u>	

Solutions	Marks	Remarks
9. (a) (i) $f(x) = x^2 + 4x + 1$ $= (x + 2)^2 - 3$ Vertex of C_1 is $(-2, -3)$	1A 1A	
(ii) $x^2 + 4x + 1 = 0$ $x = -2 \pm \sqrt{3}$ $PQ = (-2 + \sqrt{3}) - (-2 - \sqrt{3})$ $= 2\sqrt{3}$	1A 1M 1A	Answer in decimal - no mark For subtraction Accept $PQ = \sqrt{12}$
<u>Alt. Solution</u> $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$ $= (-4)^2 - 4 = 12$ $PQ = \alpha - \beta = 2\sqrt{3}$	1M 1A 1A	Accept $PQ = \alpha - \beta$
<hr style="width: 10%; margin: auto;"/> 5 <hr style="width: 10%; margin: auto;"/>		
(b) (i) Vertex of C_2 is $(-2, -3 - m)$ $g(x) = (x + 2)^2 - (3 + m)$	1M 1A	$x^2 + 4x + 1 - m$
(ii) $x^2 + 4x + 1 - m = 0$ $x = -2 \pm \sqrt{m + 3}$ $P'Q' = 2\sqrt{m + 3}$	1A 1A	Accept $P'Q' = \sqrt{4m + 12}$
<u>Alt. Solution</u> $(\alpha' - \beta')^2 = 4m + 12$ $P'Q' = \alpha' - \beta' = 2\sqrt{m + 3}$	1A 1A	
(iii) $2\sqrt{m + 3} = 2(2\sqrt{3})$ $m = 9$	1A 1A <hr style="width: 10%; margin: auto;"/> 6 <hr style="width: 10%; margin: auto;"/>	
(c) (i) Vertex of C_3 is $(-2 + n, -3)$ $h(x) = (x + 2 - n)^2 - 3$	1M 1A	
(ii) $h(0) = 0$ $0 = (2 - n)^2 - 3$ $n = 2 \pm \sqrt{3}$	1M 1A+1A <hr style="width: 10%; margin: auto;"/> 5 <hr style="width: 10%; margin: auto;"/>	3.73, 0.268

RESTRICTED 内部文件

Solutions	Marks	Remarks
10. (a) (i) $\frac{dy}{dx} = \frac{2(x^2 + 2) - 2x(2x + 1)}{(x^2 + 2)^2}$ $= \frac{-2(x^2 + x - 2)}{(x^2 + 2)^2}$ $\frac{-2(x^2 + x - 2)}{(x^2 + 2)^2} < 0$ $x^2 + x - 2 > 0$ $x > 1 \quad \text{or} \quad x < -2$	1A 1M 1A 1A	≤ 0 , no mark.
(ii) $\frac{-2(x^2 + x - 2)}{(x^2 + 2)^2} = 0$ $x = 1 \quad \text{or} \quad -2$ $x = 1, y = 1 \quad (1, 1) \text{ is a maximum point.}$ $x = -2, y = \frac{-1}{2} \quad (-2, \frac{-1}{2}) \text{ is a minimum point.}$	1M 1A 1A $\frac{1A}{8}$	
(b) Curve C_1 : Shape Intercepts End points Turning points	1A 1A 1A $\frac{1A}{4}$	Curve not labelled but position correct - deduct 1 mark only. Pure plotting without part (a) - no mark.
(c) Curve C_2 : Shape Intercepts End points Turning points $(-2, \frac{1}{2})$, $(1, 0)$	1A 1A 1A $\frac{1A}{4}$	

Candidate Number

Centre Number

Seat Number

Total Marks on this page

10. If you attempt Question 10, fill in the details in the first three boxes above and tie this sheet into your answer book.

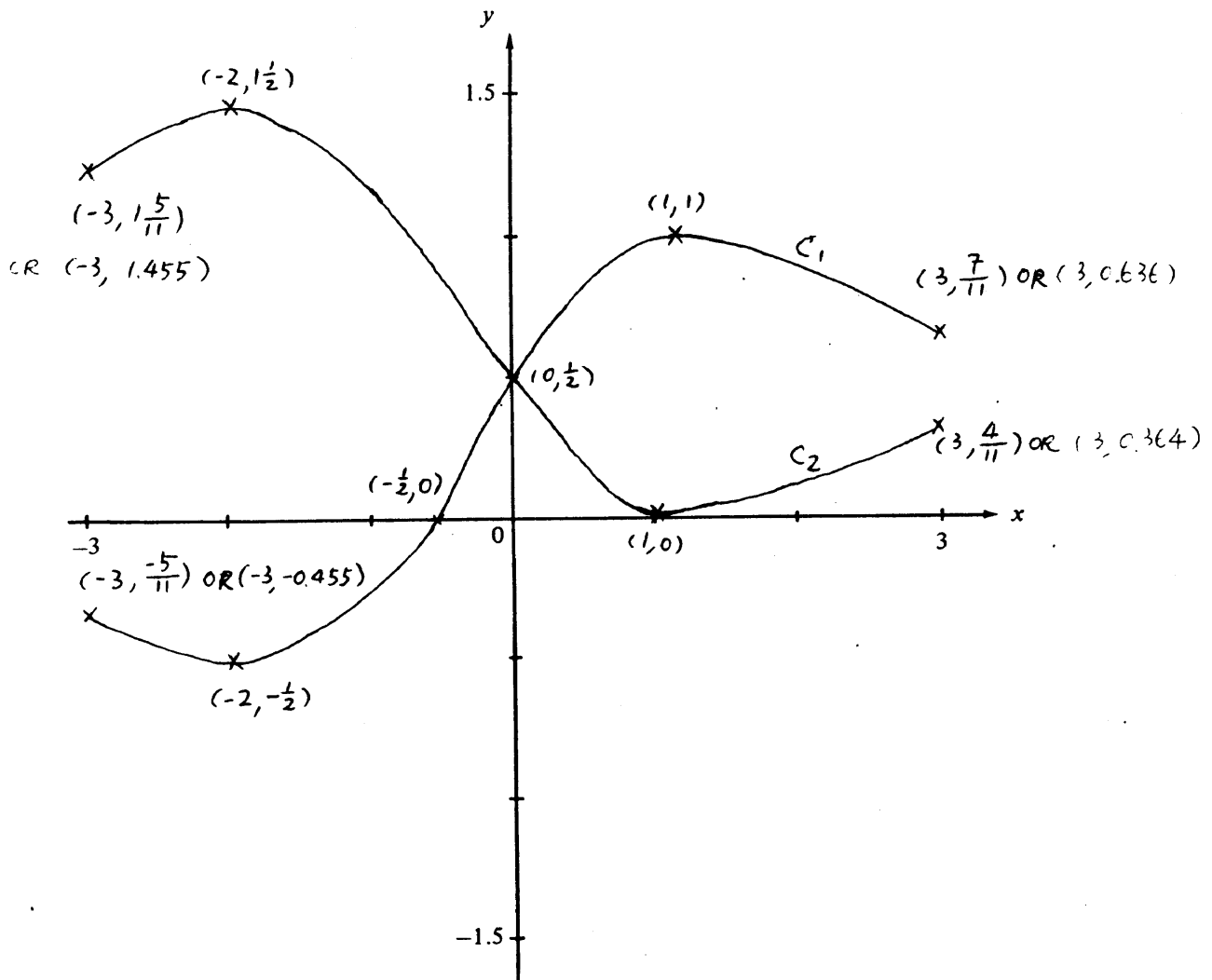


Figure 3

Solutions	Marks	Remarks
11. (a) By Sine Law,		
$\frac{x}{\sin(\pi - \frac{2\pi}{3} - \theta)} = \frac{3}{\sin \frac{2\pi}{3}}$	1M	
$x = 2\sqrt{3}\sin(\frac{\pi}{3} - \theta)$	$\frac{1A}{2}$	$x = 3\cos\theta - \sqrt{3}\sin\theta$
(b) $S = \frac{1}{2}3x\sin\theta$	1A	
$= 3\sqrt{3}\sin(\frac{\pi}{3} - \theta)\sin\theta$	1A	$S = \frac{9}{2}\sin\theta\cos\theta - \frac{3\sqrt{3}}{2}\sin^2\theta$ $= -\frac{3\sqrt{3}}{4} + \frac{3\sqrt{3}}{2}\cos(\frac{\pi}{3} - 2\theta)$
$\frac{dS}{d\theta} = 3\sqrt{3}[\cos\theta\sin(\frac{\pi}{3} - \theta) - \sin\theta\cos(\frac{\pi}{3} - \theta)]$ $= 3\sqrt{3}\sin(\frac{\pi}{3} - 2\theta)$	1	
$\frac{dS}{d\theta} = 0 \text{ when } \theta = \frac{\pi}{6} \quad (\because 0 \leq \theta \leq \frac{\pi}{3})$	1M+1A	Accept omitting $0 \leq \theta \leq \frac{\pi}{3}$
$\frac{d^2S}{d\theta^2} = -6\sqrt{3}\cos(\frac{\pi}{3} - 2\theta)$	1A	
$\left. \frac{d^2S}{d\theta^2} \right _{\theta = \frac{\pi}{6}} = -6\sqrt{3} \quad \therefore \text{max.}$	1M	Awarded only when the 2nd derivative is correct.
<u>Alt. Solution for checking maximum</u>		
$\frac{dS}{d\theta} > 0 \text{ for } 0 < \theta < \frac{\pi}{6}$	1A	for correct ranges of θ
$\frac{dS}{d\theta} < 0 \text{ for } \frac{\pi}{6} < \theta < \frac{\pi}{3}$		
$\theta = \frac{\pi}{6} \text{ is a maximum}$	1M	for slope change from +ve to -ve.
$S_{\max} = 3\sqrt{3}\sin(\frac{\pi}{3} - \frac{\pi}{6})\sin\frac{\pi}{6}$ $= \frac{3\sqrt{3}}{4}$	$\frac{1A}{8}$	Only awarded if if max. is checked.

Solutions	Marks	Remarks
<p>(c) (i) $x = 2\sqrt{3}\sin(\frac{\pi}{3} - \theta)$</p> <p>$\frac{dx}{dt} = -2\sqrt{3}\cos(\frac{\pi}{3} - \theta)\frac{d\theta}{dt}$</p> <p>Since $\frac{dx}{dt} = -\frac{\sqrt{3}}{3}$</p> <p>$\frac{d\theta}{dt} = \frac{1}{6\cos(\frac{\pi}{3} - \theta)}$</p>	<p>1A</p> <p>1A</p> <p>1</p>	<p>Omit -ve sign (no mark)</p>
<p>(ii) $\therefore 0 \leq \theta \leq \frac{\pi}{3}$</p> <p>$\frac{1}{2} \leq \cos(\frac{\pi}{3} - \theta) \leq 1$</p>	<p>1A</p> <p>1A</p>	
<p>\therefore greatest value of $\frac{d\theta}{dt} = \frac{1}{3}$</p> <p>least value of $\frac{d\theta}{dt} = \frac{1}{6}$</p>	<p>} 1A</p> <hr style="width: 50%; margin: 0 auto;"/> <p style="text-align: center;">6</p>	

RESTRICTED 内部文件

Solutions	Marks	Remarks
12. (a) Let $z = x + yi$		
(i) $z\bar{z} = (x + yi)(x - yi) = x^2 + y^2 \quad \therefore, \text{real}$	1	
(ii) $z + \bar{z} = (x + yi) + (x - yi) = 2x = 2\text{Re}(z)$	$\frac{1}{2}$	
	<hr style="width: 100%; border: 0.5px solid black;"/>	
(b) (i) (1) By (a) (ii)		
$\text{Re}(p\bar{r}) = \frac{1}{2}(p\bar{r} + \overline{p\bar{r}})$	1A	
$= \frac{1}{2}(p\bar{r} + \bar{p}r)$	1A	
$= 0$	1	
(2) $\text{Re}\left(\frac{p}{r}\right) = \frac{1}{2}\left(\frac{p}{r} + \overline{\left(\frac{p}{r}\right)}\right)$	1A	$\text{Re}\left(\frac{p}{r}\right) = \text{Re}\left(\frac{p\bar{r}}{r\bar{r}}\right)$ 1A
$= \frac{1}{2}\left(\frac{p}{r} + \frac{\bar{p}}{\bar{r}}\right)$		$= \frac{\text{Re}(p\bar{r})}{r\bar{r}}$ 1A
$= \frac{1}{2} \frac{p\bar{r} + \bar{p}r}{r\bar{r}}$	1A	$= 0$ 1
$= 0$	1	
<p><u>Alt. Solution</u></p> <p>Let $p = a + bi, r = c + di$</p> <p>(1) $p\bar{r} + \bar{p}r = 0$</p> <p style="padding-left: 40px;">$(a + bi)(c - di) + (a - bi)(c + di) = 0$</p> <p style="padding-left: 40px;">$ac + bd = 0$</p> <p style="padding-left: 40px;">$\text{Re}(p\bar{r}) = ac + bd$</p> <p style="padding-left: 40px;">$= 0$</p> <p>(2) $\frac{p}{r} = \frac{a + bi}{c + di} \cdot \frac{c - di}{c - di}$</p> <p style="padding-left: 40px;">$= \frac{(ac + bd) + (bc - ad)i}{c^2 + d^2}$</p> <p style="padding-left: 40px;">$\text{Re}\left(\frac{p}{r}\right) = \frac{ac + bd}{c^2 + d^2}$</p> <p style="padding-left: 40px;">$= 0$</p>		

RESTRICTED 内部文件

Solutions	Marks	Remarks
<p>(b) (ii) <u>Method 1</u></p> <p>$\therefore \operatorname{Re}\left(\frac{p}{r}\right) = 0$</p> <p>$\arg\left(\frac{p}{r}\right) = \pm\frac{\pi}{2}$</p> <p>$\arg p - \arg r = \pm\frac{\pi}{2} \text{ or } \pm\frac{3\pi}{2}$</p> <p>$\therefore OA \perp OC$</p> <p>$\therefore OABC \text{ is a rectangle}$</p>	<p>1A</p> <p>1A</p> <p>1</p>	<p>Accept omitting \pm sign</p> <p>Accept omitting $\pm \frac{3\pi}{2}$</p>
<p><u>Method 2</u></p> <p>$AC ^2 = (p - r)(\overline{p - r})$ $= p\bar{p} - p\bar{r} - \bar{p}r + r\bar{r}$ $= p\bar{p} + r\bar{r}$</p> <p>$OB ^2 = q\bar{q} = (p + r)(\overline{p + r}) = p\bar{p} + r\bar{r}$</p> <p>$AC = OB$</p> <p>$\therefore OABC \text{ is a rectangle}$</p>	<p>1A</p> <p>1A</p> <p>1</p>	
<p><u>Alt. Solution</u></p> <p><u>Method 1</u></p> <p>Slope of $OC = \frac{d}{c}$ ($p = a + bi, r = c + di$)</p> <p>Slope of $OA = \frac{b}{a}$</p> <p>Product of slope = $\frac{d}{c} \cdot \frac{b}{a}$ $= \frac{-ac}{ac} \text{ (from (1))}$ $= -1$</p> <p>$OC \perp OA \therefore OABC \text{ is a rectangle}$</p> <p><u>Method 2</u></p> <p>$OA^2 = a^2 + b^2, OC^2 = c^2 + d^2$</p> <p>$AC^2 = (a - c)^2 + (b - d)^2$ $= a^2 + c^2 + b^2 + d^2 - 2(ac + bd)$ $= (a^2 + b^2) + (c^2 + d^2)$ $= OA^2 + OC^2$</p> <p>$OA \perp OC$ (Converse of Pythagoras Theorem)</p> <p>$\therefore OABC \text{ is a rectangle.}$</p>	<p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1</p>	<p>Accept the negligence of considering $a = 0$ or $c = 0$</p>

Solutions	Marks	Remarks
<p>(b) (iii) $p = 2ri$</p> $\frac{p - r}{p + r} = \frac{2ri - r}{2ri + r}$ $= \frac{-1 + 2i}{1 + 2i}$ $= \frac{3}{5} + \frac{4}{5}i$ <p>$\arg\left(\frac{p - r}{p + r}\right) = \theta$</p> $\tan \theta = \frac{4/5}{3/5}$ $= \frac{4}{3}$	<p>1A</p> <p>1A</p> <p>1A</p> <p>1M</p> <p>1A</p> <hr style="width: 50%; margin-left: auto; margin-right: 0;"/> <p style="text-align: right;">14</p>	<p>Accept $\frac{3 + 4i}{5}$</p> <p>$\arg(p - r) -$ $\arg(p + r) = \theta$ (can be omitted)</p>

Solution	Marks	Remarks
<p>5. $2\sin\frac{x}{2}\sin\frac{3x}{2} = 1$</p> <p>$\cos x - \cos 2x = 1$</p> <p>$\cos x - (2\cos^2 x - 1) = 1$</p> <p>$2\cos^2 x - \cos x = 0$</p> <p>$\cos x = 0$ or $\frac{1}{2}$</p> <p>$x = 2n\pi \pm \frac{\pi}{2} \quad (\frac{2n+1}{2}\pi)$</p> <p>or $2n\pi \pm \frac{\pi}{3}$ where $n \in \mathbb{Z}$</p>	<p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p> <p style="border-top: 1px solid black; border-bottom: 3px double black;">5</p>	<p>360n° ± 90°, (2n + 1) 90° 360n° ± 60° use different units (pp - 1)</p>
<p><u>Alt. Solution</u></p> <p>Let $\sin\frac{x}{2} = t$</p> <p>$t(3t - 4t^3) = \frac{1}{2}$</p> <p>$8t^4 - 6t^2 + 1 = 0$</p> <p>$(2t^2 - 1)(4t^2 - 1) = 0$</p> <p>$t = \pm\frac{\sqrt{2}}{2}$ or $\pm\frac{1}{2}$</p> <p>$\frac{x}{2} = n\pi \pm \frac{\pi}{4}$ or $n\pi \pm \frac{\pi}{6}$</p> <p>$x = 2n\pi \pm \frac{\pi}{2}$ or $2n\pi \pm \frac{\pi}{3}$</p>	<p>1A</p> <p>1A</p> <p>1A</p> <p>1A+1A</p>	
<p>6. (a) $r = \sqrt{1^2 + (\sqrt{3})^2} = 2$</p> <p>$\tan \alpha = \sqrt{3} \quad \therefore \alpha = 60^\circ$</p> <p>(b) $x = \frac{1}{2\cos(\theta - 60^\circ) + 5}$</p> <p>$-1 \leq \cos(\theta - 60^\circ) \leq 1$</p> <p>$\therefore \frac{1}{7} \leq x \leq \frac{1}{3}$</p>	<p>1A</p> <p>1A</p> <p>1M</p> <p style="border-top: 1px solid black; border-bottom: 3px double black;">1A+1A 5</p>	<p>no mark if in radian</p>

Solution	Marks	Remarks
7. Equation of CD : $y = mx+1$ ----- (1)	1A	
Equation of AB : $\frac{x}{3} + \frac{y}{5} = 1$ ----- (2)	1A	
Subs. (1) into (2) : $\frac{x}{3} + \frac{mx+1}{5} = 1$		
$x = \frac{12}{5+3m}$	1A	
Area of $\triangle BCD = \frac{1}{2} (5-1) \left(\frac{12}{5+3m}\right)$	1A	
$\frac{24}{5+3m} = \frac{1}{2} \cdot \frac{15}{2} = \frac{15}{4}$	1M	
$m = \frac{7}{15}$		
\therefore Equation of CD is $y = \frac{7x}{15} + 1$	1A	$7x - 15y + 15 = 0$
	<u>6</u>	

<u>Alt. Solution</u>	
Let coordinates of D be (x, y)	
$\frac{4x}{2} = \frac{1}{2} \cdot \frac{15}{2}$	1M
$x = \frac{15}{8}$	1A
Equation of AB : $\frac{x}{3} + \frac{y}{5} = 1$	1A
Subs. $x = \frac{15}{8}$, $y = \frac{15}{8}$	1A
\therefore Equation of CD	
$\frac{y-1}{x} = \frac{\frac{15}{8}-1}{\frac{15}{8}}$	1M
$y = \frac{7}{15}x + 1$	1A

$\frac{\frac{y}{15} - 3}{8} = \frac{5}{-3}$ 1A

$y = \frac{15}{8}$ 1A

Solution	Marks	Remarks
<p>8. Let coordinates of S and T be (a, 0), (b, b) respectively</p> <p>coordinates of mid-point is $(\frac{a+b}{2}, \frac{b}{2})$</p> <p>Let $x = \frac{a+b}{2}$, $y = \frac{b}{2}$</p> <p>$b = 2y$, $a = 2(x - y)$</p> <p>$(a - b)^2 + (b - 0)^2 = 4$</p> <p>$(2x - 4y)^2 + (2y)^2 = 4$</p> <p>$(x - 2y)^2 + y^2 = 1$</p> <p>$x^2 - 4xy + 5y^2 - 1 = 0$</p>	<p>1A</p> <p>1A</p> <p>1M</p> <p>1M</p> <p>1A</p> <p><u>1A</u></p> <p><u>6</u></p>	<p></p> <p></p> <p></p> <p>For making a, b as subjects</p> <p></p> <p></p> <p></p>
<p><u>Alt. Solution</u></p> <p>Let coordinates of P be (x, y)</p> <p>then coordinates of T is (2y, 2y)</p> <p>coordinates of S is (2x - 2y, 0)</p> <p>$(2x - 4y)^2 + 4y^2 = 4$</p> <p>$x^2 - 4xy + 5y^2 - 1 = 0$</p>	<p></p> <p>1A</p> <p>2A</p> <p>1M+1A</p> <p>1A</p>	<p></p> <p></p> <p></p> <p></p> <p></p>

	Solution	Marks	Remarks
9. (a) (i)	$\int_0^{\pi} \cos^2 x dx = \int_0^{\pi} \frac{1}{2}(1 + \cos 2x) dx$ $= \left[\frac{1}{2} \left(x + \frac{\sin 2x}{2} \right) \right]_0^{\pi}$ $= \pi/2$	1A 1A 1A	
(ii)	<p>Put $x = \pi - y$</p> $\int_0^{\pi} x \cos^2 x dx = \int_{\pi}^0 (\pi - y) \cos^2(\pi - y) - dy$ $= \pi \int_0^{\pi} \cos^2 y dy - \int_0^{\pi} y \cos^2 y dy$	1A 1M	For separating into 2 integrals
	$2 \int_0^{\pi} x \cos^2 x dx = \pi \int_0^{\pi} \cos^2 x dx$ $= \pi^2/2$	1M	
	$\therefore \int_0^{\pi} x \cos^2 x dx = \pi^2/4$	<u>1A</u> <u>7</u>	
(b) (i)	<p>Put $x = \pi + y$</p> $\int_{\pi}^{2\pi} x \cos^2 x dx = \int_0^{\pi} (\pi + y) \cos^2(\pi + y) dy$ $= \pi \int_0^{\pi} \cos^2 y dy + \int_0^{\pi} y \cos^2 y dy$ $= \pi \int_0^{\pi} \cos^2 x dx + \int_0^{\pi} x \cos^2 x dx$	1A 1A 1	
(ii)	$\int_0^{2\pi} x \cos^2 x dx = \int_0^{\pi} x \cos^2 x dx + \int_{\pi}^{2\pi} x \cos^2 x dx$ $= \int_0^{\pi} x \cos^2 x dx + \pi \int_0^{\pi} \cos^2 x dx$ $+ \int_0^{\pi} x \cos^2 x dx$ $= \frac{\pi^2}{4} + \pi \left(\frac{\pi}{2} \right) + \frac{\pi^2}{4}$ $= \pi^2$	1A 1M <u>1</u> <u>6</u>	For subs. (b) (i)
(c)	<p>Put $x^2 = y$</p> $2x dx = dy$ $\int_0^{\sqrt{2\pi}} x^3 \cos^2 x^2 dx = \int_0^{2\pi} y \cos^2 y \cdot \frac{1}{2} dy$ $= \frac{1}{2} \int_0^{2\pi} y \cos^2 y dy$ $= \frac{\pi^2}{2}$	1A 1A <u>1A</u> <u>3</u>	

Solution	Marks	Remarks
<p>10. (a) $\frac{dy}{dx} \Big _{x=t} = 2t - 2$</p> <p>y-coordinates of $P = t^2 - 2t + 3$</p> <p>Equation of tangent : $y - (t^2 - 2t + 3)$</p> <p style="padding-left: 100px;">$= (2t - 2)(x - t)$</p> <p style="padding-left: 100px;">$y = (2t - 2)x - t^2 + 3 \dots (*)$</p>	<p style="text-align: center;">1A</p> <p style="text-align: center;">1A</p> <p style="text-align: center;">1M</p> <p style="text-align: center;"><u>1A</u></p> <p style="text-align: center;"><u>4</u></p>	
<p><u>Alt. Solution</u></p> <p>Using the formula $\frac{y + y_1}{2} = xx_1 - (x + x_1) + 3$</p> <p>Equation of tangent : $\frac{y + (t^2 - 2t + 3)}{2}$</p> <p style="padding-left: 100px;">$= tx - (t + x) + 3$</p> <p style="padding-left: 100px;">$y = (2t - 2)x - t^2 + 3$</p>	<p style="text-align: center;">1M+1A</p> <p style="text-align: center;">+1A</p> <p style="text-align: center;">1A</p>	<p>1A for $y_1 = t^2 - 2t + 3$</p>
<p>(b) (i) Put $t = \frac{1}{3}$ in $(*)$</p> <p>Equation of T_1 : $y = \frac{-4}{3}x + \frac{26}{9}$</p> <p>(ii) Coordinates of C : (1, 2)</p> <p>Coordinates of D : $(1, \frac{14}{9})$</p> <p>(iii) Subs. $(1, \frac{14}{9})$ into $(*)$</p> <p style="padding-left: 20px;">$\frac{14}{9} = 2t - 2 - t^2 + 3$</p> <p style="padding-left: 20px;">$9t^2 - 18t + 5 = 0$</p> <p style="padding-left: 20px;">$t = \frac{1}{3} \text{ or } \frac{5}{3}$</p> <p style="padding-left: 20px;">\therefore x-coordinate of B = $\frac{5}{3}$</p> <p style="padding-left: 20px;">y-coordinate of B = $(\frac{5}{3})^2 - 2(\frac{5}{3}) + 3 = \frac{22}{9}$</p> <p style="padding-left: 20px;">Coordinates of B = $(\frac{5}{3}, \frac{22}{9})$</p>	<p style="text-align: center;">1A</p> <p style="text-align: center;">1A</p> <p style="text-align: center;">1M</p> <p style="text-align: center;">1A</p> <p style="text-align: center;"><u>1A</u></p> <p style="text-align: center;"><u>6</u></p>	

Solution

Marks

Remarks

Alt. Solution

(iii) Since S is symmetrical about $x = 1$ and
 x-coordinate of A = $\frac{1}{3}$, \therefore by symmetry x
 coordinate of B = $1 + (1 - \frac{1}{3}) = \frac{5}{3}$

1M+1A

coordinates of B = $(\frac{5}{3}, \frac{22}{9})$

1A

(c) Centre of circle lies on $x = 1$

let its coordinates be $(1, a)$

1A

Radius = Distance to T_1

$$= \left| \frac{-\frac{4}{3} - a + \frac{26}{9}}{\sqrt{1 + (\frac{4}{3})^2}} \right|$$

$$= \left| \frac{14 - 9a}{15} \right|$$

1A

Since the circles pass through C $(1, 2)$

$$\text{Radius} = |2 - a|$$

1M

$$|2 - a| = \left| \frac{14 - 9a}{15} \right|$$

1M

$$a = \frac{8}{3} \text{ or } \frac{11}{6}$$

Coordinates of centres are $(1, \frac{8}{3})$ or $(1, \frac{11}{6})$

$\frac{1A+1A}{6}$

	Solution	Marks	Remarks
(b) (i)	Let equation of L_1 be $y = mx + 2$	1A	
	centre of C_1 is $(-\frac{1}{2}, -\frac{1}{2})$, radius $r = \sqrt{7}$	1A	
	Distance from centre to L_1		
	$d = \left \frac{m(-\frac{1}{2}) - (-\frac{1}{2}) + 2}{\sqrt{1 + m^2}} \right $	1M	
	$= \left \frac{5 - m}{2\sqrt{1 + m^2}} \right $		
	Since $d^2 = r^2 - (\frac{\sqrt{2}}{2})^2$	1M	
	$\left(\frac{5 - m}{2\sqrt{1 + m^2}} \right)^2 = (\sqrt{7})^2 - \left(\frac{\sqrt{2}}{2} \right)^2$		
	$25m^2 + 10m + 1 = 0$		
	$(5m + 1)^2 = 0$		
	$m = -\frac{1}{5}$		
	Equation of L_1 is $y = -\frac{1}{5}x + 2$	1A	$x + 5y - 10 = 0$

	Alt. Solution	Marks	
	Let equation of L_1 be $y = mx + 2$	1A	
	Subs. into C_1		
	$2x^2 + 2(mx + 2)^2 + 2x + 2(mx + 2) - 13 = 0$	1M	
	$(2m^2 + 2)x^2 + (10m + 2)x - 1 = 0$		
	Let coordinates of intersecting points be $(x_1, y_1), (x_2, y_2)$		
	$x_1 + x_2 = \frac{-(5m + 1)}{1 + m^2}, x_1 x_2 = \frac{-1}{2(1 + m^2)}$	1M	
	$AB^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$		
	$= (1 + m^2)(x_1 - x_2)^2$	1A	
	$= (1 + m^2)[(x_1 + x_2)^2 - 4x_1 x_2]$		
	$= \frac{(5m + 1)^2}{1 + m^2} + 2 = 2$		
	$m = -\frac{1}{5}$		
	Equation of L_1 is $y = -\frac{1}{5}x + 2$	1A	

Solution	Marks	Remarks
<p>(ii) The locus is the perpendicular bisector of AB.</p> <p>Since AB is a chord of C_1, the perpendicular bisector of AB passes through centre of $C_1(-\frac{1}{2}, -\frac{1}{2})$</p> <p>Equation of locus is $y + \frac{1}{2} = 5(x + \frac{1}{2})$</p> <p style="text-align: center;">$y = 5x + 2$</p>	<p>2M</p> <p>2M</p> <p><u>1A</u></p> <p style="text-align: center;"><u>10</u></p>	
<p><u>Alt. Solution</u></p> <p>$x^2 + y^2 + x + y - \frac{13}{2} + k(\frac{1}{5}x + y - 2) = 0$</p> <p>$x^2 + y^2 + (1 + \frac{k}{5})x + (1 + k)y - (2k + \frac{13}{2}) = 0$</p> <p>coordinate of centre is $(\frac{-(1 + \frac{k}{5})}{2}, \frac{-(k + 1)}{2})$</p> <p>Let coordinates of centre be (x, y)</p> <p>$\left\{ \begin{array}{l} x = -\frac{1}{2}(1 + \frac{k}{5}) \\ y = -\frac{1}{2}(k + 1) \end{array} \right.$</p> <p>Eliminating k,</p> <p style="text-align: center;">$y = 5x + 2$</p>	<p>1M</p> <p>1M+1A</p> <p>1M</p> <p>1A</p>	

Solution	Marks	Remarks
13. (a) By Sine Law		
$\frac{AB}{\sin\theta} = \frac{AQ}{\sin\angle ABQ}$		
$\sin\angle ABQ = \frac{AQ}{AB}\sin\theta$	1A	
$\sin\angle APQ = \frac{AQ}{PQ}$	1A	
$\angle APQ = \angle ABQ$	1A	
$\therefore \frac{AQ}{AB}\sin\theta = \frac{AQ}{PQ}$		
$PQ = \frac{AB}{\sin\theta}$	<u>1A</u> <u>4</u>	
(b) By Cosine Law,		
$AB^2 = AP^2 + BP^2 - 2AP \cdot BP\cos(\pi - \theta)$	1M	
$= AP^2 + BP^2 + 2AP \cdot BP\cos\theta$	1A	
$\therefore PQ = \frac{\sqrt{AP^2 + BP^2 + 2AP \cdot BP\cos\theta}}{\sin\theta}$	<u>1</u> <u>3</u>	
(c) (i) $\cot^2\phi = \frac{PQ^2}{VP^2}$	1A	
$= \frac{AP^2 + BP^2 + 2AP \cdot BP\cos\theta}{VP^2\sin^2\theta}$	1M	
$= \frac{1}{\sin^2\theta} \left[\left(\frac{AP}{VP}\right)^2 + \left(\frac{BP}{VP}\right)^2 + 2\left(\frac{AP}{VP}\right)\left(\frac{BP}{VP}\right)\cos\theta \right]$	1M	
$= \frac{\cot^2\alpha + \cot^2\beta + 2\cot\alpha\cot\beta\cos\theta}{\sin^2\theta}$	1	
(ii) $\cot^2\frac{\pi}{6} = \frac{1}{\sin^2\theta} \left(\cot^2\frac{\pi}{4} + \cot^2\frac{\pi}{3} \right.$		
$\left. + 2\cot\frac{\pi}{4}\cot\frac{\pi}{3}\cos\theta \right)$		
$3\sin^2\theta = \frac{4}{3} + \frac{2}{\sqrt{3}}\cos\theta$	1A	
$9\cos^2\theta + 2\sqrt{3}\cos\theta - 5 = 0$	1A	
$\cos\theta = \frac{\sqrt{3}}{3} \quad \text{or} \quad \frac{-5\sqrt{3}}{9}$	1A	
$\theta = 0.955 \quad \text{or} \quad 2.87 \text{ (rejected)}$	1A	
$\therefore \theta = 0.955$	<u>1A</u> <u>9</u>	