Solution	Marks	Remarks
$\frac{dy}{dx} = \sin 5x + 5x\cos 5x$	2A	
$\frac{d^2y}{dx^2} = 5\cos 5x + 5\cos 5x - 25x\sin 5x$		
$= 10\cos 5x - 25x\sin 5x$	1A	
$\frac{d^2y}{dx^2} + 25y = 10\cos 5x$	1A	
	4	

Solution	Marks	Remarks
2. (a) $\vec{OC} = \frac{\underline{i} + 3\underline{j} + k(4\underline{i} - \underline{j})}{k+1}$	1A	omit vector sign (pp-1)
$= \frac{1}{k+1}[(4k+1)\underline{i} + (3-k)\underline{j}]$	1A	
(b) $\vec{AB} = 3\underline{i} - 4\underline{j}$	1A	Alt. Solution:
$OC \perp AB$		slope of AB $= -\frac{4}{3}$
$\vec{OC} \cdot \vec{AB} = 0$ (omit dot sign pp-1)		slope of OC $= \frac{3-k}{4k+1}$
$3 \cdot \frac{4k+1}{k+1} + (-4)\left(\frac{3-k}{k+1}\right) = 0$	1M	$\left(-\frac{4}{3}\right)\left(\frac{3-k}{4k+1}\right) = -1$ 1M
$k = \frac{9}{16}$	1A	$k = \frac{9}{16}$ 1A
	5	

Solution	Marks	Remarks
3. $y = x^3$		
$y' = 3x^2$		
$3x^2 = \frac{3}{4}$	1M	
$x = \pm\frac{1}{2}$		
$x = \frac{1}{2}$, $y = \frac{1}{8}$	1A	
$x = -\frac{1}{2}$, $y = -\frac{1}{8}$	1A	
$\frac{y - \frac{1}{8}}{x - \frac{1}{2}} = \frac{3}{4}$		
$y = \frac{3}{4}x - \frac{1}{4}$ or $3x - 4y - 1 = 0$	1A	
$\frac{y + \frac{1}{8}}{x + \frac{1}{2}} = \frac{3}{4}$		
$y = \frac{3}{4}x + \frac{1}{4}$ or $3x - 4y + 1 = 0$	1A	
	5	

Solution	Marks	Remarks
$[\tan(\theta - x)]$		
$= \frac{\tan\theta - \tan x}{1 + \tan\theta\tan x}$	1A	
$= \frac{\tan x}{1 + 2\tan^2 x}$	1A	
(b) $\frac{dy}{dx} = \frac{(1 + 2\tan^2 x)\sec^2 x - \tan x \cdot 4\tan x\sec^2 x}{(1 + 2\tan^2 x)^2}$	1M	For quotient rule.
$= \frac{\sec^2 x - 2\tan^2 x\sec^2 x}{(1 + 2\tan^2 x)^2}$		
$= 0$		
$\tan^2 x = \frac{1}{2}$	1A	
(or equivalent answers such as $\sec^2 x = \frac{3}{2}$, $\cos^2 x = \frac{2}{3}$, $\sin^2 x = \frac{1}{3}$)		
$x = 0.615$ (0.61548) 0.196π	1A	Do not accept answers given in degrees.
	5	

Solution	Marks	Remarks
5. $\frac{x^2 + 5x + 1}{x^2 - x + 1} = r$		
$(r - 1)x^2 - (r + 5)x + (r - 1) = 0$	1A	
or $(1 - r)x^2 + (r + 5)x + (1 - r) = 0$	(1A)	
$D = (r + 5)^2 - 4(1 - r)^2$		
For real values of x, $(r + 5)^2 - 4(1 - r)^2 \geq 0$	1M	(1M for using $D \geq 0$)
$r^2 + 10r + 25 - 4r^2 + 8r - 4 \geq 0$		
$3r^2 - 18r - 21 \leq 0$		
$r^2 - 6r - 7 \leq 0$		
$(r + 1)(r - 7) \leq 0$ or $(1 + r)(7 - r) \geq 0$	1A	
$7 \geq r \geq -1$	1A	
	5	

Solution	Marks	Remarks
6. $(p + qi)^2 = 21 - 20i$		
$p^2 + 2pqi - q^2 = 21 - 20i$		
$p^2 - q^2 = 21$, $2pq = -20$ }	1M+1A	
Solving,		Alt. Solution:
$p^2 - \frac{100}{p^2} = 21$		$\frac{100}{q^2} - q^2 = 21$
$p^4 - 21p^2 - 100 = 0$	1A	$q^4 + 21q^2 - 100 = 0$
$(p^2 + 4)(p^2 - 25) = 0$		$(q^2 - 4)(q^2 + 25) = 0$
$p = \pm 5$	1A	$q = \pm 2$
$p = 5$, $q = -2$; $p = -5$, $q = 2$ }	1A	
The two square roots are $5 - 2i$ and $-5 + 2i$.	1A	
	6	

Solutions	Marks	Remarks
7. a) $\frac{1-\sin\theta+i\cos\theta}{1-\sin\theta-i\cos\theta}$		Alt. Solution:
$=\frac{(1-\sin\theta+i\cos\theta)(1-\sin\theta+i\cos\theta)}{(1-\sin\theta-i\cos\theta)(1-\sin\theta+i\cos\theta)}$ (for correct conjugate only)	1M	$(1-s-ic)i(c+is)$ 1M
$=\frac{(1-\sin\theta)^2-\cos^2\theta+2\cos\theta(1-\sin\theta)i}{(1-\sin\theta)^2+\cos^2\theta}$		$=(-s+s^2+c^2)+i(c-cs+sc)$
$=\frac{(2\sin^2\theta-2\sin\theta)+2\cos\theta(1-\sin\theta)i}{2-2\sin\theta}$		$=1-s+ci$ 1
$=-\sin\theta+i\cos\theta$ $=i(\cos\theta+i\sin\theta)$)	1	
b) $\left(\frac{1-\sin\frac{7\pi}{36}+i\cos\frac{7\pi}{36}}{1-\sin\frac{7\pi}{36}-i\cos\frac{7\pi}{36}}\right)^6 = i^6(\cos\frac{7\pi}{36}+i\sin\frac{7\pi}{36})^6$	1A	
$=-(\cos\frac{7\pi}{6}+i\sin\frac{7\pi}{6})$	1M+1A	1M for DeMoivre's Thm. 1A for $(i)^6=-1$
$=\frac{\sqrt{3}}{2}+\frac{1}{2}i$	1	
	6	
8. $(x-2)^2-5\|x-2\|-6=0$		
Solution (1):		
$(x-2)^2=\|x-2\|^2$	1M	
$\|x-2\|^2-5\|x-2\|-6=0$		
$(\|x-2\|+1)(\|x-2\|-6)=0$	1A	
$\|x-2\|=-1$ or $\|x-2\|=6$		
No solution or ($x=-4$ or 8) $\therefore x=-4$ or 8	2A+1A +1A	
	6	
Solution (2):		
2 cases, (i) $x\geq 2$ (ii) $x<2$	1M	
Case (i) $x\geq 2$,		Notes: (1) $x\geq 2$, $x\leq 2$ (deduct no mark) (2) $x>2$, $x<2$ (pp-1) (3) missing 2 cases, ~~(pp-1)~~ (4) only 1 case without stating range of x (no mark)
$(x-2)^2-5(x-2)-6=0$	1A	
$[(x-2)-6][(x-2)+1]=0$ or $x^2-9x+8=0$		
$x=1$ or 8	1A	
Rejecting $x=1$, $x=8$	1A	
Case (ii) $x<2$,		
$(x-2)^2+5(x-2)-6=0$	1A	
$[(x-2)+6][(x-2)-1]=0$		
$x=3$ or -4		
Rejecting $x=3$, $x=-4$	1A	
$x=-4$ or 8		

Solutions	Marks	Remarks
8. Solution (3):		
$(x-2)^2-5\|x-2\|-6=0$		
$x-2=u$		
$u^2-5\|u\|-6=0$		
$u^2-6=5\|u\|$		
$u^4-12u^2+36=25u^2$	1M	
$u^4-37u^2+36=0$		
$(u^2-1)(u^2-36)=0$	1A	
$u=\pm 1$, $u=\pm 6$		
$x=2+u$		
$x=3$ or 1 $x=8$ or -4)	2A	
Rejecting, $x=8$ or -4 ($x\neq 1, -4$)	2A	

$\alpha\beta = q$ ... 1A

(i) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= p^2 - 2q$ — 1A

(ii) $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$ — 1A — $(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$= -p(p^2 - 2q - q)$ — $= -p^3 + 3pq$

$= -p(p^2 - 3q)$ — 1A — $= -p(p^2 - 3q)$

(iii) $(\alpha^2 - \beta - 1)(\beta^2 - \alpha - 1)$

$= \alpha^2\beta^2 - \alpha^3 - \alpha^2 - \beta^3 + \alpha\beta + \beta - \beta^2 + \alpha + 1$

$= (\alpha\beta)^2 - (\alpha^3 + \beta^3) - (\alpha^2 + \beta^2) + \alpha\beta + (\alpha + \beta) + 1$ — 1M

$= q^2 + p(p^2 - 3q) - (p^2 - 2q) + q - p + 1$ — 1A

$= p^3 - 3pq + q^2 - p^2 + 3q - p + 1$

$= q^2 - 3(p - 1)q + p^3 - p^2 - p + 1$

$= q^2 - 3(p - 1)q + (p - 1)^2(p + 1)$ — 6

(b) If the square of one root of (*) minus the other root equals 1,

i.e. $\alpha^2 - \beta = 1$ or $\beta^2 - \alpha = 1$ — 1A — Accept omitting either one

$(\alpha^2 - \beta - 1)(\beta^2 - \alpha - 1) = 0$ — 1A

From (a)(iii), $q^2 - 3(p - 1)q + (p - 1)^2(p + 1) = 0$ — 1, 3

(c) $D = 9(p - 1)^2 - 4(p - 1)^2(p + 1)$ — 1A

≥ 0 — 1M — no mark if omitting equality sign

$(p - 1)^2 \geq 0$

$\therefore 9 - 4(p + 1) \geq 0$ — 1A

$p \leq \frac{5}{4}$ — 1A, 4

(d) $4x^2 + 5x + k = 0$

$x^2 + \frac{5}{4}x + \frac{k}{4} = 0$ or $p = \frac{5}{4}$, $q = \frac{k}{4}$ — 1A

(**) becomes $(\frac{k}{4})^2 - (\frac{3}{4})(\frac{k}{4}) + (\frac{1}{4})^2(\frac{9}{4}) = 0$

or $k^2 - 3k + \frac{9}{4} = 0$ — 1A

$k = \frac{3}{2}$ — 1A

13. (a) (i) $T = k\sqrt{a^2 + x^2} + (3a - x)$ — 2A

(ii) $\frac{dT}{dx} = \frac{kx}{\sqrt{a^2 + x^2}} - 1$ — 1A

$= 0$ — 1M

$k^2x^2 = a^2 + x^2$

$x^2 = \frac{a^2}{k^2 - 1}$

$x = \frac{a}{\sqrt{k^2 - 1}}$ — 1A

$k = 2$, $\frac{d^2T}{dx^2} = \frac{2\sqrt{x^2 + a^2} - \frac{2x^2}{\sqrt{a^2 + x^2}}}{a^2 + x^2}$ — 1M

$= \frac{2a^2}{(a^2 + x^2)^{3/2}} > 0$ — 1A

$T_{min} = 2\sqrt{a^2 + \frac{a^2}{3}} + 3a - \frac{a}{\sqrt{3}}$

$= \frac{4}{\sqrt{3}}a + 3a - \frac{a}{\sqrt{3}}$

$= \sqrt{3}a + 3a$ — 1A

Alt. Solution:

$\frac{dT}{dx} = \frac{2x}{\sqrt{a^2 + x^2}} - 1 = 0$ ...1M+1A

$4x^2 = a^2 + x^2$

$x = \frac{a}{\sqrt{3}}$ 1A

$\frac{d^2T}{dx^2} = \frac{2a^2}{(a^2 + x^2)^{3/2}}$ 1M+1A

> 0

(iii) $\frac{a}{\sqrt{k^2 - 1}} > 3a$

$\frac{1}{k^2 - 1} > 9$

$k^2 - 1 < \frac{1}{9}$ — 1A

$k^2 < \frac{10}{9}$

$(1 <)\ k < \frac{\sqrt{10}}{3}$ — 1A — Accept $k < \frac{\sqrt{10}}{3}$; Accept $\frac{-\sqrt{10}}{3} < k < \frac{\sqrt{10}}{3}$

It would cost more to transport goods via P if the minimum point is beyond B, — 1M

i.e. $\frac{a}{\sqrt{k^2 - 1}} > 3a$

$(1 <)\ k < \frac{\sqrt{10}}{3}$ — 1A, 12

(b) $k = 2$

(i) $b = 2a$, $T = 2\sqrt{a^2 + x^2} + (2a - x)$

$\frac{dT}{dx} = \frac{2x}{\sqrt{a^2 + x^2}} - 1$

$= 0$

$x = \frac{a}{\sqrt{3}}$ — 1A

$T_{min} = 2a + \sqrt{3}a$ — 1A

Alt. Solution: This exp. is the same as the exp. for T in (a)(i) except for the constant 2a and this will not affect the value of x for $\frac{dT}{dx} = 0$

$\therefore x = \frac{a}{\sqrt{3}}$

(ii) $b = \frac{1}{2}a < \frac{a}{\sqrt{3}}$

For minimum value of T, go directly from A to B.

i.e. $x = \frac{1}{2}a$ — 1A

$T_{min} = 2\sqrt{a^2 + (\frac{1}{2}a)^2}$

$= \sqrt{5}a$ — 1A, 4

Solution	Marks	Remarks
13. (a) $\arg(z - k) = \angle OKP$	1A	Can be omitted
$\arg\left(\frac{z-h}{z-k}\right) = \arg(z-h) - \arg(z-k)$	1M	
$= \angle HPK$	1A	can be omitted
$= \pm 90°$	1A	($\pm$ can be omitted)
$\therefore$ Real part of $\frac{z-h}{z-k} = 0$	1 5	
Alt. Solution:		
$\frac{z-h}{z-k} = \frac{x+iy-h}{x+iy-k}$		
$= \frac{[(x-h)+iy][(x-h)-iy]}{[(x-k)+iy][(x-k)-iy]}$	1M	
$= \frac{[(x-h)(x-k)+y^2] + i[(x-k)y-(x-h)y]}{(x-k)^2+y^2}$		
Real part of $\frac{z-h}{z-k} = \frac{(x-h)(x-k)+y^2}{(x-k)^2+y^2}$	1A	
P lies on the circle with HK as diameter.		
$\therefore \frac{y-0}{x-h} \cdot \frac{y-0}{x-k} = -1$	1A	
$y^2 + (x-h)(x-k) = 0$	1A	
$\therefore$ Real part of $\frac{z-h}{z-k} = 0$	1	
(b) (i) $x^2 - 2x + 2 = 0$		
$x = 1 \pm i$	1A	
Since $-\pi < \arg z_2 < \arg z_1 < \pi$		
$z_1 = 1 + i$ $z_2 = 1 - i$)	1A	
(ii) $x^2 + 2tx - 4 = 0$		
$D = (2t)^2 + 16$	1A	
$\because D > 0 \quad \therefore \alpha$ and β are real and distinct	1	
$\because \alpha\beta = -4 < 0 \quad \therefore$ opp. sign	1	
(iii) $\frac{z_1 - \alpha}{z_1 - \beta} = \frac{1+i-\alpha}{1+i-\beta}$		
$= \frac{[(1-\alpha)+i][(1-\beta)-i]}{[(1-\beta)+i][(1-\beta)-i]}$		
$= \frac{(1-\alpha)(1-\beta)+1+(\alpha-\beta)i}{(1-\beta)^2+1}$	1	
$\frac{z_1 - \alpha}{z_1 - \beta} = \frac{[(1-\alpha)(1-\beta)+1] + (\beta-\alpha)i}{(1-\beta)^2+1}$	1A	
(iv) C lies on the circle with AB as diameter.		
From (a), real part of $\frac{z_1 - \alpha}{z_1 - \beta} = 0$.	1M	
$\frac{(1-\alpha)(1-\beta)+1}{(1-\beta)^2+1} = 0$		
$(1-\alpha)(1-\beta)+1 = 0$	1	
$1 - (\alpha+\beta) + \alpha\beta + 1 = 0$		
$1 + 2t - 4 + 1 = 0$	1A	For $\alpha + \beta = -2t$, $\alpha\beta = -4$
$t = 1$	1A 11	
Alt. Solution:		
$\alpha = -t + \sqrt{t^2+4}$, $\beta = -t - \sqrt{t^2+4}$		
$(1 + t - \sqrt{t^2+4})(1 + t + \sqrt{t^2+4}) + 1 = 0$	1A	
$(1+t)^2 - (t^2+4) + 1 = 0$		
$t = 1$	1A	

(a) (i) $\overrightarrow{OM} = \frac{1}{2}(\underline{a} + \underline{b})$ — 1A

$\overrightarrow{OD} = \frac{1}{3}\underline{a}$ — 1A

(ii) $\overrightarrow{OK} = \lambda\overrightarrow{OD} + (1 - \lambda)\overrightarrow{OB}$ — 1M

$= \lambda \cdot \frac{1}{3}\underline{a} + (1 - \lambda)\underline{b}$ — 1A

$\overrightarrow{OK} = \mu\overrightarrow{OM}$

$= \frac{\mu}{2}\underline{a} + \frac{\mu}{2}\underline{b}$

$\therefore \quad \frac{\lambda}{3} = \frac{\mu}{2}$ and $(1 - \lambda) = \frac{\mu}{2}$ — 1M+1A

$\lambda = \frac{3}{4}$)

$\mu = \frac{1}{2}$) .... — 1A

7

(b) (i) $\overrightarrow{OM} = 7\underline{i} + 4\underline{j}$ — 1A

$\overrightarrow{DB} = \overrightarrow{OB} - \overrightarrow{OD}$

$= (2\underline{i} + 8\underline{j}) - \frac{1}{3}(12\underline{i})$

$= -2\underline{i} + 8\underline{j}$ — 1A

(ii) $\overrightarrow{OM} \cdot \overrightarrow{DB} = (-2)(7) + (8)(4)$ — 1M

$= 18$ — 1A

$\cos \angle BKM = \frac{\overrightarrow{OM} \cdot \overrightarrow{DB}}{|\overrightarrow{OM}||\overrightarrow{DB}|}$ — 1M

$= \frac{18}{\sqrt{7^2 + 4^2}\sqrt{8^2 + (-2)^2}}$

$= 0.2707$

$\angle BKM = 74.3°$ (or 1.30 rad.) — 1A — 74.294°, 74°18' (1.2967 rad.)

(iii) $\overrightarrow{AP} = \frac{\overrightarrow{AB} + 2\overrightarrow{AO}}{3}$

$= \frac{(-10\underline{i} + 8\underline{j}) + 2(-12\underline{i})}{3}$

$= \frac{-34\underline{i} + 8\underline{j}}{3}$ — 1A

Alt. Solution:

$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA}$

$= \frac{1}{3}(2\underline{i} + 8\underline{j}) - 12\underline{i}$

$= \frac{1}{3}(-34\underline{i} + 8\underline{j})$ 1A

$\overrightarrow{AK} = \overrightarrow{OK} - \overrightarrow{OA}$

$= \frac{1}{2}\overrightarrow{OM} - \overrightarrow{OA}$

$= \frac{1}{2}(7\underline{i} + 4\underline{j}) - 12\underline{i}$

$= -\frac{17}{2}\underline{i} + 2\underline{j}$ — 1A

$\overrightarrow{AK} = \frac{3}{4}\overrightarrow{AP}$)

$\therefore$ A, P, K are collinear.) .... — 1A

Alt. Solution:

1. calculate $\angle PAK = 0$
2. show slope of AP $=$ slope of AK $= \frac{-4}{17}$

9

10. (a) $\ell + 2\pi r = 1$ — 1A

$V = \pi r^2 \ell$

$= \pi r^2(2 - 2\pi r)$ — 1A

$\frac{dV}{dr} = 2\pi(2r - 3\pi r^2)$ — 1A

$= 0$ — 1M

$r \neq 0, \quad r = \frac{2}{3\pi}$ — 1A

$\frac{d^2V}{dr^2} = 2\pi(2 - 6\pi r)$)

When $r = \frac{2}{3\pi}$, $\frac{d^2V}{dr^2} = 2\pi(-2) < 0$) .... — 1M

V is a max. when $r = \frac{2}{3\pi}$

$V_{max} = \frac{8}{27\pi}$ (m³) — 1A — no unit (pp-1)

7

(b) (i) $S = 2\pi r^2 + 2\pi r\ell$

$= 2\pi r^2 + 2\pi r(2 - 2\pi r)$

$= (2\pi - 4\pi^2)r^2 + 4\pi r$ — 1A

(ii) $\frac{dS}{dr} = (4\pi - 8\pi^2)r + 4\pi$ — 1A

$= 0$

$r = \frac{1}{2\pi - 1}$ (or 0.189) — 1A

$\frac{d^2S}{dr^2} = 4\pi - 8\pi^2 < 0$ — 1M

$\therefore$ S is a max. when $r = \frac{1}{2\pi - 1}$

Alt. Solution:

S is quadratic }
$2\pi - 4\pi^2 < 0$ } 1M

x-coord. of centre

$= \frac{-4\pi}{2(2\pi - 4\pi^2)}$ 1A

$= \frac{1}{2\pi - 1}$

This corresponds to a max.

$\therefore$ S is a max. when

$r = \frac{1}{2\pi - 1}$ 1A

(iii) $0.15 \leq r \leq 0.25$

(1) S is increasing, $\frac{dS}{dr} \geq 0$

$(4\pi - 8\pi^2)r + 4\pi \geq 0$ — 1M

$0.15 \leq r \leq \frac{1}{2\pi - 1}$ — 1A — Accept no equality sign. Accept $r \leq \frac{1}{2\pi - 1}$

(2) S is decreasing, $(4\pi - 8\pi^2)r + 4\pi \leq 0$

$\frac{1}{2\pi - 1} \leq r \leq 0.25$ — 1A — Accept no equality sign. Accept $r \geq \frac{1}{2\pi - 1}$

$S_{r=0.25} = 1.07$

$S_{r=0.15} = 1.14$

From above, S increasing for $0.15 \leq r \leq 0.189$ and decreasing for $0.189 \leq r \leq 0.25$ — 1M

[Alternatively, consider the shape of the graph of $S = (2\pi - 4\pi^2)r^2 + 4\pi r$ which is a quadratic in r.]

Smallest value of S occurs when $r = 0.25$

Smallest value of $S = 1.07$ or $(\frac{9\pi}{8} - \frac{\pi^2}{4})$ — 1A

9

香 港 考 試 局
HONG KONG EXAMINATIONS AUTHORITY

一 九 八 九 年 香 港 中 學 會 考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1989

附 加 數 學 （卷 二）
Additional Mathematics (Paper II)

評卷參考
Marking Scheme

致在學校任教之閱卷員特別留意

本評卷參考並非標準答案，故極不宜落於學生手中，以免引起誤會。

遇有學生求取此文件時，閱卷員應嚴予拒絕。閱卷員在任何情況下披露本評卷參考內容，均有違閱卷員守則及「一九七七年香港考試局法例」。

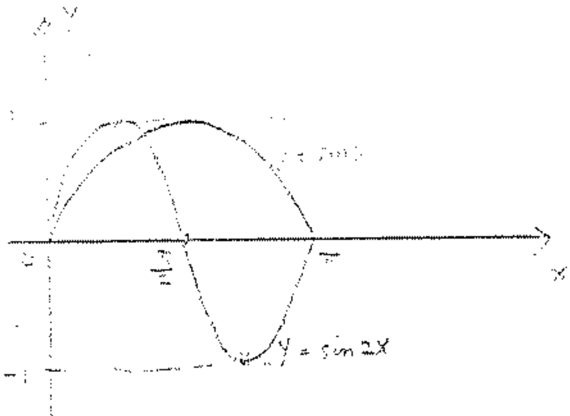
Special Notes for Teacher Markers

It is highly undesirable that this marking scheme should fall into the hands of students. They are likely to regard it as a set of model answers, which it certainly is not.

Markers should therefore resist pleas from their students to have access to this document. Making it available would constitute mis-conduct on the part of the marker and is, moreover in breach of the 1977 Hong Kong Examinations Authority Ordinance.

Solution	Marks	Remarks
1. $(1+x)^{10}(1-\frac{2}{x})^3$		
$= (1 + 10x + 45x^2 + 120x^3 + \ldots)(1 - \frac{6}{x} + \frac{12}{x^2} - \frac{8}{x^3})$	1A+1A	Accept nCr form, deduct 1 mark for missing ...
constant term		
$= 1(1) + 10(-6) + 45(12) + 120(-8)$	1M	[illegible]
$= 1 - 60 + 540 - 960$		
$= -479$	2A	
	5	
2. For $n = 1$, L.H.S. $= 1 \cdot 2 \cdot 3 = 6$		[illegible]
R.H.S. $= \frac{1(2)(3)(4)}{4} = 6$		the statement is true for n=1 [illegible]
$\therefore$ the equality holds for $n = 1$.	1	
Assume the equality holds for some integer k.	1	② assume n=k is true [illegible]
For $n = k+1$, L.H.S.		③ [illegible]
$= (1)(2)(3) + (2)(3)(4) + \ldots + k(k+1)(k+2) + (k+1)(k+2)(k+3)$		
$= \frac{k(k+1)(k+2)(k+3)}{4} + (k+1)(k+2)(k+3)$	[illegible]	
$= \frac{1}{4}(k+1)(k+2)(k+3)(k+4)$	[illegible]	
$\therefore$ the equality holds for $n = k+1$.	[illegible]	
By the principle of Mathematical Induction, the equality holds for all +ve integers n.	1	
	5	
3. $u = 2x^2 + 1$	1A	
$du = 4x\,dx$		
When $x = 0$, $u = 1$; $x = 2$, $u = 9$)	1A	
$\int_0^2 \frac{8x^3}{\sqrt{2x^2+1}}\,dx = \int_1^9 \frac{u-1}{\sqrt{u}}\,du$	1A	Integrand must be in terms of u.
$= \int_1^9 (\sqrt{u} - \frac{1}{\sqrt{u}})\,du$		
$= [\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}]_1^9$	1A	
$= \frac{40}{3}$ or 13.3	1A	
	5	

Solution	Marks	Remarks
(a) $\int \cos^2 2x\,dx$		
$= \int \frac{1}{2}(1+\cos 4x)\,dx$	1A	
$= \frac{1}{2}x + \frac{1}{8}\sin 4x + c$	1A+1A	Deduct 1 mark for omitting c
(b) $\int \sin^2 2x\,dx$		
$= \int (1-\cos^2 2x)\,dx$	1A	No mark if using $\int \frac{1}{2}(1 + \cos 4x)dx$
$= x - \int \cos^2 2x\,dx$		
$= \frac{1}{2}x - \frac{1}{8}\sin 4x + c'$	1A	
	5	
5. (a) $r = \sqrt{5^2 + (-12)^2} = 13$	1A	
$p = 7$	1A	
$\tan\alpha = \frac{12}{5}$		
$\alpha = 67.4^\circ\ (67^\circ 23')$	1A	
$y = 13\sin(\theta - 67.4^\circ) + 7$		
(b) Least value of $y = 13(-1) + 7$	1M	for putting $\sin(\theta - 67.4^\circ) = -1$
$= -6$	1A	
	5	
6. $2\cos 2\theta + 5\sin\theta - 3 = 0$		
$2(1 - 2\sin^2\theta) + 5\sin\theta - 3 = 0$	1A	
$4\sin^2\theta - 5\sin\theta + 1 = 0$	1A	
$(4\sin\theta - 1)(\sin\theta - 1) = 0$		
$\sin\theta = \frac{1}{4}$ or 1	1A	
$\theta = 180k^\circ + (-1)^k 14.5^\circ\ (14^\circ 29')$	1A	$k\pi + (-1)^k(0.253)$
or $180k^\circ + (-1)^k 90^\circ$	1A	$k\pi + (-1)^k \frac{\pi}{2}$, $360k^\circ + 90^\circ$
where $k \in Z$		use different units (pp-1)
	5	

Solution	Marks	Remarks
7. (a) Slope of $L_2 = \frac{17}{7}$		
$\left\lvert \frac{m - \frac{17}{7}}{1 + m(\frac{17}{7})} \right\rvert = \tan 45°$	1M	Accept formula with no absolute value sign or ± sign
$m = \frac{5}{12}$ or $-\frac{12}{5}$	1A+1A	
(b) For $m = \frac{5}{12}$, $L_1 : y = \frac{5}{12}x + c$		
$5x - 12y + 12c = 0$		
$\frac{5 - 12(2) + 12c}{\sqrt{5^2 + (-12)^2}} = \pm 5$	1M	Accept formula without ±
$-19 + 12c = \pm 65$		
$c = 7$ or $-\frac{23}{6}$	1A+1A	
	6	
8. (a) correct graph	1A	
 		
$\sin 2x = \sin x$		
$2\sin x\cos x = \sin x$		
$\sin x = 0$ or $\cos x = \frac{1}{2}$	1A	
$x = 0, \pi$ or $\frac{\pi}{3}$	1A	not acceptable if in degree
(b) Required area =		
$\int_0^{\frac{\pi}{3}}(\sin 2x - \sin x)dx + \int_{\frac{\pi}{3}}^{\pi}(\sin x - \sin 2x)dx$	1M	for $\int_a^b (f_1(x)-f_2(x))dx$
$= \left[-\frac{1}{2}\cos 2x + \cos x\right]_0^{\frac{\pi}{3}} + \left[-\cos x + \frac{1}{2}\cos 2x\right]_{\frac{\pi}{3}}^{\pi}$		
$= (\frac{1}{4} + \frac{1}{2} + \frac{1}{2} - 1) + (1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{4})$		
$= \frac{1}{4} + 2\frac{1}{4}$		
$= 2\frac{1}{2}$	2A	
	6	

Solution	Marks	Remarks
1. (a) $x = \tan\theta$		
$dx = \sec^2\theta\, d\theta$	1A	
$x = 0,\ \theta = 0$; $x = 1,\ \theta = \frac{\pi}{4}$	1A	
$\int_0^1 \frac{dx}{1+x^2} = \int_0^{\frac{\pi}{4}} \frac{\sec^2\theta}{1+\tan^2\theta}\, d\theta$		
$= \int_0^{\frac{\pi}{4}} d\theta$	1A	
$= \frac{\pi}{4}$	1A	
	4	
(b) $\frac{d}{dx}\left[\frac{x}{(1+x^2)^{n-1}}\right]$		
$= \frac{(1+x^2)^{n-1} - 2x^2(n-1)(1+x^2)^{n-2}}{(1+x^2)^{2(n-1)}}$	2A	
$= \frac{1}{(1+x^2)^{n-1}} - 2(n-1)\frac{x^2}{(1+x^2)^n}$		
Integrating both sides with respect to x,	1A	
$\frac{x}{(1+x^2)^{n-1}} = \int \frac{dx}{(1+x^2)^{n-1}} - 2(n-1)\int \frac{x^2}{(1+x^2)^n}\, dx$		
$\int \frac{x^2}{(1+x^2)^n}\, dx = \frac{1}{2(n-1)}\left[\int \frac{dx}{(1+x^2)^{n-1}} - \frac{x}{(1+x^2)^{n-1}}\right]$		
	4	
(c) $\int \frac{dx}{(1+x^2)^n} = \int \frac{dx}{(1+x^2)^{n-1}} - \int \frac{x^2}{(1+x^2)^n}\, dx$	1A	
$= \int \frac{dx}{(1+x^2)^{n-1}} - \frac{1}{2(n-1)}\int \frac{dx}{(1+x^2)^{n-1}} + \frac{1}{2(n-1)}\cdot\frac{x}{(1+x^2)^{n-1}}$	1A	
$= \frac{2n-3}{2n-2}\int \frac{dx}{(1+x^2)^{n-1}} + \frac{1}{2(n-1)}\cdot\frac{x}{(1+x^2)^{n-1}}$		
	3	
(d) (i) $\int_0^1 \frac{dx}{(1+x^2)^2}$		
$= \frac{1}{2}\int_0^1 \frac{dx}{1+x^2} + \frac{1}{2}\left[\frac{x}{1+x^2}\right]_0^1$	1A+1A	
$= \frac{1}{8}(\pi + 2)$ $\left(\frac{\pi}{8} + \frac{1}{4}\right)$	1A	(= 0.643)
(ii) $\int_0^1 \frac{dx}{(1+x^2)^3}$		
$= \frac{3}{4}\int_0^1 \frac{dx}{(1+x^2)^2} + \frac{1}{4}\left[\frac{x}{(1+x^2)^2}\right]_0^1$	[illegible]	
$= \frac{1}{32}(3\pi + 8)$	[illegible]	(= 0.545)
$\left(\frac{3\pi}{32} + \frac{1}{4}\right)$	5	

Solution	Marks	Remarks
9. (d)		
Alt. Solution:		
(i) $\int_0^1 \frac{dx}{(1+x^2)^2}$		
$= \int_0^{\frac{\pi}{4}} \cos^2\theta \, d\theta$		
$= \int_0^{\frac{\pi}{4}} \frac{1}{2}(1 + \cos 2\theta)\, d\theta$	2A	(integrable form)
$= \frac{1}{8}(\pi + 2)$	1A	
(ii) $\int_0^1 \frac{dx}{(1+x^2)^3}$		
$= \int_0^{\frac{\pi}{4}} \cos^4\theta \, d\theta$		
$= \int_0^{\frac{\pi}{4}} \frac{1}{4}\left(1 + 2\cos 2\theta + \frac{1}{2}(1 + \cos 4\theta)\right) d\theta$		or $\int_0^{\frac{\pi}{4}} \left(\frac{3}{8} + \frac{1}{2}\cos 2\theta + \frac{1}{8}\cos 4\theta\right)$
$= \frac{1}{32}(3\pi + 8)$	2A	

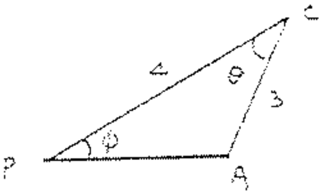
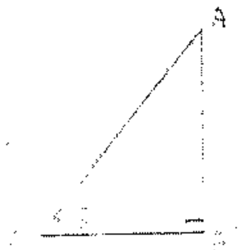
Solution	Marks	Remarks
(a) $x^2 + y^2 - 6x - 8y + 21 + k(x^2+y^2-18x-14y+105) = 0$	1A	→ $k = \frac{1}{2}$
or $k(x^2+y^2-6x-8y+21) + (x^2+y^2-18x-14y+105) = 0$		→ $k = 2$
or $(x^2+y^2-6x-8y+21) + k(2x+y-14) = 0$		→ $k = -2$
etc.		→ $k = -\frac{1}{5}$
C_3 passes through (5, 6).		
$25 + 36 - 30 - 48 + 21 + k(25+36-90-84+105) = 0$	1M	
$k = \frac{1}{2}$	1A	
$C_3 : \frac{3}{2}x^2 + \frac{3}{2}y^2 - 15x - 15y + \frac{147}{2} = 0$ $x^2 + y^2 - 10x - 10y + 49 = 0$ or $(x - 5)^2 + (y - 5)^2 = 1$	1A	
Centre is at (5, 5) and it lies on $y = x$	1	
	5	
(b) Let the equation of tangent be $y = mx$.	1A	
Sub. in equation of C_3 : $x^2 + m^2x^2 - 10x - 10mx + 49 = 0$ $(1 + m^2)x^2 - 10(1 + m)x + 49 = 0$	1M	Alt. Solution: Distance from centre (5, 5) to the line = radius $\left\|\frac{5m - 5}{\sqrt{m^2 + 1}}\right\| = 1$ 1M+1M ⋮ $12m^2 - 25m + 12 = 0$
For tangency, D = 0. $100(1 + m)^2 - 4(49)(1 + m^2) = 0$	1M	
$12m^2 - 25m + 12 = 0$		
$(4m - 3)(3m - 4) = 0$		
$m = \frac{3}{4}$ or $\frac{4}{3}$	2A	
Equations of tangents : $y = \frac{3}{4}x$, $y = \frac{4}{3}x$		
Length of tangents $= \sqrt{(\text{Dist. from } (0, 0) \text{ to centre})^2 - (\text{radius})^2}$	1M	
$= \sqrt{(5^2 + 5^2) - 1^2}$		
$= 7$	1A	
	7	

GA 29

Solution	Marks	Remarks
10. (b) <u>Alt. Solution (1):</u>		
Sub. (0, 0) into equation of C_3.		
Length of tangent $= \sqrt{0^2 + 0^2 - 10(0) - 10(0) + 49}$	1M	
$= 7$	1A	
<u>Alt. Solution (2):</u>		
For points of contact:		
$x = \dfrac{10(m+1)}{2(m^2+1)} = \dfrac{5(m+1)}{m^2+1}$	1M	
$= \dfrac{28}{5}$ or $\dfrac{21}{5}$		
When $x = \dfrac{28}{5}$, $y = \dfrac{21}{5}$		
$x = \dfrac{21}{5}$, $y = \dfrac{28}{5}$		
Length of tangent $= \sqrt{(\frac{28}{5})^2 + (\frac{21}{5})^2}$		
$= 7$	1A	
(c) $x' = \frac{1}{3}a$, $y' = \frac{1}{3}b$	1A	
$a = 3x'$, $b = 3y'$	1M	for making a, b as subjects
P is a variable point on C_3.		
Therefore,		
$(3x')^2 + (3y')^2 - 10(3x') - 10(3y') + 49 = 0$	1M	
Equation of locus of M:		
$9x^2 + 9y^2 - 30x - 30y + 49 = 0$	1A / 4	

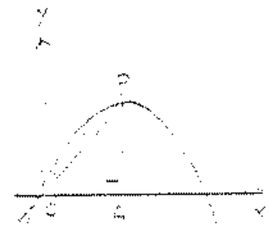
Solution	Marks	Remarks
(a) $y^2 = 8x$		$y = \sqrt{8x}$ without $\pm$ sign, max. 2 marks for part (a)
$2y \cdot \frac{dy}{dx} = 8$	1A	
$\frac{dy}{dx} = \frac{4}{y}$	1A	
Equation of tangent: $\frac{y - y_0}{x - x_0} = \frac{4}{y_0}$	1M	
$y_0y - y_0^2 = 4x - 4x_0$		
$y_0y = 4x + y_0^2 - 4x_0$		
$y_0y = 4x + 8x_0 - 4x_0$ }		
$y_0y = 4x + 4x_0$ }	1	
	4	
(b) Since $y_0 \neq 0$,		
$y = \frac{4}{y_0}x + \frac{4x_0}{y_0}$	1A	
Put $\frac{4}{y_0} = m$	1A	
$\frac{4x_0}{y_0} = \frac{4}{y_0} \cdot \frac{y_0^2}{8}$	1M	
$= \frac{y_0}{2} = \frac{1}{2} \cdot \frac{4}{m}$		
$= \frac{2}{m}$	1	
$\therefore$ Equation of tangent is $y = mx + \frac{2}{m}$	4	
(c) Equation of tangent: $y = mx + \frac{2}{m}$		
This passes through $(-4, -2)$.		
$-2 = -4m + \frac{2}{m}$	1A	
$2m^2 - m - 1 = 0$		
$m = 1$ or $-\frac{1}{2}$	1A	
	2	
(d) $a = -12m + \frac{2}{m}$		Alt. Solution:
$12m^2 + am - 2 = 0$	1A	$12m^2 + am - 2 = 0$
$m_1m_2 = -\frac{1}{6}$ }		$m = \frac{-a \pm \sqrt{a^2+96}}{24}$
$m_1 + m_2 = -\frac{a}{12}$ }	1A	$m_1 - m_2 = \frac{\pm\sqrt{a^2+96}}{12}$
$(m_1 - m_2)^2 = (-\frac{a}{12})^2 - 4(-\frac{1}{6})$	1M	$\pm\frac{\sqrt{a^2+96}}{12(1-1/6)} = \tan 45°$
$= \frac{a^2}{144} + \frac{2}{3}$		...1M+1A
$\tan 45° = \pm\frac{\sqrt{\frac{a^2}{144} + \frac{2}{3}}}{1 + (-\frac{1}{6})}$	1M+1A	$a = \pm 2$
$\frac{a^2}{144} + \frac{2}{3} = (\frac{5}{6})^2$		
$a = \pm 2$	1A	
	6	

Solution	Marks	Remarks
3. (a) $\triangle ABC = \frac{1}{2}(3)(15)\sin 2\theta$		
$\triangle APC = \frac{1}{2}(3)(4)\sin\theta$	1A+1A	all correct 2A one or two correct 1A
$\triangle BPC = \frac{1}{2}(4)(15)\sin\theta$		
$\frac{45}{2}\sin 2\theta = 6\sin\theta + 30\sin\theta$	1M	
$\frac{45}{2}(2)\cos\theta\sin\theta = 36\sin\theta$	1M	
$\cos\theta = \frac{36}{45}$		
$= \frac{4}{5}$	1 ― 5	
(b) $AA' = 3\sin\theta = \frac{9}{5}$ (cm)	1A	
$BB' = 15\sin\theta = 9$ (cm)	1A	
$A'B' = 15\cos\theta - 3\cos\theta$		
$= 12\cos\theta$		
$= \frac{48}{5}$ (cm)	1A	
$AB^2 = (AA')^2 + (A'B)^2$	1M	or $(BB')^2 + (AB')^2$
$= (AA')^2 + (BB')^2 + (A'B')^2$	1M	
$= (\frac{9}{5})^2 + 9^2 + (\frac{48}{5})^2$		
$= \frac{882}{5}$		
$AB = \sqrt{\frac{882}{5}}$		deduct 1 mark for answers without units
$= 13.28$		
≈ 13.3 (cm)	1A ― 6	
(c) $BP^2 = 15^2 + 4^2 - 2(15)(4)\cos\theta$		
$= 225 + 16 - 96$		
$= 145$	1A	
$AP^2 = 3^2 + 4^2 - 2(3)(4)\cos\theta$		
$= 9 + 16 - \frac{96}{5}$		
$= \frac{29}{5}$	1A	
$AB = 15 - 3$		
$= 12$	1A	
$\cos\angle APB = \dfrac{145 + \frac{29}{5} - 12^2}{2\sqrt{145}\sqrt{\frac{29}{5}}}$	1M	
$= 0.117$		
$\angle APB = 83.3°$	1A ― 5	

Solution	Marks	Remarks
13. (b)		
Alt. Solution (1): 		
$\angle APB = 180° - 2\phi$	1M	
$AP^2 = \frac{29}{5}$	1A	
$\frac{\sin\theta}{\sqrt{\frac{29}{5}}} = \frac{\sin\phi}{3}$	1M	
$\phi = 48.37°$	1A	
$\angle APB = (180° - 2\phi)$		
$= \underline{83.3°}$	1A	
Alt. Solution (2): 		
$\angle APB = 180° - 2\phi$	1M	
$AA' = \frac{9}{5}$		
$CA' = 3\cos\theta = \frac{12}{5}$		
$PA' = 4 - \frac{12}{5} = \frac{8}{5}$	1A	
$\tan\phi = \frac{\frac{9}{5}}{\frac{8}{5}} = \frac{9}{8}$	1M	
$\phi = 48.37°$	1A	
$\angle APB = 180° - 2\phi$		
$= \underline{83.3°}$	1A	

Solution	Marks	Remarks
(a) $\frac{dy}{dx} = -4x + k$		
$y = \int (-4x + k)dx$		
$= -2x^2 + kx + c$	1A	No mark for part (a) if omitting c.
The curve C passes through (0, 0) and (5, 10).		
Sub. (0, 0) and (5, 10) into equation of C.	1M	
$c = 0$	1A	
$10 = -2(5)^2 + 5k$		
$k = 12$	1A	
$y = -2x^2 + 12x$	4	
(b) $y = 0$		
$x = 0$ or 6	1A	
Area $= \int_0^6 (-2x^2 + 12x)dx$	1M	for $\int_a^b y dx$
$= [-\frac{2}{3}x^3 + \frac{12x^2}{2}]_0^6$		
$= 72$	1A	
	3	
(c) (i) OP : $y = \frac{b}{a}x$	1A	
Area of shaded region		
$= \int_0^a [(-2x^2 + 12x) - \frac{b}{a}x]dx$	1M	for $\int_a^b (f_1(x) - f_2(x))dx$
$= \int_0^a (-2x^2 + 12x - \frac{-2a^2 + 12a}{a}x)dx$	1M	for substituting $b = -2a^2 + 12a$
$= [-\frac{2}{3}x^3 + 6x^2 + (2a - 12)\frac{x^2}{2}]_0^a$		
$= \frac{1}{3}a^3$	1A	
$\frac{1}{3}a^3 = \frac{1}{8}(72)$		
$a = 3$	1A	

RESTRICTED 內部文件

Solution	Marks	Remarks
3. (c) (i) **Alt. Solution:**		
Area of $\triangle OAP = \frac{1}{2}ab$	1A	
$= \frac{1}{2}a(-2a^2 + 12a)$	1M	for substituting $b = -2a^2 + 12a$
$= 6a^2 - a^3$		
Area of shaded region + $\triangle OAP = \int_0^a (-2x^2 + 12x)\,dx$		
$= 6a^2 - \frac{2}{3}a^3$		
Area of shaded region $= (6a^2 - \frac{2}{3}a^3) - (6a^2 - a^3)$	1M	
$= \frac{1}{3}a^3$	1A	
$\frac{1}{3}a^3 = \frac{1}{8}(72)$		
$a = 3$	1A	
(ii) Vol. of solid generated by shaded region and $\triangle OAP$		
$= \int_0^3 \pi y^2\,dx$		
$= \int_0^3 \pi(12x - 2x^2)^2\,dx$	1M+1M	1M for $\int_a^b \pi y^2\,dx$
$= 4\pi\left[\frac{x^5}{5} - \frac{12x^4}{4} + \frac{36x^3}{3}\right]_0^3$		
$= \frac{32\pi(3)^4}{5}$		
$= 518.4\pi$ (or 1628.60)		
Vol. of cone $= \frac{1}{3}\pi ab^2$		
$= \frac{1}{3}\pi(3)18^2$		
$= 324\pi$		
Required volume $= 518.4\pi - 324\pi$	1M	
$= 194.4\pi$ (or 610.73)	1A	
Alt. Solution:		
Required volume $= \int_0^3 \pi[(12x - 2x^2)^2 - (6x)^2]\,dx$	1M+1M+1A	1M for $\int_a^b \pi y^2\,dx$, 1M for difference of volume
$= 4\pi\int_0^3 (27x^2 - 12x^3 + x^4)\,dx$		
$= 4\pi\left[\frac{27x^3}{3} - \frac{12x^4}{4} + \frac{x^5}{5}\right]_0^3$		
$= 194.4\pi$ (or 610.73)	1A	