89-CE
A MATHS
PAPER I

HONG KONG EXAMINATIONS AUTHORITY
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1989

ADDITIONAL MATHEMATICS PAPER I

8.30 am-10.30 am (2 hours)
This paper must be answered in English

Answer ALL questions in Section A and any THREE questions from Section B.

All working must be clearly shown.

Unless otherwise specified in a question, it is sufficient for numerical answers to be given correct to three significant figures.

SECTION A (42 marks)
Answer ALL questions in this section.

1. Let $y = x \sin 5x$.

   Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

   Hence find $\frac{d^2y}{dx^2} + 25y$. (4 marks)

2. Let $\overrightarrow{OA} = \mathbf{i} + 3\mathbf{j}$, $\overrightarrow{OB} = 4\mathbf{i} - \mathbf{j}$ and C be a point dividing AB internally in the ratio $k : 1$.

   (a) Express $\overrightarrow{OC}$ in terms of k , $\mathbf{i}$ and $\mathbf{j}$.

   (b) If OC is perpendicular to AB , find the value of k . (5 marks)

3. Find the coordinates of the two points on the curve $y = x^3$ at which the tangents to the curve have a slope of $\frac{3}{4}$.

   Hence find the equations of the two tangents to the curve $y = x^3$ which are parallel to the line $3x - 4y = 0$. (5 marks)

4. Let $\tan\theta = 2\tan x$ and $y = \tan(\theta - x)$ where $0 \leqslant x < \frac{\pi}{2}$.

   (a) Express y in terms of $\tan x$.

   (b) When $\frac{dy}{dx} = 0$, find the value of x . (5 marks)

5. Let $\dfrac{x^2 + 5x + 1}{x^2 - x + 1} = r$ ....(*).

Express (*) in the form $ax^2 + bx + c = 0$.

Hence find the range of the values of r for real values of x . (5 marks)

6. p and q are real numbers such that $(p + qi)^2 = 21 - 20i$.

Find the values of p and q .

Hence write down the two square roots of $21 - 20i$. (6 marks)

7. Show that $\dfrac{1 - \sin\theta + i\cos\theta}{1 - \sin\theta - i\cos\theta} = i(\cos\theta + i\sin\theta)$.

Hence show that $\left(\dfrac{1 - \sin\frac{7\pi}{36} + i\cos\frac{7\pi}{36}}{1 - \sin\frac{7\pi}{36} - i\cos\frac{7\pi}{36}}\right)^6 = \dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i$. (6 marks)

8. Solve $(x - 2)^2 - 5|x - 2| - 6 = 0$. (6 marks)

SECTION B (48 marks)
Answer any THREE questions from this section.
Each question carries 16 marks.

9.

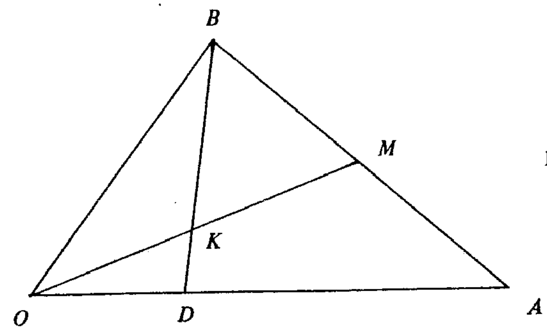

Figure 1

In Figure 1, M is the mid-point of AB and D is the point on OA such that $OD : DA = 1 : 2$. OM intersects BD at K . Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) (i) Express $\overrightarrow{OM}$ and $\overrightarrow{OD}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(ii) Suppose $BK : KD = \lambda : 1 - \lambda$.

Express $\overrightarrow{OK}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and λ .

Let $\overrightarrow{OK} = \mu\overrightarrow{OM}$. Find the values of λ and μ . (7 marks)

(b) Suppose $\mathbf{a} = 12\mathbf{i}$ and $\mathbf{b} = 2\mathbf{i} + 8\mathbf{j}$.

(i) Find $\overrightarrow{OM}$ and $\overrightarrow{DB}$ in terms of $\mathbf{i}$ and $\mathbf{j}$.

(ii) Evaluate $\overrightarrow{OM} \cdot \overrightarrow{DB}$ and hence find $\angle BKM$.

(iii) Suppose P is the point on OB such that $OP : PB = 1 : 2$.

Find $\overrightarrow{AP}$ and $\overrightarrow{AK}$, and hence show that A , K , P are collinear. (9 marks)

10. A solid right circular cylinder has length ℓ metres and base radius r metres. The sum of its length and the circumference of its cross-section is 2 metres.

(a) Find the maximum volume of the cylinder. (7 marks)

(b) Let the total surface area of the cylinder be S square metres.

(i) Express S in terms of r.

(ii) Find the value of r such that S is a maximum.

(iii) Suppose $0.15 \leqslant r \leqslant 0.25$.

Determine the range of the values of r for which S is

(1) increasing,

(2) decreasing.

Hence or otherwise, find the smallest value of S. (9 marks)

11. (a) Let α, β be the roots of the equation

$$x^2 + px + q = 0 \quad \text{.....................} (*),$$

where p and q are real constants.

Find, in terms of p and q,

(i) $\alpha^2 + \beta^2$,

(ii) $\alpha^3 + \beta^3$,

(iii) $(\alpha^2 - \beta - 1)(\beta^2 - \alpha - 1)$. (6 marks)

(b) If the square of one root of (*) minus the other root equals 1, use (a), or otherwise, to show that

$$q^2 - 3(p-1)q + (p-1)^2(p+1) = 0 \quad \text{.....................} (**).$$

(3 marks)

(c) Find the range of values of p such that the quadratic equation (**) in q has real roots. (4 marks)

(d) Suppose k is a real constant. If the square of one root of $4x^2 + 5x + k = 0$ minus the other root equals 1, use the result in (b), or otherwise, to find the value of k. (3 marks)

12.

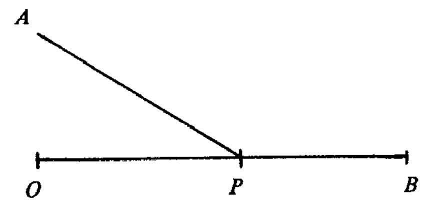

Figure 2

In Figure 2, B, due east of O, is the terminus of a railway OB of length b km, and A is a town a km north of O. A road AP is to be built connecting A to the railway at P so that goods can be transported from A to B via P. The cost of transporting 1 tonne of goods per km by road is $\$k$ $(k > 1)$ and $\$1$ per km by railway. Let $OP = x$ km, where $0 \leqslant x \leqslant b$, and let the cost of transporting 1 tonne of goods from A to B via P be $\$T$.

(a) Suppose $b = 3a$.

(i) Find T in terms of x, a and k.

(ii) If $k = 2$, find, in terms of a, the minimum value of T.

(iii) Find the range of values of k for which $\dfrac{a}{\sqrt{k^2 - 1}} > 3a$.

Hence determine the range of values of k for which it would cost more to transport goods from A to B via P than directly from A to B without using railway. (12 marks)

(b) Let $k = 2$. Find, in terms of a, the minimum value of T for

(i) $b = 2a$,

(ii) $b = \frac{1}{2}a$.

(4 marks)

13. (a) In Figure 3, the points P, H and K represent respectively the complex number $z = x + iy$ and the real numbers h and k.

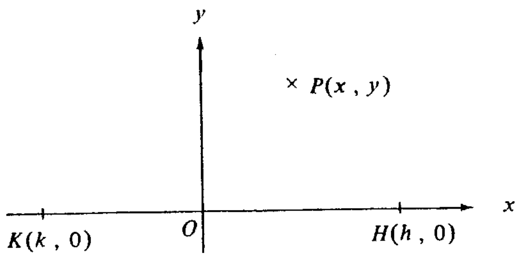

Figure 3

If P lies on the circle with HK as diameter, show that the real part of $\left(\frac{z-h}{z-k}\right)$ is 0 .

(5 marks)

(b) z_1 and z_2 are the roots of $x^2 - 2x + 2 = 0$, where $-\pi < \arg z_2 < \arg z_1 < \pi$. α and β are the roots of $x^2 + 2tx - 4 = 0$, where t is a real number.

(i) Find z_1 and z_2.

(ii) Show that α and β are real and distinct and that they have opposite signs.

(iii) Show that $\frac{z_1 - \alpha}{z_1 - \beta} = \frac{(1-\alpha)(1-\beta) + 1 + (\alpha - \beta)i}{(1-\beta)^2 + 1}$ and obtain a similar expression for $\frac{z_2 - \alpha}{z_2 - \beta}$.

(iv) Suppose $\alpha > \beta$ and α, β, z_1, z_2 are represented respectively by the points A, B, C, D on the Argand plane. In addition, C and D lie on the circle with AB as diameter.

Show that $(1-\alpha)(1-\beta) + 1 = 0$, and hence find the value of t.

(11 marks)

END OF PAPER

89-CE
A MATHS
PAPER II

HONG KONG EXAMINATIONS AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1989

ADDITIONAL MATHEMATICS PAPER II

11.15 am-1.15 pm (2 hours)

This paper must be answered in English

Answer ALL questions in Section A and any THREE questions from Section B.

All working must be clearly shown.

Unless otherwise specified in a question, it is sufficient for numerical answers to be given correct to three significant figures.

SECTION A (42 marks)

Answer ALL questions in this section.

1. Find the constant term in the expansion of $(1+x)^{10}\left(1-\frac{2}{x}\right)^3$.

(5 marks)

2. Prove, by mathematical induction, that

$$1\times2\times3+2\times3\times4+\ldots+n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$$

for all positive integers n .

(5 marks)

3. Using the substitution $u=2x^2+1$, evaluate $\int_0^2 \frac{8x^3}{\sqrt{2x^2+1}}\,dx$.

(5 marks)

4. (a) Find $\int \cos^2 2x\,dx$.

(b) Using the result in (a), find $\int \sin^2 2x\,dx$.

(5 marks)

5. Let $y=5\sin\theta-12\cos\theta+7$.

(a) Express y in the form $r\sin(\theta-\alpha)+p$ where r , α and p are constants and $0^\circ\leqslant\alpha\leqslant90^\circ$.

(b) Using the result in (a), find the least value of y .

(5 marks)

6. Find the general solution of $2\cos2\theta+5\sin\theta-3=0$.

(5 marks)

7. A straight line $L_1: y=mx+c$, where m and c are constants, makes an angle of 45° with the line $L_2: 17x-7y+14=0$.

(a) Find the two values of m .

(b) If the distance from the point (1, 2) to L_1 is 5, and m takes the greater of the two values obtained in (a), find the two values of c .

(6 marks)

8.

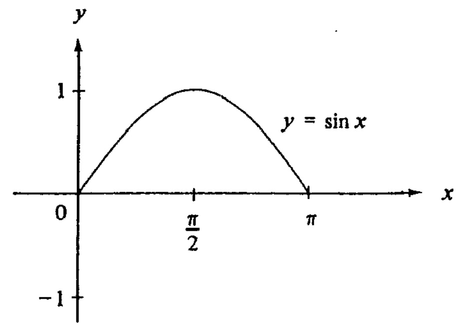

Figure 1

Figure 1 shows the graph of $y=\sin x$ for $0\leqslant x\leqslant\pi$.

(a) Copy Figure 1 into your answer book and sketch the graph of $y=\sin 2x$ for $0\leqslant x\leqslant\pi$ on the same figure.

Calculate the x-coordinates of the intersecting points of the two curves for $0\leqslant x\leqslant\pi$.

(b) Find the area bounded by the two curves for $0\leqslant x\leqslant\pi$.

(6 marks)

SECTION B (48 marks)
Answer any THREE questions from this section.
Each question carries 16 marks.

9. Let n be an integer greater than 1.

(a) Using the substitution $x = \tan\theta$, evaluate $\int_0^1 \frac{dx}{1+x^2}$. (4 marks)

(b) By differentiating $\frac{x}{(1+x^2)^{n-1}}$ with respect to x, show that

$$\int \frac{x^2}{(1+x^2)^n}\,dx = \frac{1}{2(n-1)}\left[\int \frac{dx}{(1+x^2)^{n-1}} - \frac{x}{(1+x^2)^{n-1}}\right].$$

(4 marks)

(c) Using the identity $\frac{1}{(1+x^2)^n} \equiv \frac{1}{(1+x^2)^{n-1}} - \frac{x^2}{(1+x^2)^n}$,

show that

$$\int \frac{dx}{(1+x^2)^n} = \frac{2n-3}{2n-2}\int \frac{dx}{(1+x^2)^{n-1}} + \frac{1}{2(n-1)}\cdot\frac{x}{(1+x^2)^{n-1}}.$$

(3 marks)

(d) Using the above results or otherwise, evaluate

(i) $\int_0^1 \frac{dx}{(1+x^2)^2}$,

(ii) $\int_0^1 \frac{dx}{(1+x^2)^3}$. (5 marks)

10. Two circles $C_1 : x^2 + y^2 - 6x - 8y + 21 = 0$

and $C_2 : x^2 + y^2 - 18x - 14y + 105 = 0$

intersect at A and B. C_3 is another circle passing through A, B and the point (5, 6).

(a) Write down the equation of the family of circles passing through A and B.

Hence find the equation of C_3 and show that its centre lies on the line $y = x$. (5 marks)

(b) Find the length and the equations of the two tangents from the origin O to C_3. (7 marks)

(c) P is the point (a, b) and $M(x', y')$ is the point dividing OP in the ratio 1 : 2.

Express a and b in terms of x' and y'.

If P is a variable point on C_3, find the equation of the locus of M. (4 marks)

11. (x_0, y_0) is a point on the parabola $y^2 = 8x$ where $x_0 \neq 0$.

(a) Find $\frac{dy}{dx}$.

Hence show that the equation of the tangent to the parabola at (x_0, y_0) is $y_0 y = 4x + 4x_0$. (4 marks)

(b) Using the result in (a), show that the equation of the tangent of slope m to the parabola is $y = mx + \frac{2}{m}$. (4 marks)

(c) Find the slopes of the two tangents from the point $(-4, -2)$ to the parabola. (2 marks)

(d) If the angle between the two tangents from the point $(-12, a)$ to the parabola is $45°$, find a. (6 marks)

12.

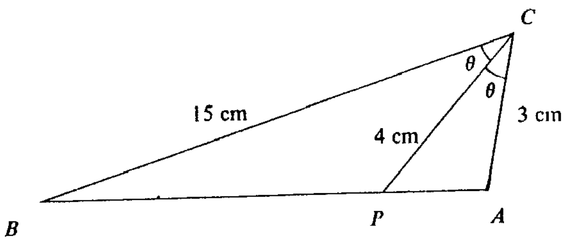

Figure 2(a)

In Figure 2(a), ABC is a triangular piece of paper. P is the point on AB such that CP bisects $\angle ACB$. $\angle ACP = \theta$, $AC = 3$ cm, $BC = 15$ cm and $CP = 4$ cm.

(a) By considering the areas of ΔABC, ΔAPC and ΔBPC, show that $\cos\theta = \frac{4}{5}$. (5 marks)

(b) ΔABC is folded along CP so that the planes APC and BPC are perpendicular as shown in Figure 2(b). A' and B' are respectively the feet of the perpendiculars from A and B to CP and CP produced.

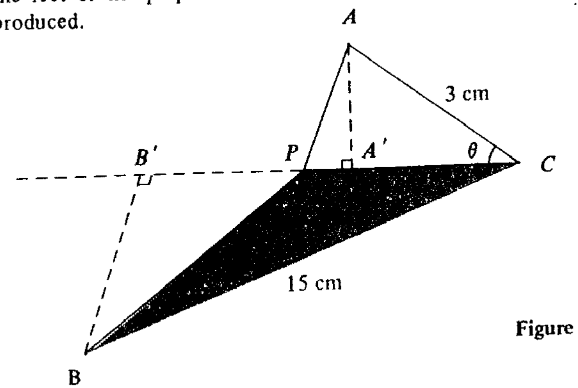

Figure 2(b)

(i) Find AA', BB' and $A'B'$.

(ii) Find the distance between A and B. (6 marks)

12. (c) The paper is further folded along CP until CA lies along CB as shown in Figure 2(c). Find $\angle APB$. (5 marks)

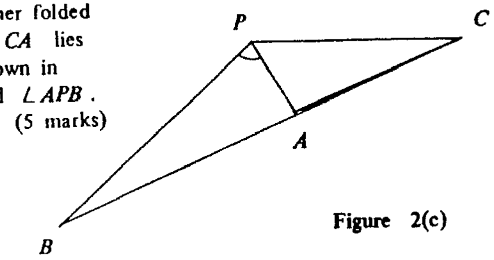

Figure 2(c)

13. The slope, at any point (x, y), of a curve C is given by $\frac{dy}{dx} = -4x + k$, where k is a constant. The curve passes through the origin and the point (5, 10).

(a) Find the value of k and the equation of the curve C. (4 marks)

(b) Find the area of the region bounded by the curve C and the x-axis. (3 marks)

(c) In Figure 3, $P(a, b)$ is a point on the curve C. The area of the region bounded by the curve and the chord OP (the shaded region in the figure) is $\frac{1}{8}$ the area obtained in (b).

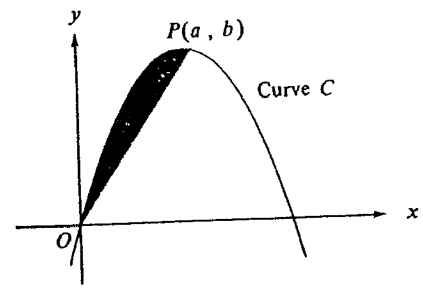

Figure 3

(i) Find the value of a.

(ii) If the shaded region is revolved about the x-axis, find the volume of the solid generated. (9 marks)

END OF PAPER

1989

Additional Mathematics I

1. $\sin 5x + 5x\cos 5x$

   $10\cos 5x - 25x\sin 5x$

   $10\cos 5x$

2. (a) $\frac{1}{k+1}[(4k+1)\mathbf{i} + (3-k)\mathbf{j}]$

   (b) $\frac{9}{16}$

3. $(\frac{1}{2}, \frac{1}{8})$, $(-\frac{1}{2}, -\frac{1}{8})$

   $3x - 4y - 1 = 0$

   $3x - 4y + 1 = 0$

4. (a) $y = \frac{\tan x}{1 + 2\tan^2 x}$

   (b) 0.615

5. $(r-1)x^2 - (r+5)x + (r-1) = 0$

   $7 > r > -1$

6. $p = 5$, $q = -2$

   $p = -5$, $q = 2$

   $5 - 2i$, $-5 + 2i$

8. -4 , 8

9. (a) (i) $\vec{OM} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$

   $\vec{OD} = \frac{1}{3}\mathbf{a}$

   (ii) $\vec{OK} = \frac{1}{3}\lambda\mathbf{a} + (1-\lambda)\mathbf{b}$

   $\lambda = \frac{3}{4}$, $\mu = \frac{1}{2}$

   (b) (i) $\vec{OM} = 7\mathbf{i} + 4\mathbf{j}$

   $\vec{DB} = -2\mathbf{i} + 8\mathbf{j}$

   (ii) 18 , 74.3°

9. (b) (iii) $\vec{AP} = \frac{2}{3}(-17\mathbf{i} + 4\mathbf{j})$

   $\vec{AK} = \frac{1}{2}(-17\mathbf{i} + 4\mathbf{j})$

10. (a) $\frac{8}{27\pi}$ m³

    (b) (i) $S = (2\pi - 4\pi^2)r^2 + 4\pi r$

    (ii) $r = \frac{1}{2\pi - 1}$

    (iii) (1) $0.15 < r < \frac{1}{2\pi - 1}$

    (2) $\frac{1}{2\pi - 1} < r < 0.25$

    Smallest value of $S = 1.07$

11. (a) (i) $p^2 - 2q$

    (ii) $-p(p^2 - 3q)$

    (iii) $q^2 - 3(p-1)q + (p-1)^2(p+1)$

    (c) $p < \frac{5}{4}$

    (d) $\frac{3}{2}$

12. (a) (i) $T = k\sqrt{a^2 + x^2} + (3a - x)$

    (ii) $(\sqrt{3} + 3)a$

    (iii) $1 < k < \frac{\sqrt{10}}{3}$

    $1 < k \leqslant \frac{\sqrt{10}}{3}$

    (b) (i) $(2 + \sqrt{3})a$

    (ii) $\sqrt{5}a$

13. (b) (i) $z_1 = 1 + i$

    $z_2 = 1 - i$

    (iii) $\frac{[(1-\alpha)(1-\beta) + 1] + (\beta - \alpha)i}{(1-\beta)^2 + 1}$

    (iv) $t = 1$

1989

Additional Mathematics II

1. -479

3. $\frac{40}{3}$

4. (a) $\frac{1}{2}x + \frac{1}{8}\sin 4x + c$

   (b) $\frac{1}{2}x - \frac{1}{8}\sin 4x + c'$

5. (a) $y = 13\sin(\theta - 67.4°) + 7$

   (b) -6

6. $180k° + (-1)^k\, 14.5°$

   $180k° + (-1)^k\, 90°$

7. (a) $\frac{5}{12}$, $-\frac{12}{5}$

   (b) 7 , $-\frac{23}{6}$

8. (a) 0 , π , $\frac{\pi}{3}$

   (b) $2\frac{1}{2}$

9. (a) $\frac{\pi}{4}$

   (d) (i) $\frac{1}{8}(\pi + 2)$

   (ii) $\frac{1}{32}(3\pi + 8)$

10. (a) $x^2 + y^2 - 6x - 8y + 21 + k(x^2 + y^2 - 18x - 14y + 105) = 0$

    $x^2 + y^2 - 10x - 10y + 49 = 0$

    (b) 7

    $y = \frac{3}{4}x$, $y = \frac{4}{3}x$

    (c) $a = 3x'$, $b = 3y'$

    $9x^2 + 9y^2 - 30x - 30y + 49 = 0$

11. (a) $\frac{dy}{dx} = \frac{4}{y}$

    (c) 1 , $-\frac{1}{2}$

    (d) ± 2

12. (b) (i) $\frac{9}{5}$ cm , 9 cm , $\frac{48}{5}$ cm

    (ii) 13.3 cm

    (c) 83.3°

13. (a) 12

    $y = -2x^2 + 12x$

    (b) 72

    (c) (i) 3

    (ii) 194.4π