香 港 考 試 局
HONG KONG EXAMINATIONS AUTHORITY

一 九 八 八 年 香 港 中 學 會 考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1988

附 加 數 學 （卷 一）
Additional Mathematics (Paper I)

評卷參考
Marking Scheme

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<u>請在學校任教之閱卷員特別留意</u>

本評卷參考並非標準答案，故極不宜落於學生手中，以免引起誤會。

遇有學生求取此文件時，閱卷員應嚴予拒絕。閱卷員在任何情況下披露本評卷參考內容，均有違閱卷員守則及「一九七七年香港考試局法例」。

<u>Special Notes for Teacher Markers</u>

It is highly undesirable that this marking scheme should fall into the hands of students. They are likely to regard it as a set of model answers, which it certainly is not.

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RESTRICTED 內部文件

SOLUTIONS	MARKS	REMARKS
1. (a) $(\sqrt{x+1+\Delta x} - \sqrt{x+1})(\sqrt{x+1+\Delta x} + \sqrt{x+1})$		
$= (x + 1 + \Delta x) - (x + 1)$		
$= \Delta x$	1A	
(b) $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$	1A	
$= \lim_{\Delta x \to 0} \left(\frac{\sqrt{x+1+\Delta x} - \sqrt{x+1}}{\Delta x} \right)$	1A	
$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+1+\Delta x} + \sqrt{x+1}}$	1A	This can't be omitted.
$= \frac{1}{2\sqrt{x+1}}$	1A	
	5	
2. Differentiating $y^2 = x^2y + 2$		
$(2y)(y') = x^2y' + 2xy$	1M	
When $x = 1$, $y^2 = y + 2$		
$y = 2$ or -1		
At $(1, 2)$, $y' = \frac{4}{3}$	1A	
$4x - 3y + 2 = 0$	1A	
At $(1, -1)$, $y' = \frac{2}{3}$	1A	
$2x - 3y - 5 = 0$	1A	
	5	
3. (a) $z_1 = 1 + 2i$, $z_2 = 1 + i$, $z_3 = 3 + 2i$	1A	
$\frac{z_1 z_2}{z_3} = \frac{-1+3i}{3+2i}$ or $\frac{7+4i}{13} \cdot (1 + i)$	1A	For z_1z_2 or $\frac{z_1}{z_3}$
$= \frac{3}{13} + \frac{11}{13}i$	1A	Accept $\frac{3+11i}{13}$
(b) $\angle AOD + \angle BOD - \angle COD$		
$= \arg z_1 + \arg z_2 - \arg z_3$		
$= \arg\left(\frac{z_1 z_2}{z_3}\right)$	1M	
$= \tan^{-1} \frac{11}{3}$		
$= 74.7°$ (or 1.30 rad.)	1A	
	5	

SOLUTIONS	MARKS	REMARKS
4. (a) $y' = \cos x + 2\sin x$	1A	
$y'' = -\sin x + 2\cos x$	1A	
(b) $y' = 0$	1M	
$\tan x = -\frac{1}{2}$		
$x = 2.68$ or 5.82	1A	Accept $x = 153°$ or $333°$ but pp-1.
Testing for min.	1M	
$x = 5.82$ corr. to a min.		
Minimum value of $y = -2.24$	1A	
	6	
5. $D = (4m)^2 - 4(4m + 15)$	1A	
$= 16m^2 - 16m - 60$	1A	
$f(x) > 0$ for all values of x		$f(x) = (x+2m)^2+(15+4m-4m^2) > 0$
$D < 0$	1M	$(15 + 4m - 4m^2) > 0$ 1
$16m^2 - 16m - 60 < 0$		
$(2m + 3)(2m - 5) < 0$	1A	
$-\frac{3}{2} < m < \frac{5}{2}$	1A	
	5	
6. $\underline{a} \cdot \underline{c} = 6 + 4k$	1A	If vector sign omitted, pp-
$\underline{b} \cdot \underline{c} = 16 + 6k$	1A	
Let θ be the angle between $\underline{a}$ and $\underline{c}$		
$\underline{a} \cdot \underline{c} = \lvert\underline{a}\rvert\lvert\underline{c}\rvert\cos\theta$		Alt. Solution (1):
$= 5\sqrt{4 + k^2}\cos\theta$	1A	
$\underline{b} \cdot \underline{c} = 10\sqrt{4 + k^2}\cos\theta$	1A	OA: $4x - 3y = 0$ OB: $6x - 8y = 0$ $3x - 4y = 0$
$\frac{6 + 4k}{5\sqrt{4 + k^2}} = \frac{16 + 6k}{10\sqrt{4 + k^2}}$	1M	$\frac{8 - 3k}{5} = \pm\frac{6 - 4k}{5}$ 1M+1
$k = 2$	1A	$k = \pm 2$ 1
	6	rejecting $k = -2$, $k = 2$ 1

Alt. Solution (2):

$\angle AOX = \tan^{-1}\frac{4}{3} = 53.13°$)
$\angle BOX = \tan^{-1}\frac{3}{4} = 36.87°$) 1A

$\angle COX = \frac{\angle AOX + \angle BOX}{2}$ 1M

$= 45°$ 1A

$k = 2$ 1A

SOLUTIONS	MARKS	REMARKS
7. (i) $x \geqslant 3$, $\frac{x-3}{2x} < 1$		
$x - 3 < 2x$		
$x > -3$	1A	
Therefore, $x \geqslant 3$	1A	
(ii) $3 > x > 0$, $\frac{3-x}{2x} < 1$		
$3 - x < 2x$		
$x > 1$	1A	
Therefore, $3 > x > 1$	1A	
(iii) $x < 0$, $\frac{3-x}{2x} < 1$		
$3 - x > 2x$		
$1 > x$	1A	
Therefore, $x < 0$	1A	
Combining the 3 cases, $x < 0$ or $x > 1$	1A	
	7	

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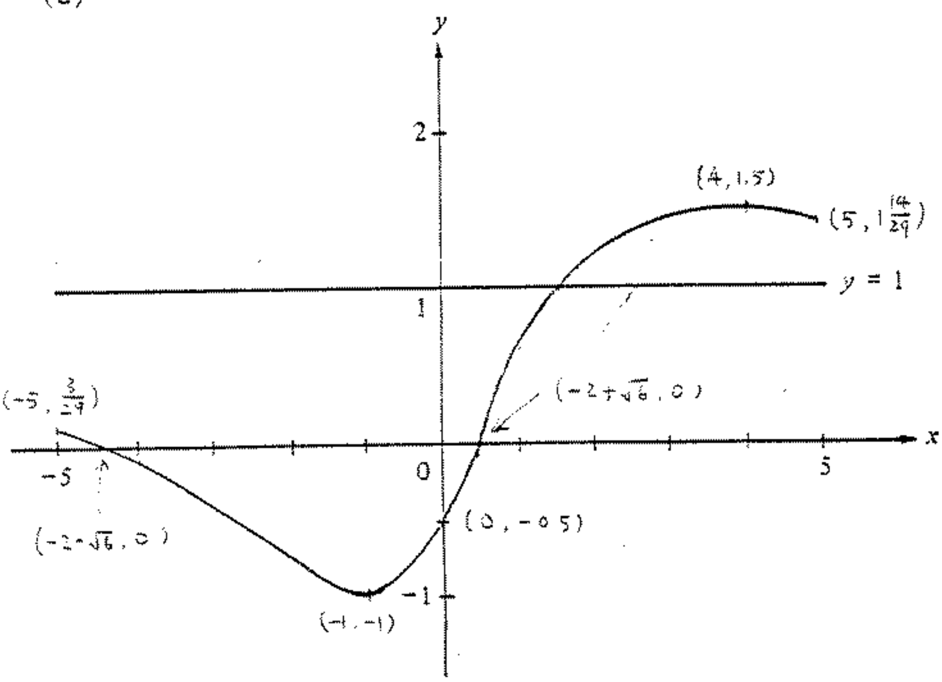
SOLUTIONS	MARKS	REMARKS
8. (a) Solving $y = \dfrac{x^2 + 4x - 2}{x^2 + 4}$ and $y = 1$,		
$x^2 + 4x - 2 = x^2 + 4$		
$4x = 6$		
$x = 1.5$		
P is the point (1.5, 1)	1A	
	1	
(b) (i) put $y = 0$		
$x^2 + 4x - 2 = 0$		
$x = -2 \pm \sqrt{6}$ (or -4.45, 0.45)	1A+1A	
put $x = 0$		
$y = -0.5$	1A	
(ii) $y' = \dfrac{(x^2 + 4)(2x + 4) - (x^2 + 4x - 2)(2x)}{(x^2 + 4)^2}$	1M	
$= \dfrac{-4x^2 + 12x + 16}{(x^2 + 4)^2}$	1A	
$= \dfrac{-4(x - 4)(x + 1)}{(x^2 + 4)^2}$		
< 0	1M	Putting $y' < 0$
$x > 4$ or $x < -1$	2A	
(iii) $y' = 0$	1M	
$x = 4$ or $x = -1$		
(4, 1.5) is a maximum point.	1A	
(−1, −1) is a minimum point.	1A	
	11	
(c)	2	Shape
	1A	intercepts, end-points (3 out of 5)
	1A	max. & min. points
	4	



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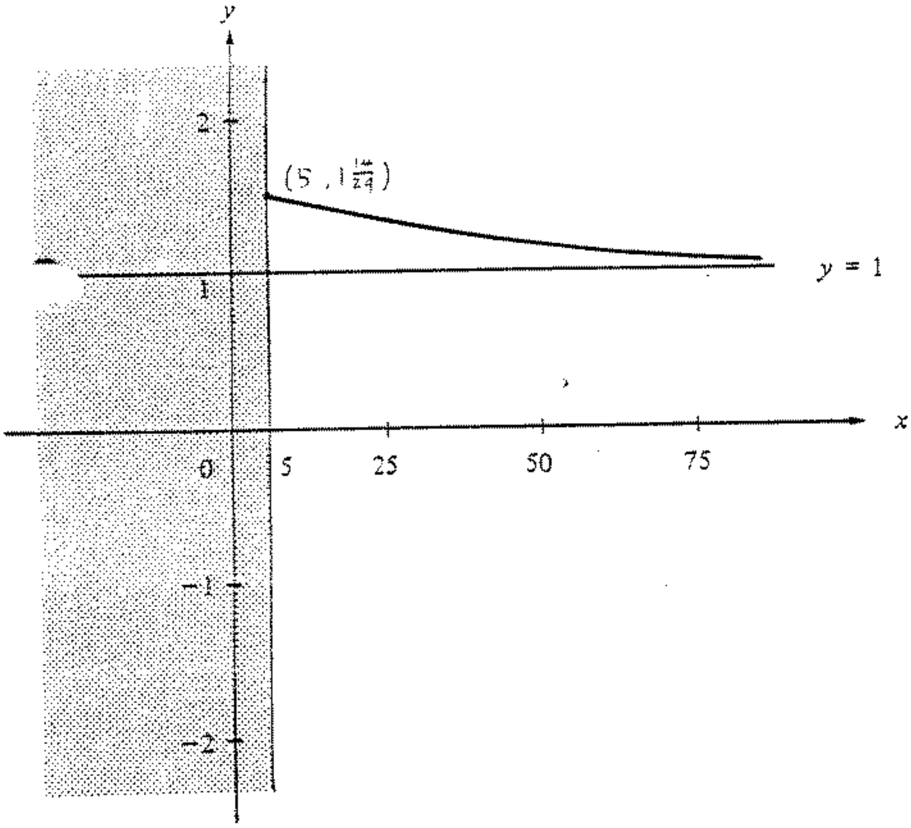
SOLUTIONS	MARKS	REMARKS
8. (d) (i) $y = 1 + \frac{4x - 6}{x^2 + 4}$	1A	
If $x > 1\frac{1}{2}$, $4x - 6 > 0$ Therefore, $\frac{4x - 6}{x^2 + 4} > 0$ $y > 1$	1	
(ii)	2	Curve cutting y = 1, deduc 1 mark. Wrong position of starting point, deduct 1 mark.
	4	



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SOLUTIONS	MARKS	REMARKS
9. (a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$	1A	If vector sign ommitted, pp
$= -7\underline{i} - 4\underline{j}$	1A	Alt. Solution:
$\overrightarrow{AB} - \overrightarrow{BC} = -12\underline{i} + 6\underline{j}$		$\overrightarrow{AB} - \overrightarrow{BC}$
$\overrightarrow{BC} = (-7\underline{i} - 4\underline{j}) - (-12\underline{i} + 6\underline{j})$	1M	$= (\overrightarrow{OB} - \overrightarrow{OA}) - (\overrightarrow{OC} - \overrightarrow{OB})$
$= 5\underline{i} - 10\underline{j}$		$= -12\underline{i} + 6\underline{j}$
$\overrightarrow{OC} - \overrightarrow{OB} = 5\underline{i} - 10\underline{j}$		$\overrightarrow{OC} = 2\overrightarrow{OB} - \overrightarrow{OA} - (-12\underline{i} + 6\underline{j})$
$\overrightarrow{OC} = -\underline{i} - 12\underline{j}$	2A 5	$= -\underline{i} - 12\underline{j}$ ....
(b) (i) $\overrightarrow{AX} = k\overrightarrow{OX}$		
$\overrightarrow{OX} - \overrightarrow{OA} = k\overrightarrow{OX}$		
$(1 - k)\overrightarrow{OX} = \overrightarrow{OA}$	1A	
$k \neq 1, \quad \overrightarrow{OX} = \frac{1}{1-k}(\underline{i} + 2\underline{j})$	1A	Accept omitting $k \neq 1$.
(ii) $\overrightarrow{BX} = \overrightarrow{OX} - \overrightarrow{OB}$		
$= \left(\frac{1}{1-k} + 6\right)\underline{i} + \left(\frac{2}{1-k} + 2\right)\underline{j}$	1A	
$OX \perp BX$		
$\frac{1}{1-k}\left(\frac{1}{1-k} + 6\right) + \frac{2}{1-k}\left(\frac{2}{1-k} + 2\right) = 0$	1M	
$(7 - 6k) + 2(4 - 2k) = 0$		
$k = 1\frac{1}{2}$	2A	
$\overrightarrow{OX} = -2\underline{i} - 4\underline{j}$	1A	Alt. Solution:
$\overrightarrow{AX} + \overrightarrow{BX} + \overrightarrow{CX}$		$\overrightarrow{AX} = -3\underline{i} - 6\underline{j}$)
$= (\overrightarrow{OX} - \overrightarrow{OA}) + (\overrightarrow{OX} - \overrightarrow{OB}) + (\overrightarrow{OX} - \overrightarrow{OC})$		$\overrightarrow{BX} = 4\underline{i} - 2\underline{j}$).....
$= 3\overrightarrow{OX} - (\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC})$	2A	$\overrightarrow{CX} = -\underline{i} + 8\underline{j}$ ....
$= (-6\underline{i} - 12\underline{j}) - (-6\underline{i} - 12\underline{j})$		$\overrightarrow{AX} + \overrightarrow{BX} + \overrightarrow{CX} = \vec{0}$ ....
$= \vec{0}$	1A	
$\overrightarrow{AC} = -2\underline{i} - 14\underline{j}$, $\overrightarrow{AB} = -7\underline{i} - 4\underline{j}$		Alt. Solution:
$\overrightarrow{AM} = \frac{1}{2}(\overrightarrow{AC} + \overrightarrow{AB})$	1M	M is the point $(-\frac{7}{2}, -7)$
$= -\frac{9}{2}\underline{i} - 9\underline{j}$	1A	$\overrightarrow{AM} = -\frac{9}{2}\underline{i} - 9\underline{j}$ ....
$\overrightarrow{AX} = -3\underline{i} - 6\underline{j}$	1A	Alt. Solution: Slope of $AX = \frac{-6}{-3} = 2$
$= \frac{2}{3}(-\frac{9}{2}\underline{i} - 9\underline{j})$		Slope of $AM = \frac{-9}{-\frac{9}{2}} = 2$
$= \frac{2}{3}\overrightarrow{AM}$	1A	
Therefore, X lies on AM.	1M 15	Slope of AX = slope of AM ∴ X lies on AM ....

SOLUTIONS	MARKS	REMARKS
10.(a) (i) $f(x) = 0$		Alt. Solution:
$x^2 + 2x - 1 = 0$	1M	$x^2 + 2x - 1 = 0$
$x = -1 \pm \sqrt{2}$ (Accept −2.41 or 0.41)	1A	$PQ = \lvert x_1 - x_2 \rvert$
$PQ = 2\sqrt{2}$ (Accept $\sqrt{8}$ or 2.83)	1A	$= \sqrt{(x_1 - x_2)^2}$
$g(x) = 0$		$= \sqrt{(x_1+x_2)^2 - 4x_1x_2}$
$-x^2 + 2kx - k^2 + 6 = 0$		$= \sqrt{(-2)^2 - 4(-1)}$
$x^2 - 2kx + k^2 - 6 = 0$		$= 2\sqrt{2}$ ....
$x = k \pm \sqrt{k^2 - (k^2 - 6)}$		$g(x) = 0$
$= k \pm \sqrt{6}$	1A	$RS = \sqrt{(2k)^2 - 4(k^2 - 6)}$
$RS = 2\sqrt{6}$ (Accept $\sqrt{24}$ or 4.90)	1A	$= 2\sqrt{6}$ ....
(ii) x-coordinate of the mid-point of RS		
$= \frac{(k + \sqrt{6}) + (k - \sqrt{6})}{2}$	1M	This can be omitted.
$= k$	1A	
x-coordinate of the mid-point of PQ = −1	1A	This can be omitted.
$k = -1$	1A	
	9	
(b) $x^2 + 2x - 1 = -x^2 + 2kx - k^2 + 6$	1	
$2x^2 + (2 - 2k)x + (k^2 - 7) = 0$		
$D = 0$		
$(2 - 2k)^2 - 4(2)(k^2 - 7) = 0$	1M	
$k^2 + 2k - 15 = 0$		
$k = 3$ or -5	2A	
For $k = 3$, $2x^2 - 4x + 2 = 0$		
$x^2 - 2x + 1 = 0$		
$x = 1$) $y = 2$)	1A	
The point is (1, 2).		
For $k = -5$, $2x^2 + 12x + 18 = 0$		
$x^2 + 6x + 9 = 0$		
$x = -3$) $y = 2$)	1A	
The point is (−3, 2).		
	6	

SOLUTIONS	MARKS	REMARKS
10.(c) $f(x) > g(x)$		
$x^2 + 2x - 1 > -x^2 + 2kx - k^2 + 6$		
$2x^2 + (-2k + 2)x + k^2 - 7 > 0$	1A	
This is true for any real value of x,		
$(2 - 2k)^2 - 4(2)(k^2 - 7) < 0$	2M	
$k^2 + 2k - 15 > 0$		
$k > 3$ or $k < -5$	2A	
	5	

Alt. Solution:

$f(x) > g(x)$

$x^2 + 2x - 1 > -x^2 + 2kx - k^2 + 6$

$2x^2 + (-2k + 2)x + k^2 - 7 > 0$ 1A

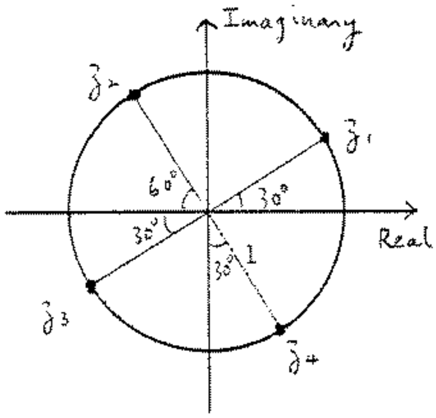
$x^2 + (1 - k)x + \frac{1}{2}(k^2 - 7) > 0$

$(x + \frac{1 - k}{2})^2 + \frac{1}{2}(k^2 - 7) - (\frac{1 - k}{2})^2 > 0$ 1M

$\frac{1}{2}(k^2 - 7) - (\frac{1 - k}{2})^2 > 0$ 1M

$k^2 + 2k - 15 > 0$

$k > 3$ or $k < -5$ 2A

SOLUTIONS	MARKS	REMARKS
11.(a) (i) $z = \cos\theta + i\sin\theta$		
$z^n = \cos n\theta + i\sin n\theta$	1A	
$\frac{1}{z^n} = \cos n\theta - i\sin n\theta$	1A	Accept $\frac{1}{z^n} = z^{-n}$ $= \cos(-n\theta) + i\sin(-n\theta)$
$z^n + \frac{1}{z^n} = 2\cos n\theta$) $z^n - \frac{1}{z^n} = 2i\sin n\theta$)	1	
(ii) $\frac{(z^2 - \frac{1}{z^2})i}{z^2 + \frac{1}{z^2}} = \frac{2i^2\sin 2\theta}{2\cos 2\theta}$	1A	
$= -\tan 2\theta$	1A	
$\tan 2\theta = \sqrt{3}$	1M	
$2\theta = n\pi + \frac{\pi}{3}$		
$\theta = \frac{n\pi}{2} + \frac{\pi}{6}$		
$z = \cos(\frac{n\pi}{2} + \frac{\pi}{6}) + i\sin(\frac{n\pi}{2} + \frac{\pi}{6})$) where $n = 0, 1, 2, 3$) ..	2A	NOTE: If one or more roots missing, award 1 ma
[Accept n = 0, 1, 2, 3, .../n is an integer.]		
OR $z = \frac{\sqrt{3}}{2} + \frac{i}{2}, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{\sqrt{3}}{2} - \frac{i}{2}, \frac{1}{2} - \frac{\sqrt{3}}{2}i$.		
 Positions should be either specified by coordinates or unit modulus and angles.	1A+1A	1A For two points. 2A For four points. NOTE: If positions not specified, deduct 1 mark.
	10	
(b) (i) $x = \frac{1 \pm \sqrt{1-4}}{2}$		
$= \frac{1 \pm \sqrt{3}i}{2}$		
$= \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$	1A	
or $\cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3})$ [Accept $\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}$]	1A	Accept $\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}$

SOLUTIONS	MARKS	REMARKS
11.(b) (ii) Product of roots $= \left(\frac{\alpha}{\beta}\right)^k \left(\frac{\beta}{\alpha}\right)^k$		
$= 1$	1A	
$\frac{\alpha}{\beta} = \frac{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}{\cos(\frac{-\pi}{3}) + i\sin(\frac{-\pi}{3})}$ or $\frac{\cos(\frac{-\pi}{3}) + i\sin(\frac{-\pi}{3})}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$		
$= \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$ or $\cos(\frac{-2\pi}{3}) + i\sin(\frac{-2\pi}{3})$	1A	
Sum of roots $= \left(\frac{\alpha}{\beta}\right)^k + \left(\frac{\beta}{\alpha}\right)^k$		
$= z^k + \left(\frac{1}{z}\right)^k$		
$= 2\cos\frac{2k\pi}{3}$	2A	
Required equation is:		
$x^2 - 2\cos\frac{2k\pi}{3} \cdot x + 1 = 0$	1A	This can be omitted.
(1) When $k = 3n$,		
$\frac{2k\pi}{3}$ is a multiple of 2π,		
equation becomes $x^2 - 2x + 1 = 0$.	1A	
(2) When $k = 3n + 1$,		
$\cos\frac{2k\pi}{3} = \cos(2n\pi + \frac{2\pi}{3}) = -\frac{1}{2}$		
equation becomes $x^2 + x + 1 = 0$	1A	
(3) When $k = 3n + 2$,		
$\cos\frac{2k\pi}{3} = \cos(2n\pi + \frac{4\pi}{3}) = -\frac{1}{2}$		
equation becomes $x^2 + x + 1 = 0$	1A	
	10	

Alt. Solution:

(b) (ii) (1) $k = 3n$,	
product of roots $= 1$	1A
sum of roots $= \left(\frac{\alpha}{\beta}\right)^{3n} + \left(\frac{\beta}{\alpha}\right)^{3n}$	
$= 2\cos\frac{2(3n\pi)}{3}$	1A
$= 2$	1A
Equation: $x^2 - 2x + 1 = 0$	1A
(2) $k = 3n + 1$,	
sum of roots $= -1$	1A
Equation: $x^2 + x + 1 = 0$	1A
(3) $k = 3n + 2$,	
sum of roots $= -1$	1A
Equation: $x^2 + x + 1 = 0$	1A

SOLUTIONS	MARKS	REMARKS
12.(a) (i) $2\pi r = \ell\phi$		
$r = \frac{\ell\phi}{2\pi}$	1A	
(ii) Let h be the height of the cone.		
$h^2 = \ell^2 - r^2$		
$= \ell^2 - \frac{\ell^2\phi^2}{4\pi^2}$	1M	For Pythagoras' Theorem.
Volume of the cone $= \frac{1}{3}\pi r^2 h$		
$= \frac{1}{3}\pi \cdot \frac{\ell^2\phi^2}{4\pi^2} \cdot \sqrt{\ell^2 - \frac{\ell^2\phi^2}{4\pi^2}}$	1M	For substitution.
$V^2 = \frac{\ell^6}{576\pi^4}[4\pi^2\phi^4 - \phi^6]$		
$= k(4\pi^2\phi^4 - \phi^6)$	1	
(iii) $\frac{d(V^2)}{d\phi} = k(16\pi^2\phi^3 - 6\phi^5)$	1A	
$= 0$	1M	
$\phi \neq 0, \quad \phi^2 = \frac{8\pi^2}{3}$		
$\phi > 0, \quad \phi = \frac{2\sqrt{6}}{3}\pi$ (or 5.13) (or 1.63π)	2A	Accept $\phi = 0$ or $\pm\frac{2\sqrt{6}}{3}\pi$
$\frac{d^2(V^2)}{d\phi^2} = k(48\pi^2\phi^2 - 30\phi^4)$		
$= 6k\phi^2(8\pi^2 - 5\phi^2)$		
$\left.\frac{d^2(V^2)}{d\phi^2}\right\vert_{\phi^2=\frac{8\pi^2}{3}} = 6k \cdot \frac{8\pi^2}{3}(8\pi^2 - 5\cdot\frac{8\pi^2}{3})$		
< 0	1M	
V^2 is a maximum when $\phi = \frac{2\sqrt{6}}{3}\pi$)		
)	1A	
V is a maximum when $\phi = \frac{2\sqrt{6}}{3}\pi$)	10	
(b) (i) $\ell = r - r\cos\theta$	1A	
(ii) $A = \frac{1}{2}(r^2)(2\theta) - \frac{1}{2}r^2\sin 2\theta$		
$= r^2\theta - \frac{1}{2}r^2\sin 2\theta$	2A	or $r^2\theta - r^2\sin\theta\cos\theta$
(iii) $\frac{dA}{d\theta} = r^2 - r^2\cos 2\theta$	1A	or $2r^2\sin^2\theta$
$\frac{d\theta}{d\ell} = \frac{1}{r\sin\theta}$	1A	
$\frac{dA}{dt} = \frac{dA}{d\theta}\cdot\frac{d\theta}{d\ell}\cdot\frac{d\ell}{dt}$	1M	
$= (r^2 - r^2\cos 2\theta)\cdot\frac{1}{r\sin\theta}\cdot u$	1M	
$= \frac{ru(1-\cos 2\theta)}{\sin\theta}$	1A	Accept $\frac{u(r - r\cos 2\theta)}{\sin\theta}$
$= 2ru\sin\theta$	1A	
When $\theta = \frac{\pi}{6}$, $\frac{dA}{dt} = 2ru\sin\frac{\pi}{6}$		
$= ru$	1	
	10	

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一九八八年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1988

附加數學（卷二）
Additional Mathematics (Paper II)

評卷參考
Marking Scheme

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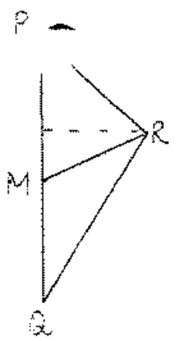
SOLUTIONS	MARKS	REMARKS
1. $(1+3x)^4(1-2x)^5$		
$= [1+4(3x)+6(3x)^2+\ldots][1+5(-2x)+10(-2x)^2+\ldots]$	1A+1A	If '...' omitted, withhold 1 mark.
$= [1+12x+54x^2+\ldots][1-10x+40x^2+\ldots]$		
$a = 2$	1A	
$b = 40+54-120$		
$= -26$	2A	
	5	
2. (a) P is the point $\left(\frac{7k+1}{k+1}, \frac{4k+2}{k+1}\right)$	1A+1A	Alt. Solution:
(b) Sub. the coordinates of P in $7x-3y-28=0$,		Intersection of AB and $7x-3y-28=0$ is $Q(\frac{11}{2}$,
$7\left(\frac{7k+1}{k+1}\right) - 3\left(\frac{4k+2}{k+1}\right) - 28 = 0$	2M	$\frac{7k+1}{k+1} = \frac{11}{2}$ or $\frac{4k+2}{k+1} = \frac{7}{2}$ ...
$7+49k-6-12k-28k-28=0$		$k = 3$ ....
$9k = 27$		NOTE: No marks awarded fo[r]
$k = 3$	1A	$\frac{AQ}{QB} = \frac{3\sqrt{10}/2}{\sqrt{10}/2} = 3$
The ratio is $3:1$.		
	5	
3. The two curves intersect when		
$x(x+3) = x(5-x)$		
$x = 0$ or $x = 1$	1A	
Area of shaded region $= \int_0^1 [x(5-x)-x(x+3)]dx$	1M+1A	1M for $\int_a^b (f_2(x)-f_1(x))d[x]$
$= \int_0^1 (2x-2x^2)dx$		Alt. Solution:
		$\int_0^1 x(5-x)dx = \frac{13}{6}$
$= [x^2 - \frac{2}{3}x^3]_0^1$	1A	$\int_0^1 x(x+3)dx = \frac{11}{6}$ ......
$= \frac{1}{3}$	1A	Area $= \frac{13}{6} - \frac{11}{6}$
		$= \frac{1}{3}$ ....
4. $\sqrt{x^2+y^2} + \sqrt{(x-6)^2+y^2} = 10$	1A	
$(10-\sqrt{x^2+y^2})^2 = (\sqrt{(x-6)^2+y^2})^2$	1M	
$100 - 20\sqrt{x^2+y^2} + x^2+y^2 = (x-6)^2+y^2$		
$20\sqrt{x^2+y^2} = 12x+64$		
$(5\sqrt{x^2+y^2})^2 = (3x+16)^2$	1M	
$16x^2+25y^2-96x-256=0$	2A	
	5	

SOLUTIONS	MARKS	REMARKS
5. $n = 1$, L.S. $= 1^2 = 1$		
R.S. $= \dfrac{1(2-1)(2+1)}{3} = 1$		
The equality holds for $n = 1$.	1	
Assume $1^2+3^2+\ldots+(2k-1)^2 = \dfrac{k(2k-1)(2k+1)}{3}$ for some positive integer k.	1	
$n = k + 1$,		Alt. Solution:
L.S. $= 1^2 + 3^2 + \ldots + (2k-1)^2 + [(2(k+1) - 1]^2$	1	L.S. $= \ldots$
$= \dfrac{k(2k-1)(2k+1)}{3} + (2k+1)^2$	1	$= \dfrac{4k^3+12k^2+11k+3}{3}$
$= \dfrac{(2k+1)}{3}[2k^2 - k + 3(2k+1)]$		R.S. $= \dfrac{1}{3}(k+1)(2k+1)(2k+3)$
$= \frac{1}{3}(2k+1)(2k^2+5k+3)$		$= \dfrac{1}{3}(4k^3+12k^2+11k$
$= \frac{1}{3}(2k+1)(2k+3)(k+1)$		$=$ L.S.
$= \frac{1}{3}(k+1)(2k+1)(2k+3)$	1	
Therefore equality holds for $n = k + 1$.		
By the Principle of Mathematical Induction, the equality holds for all positive integers n.	1 6	Award this mark only if a candidate has scored the first 5 marks.
		Alt. Solution:
6. Put $u = 9 - x^3$	1A	Put $v^2 = 9 - x^3$
$du = -3x^2dx$	1A	$2vdv = -3x^2dx$ 1
When $x = 0$, $u = 9$) $x = 2$, $u = 1$)	1A	When $x = 0$, $v = 3$) $x = 2$, $v = 1$)
$\int_0^2 \dfrac{x^2dx}{\sqrt{9-x^3}}$		$\int_0^2 \dfrac{x^2dx}{\sqrt{9-x^3}} = \int_3^1 \left(-\dfrac{2}{3}\right)dv$
		$= [\frac{2}{3}v]_1^3$ 1
$= \int_9^1 \dfrac{-du}{3\sqrt{u}}$	1A	$= \dfrac{4}{3}$ 1
		Alt. Solution:
$= \frac{1}{3}[\frac{\sqrt{u}}{\frac{1}{2}}]_1^9$	1A	Put $x^3 = 9\sin^2\theta$ 1
		$3x^2dx = 18\sin\theta\cos\theta d\theta$ 1
$= \frac{4}{3}$	1A 6	When $x = 0$, $\theta = 0$ $x = 2$, $\theta = 1.231$ } 1
		$\int_0^2 \dfrac{x^2dx}{\sqrt{9-x^3}}$
		$= \int_0^{1.231} 2\sin\theta d\theta$ 1
		$= [-2\cos\theta]_0^{1.231}$ 1
		1.33 1

SOLUTIONS	MARKS	REMARKS
7. (a) $\cos\frac{3\pi}{10} = \sin(\frac{\pi}{2} - \frac{3\pi}{10})$	1A	**Alt. Solution:**
$= \sin\frac{2\pi}{10}$	1	$\cos 3\theta = \sin 2\theta$
$\frac{\pi}{10}$ is a root of $\cos 3\theta = \sin 2\theta$.		$\cos 3\theta = \cos(\frac{\pi}{2} - 2\theta)$ 1
Alt. Solution: $\sin\frac{2\pi}{10} = \cos(\frac{\pi}{2} - \frac{2\pi}{10})$ 1A		$3\theta = 2n\pi \pm (\frac{\pi}{2} - 2\theta)$
$= \cos\frac{3\pi}{10}$ 1		$5\theta = 2n\pi + \frac{\pi}{2}$
		$\theta = 2n\pi - \frac{\pi}{2}$
		$\theta = \frac{\pi}{10}$ 1
(b) $\cos 3\theta = \sin 2\theta$		
$4\cos^3\theta - 3\cos\theta = 2\sin\theta\cos\theta$	1A	
$\cos\theta \neq 0$ for $\theta = \frac{\pi}{10}$.		
Therefore, $4\cos^2\theta - 3 - 2\sin\theta = 0$		
$4\sin^2\theta + 2\sin\theta - 1 = 0$	2A	
$\sin\theta = \frac{-2 \pm \sqrt{4 + 16}}{8}$	1A	
As $\sin\frac{\pi}{10} > 0$,		
$\sin\frac{\pi}{10} = \frac{\sqrt{5} - 1}{4}$	1A	
	7	
8. (a) $u = \sin x$		
$du = \cos x\, dx$	1A	
$x = 0$, $u = 0$)) .... $x = \frac{\pi}{2}$, $u = 1$)	1A	
$\int_0^{\frac{\pi}{2}} \cos^7 x dx = \int_0^1 (1 - u^2)^3 du$	1A	
$= \int_0^1 (1 - 3u^2 + 3u^4 - u^6) du$	1M	For expanding integrand.
$= [\, u - u^3 + \frac{3}{5}u^5 - \frac{1}{7}u^7 \,]_0^1$		
$= \frac{16}{35}$	1A	
	5	
(b) $\frac{dy}{dx} = \cos^{2n}x + (2n - 1)\cos^{2n-2}x\,(-\sin x)\,\sin x$	1A	
$= \cos^{2n}x - (2n - 1)\cos^{2n-2}x\sin^2 x$	1A	
Integrating,		
$\int [\cos^{2n}x - (2n-1)\cos^{2n-2}x\sin^2 x] dx = \sin x\cos^{2n-1}x + C$	1M	
$\int [\cos^{2n}x - (2n-1)\cos^{2n-2}x(1-\cos^2 x)] dx = \sin x\cos^{2n-1}x + C$		
$2n\int \cos^{2n}x dx - (2n-1)\int \cos^{2n-2}x dx = \sin x\cos^{2n-1}x + C$	1	
	4	

SOLUTIONS	MARKS	REMARKS
8. (c) (i) From (b),		
$2n\int_0^{\frac{\pi}{2}}\cos^{2n}x\,dx - (2n-1)\int_0^{\frac{\pi}{2}}\cos^{2n-2}x\,dx$		
$= [\sin x\cos^{2n-1}x]_0^{\frac{\pi}{2}}$	1A	OR $[\sin x\cos^{2n-1}x + C]_0^{\frac{\pi}{2}}$
$2n\int_0^{\frac{\pi}{2}}\cos^{2n}x\,dx - (2n-1)\int_0^{\frac{\pi}{2}}\cos^{2n-2}x\,dx = 0$	1A	For R.S.
$\int_0^{\frac{\pi}{2}}\cos^{2n}x\,dx = \frac{2n-1}{2n}\int_0^{\frac{\pi}{2}}\cos^{2n-2}x\,dx$	1	
(ii) $\int_0^{\frac{\pi}{2}}\cos^6 x\,dx = \frac{5}{6}\int_0^{\frac{\pi}{2}}\cos^4 x\,dx$	1A	
$= \frac{5}{6}\cdot\frac{3}{4}\int_0^{\frac{\pi}{2}}\cos^2 x\,dx$	1A	Alt. Solution:
$\int_0^{\frac{\pi}{2}}\cos^2 x\,dx = \frac{1}{2}\int_0^{\frac{\pi}{2}}dx$		$\int_0^{\frac{\pi}{2}}\cos^2 x\,dx = \int_0^{\frac{\pi}{2}}\frac{1}{2}(\cos 2x+1)d$
$= \frac{1}{2}[x]_0^{\frac{\pi}{2}}$		$= \frac{1}{2}[\frac{1}{2}\sin 2x + x]_0^{\frac{\pi}{2}}$
$= \frac{\pi}{4}$	1A	$= \frac{\pi}{4}$ 1
Therefore, $\int_0^{\frac{\pi}{2}}\cos^6 x\,dx = \frac{5}{6}\cdot\frac{3}{4}\cdot\frac{\pi}{4}$		
$= \frac{5}{32}\pi$	1A 7	
(d) Put $v = \frac{\pi}{2} - x$	1A	
$dv = -dx$; $x = 0, v = \frac{\pi}{2}$; $x = \frac{\pi}{2}, v = 0$	1A	
$\int_0^{\frac{\pi}{2}}\sin^6 x\,dx = \int_{\frac{\pi}{2}}^{0}\sin^6(\frac{\pi}{2} - v)(-dv)$		NOTE: If a cand. claims
$= \int_0^{\frac{\pi}{2}}\cos^6 v\,dv$	1A	$\int_0^{\frac{\pi}{2}}\sin^6 x\,dx = \int_0^{\frac{\pi}{2}}\cos^6 x\,dx$ 1
$= \frac{5}{32}\pi$	1A 4	$= \frac{5}{32}\pi$........ 1
Alt. Solution:		
$\int_0^{\frac{\pi}{2}}\sin^6 x\,dx = \int_0^{\frac{\pi}{2}}(1 - \cos^2 x)^3 dx$	1A	
$= \int_0^{\frac{\pi}{2}}(1 - 3\cos^2 x + 3\cos^4 x - \cos^6 x)dx$	1A	
$= [x]_0^{\frac{\pi}{2}} - \frac{3}{2}\cdot\frac{\pi}{2} + 3\cdot\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2} - \frac{5}{32}\pi$	1M	For using (c).
$= \frac{5}{32}\pi$	1A	

SOLUTIONS	MARKS	REMARKS
9. (a) In ΔPQR, $\frac{PR}{\sin\beta^\circ} = \frac{c}{\sin\angle PRQ}$	2A	Accept expressions with no degree measure
$\sin\angle PRQ = \sin(180^\circ - \alpha^\circ - \beta^\circ)$		
$= \sin(\alpha^\circ + \beta^\circ)$	2A	
In ΔPBR, $h = PR\tan\theta^\circ$	1M	
$= \frac{c\tan\theta^\circ\sin\beta^\circ}{\sin(\alpha^\circ+\beta^\circ)}$	1	
	6	
(b) (i) In ΔPQR, $\frac{QR}{\sin\alpha^\circ} = \frac{c}{\sin\angle PRQ}$	1A	
$QR = \frac{c\sin 54^\circ}{\sin 80^\circ}$ (= 0.8215c)		
$\tan\angle BQR = \frac{h}{QR}$	2M	h = 0.6129c
$= \frac{c\sin 46^\circ\tan 40^\circ}{\sin 100^\circ} \cdot \frac{\sin 80^\circ}{c\sin 54^\circ}$		
$\angle BQR = 36.7^\circ$	2A	
(ii) In ΔQMR, $MR^2 = QM^2 + QR^2 - 2QM \cdot QR \cdot \cos 46^\circ$	2M	Alt. Solution:
$MR = 0.5951c$		In ΔPQR, $PR = \frac{c\sin 46^\circ}{\sin 80^\circ}$
		$MR^2 = PM^2 + PR^2 - 2PM \cdot PR\cos 54^\circ$ 2
$\tan\angle BMR = \frac{BR}{MR}$	1M	
$= \frac{c\tan 40^\circ\sin 46^\circ}{\sin 100^\circ} \cdot \frac{1}{0.5951c}$		
$\angle BMR = 45.8^\circ$	2A	
In ΔPMR, $\frac{\sin\angle PMR}{PR} = \frac{\sin 54^\circ}{MR}$	1M	for $\angle PMR$
$\sin\angle PMR = \sin 54^\circ \cdot \frac{0.7304c}{0.5951c}$		
$\angle PMR = 83.2^\circ$ or 96.8° (rejected) (Accept $\angle PMR = 83.2^\circ$)	2A	
The bearing of B from M is N83.2°E.	1A	
	14	



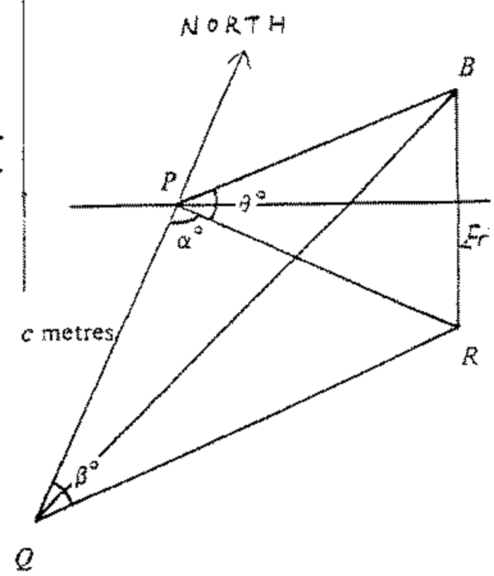
Alt. Solution:

In ΔQMR,

$\frac{\sin\angle QMR}{QR} = \frac{\sin 46^\circ}{MR}$ 1M

$\angle QMR = 96.8^\circ$ or 83.2° 2A

Rejecting 83.2°, the bearing of B from M is N83.2°E. 1A



SOLUTIONS	MARKS	REMARKS
10.(a) $x = a\sin\theta$		
$dx = a\cos\theta d\theta$	1A	
When $x = 0$, $\theta = 0$; $x = a$, $\theta = \frac{\pi}{2}$	1A	
$\int_0^a \sqrt{a^2 - x^2}\, dx = \int_0^{\frac{\pi}{2}} a^2\cos^2\theta d\theta$	1A	
$= \frac{a^2}{2}\int_0^{\frac{\pi}{2}} (1 + \cos 2\theta) d\theta$		
$= \frac{1}{2}a^2 [\theta + \frac{1}{2}\sin 2\theta]_0^{\frac{\pi}{2}}$		
$= \frac{\pi a^2}{4}$	1A	
Area of ellipse $= 2\int_{-a}^{a} y dx$	1A	
$= 4\int_0^a b\sqrt{1 - \frac{x^2}{a^2}}\, dx$		
$= \frac{4b}{a}\int_0^a \sqrt{a^2 - x^2}\, dx$		
$= \pi ab$	1A 6	
(b) (i) Volume of pebble $= \int_{-1}^{1} \pi y^2 dx$	1A+1M	1A for limits 1M for $\int_a^b \pi y^2 dx$
$= \int_{-1}^{1} \pi(\frac{3}{4})^2(1 - x^2)dx$	1A	
$= \frac{9}{16}\pi[x - \frac{x^3}{3}]_{-1}^{1}$		
$= \frac{3}{4}\pi$	1A	
(ii) (1) $V = \int_{-b}^{-(b-h)} \pi x^2 dy$	1M+1A	1A for limits 1M for $\int_a^b \pi x^2 dy$
$= \int_{-b}^{-b+h} \pi\cdot 4b^2(1 - \frac{y^2}{b^2})dy$	1A	
$= 4\pi b^2[y - \frac{y^3}{3b^2}]_{-b}^{-b+h}$		
$= 4\pi b^2[-b + h - \frac{(-b+h)^3}{3b^2} + b - \frac{b^3}{3b^2}]$		
$= 4\pi b^2[h - \frac{b}{3} + \frac{b^3 - 3b^2h + 3bh^2 - h^3}{3b^2}]$	1M	for expanding $(b-h)^3$
$= 4\pi b^2[\frac{3bh^2 - h^3}{3b^2}]$		
$= \frac{4\pi h^2}{3}(3b - h)$	1	
$\frac{dV}{dh} = 8\pi bh - 4\pi h^2$	1A	
When $h = \frac{b}{2}$, $\frac{dV}{dh} = 4\pi b^2 - \pi b^2$		
$= 3\pi b^2$	1A	
(2) $\delta V \approx \frac{dV}{dh}\cdot\delta h$	1M	
$\frac{3}{4}\pi \approx 3\pi(5)^2\cdot\delta h$	1M	For δV = vol. of pebble in (b)(i)
$\delta h \approx 0.01$ (unit)	1 14	

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SOLUTIONS	MARKS	REMARKS
11. (a) S lies on the perpendicular through K,		
slope of KS = −5	1A	
$\frac{y-12}{x-1} = -5$	1A	
$5x + y - 17 = 0$		Alt. Solution:
S also lies on the perpendicular bisector of HK:		HS = KS
Mid-point of HK is (−1, 9))		$\sqrt{(x+3)^2 + (y-6)^2}$
Slope of HK = $\frac{12-6}{1-(-3)} = \frac{3}{2}$)	1A	$= \sqrt{(x-1)^2 + (y-12)^2}$ 1M+
$\frac{y-9}{x+1} = -\frac{2}{3}$	1M+1A	$8x + 12y - 100 = 0$ ....
$-3y + 27 = 2x + 2$		$2x + 3y - 25 = 0$
$2x + 3y - 25 = 0$		
Solving the two equations,		
$x = 2,\ y = 7$	1A	
S is the point (2, 7).		
Equation of C: $(x-2)^2 + (y-7)^2 = (2-1)^2 + (7-12)^2$	1M	
$(x-2)^2 + (y-7)^2 = 26$	1A	
$x^2 + y^2 - 4x - 14y + 27 = 0$	8	

Alt. Solution:

Let the equation of C be

$x^2 + y^2 + 2gx + 2fy + c = 0$ 1M

This passes through (1, 12) and (−3, 6).

$1^2 + 12^2 + 2g + 24f + c = 0$ 1A

$9 + 36 - 6g + 12f + c = 0$ 1A

Differentiating the equation of C,

$2x + 2yy' + 2g + 2fy' = 0$ 1M

$2 + 24(\frac{1}{5}) + 2g + 2f(\frac{1}{5}) = 0$ 1A

$5g + f + 17 = 0$

Solving the three equations,

$g = -2$)
$f = -7$) 1A
$c = 27$)

S is (2, 7). 1A

Equation of C is $x^2+y^2-4x-14y+27 = 0$ 1A

Alt. Solution:

$\frac{12+f}{1+g} = -5$ 1M+

$5g + f + 17 = 0$

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SOLUTIONS	MARKS	REMARKS

11.(b) Equation of family of circles through A and B:

$$x^2 + y^2 - 4x - 14y + 27 + k(3x-2y-5) = 0$$ 1M+1A

$$x^2 + y^2 + (3k-4)x - (14+2k)y + (27-5k) = 0$$

The centre is at $\left(\frac{4-3k}{2} , \frac{14+2k}{2} \right)$ 1A

This lies on L, therefore

$$3\left(\frac{4-3k}{2} \right) - 2\left(\frac{14+2k}{2} \right) - 5 = 0$$ 2M

$k = -2$ 1A

Required equation is $x^2+y^2-10x-10y+37 = 0$ ...(*) 1A

Total: 7

(c) Showing that S lies on the circle (*). 2A

$\angle ASB = 90°$ ($\angle$ in a semi-circle) 1

Alt. Solution (1):

A and B are (3, 2) and (7, 8).
S is (2, 7).

$AS^2 + BS^2 = (8-7)^2 + (7-2)^2 + (7-2)^2 + (2-3)^2 = 52$ 1A

$AB^2 = (7 - 3)^2 + (8 - 2)^2 = 52$ 1A

$AS^2 + BS^2 = AB^2$
$\angle ASB = 90°$) 1

Alt. Solution (2):

A(7, 8), B(3, 2)

slope of AS $= \frac{8-7}{7-2} = \frac{1}{5}$ 1A

slope of BS $= \frac{7-2}{2-3} = -5$ 1A

slope of AS   slope of BS = -1
therefore $\angle ASB = 90°$) 1

$\angle APB = \frac{1}{2} \angle ASB$ or $180° - \frac{1}{2} \angle ASB$

$= 45°$ or $135°$ 1A+1A

Total: 5

REMARKS

Alt. Solution:

Solving eqts. of L and C,

$x^2 - 10x + 21 = 0$
$(x - 3)(x - 7) = 0$

$\begin{cases} x = 3 \\ y = 2 \end{cases}$ or $\begin{cases} x = 7 \\ y = 8 \end{cases}$

A, B are (3, 2), (7, 8).

Radius $= \sqrt{\left(\frac{3k-4}{2}\right)^2 + \left(\frac{14+2k}{2}\right)^2 - (27-5k)}$

$= \frac{1}{2}AB$

$= \frac{1}{2}\sqrt{(7-3)^2 + (8-2)^2}$

$= \sqrt{13}$

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SOLUTIONS	MARKS	REMARKS

12. (a) Equation of L: $\frac{y - 0}{x + 2} = m$ — 1A

$y = m(x + 2)$

$y = mx + 2m$

Since A and B are the intersecting points of L and the parabola $y^2 = 8x$, the coordinates of A and B satisfies the equations of L and the parabola, i.e.

$y = mx + 2m$ and $y^2 = 8x$.

Eliminating y, $(mx + 2m)^2 = 8x$ 1M

$m^2x^2 + (4m^2 - 8)x + 4m^2 = 0$ — 1

3

$$\left.\begin{aligned} x_1 + x_2 &= \frac{8 - 4m^2}{m^2} \\ x_1x_2 &= 4 \end{aligned}\right)$$ 1A

$(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2$ — 1A

$= \left(\frac{8 - 4m^2}{m^2}\right)^2 - 16$ .... 1M+1A

$= 16\left[\frac{(2 - m^2)}{m^2}\right]^2 - 16$

$= \frac{16(4 - 4m^2)}{m^4}$ 1A

$= \frac{64(1 - m^2)}{m^4}$

5

Alt. Solution:

$x = \frac{-(4m^2-8) \pm \sqrt{(4m^2-8)^2 - 4(4m^2)(m^2)}}{2m^2}$ — 1A

$(x_1 - x_2)^2 = \left[\frac{2\sqrt{(4m^2-8^2)^2 - 16m^4}}{2m^2}\right]^2$ — 2M+1A

$= \frac{64(1 - m^2)}{m^4}$ — 1A

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SOLUTIONS	MARKS	REMARKS
12. (c) $y_1 = mx_1 + 2m$, $y_2 = mx_2 + 2m$	1A	Can be omitted.
$y_1 - y_2 = m(x_1 - x_2)$	2A	
$AB^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$		
$= (x_1 - x_2)^2 + m^2((x_1 - x_2)^2$	1M	
$= (1 + m^2)(x_1 - x_2)^2$, $= \dfrac{64(1 + m^2)(1 - m^2)}{m^4}$	1	
	5	

Alt. Solution:

Eliminating x from $y = mx + 2m$ and $y^2 = 8x$.

$my^2 - 8y + 16m = 0$ 1A

$y_1 + y_2 = \dfrac{8}{m}$, $y_1 y_2 = 16$ 1A

$(y_1 - y_2)^2 = (y_1 + y_2)^2 - 4y_1y_2$

$= (\dfrac{8}{m})^2 - 64$ 1M

$= \dfrac{64(1 - m^2)}{m^2}$ 1A

$AB^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

$= \dfrac{64(1 - m^2)}{m^4} + \dfrac{64(1 - m^2)}{m^2}$

$= \dfrac{64(1 - m^2)(1 + m^2)}{m^4}$ 1

SOLUTIONS	MARKS	REMARKS
(d) From (c), $AB^2 = 0$ or from (a), D= 0.	1M	
$m^2 - 1 = 0$		
$m = \pm 1$	1A+1A	
	3	
(e) L: $y = \dfrac{\sqrt{3}}{3}x + \dfrac{2\sqrt{3}}{3}$		
$x - \sqrt{3}y + 2 = 0$		
Distance from C to L = $\left\lvert \dfrac{2 - \sqrt{3}(0) + 2}{\sqrt{1 + 3}} \right\rvert$	1M	Absolute value sign optional
$= 2$	1A	
Length of AB = $\sqrt{\dfrac{64(1 + \frac{1}{3})(1 - \frac{1}{3})}{\frac{1}{9}}}$		
$= 16\sqrt{2}$	1A	
$\Delta ABC = \frac{1}{2}(2)(16\sqrt{2})$		
$= 16\sqrt{2}$	1A	
	4	