88-CE
A MATHS
PAPER I

HONG KONG EXAMINATIONS AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1988

附加數學 試卷一

ADDITIONAL MATHEMATICS PAPER I

8.30 am–10.30 am (2 hours)
This paper must be answered in English

Answer **ALL** questions in Section A and any **THREE** questions from Section B.

All working must be clearly shown.

Unless otherwise specified in a question, it is sufficient for numerical answers to be given correct to three significant figures.

SECTION A (39 marks)

Answer ALL questions in this section.

1. (a) Simplify $(\sqrt{x+1+\Delta x} - \sqrt{x+1})(\sqrt{x+1+\Delta x} + \sqrt{x+1})$.

   (b) Let $y = \sqrt{x+1}$.

   Find $\frac{dy}{dx}$ from first principles.

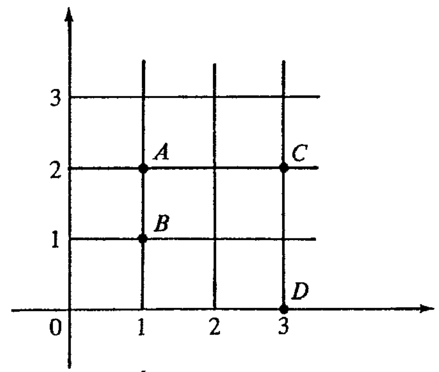
(5 marks)

2. Find the equations of the two tangents to the curve $y^2 = x^2y + 2$ at the points where $x = 1$.

(5 marks)

3.

Figure 1



A, B, C and D are four points in the Argand diagram (see Figure 1), and A, B, C represent the three complex numbers z_1, z_2 and z_3 respectively.

(a) Express $\frac{z_1 z_2}{z_3}$ in the form $a + bi$ where a and b are rational.

(b) Using the result of (a), find $\angle AOD + \angle BOD - \angle COD$.

(5 marks)

4. Let $y = \sin x - 2\cos x$ where $0 \leqslant x \leqslant 2\pi$.

Find (a) $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$,

(b) the minimum value of y.

(6 marks)

5. Let $f(x) = x^2 + 4mx + 4m + 15$, where m is a constant.

Find the discriminant of the equation $f(x) = 0$.

Hence, or otherwise, find the range of values of m so that $f(x) > 0$ for all real values of x.

(5 marks)

6. Let $\mathbf{a} = 3\mathbf{i} + 4\mathbf{j}$, $\mathbf{b} = 8\mathbf{i} + 6\mathbf{j}$ and $\mathbf{c} = 2\mathbf{i} + k\mathbf{j}$ where k is a constant.

Find $\mathbf{a} \cdot \mathbf{c}$ and $\mathbf{b} \cdot \mathbf{c}$ in terms of k.

If $\mathbf{c}$ makes equal angles with $\mathbf{a}$ and $\mathbf{b}$, evaluate k.

(6 marks)

7. Solve the inequality $\frac{|x-3|}{2x} < 1$ by considering each of the following cases:

(i) $x \geqslant 3$,

(ii) $3 > x > 0$,

(iii) $0 > x$.

(7 marks)

SECTION B (60 marks)

Answer any THREE questions from this section.
Each question carries 20 marks.

8. The curve $C : y = \frac{x^2 + 4x - 2}{x^2 + 4}$ cuts the line $y = 1$ at P.

(a) Find the coordinates of P.

(1 mark)

(b) For the curve C, find

(i) the x- and y- intercepts;

(ii) the range of values of x for which the slope is negative;

(iii) the turning points; and for each point, state whether it is a maximum point or a minimum point.
(Testing for maximum/minimum is not required.)

(11 marks)

(c) In Figure 2(a), sketch the curve C for $-5 \leqslant x \leqslant 5$.

(4 marks)

(d) (i) Express the equation of the curve C in the form $y = a + \frac{bx + c}{x^2 + 4}$ (a, b, c are constants).

Hence show that if $x > 1\frac{1}{2}$ then $y > 1$.

(ii) In Figure 2(b), sketch the curve C for $x \geqslant 5$.

(4 marks)

Candidate Number | Centre Number | Seat Number | Total Marks on this page

8. **If you attempt Question 8, fill in the details in the first three boxes above and tie this sheet into your answer book.**

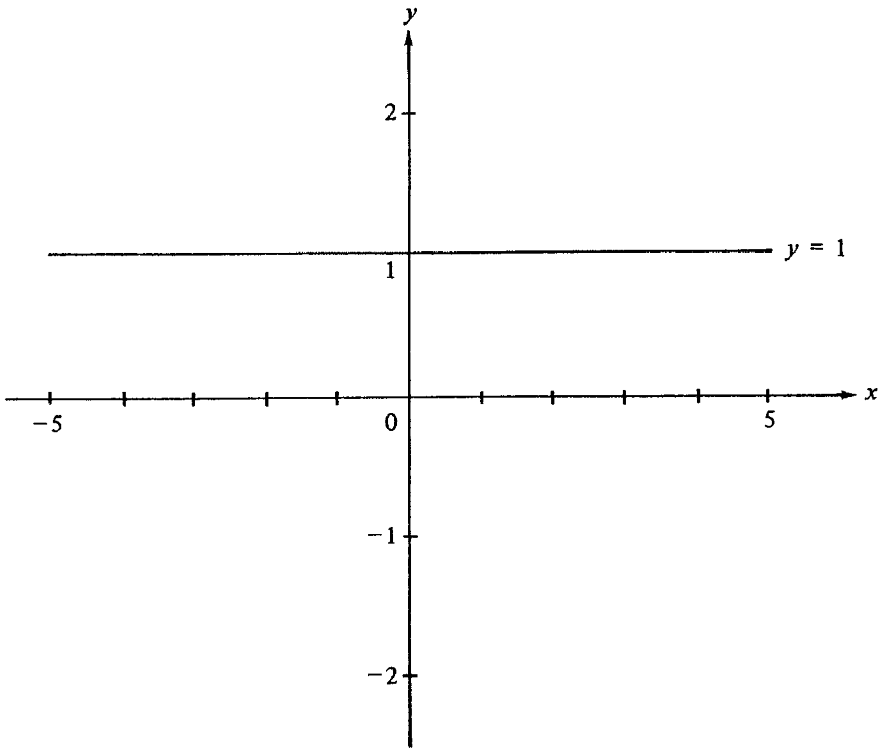


Figure 2(a)

Candidate Number | Centre Number | Seat Number | Total Marks on this page

8. **If you attempt Question 8, fill in the details in the first three boxes above and tie this sheet into your answer book.**

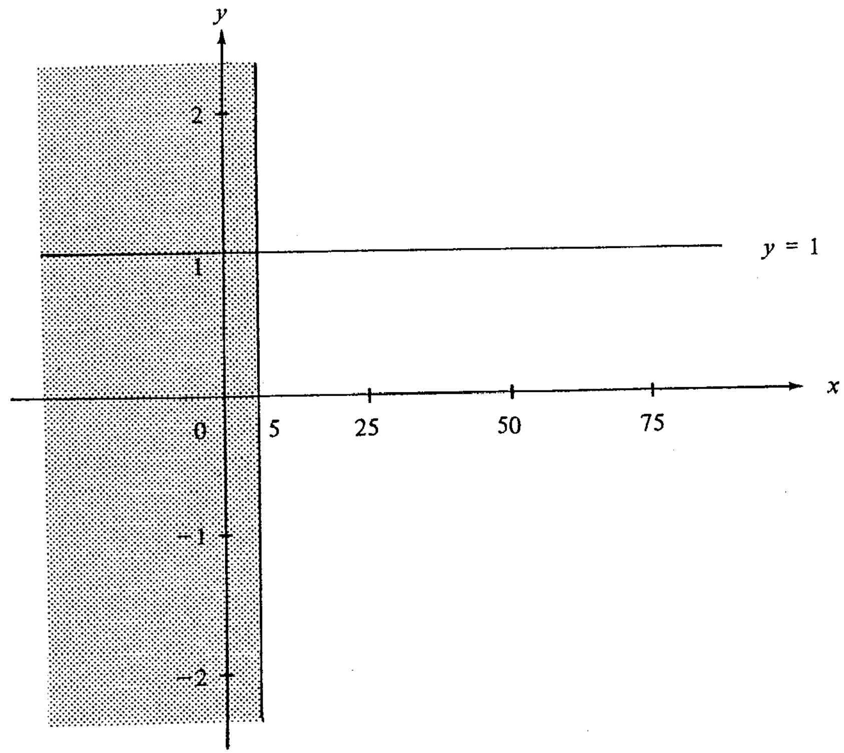


Figure 2(b)

9. A , B and C are three points on a plane such that

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} \, ,$$

$$\overrightarrow{OB} = -6\mathbf{i} - 2\mathbf{j}$$

and $\overrightarrow{AB} - \overrightarrow{BC} = -12\mathbf{i} + 6\mathbf{j}$,

where O is the origin.

(a) Find $\overrightarrow{AB}$ and $\overrightarrow{OC}$.

(5 marks)

(b) X is a point on the plane such that $\overrightarrow{AX} = k\overrightarrow{OX}$.

(i) Express $\overrightarrow{OX}$ in terms of k , $\mathbf{i}$ and $\mathbf{j}$.

(ii) If $OX \perp BX$, find the value of k and hence find $\overrightarrow{AX} + \overrightarrow{BX} + \overrightarrow{CX}$.

Furthermore, if M is the mid-point of BC, find $\overrightarrow{AM}$ and hence show that X lies on AM .

(15 marks)

10. Let $f(x) = x^2 + 2x - 1$

and $g(x) = -x^2 + 2kx - k^2 + 6$ (where k is a constant).

(a) Suppose the graph of $y = f(x)$ cuts the x-axis at the points P and Q ; and the graph of $y = g(x)$ cuts the x-axis at the points R and S .

(i) Find the lengths of PQ and RS .

(ii) Find, in terms of k , the x-coordinate of the mid-point of RS .

If the mid-points of PQ and RS coincide with each other, find the value of k .

(9 marks)

(b) If the graphs of $y = f(x)$ and $y = g(x)$ intersect at only one point, find the possible values of k ; and for each value of k , find the point of intersection.

(6 marks)

(c) Find the range of values of k such that $f(x) > g(x)$ for any real value of x .

(5 marks)

11. (a) Let $z = \cos\theta + i\sin\theta$ where $i = \sqrt{-1}$.

(i) Show that, for any positive integer n,

$$z^n + \frac{1}{z^n} = 2\cos n\theta ,$$

and $$z^n - \frac{1}{z^n} = 2i\sin n\theta .$$

(ii) Using the results of (i), find all complex numbers z such that

$$\frac{\left(z^2 - \frac{1}{z^2}\right)i}{z^2 + \frac{1}{z^2}} = -\sqrt{3} ,$$

and represent them in an Argand diagram.

(10 marks)

(b) (i) Solve $x^2 - x + 1 = 0$, giving the roots in polar form.

(ii) Let α and β be the roots of the equation in (b)(i).

Find the quadratic equation whose roots are $(\frac{\alpha}{\beta})^k$ and $(\frac{\beta}{\alpha})^k$, where k is an integer. Write the answers when

(1) $k = 3n$,

(2) $k = 3n + 1$,

(3) $k = 3n + 2$,

where n is an integer.

(10 marks)

12. (a)

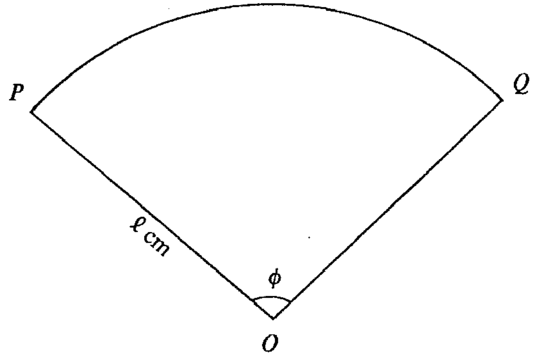


Figure 3(a)

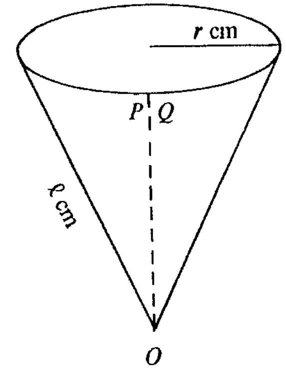


Figure 3(b)

Figure 3(a) shows a piece of paper in the shape of a sector of radius ℓ cm and $\angle POQ = \phi$ radians. It is made to form a conical vessel of radius r cm where OP coincides with OQ (see Figure 3(b)).

(i) Express r in terms of ℓ and ϕ.

Let $V\text{ cm}^3$ be the capacity of the vessel.

(ii) Show that

$$V^2 = k(4\pi^2\phi^4 - \phi^6) ,$$

where $k = \frac{\ell^6}{576\pi^4}$.

(iii) By finding $\frac{d(V^2)}{d\phi}$, determine the value of ϕ for which the capacity of the vessel is a maximum.

[Note: You may use the fact that V is a maximum when V^2 is a maximum.]

(10 marks)

12. (b)

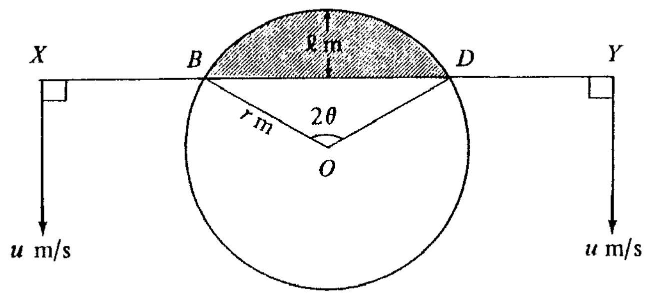


Figure 4

Figure 4 shows a circular pool of radius r metres centred at O. Two men, X and Y, holding the ends of a long rod, are walking in the direction shown at a speed of u metres per second. At a certain instant, the portion BD of the rod subtends an angle of 2θ radians at O and is at a distance ℓ metres from the mid-point C of the rim BD of the pool.

(i) Express ℓ in terms of r and θ.

Let A square metres be the area of the shaded region.

(ii) Express A in terms of r and θ.

(iii) Let $\frac{dA}{dt}$ (in $m^2 s^{-1}$) be the rate of change of the area of the shaded region with respect to time.

Express $\frac{dA}{dt}$ in terms of r, θ and u. (Hint: $\frac{d\ell}{dt} = u$.)

Hence deduce that $\frac{dA}{dt} = ru$ when $\theta = \frac{\pi}{6}$.

(10 marks)

END OF PAPER

88-CE
A MATHS
PAPER II

HONG KONG EXAMINATIONS AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1988

附加數學　試卷二

ADDITIONAL MATHEMATICS PAPER II

11.15 am–1.15 pm (2 hours)
This paper must be answered in English

Answer **ALL** questions in Section A and any **THREE** questions from Section B.

All working must be clearly shown.

Unless otherwise specified in a question, it is sufficient for numerical answers to be given correct to three significant figures.

SECTION A (39 marks)

Answer ALL questions in this section.

1. Given $(1+3x)^4(1-2x)^5 = 1 + ax + bx^2 +$ higher powers of x, find the values of the constants a and b.

(5 marks)

2. A and B are the points (1, 2) and (7, 4) respectively. P is a point on the line segment AB such that $\frac{AP}{PB} = k$.

(a) Write down the coordinates of P in terms of k.

(b) Hence find the ratio in which the line $7x - 3y - 28 = 0$ divides the line segment AB.

(5 marks)

3.

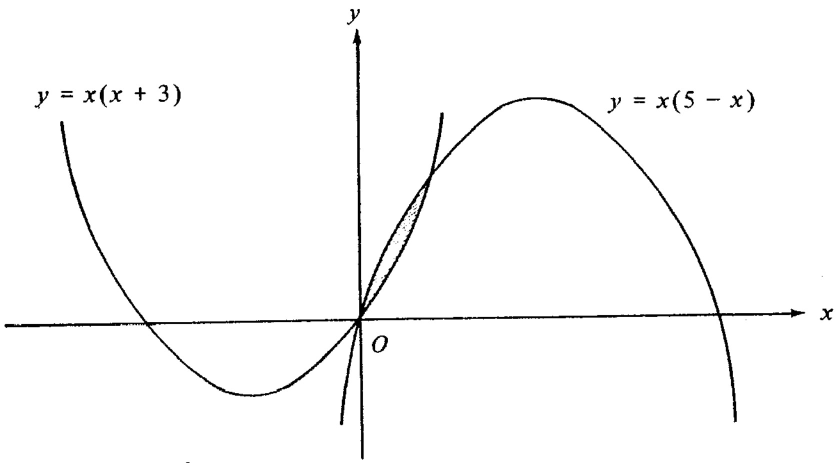


Figure 1

In Figure 1, the shaded region is bounded by the two curves $y = x(x+3)$ and $y = x(5-x)$. Find the area of the shaded region.

(5 marks)

4. O and A are the points (0, 0) and (6, 0) respectively. $P(x, y)$ is a variable point such that $PO + PA = 10$. Find the equation of the locus of P, giving the answer in the form $ax^2 + by^2 + cx + d = 0$.

(5 marks)

5. Prove, by mathematical induction, that

$$1^2 + 3^2 + \ldots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

for all positive integers n.

(6 marks)

6. Evaluate $\int_0^2 \frac{x^2\,dx}{\sqrt{9-x^3}}$.

(6 marks)

7. (a) Without using calculators, show that $\frac{\pi}{10}$ is a root of $\cos 3\theta = \sin 2\theta$.

(b) Given that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$, find the value of $\sin\frac{\pi}{10}$, expressing the answer in surd form.

(7 marks)

SECTION B (60 marks)

Answer any THREE questions from this section.
Each question carries 20 marks.

8. (a) Using the substitution $u = \sin x$, evaluate $\int_0^{\frac{\pi}{2}} \cos^7 x \, dx$.

Leave the answer as a fraction.

(5 marks)

(b) Let $y = \sin x \cos^{2n-1} x$, where n is a positive integer.

Find $\frac{dy}{dx}$.

Hence show that

$$2n\int \cos^{2n} x \, dx - (2n-1)\int \cos^{2n-2} x \, dx = \sin x \cos^{2n-1} x + C,$$

where C is a constant.

(4 marks)

(c) (i) Using (b), show that

$$\int_0^{\frac{\pi}{2}} \cos^{2n} x \, dx = \frac{2n-1}{2n} \int_0^{\frac{\pi}{2}} \cos^{2n-2} x \, dx,$$

where n is a positive integer.

(ii) Evaluate $\int_0^{\frac{\pi}{2}} \cos^6 x \, dx$ in terms of π.

(7 marks)

(d) Evaluate $\int_0^{\frac{\pi}{2}} \sin^6 x \, dx$ in terms of π.

(4 marks)

9.

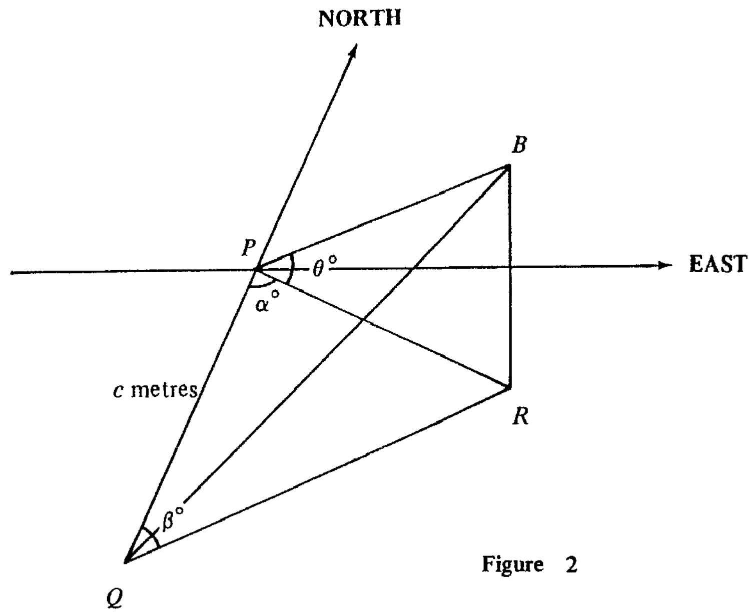


Figure 2

A balloon B is observed simultaneously from two points P and Q on a horizontal ground, P being at a distance c metres due north of Q. The bearings of the balloon from P and Q are S$\alpha°$E and N$\beta°$E respectively. The angle of elevation of B from P is $\theta°$. R is the projection of B on the ground (see Figure 2).

(a) Show that the balloon is at a height h metres where

$$h = \frac{c \tan \theta° \sin \beta°}{\sin(\alpha° + \beta°)}.$$

(6 marks)

(b) Given $\theta = 40$, $\alpha = 54$ and $\beta = 46$,

(i) find the angle of elevation of B from Q;

(ii) find the angle of elevation and the bearing of B from M, where M is the mid-point of PQ.

(14 marks)

10. (a) Using the substitution $x = a\sin\theta$, evaluate $\int_0^a \sqrt{a^2 - x^2}\,dx$, where $a > 0$.

Hence, or otherwise, find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(6 marks)

(b) (i) A pebble is in the shape of the solid of revolution of the ellipse $x^2 + \frac{y^2}{(\frac{3}{4})^2} = 1$ about the x-axis.

Find the volume of the pebble.

(ii)

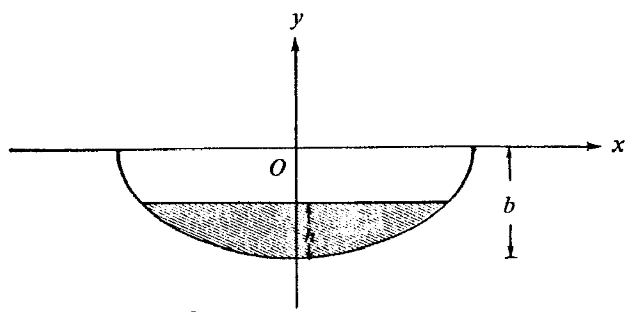


Figure 3

Figure 3 shows the cross-section of a bowl generated by revolving the lower half of the ellipse $\frac{x^2}{(2b)^2} + \frac{y^2}{b^2} = 1$ about the y-axis. The bowl contains water to a depth of h $(h < b)$.

(1) Show that the volume V of water in the bowl is given by $V = \frac{4}{3}\pi h^2(3b - h)$.

Find $\frac{dV}{dh}$ when $h = \frac{b}{2}$.

(2) Now the pebble in (i) is dropped into the bowl and is completely immersed in water. If $b = 5$ units and $h = 2.5$ units, show that the rise of water level is approximately equal to 0.01 unit.

(14 marks)

11.

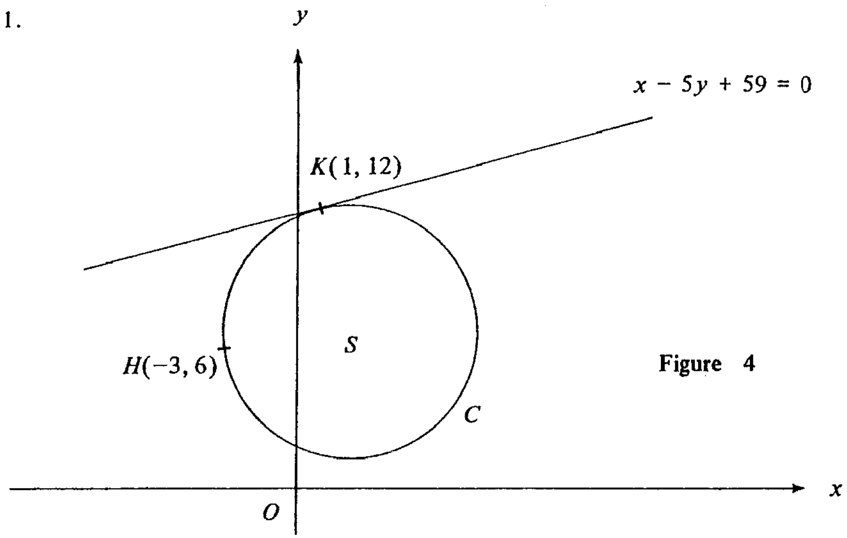


Figure 4

In Figure 4, S is the centre of the circle C which passes through $H(-3, 6)$ and touches the line $x - 5y + 59 = 0$ at $K(1, 12)$.

(a) Find the coordinates of S.

Hence, or otherwise, find the equation of the circle C.

(8 marks)

The line $L: 3x - 2y - 5 = 0$ cuts the circle C at A and B.

(b) Write down the equation of the family of circles through A and B.

Hence find the equation of the circle with AB as diameter.

(7 marks)

(c) Show that $\angle ASB = 90°$.

If P is any point on the circle C other than A or B, write down the two possible values of $\angle APB$.

(5 marks)

12. L is a line through the point $P(-2, 0)$ with slope m $(m \neq 0)$, meeting the parabola $y^2 = 8x$ at the points $A(x_1, y_1)$ and $B(x_2, y_2)$.

(a) Show that x_1 and x_2 are the roots of the equation

$$m^2x^2 + (4m^2 - 8)x + 4m^2 = 0 .$$

(3 marks)

(b) Find $(x_1 - x_2)^2$ in terms of m.

(5 marks)

(c) Show that $AB^2 = \dfrac{64(1 + m^2)(1 - m^2)}{m^4}$.

(5 marks)

(d) Find the values of m for which L touches $y^2 = 8x$.

(3 marks)

(e) If $m = \dfrac{\sqrt{3}}{3}$ and C is the point (2, 0), find by using (c), the area of ΔABC. (Leave the answer in surd form.)

(4 marks)

END OF PAPER

SECTION A

Paper I

Candidates' performance on individual questions:

Q.1 This was a straightforward question but candidates' performance was surprisingly poor. Wrongly memorizing the formula for differentiating cosec x, quite a number of candidates made mistakes as follows:

(i) $f'(x) = 2 \operatorname{cosec} 3x \operatorname{cosec} 3x \cot 3x \cdot 3$,

(ii) $f'(x) = -2 \operatorname{cosec} 3x \cot^2 3x$,

(iii) $f'(x) = -2 \operatorname{cosec} 3x \operatorname{cosec} 3x \cot 3x$.

However, those who got the correct derivative of $f(x)$ could score full marks.

Q.2 Most candidates performed well in the first part of this question. But in finding the second derivative, a great number of candidates made the following mistakes:

(i) $\dfrac{d^2y}{dx^2} = \dfrac{-(-\sin y)}{(1 + \cos y)^2}$ in which $\left(\dfrac{dy}{dx}\right)$ was missing,

(ii) $\dfrac{d^2y}{dx^2} = \dfrac{-\sin y \dfrac{dy}{dx}}{(1 + \cos y)^2}$ in which the minus sign was missing.

Q.3 Candidates' performance was far from satisfactory. Many of them could not observe the relation between the two parts in this question. They solved the second part from the very beginning by putting $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ and were thus unable to go any further.

Q.4 This question was well answered. Most candidates were able to score high marks. Others failed to do so mainly because mistakes were made in manipulation.

Q.5 The candidates, in general, performed fairly well. A few of them overlooked the fact that the roots of the given equation

1988

Additional Mathematics I

1. (a) Δx

   (b) $\dfrac{1}{2\sqrt{x+1}}$

2. $4x - 3y + 2 = 0$
   $2x - 3y - 5 = 0$

3. (a) $\dfrac{3}{13} + \dfrac{11}{13}i$

   (b) $74.7°$

4. (a) $\dfrac{dy}{dx} = \cos x + 2\sin x$
   $\dfrac{d^2y}{dx^2} = -\sin x + 2\cos x$
   -2.24

5. $16m^2 - 16m - 60$
   $-\dfrac{3}{2} < m < \dfrac{5}{2}$

6. $\mathbf{a}\cdot\mathbf{c} = 6 + 4k$
   $\mathbf{b}\cdot\mathbf{c} = 16 + 6k$
   $k = 2$

7. $x < 0$ or $x > 1$

8. (a) $(1.5, 1)$

   (b) (i) x-intercept $= -2 \pm \sqrt{6}$
   y-intercept $= -0.5$

   (ii) $x > 4$ or $x < -1$

   (iii) $(4, 1.5)$ is a maximum point.
   $(-1, -1)$ is a minimum point.

   (d) (i) $y = 1 + \dfrac{4x-6}{x^2+4}$

9. (a) $\overrightarrow{AB} = -7\mathbf{i} - 4\mathbf{j}$
   $\overrightarrow{OC} = -\mathbf{i} - 12\mathbf{j}$

   (b) (i) $\overrightarrow{OX} = \dfrac{1}{1-k}(\mathbf{i} + 2\mathbf{j})$

   (ii) $k = 1\frac{1}{2}$
   $\overrightarrow{AX} + \overrightarrow{BX} + \overrightarrow{CX} = 0$
   $\overrightarrow{AM} = -\dfrac{9}{2}\mathbf{i} - 9\mathbf{j}$

10. (a) (i) $PQ = 2\sqrt{2}$
    $RS = 2\sqrt{6}$

    (ii) x-coordinate of the mid-point of $RS = k$.
    $k = -1$

    (b) $k = 3$ or -5
    $k = 3,\ (1, 2)$
    $k = -5,\ (-3, 2)$

    (c) $k > 3$ or $k < -5$

11. (a) (ii) $z = \cos\left(\dfrac{n\pi}{2} + \dfrac{\pi}{6}\right) + i\sin\left(\dfrac{n\pi}{2} + \dfrac{\pi}{6}\right)$
    $n = 0, 1, 2, 3$

    (b) (i) $\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}$
    or $\cos\left(-\dfrac{\pi}{3}\right) + i\sin\left(-\dfrac{\pi}{3}\right)$

    (ii) (1) $x^2 - 2x + 1 = 0$
    (2) $x^2 + x + 1 = 0$
    (3) $x^2 + x + 1 = 0$

12. (a) (i) $\dfrac{\ell\phi}{2\pi}$

    (iii) $\phi = \dfrac{2\sqrt{6}}{3}\pi$

    (b) (i) $r - r\cos\theta$
    (ii) $r^2\theta - \dfrac{1}{2}r^2\sin 2\theta$
    (iii) $2ru\sin\theta$

1988

Additional Mathematics II

1. $a = 2$
   $b = -26$

2. (a) $\left(\dfrac{7k+1}{k+1},\ \dfrac{4k+2}{k+1}\right)$

   (b) $3 : 1$

3. $\dfrac{1}{3}$

4. $16x^2 + 25y^2 - 96x - 256 = 0$

6. $\dfrac{4}{3}$

7. (b) $\dfrac{\sqrt{5}-1}{4}$

8. (a) $\dfrac{16}{35}$

   (b) $2n\cos^{2n}x - (2n-1)\cos^{2n-2}x$

   (c) $\dfrac{5}{32}\pi$

   (d) $\dfrac{5}{32}\pi$

9. (b) (i) $36.7°$
   (ii) N83.2°E

10. (a) $\dfrac{\pi a^2}{4}$
    πab

    (b) (i) $\dfrac{3}{4}\pi$
    (ii) (1) $3\pi b^2$

11. (a) $(2, 7)$
    $x^2 + y^2 - 4x - 14y + 27 = 0$

    (b) $x^2 + y^2 - 4x - 14y + 27 + k(3x - 2y - 5) = 0$
    $x^2 + y^2 - 10x - 10y + 37 = 0$

    (c) $45°,\ 135°$

12. (b) $\dfrac{64(1-m^2)}{m^4}$

    (d) ± 1

    (e) $16\sqrt{2}$