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香港考試局

HONG KONG EXAMINATIONS AUTHORITY

一九八七年香港中學會考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1987

附加數學（卷一）

ADDITIONAL MATHEMATICS (Paper I)

評卷參考

MARKING SCHEME

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本評卷參考並非標準答案，故極不宜落於學生手中，以免引起誤會。

遇有學生求取此文件時，閱卷員應嚴予拒絕。閱卷員在任何情況下披露本評卷參考內容，均有違閱卷員守則及「一九七七年香港考試局法例」。

Special Note for Teacher Markers

It is highly undesirable that this marking scheme should fall into the hands of students. They are likely to regard it as a set of model answers, which it certainly is not.

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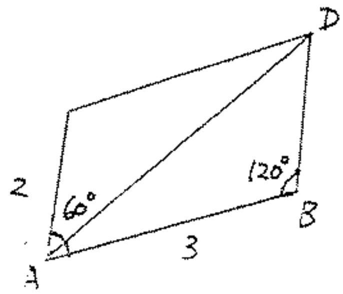
SOLUTIONS	MARKS	REMARKS
1. $f(x) = \operatorname{cosec}^2 3x$.		Alt. Solution:
$f'(x) = 2\operatorname{cosec} 3x(-\operatorname{cosec} 3x \cot 3x)\cdot 3$	2A	$f'(x) = -2(\sin 3x)^{-3}\cdot\cos 3x(3$
$= -6\operatorname{cosec}^2 3x \cot 3x$ or $\frac{-6\cos 3x}{\sin^3 3x}$		
$\therefore f'(\frac{\pi}{12}) = -6\operatorname{cosec}^2\frac{\pi}{4}\cot\frac{\pi}{4}$	1M	$= -2(\sin\frac{\pi}{4})^{-3}\cos\frac{\pi}{4}\cdot 3$
$= -12$	1A	$= -12$ ....
	4	
2. $x = y + \sin y$		Alternative solution
$\frac{dx}{dy} = 1 + \cos y$	1M+1A	Diff. both sides
$\therefore \frac{dy}{dx} = \frac{1}{1+\cos y}$	1A	$1 = y' + (\cos y)(y')$ 1M+
		$= y'(1+\cos y)$
		$y' = \frac{1}{1+\cos y}$
$\frac{d^2y}{dx^2} = \frac{-(-\sin y)}{(1+\cos y)^2}\cdot\frac{dy}{dx}$	1A	
$= \frac{\sin y}{(1+\cos y)^3}$	1A	$0 = y'' - \sin y\cdot(y')^2 + \cos y\cdot y''$
		$y'' = \frac{\sin y}{(1+\cos y)^3}$ 1
	5	
3. Let $z = x + iy$		
$z + \bar{z} = x + yi + x - yi$		
$= 2x$		
$= 2\mathrm{Re}(z)$	1	
$\|z\| = \sqrt{x^2 + y^2}$		
$\geqslant \sqrt{x^2} = \|x\| \geqslant x$		
$= \mathrm{Re}(z)$	1	
$z_1z_2 + \bar{z}_1\bar{z}_2 = z_1z_2 + \overline{z_1z_2}$		
$= 2\mathrm{Re}(z_1z_2)$	1A	
$\leqslant 2\|z_1z_2\|$	1A	
$= 2\|z_1\|\|z_2\|$	1	
	5	
Alternatively		
$z = \|z\|(\cos\theta + i\sin\theta)$		
$\bar{z} = \|z\|(\cos\theta - i\sin\theta)$		
$z + \bar{z} = 2\|z\|\cos\theta = 2\mathrm{Re}(z)$	1	
$\mathrm{Re}(z) = \|z\|\cos\theta \leqslant \|z\|$	1	
$z_1z_2 + \bar{z}_1\bar{z}_2$		
$= \|z_1\|\|z_2\|\operatorname{cis}(\theta_1+\theta_2) + \|z_1\|\|z_2\|\operatorname{cis}(-\theta_1-\theta_2)$	1A	
$= 2\|z_1\|\|z_2\|\cos(\theta_1+\theta_2)$	1A	
$\leqslant 2\|z_1\|\|z_2\|$	1	

SOLUTIONS	MARKS	REMARKS
3. Alternative Solution:		
$z_1z_2 + \bar{z}_1\bar{z}_2$		
$= (x_1 + iy_1)(x_2 + iy_2) + (x_1 - iy_1)(x_2 - iy_2)$		
$= 2(x_1x_2 - y_1y_2)$	1A	
$2\|z_1\|\|z_2\|$		
$= 2\|z_1z_2\|$		
$= 2\sqrt{(x_1x_2 - y_1y_2)^2 + (x_1y_2 + x_2y_1)^2}$		
$\geqslant 2\sqrt{(x_1x_2 - y_1y_2)^2}$	1A	
$= 2\|x_1x_2 - y_1y_2\|$		
$\geqslant 2(x_1x_2 - y_1y_2)$		
$= z_1z_2 + \bar{z}_1\bar{z}_2$	1	
4. (a) $x^2 = 3^2 + 6^2 - (2)(3)(6)\cos\theta$		
$= 45 - 36\cos\theta$	1A	$x^2 = 45 - 36\cos\theta$ 1A
(b) Differentiating with respect to time,	1M	$x = \sqrt{45 - 36\cos\theta}$
$2x\frac{dx}{dt} = 36\sin\theta\frac{d\theta}{dt}$	1A+1A	Differentiating 1M
When $\theta = \frac{\pi}{3}$, $x^2 = 45 - 36(\frac{1}{2})$		$\frac{dx}{dt} = \frac{36\sin\theta}{2\sqrt{45 - 36\cos\theta}} \cdot \frac{d\theta}{dt}$ 2A
$= 27$		$= \frac{1}{2\sqrt{45 - 18}} \cdot 36 \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{3}$
$x = \sqrt{27}$		$= 1(s^{-1})$ 2A
$\frac{dx}{dt} = (36)(\frac{\sqrt{3}}{2})(\frac{1}{3})(\frac{1}{2\sqrt{27}})$		
$= 1\ (s^{-1})$	2A	Do not deduct marks for wrong units or no units.
	6	
5. (a) $4^2 - 4p > 0$	1A	
$p < 4$	1A	
(b) $\alpha + \beta = -4$, $\alpha\beta = p$	1A	
$(\alpha^2 + \beta^2) - \alpha^2\beta^2 + 3(\alpha + \beta) - 19 = 0$		
$16 - 2p + p^2 - 12 - 19 = 0$		
$p^2 - 2p - 15 = 0$	1A	
$(p - 5)(p + 3) = 0$		
$p = 5$ or -3	1A	
but $p < 4$, $\therefore p = -3$	1A	
	6	

SOLUTIONS	MARKS	REMARKS
6. (a) $\overrightarrow{AB}\cdot\overrightarrow{AC} = 3\cdot 1\cdot\cos 60^\circ$	1A	if vector sign omitted, pp-
$= \frac{3}{2}$	1A	
(b) $\lvert\overrightarrow{AB} + 2\overrightarrow{AC}\rvert^2$		Alternative Solution:
$= (\overrightarrow{AB} + 2\overrightarrow{AC})\cdot(\overrightarrow{AB} + 2\overrightarrow{AC})$	1A	$\overrightarrow{AB} = 3\underline{i}$ 1
$= \lvert\overrightarrow{AB}\rvert^2 + 4\lvert\overrightarrow{AC}\rvert^2 + 4\overrightarrow{AB}\cdot\overrightarrow{AC}$	1A	$\overrightarrow{AC} = \frac{1}{2}\underline{i} + \frac{\sqrt{3}}{2}\underline{j}$ 1
$= 9 + 4 + 6$		$\overrightarrow{AB} + 2\overrightarrow{AC} = 4\underline{i} + \sqrt{3}\underline{j}$ 1
$= 19$	1A	$\lvert\overrightarrow{AB} + 2\overrightarrow{AC}\rvert = \sqrt{19}$ 1
$\therefore \lvert\overrightarrow{AB} + 2\overrightarrow{AC}\rvert = \sqrt{19}$	1A 6	

Alternative Solution:

$\lvert\overrightarrow{AB} + 2\overrightarrow{AC}\rvert = AD$	(1A)
$AD^2 = 3^2 + 2^2 - 2(2)(3)\cos 120^\circ$	(1A)
$= 19$	(1A)
$\lvert\overrightarrow{AB} + 2\overrightarrow{AC}\rvert = \sqrt{19}$	(1A)



SOLUTIONS	MARKS	REMARKS
7. 2 cases $[x-2 \geq 0$ or $x-2 < 0]$ or $[x-2 > 0$ or $x-2 \leq 0]$	1	Alternative Solution:
Case (i) $x - 2 \geq 0$ i.e. $x \geq 2$		L.S. < 0 1
$(x + 2)\lvert x - 2\rvert < -5$		$\therefore x + 2 < 0$ 1
$(x + 2)(x - 2) < -5$	1A	$x < -2$ 1
$x^2 - 4 < -5$		
$x^2 < -1$ impossible	1A	L.S. $= (x+2)(2-x)$
Case (ii) $x - 2 < 0$ i.e. $x < 2$		$(x+2)(2-x) < -5$ 1
$(x + 2)\lvert x - 2\rvert < -5$		$x^2 > 9$
$(x + 2)(-x + 2) < -5$	1A	$x > 3$ or $x < -3$ 1
$-x^2 + 4 < -5$		$x < -3$ 1
$x^2 > 9$		Alternative Solution:
$x > 3$ or $x < -3$	1A	$(x+2)^2(x-2)^2 > 25$ 1
$x < -3$	1A	$(x^2-1)(x^2-9) > 0$ 1
Combining (i) and (ii)		$x > 3$ or $x < -3$ 1
$\therefore x < -3$	1A 7	After checking, $x < -3$ 1

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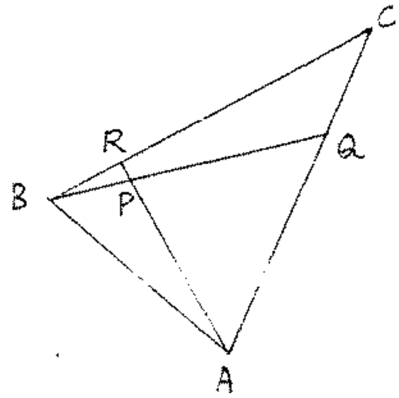
SOLUTIONS	MARKS	REMARKS
8. (a) (i) $\overrightarrow{AB} = -3\underline{i} + 3\underline{j}$	1A	if vector sign omitted, or division of vectors, pp-1
$\overrightarrow{AC} = 3\underline{i} + 6\underline{j}$	1A	
(ii) $\overrightarrow{AR} = \frac{1}{1+m}[\overrightarrow{AB} + m\overrightarrow{AC}]$	1M	
$= \frac{1}{1+m}[(-3+3m)\underline{i} + (3+6m)\underline{j}]$	1A	
	4	
(b) (i) $\overrightarrow{BC} = 6\underline{i} + 3\underline{j}$		
$AR \perp BC$, $\therefore \overrightarrow{AR}\cdot\overrightarrow{BC} = 0$		
$\frac{1}{1+m}[6(-3+3m) + 3(3+6m)] = 0$	1M+1A	
$m = \frac{1}{4}$	1	
Alternative Solution:		
slope of BC · slope of AR = −1		
$\frac{1}{2}\cdot\left(\frac{\frac{7m+4}{m+1} - 1}{\frac{7m+1}{m+1} - 4}\right) = -1$	1M+1A	
or $\frac{1}{2}\cdot\frac{3+6m}{-3+3m} = -1$		
$m = \frac{1}{4}$	1A	
(ii) $\overrightarrow{AR} = -\frac{9}{5}\underline{i} + \frac{18}{5}\underline{j}$		**Alternative Solution:**
Let $\angle QPR = \theta$		$\angle CBQ = \phi$
$\overrightarrow{BQ}\cdot\overrightarrow{AR} = \lvert\overrightarrow{BQ}\rvert\lvert\overrightarrow{AR}\rvert\cos\theta$	1M	$\overrightarrow{BQ}\cdot\overrightarrow{BC} = \lvert\overrightarrow{BQ}\rvert\lvert\overrightarrow{BC}\rvert\cos\phi$ 1
$\overrightarrow{BQ}\cdot\overrightarrow{AR} = -\frac{27}{5}$	1A	$\overrightarrow{BQ}\cdot\overrightarrow{BC} = 33$ 1
$\lvert\overrightarrow{BQ}\rvert\lvert\overrightarrow{AR}\rvert\cos\theta = \sqrt{26}\sqrt{(-\frac{9}{5})^2+(\frac{18}{5})^2}\cos\theta$	1A	$\lvert\overrightarrow{BQ}\rvert\lvert\overrightarrow{BC}\rvert\cos\phi = \sqrt{26}\cdot\sqrt{45}\cos\phi$ 1
$-\frac{27}{5} = \frac{9}{5}\cdot\sqrt{5}\cdot\sqrt{26}\cos\theta$		$\angle QBC = 15.25°$
$\theta = 105°$ (Accept answers roundable to 105°)	1A	$\theta = 105°$ 1
Alternative Solution:		
$\tan\angle CBQ = \frac{m_1 - m_2}{1 + m_1 m_2}$		
$= \frac{\frac{3}{6} - \frac{1}{5}}{1 + \frac{1}{5}\cdot\frac{3}{6}}$	1M+2A	
(Accept $\frac{1}{5} - \frac{3}{6}$ in the numerator)		
$\angle CBQ = 15.25°$		
$\theta = 105°$	1A	



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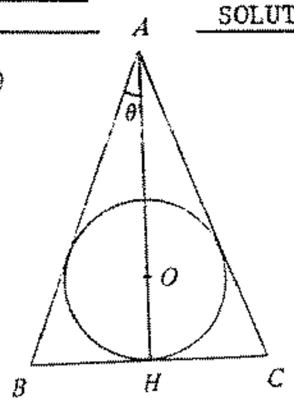
SOLUTIONS	MARKS	REMARKS
8. (b) (iii) $\overrightarrow{BQ} = \lambda\overrightarrow{BA} + \mu\overrightarrow{BC}$		
$5\underset{\sim}{i} + \underset{\sim}{j} = \lambda(3\underset{\sim}{i} - 3\underset{\sim}{j}) + \mu(6\underset{\sim}{i} + 3\underset{\sim}{j})$		
$= (3\lambda + 6\mu)\underset{\sim}{i} + (-3\lambda + 3\mu)\underset{\sim}{j}$		
$3\lambda + 6\mu = 5$, $-3\lambda + 3\mu = 1$	2M+1A	
$\lambda = \frac{1}{3}$, $\mu = \frac{2}{3}$	1A	
(iv) $\overrightarrow{BP} = \frac{1}{1+n}[\overrightarrow{BA} + n\overrightarrow{BR}]$		
$= \frac{1}{1+n}[\overrightarrow{BA} + \frac{n}{5}\overrightarrow{BC}]$	1A	
$= \frac{1}{1+n}[(3 + \frac{6}{5}n)\underset{\sim}{i} + (-3 + \frac{3}{5}n)\underset{\sim}{j}]$	1A	
$\overrightarrow{BQ} = 5\underset{\sim}{i} + \underset{\sim}{j}$		
$\overrightarrow{BP} \mathbin{/\!/} \overrightarrow{BQ}$		
$\therefore \dfrac{3 + \frac{6n}{5}}{-3 + \frac{3n}{5}} = \frac{5}{1}$	1M+1A	
$3 + \frac{6}{5}n = -15 + \frac{15}{5}n$		
$n = 10$	1A	
	16	

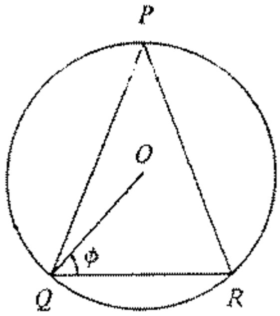
SOLUTIONS	MARKS	REMARKS
9. (a)(i)  $OA = \frac{a}{\sin\theta}$		
$AH = a + \frac{a}{\sin\theta}$	1A	
base $BC = (2)(AH\tan\theta)$	1M	
$= 2a[1 + \frac{1}{\sin\theta}]\tan\theta$	1A	
$= 2a\,\frac{(1+\sin\theta)}{\cos\theta}$		
Area, $S = \frac{1}{2}\,(a)(1 + \frac{1}{\sin\theta}) \cdot 2a\,\frac{(1+\sin\theta)}{\cos\theta}$		
$= \frac{a^2(1+\sin\theta)^2}{\sin\theta\cos\theta}$	1	
(ii) Writing $s = \sin\theta$, $c = \cos\theta$,		
$\frac{dS}{d\theta} = \frac{(sc)2(1+s)c - (1+s)^2(-s^2+c^2)}{s^2c^2} \cdot a^2$	1M	For differentiating S wit respect to θ .
$= \frac{a^2(1+s)[2sc^2 - c^2 + s^3 - sc^2 + s^2]}{s^2c^2}$		
$= \frac{a^2(1+s)}{s^2c^2}\,(s^3 + s^2 - c^2 + sc^2)$		
$= 0$	1M	
$\because\ 1 + s \neq 0$, $s^3 + s^2 - c^2 + sc^2 = 0$		
$s^3 + s^2 + (1 - s^2)(s - 1) = 0$		
$2s^2 + s - 1 = 0$	2A	
$(2s - 1)(s + 1) = 0$		
$s = \frac{1}{2}$	1A	
$\theta = 30°$ or $\frac{\pi}{6}$	1A	
	10	

NOTE: There are several alternative solutions in which S is expressed in different forms before differentiation,

e.g. $S = a^2(\frac{1}{sc} + \frac{s}{c} + \frac{2}{c})$,

$S = a^2\tan\theta(1 + \csc\theta)^2$

or $S = \frac{a^2(1+s)^2}{\sin 2\theta}$.

SOLUTIONS	MARKS	REMARKS
9. (b) (i) 		
$QR = 2b\cos\emptyset$	1A	
height $= b + b\sin\emptyset$	1A	
Area $A = \frac{1}{2}\cdot 2b\cos\emptyset\cdot b(1+\sin\emptyset)$		
$= b^2\cos\emptyset(1+\sin\emptyset)$	1	
(ii) When Δ is equilateral, $\emptyset = 30°$	1A	
$\frac{dA}{d\emptyset} = b^2[\cos^2\emptyset - \sin\emptyset(1+\sin\emptyset)]$	2A	
$= b^2[1 - 2\sin^2\emptyset - \sin\emptyset]$ or $b^2[\cos 2\emptyset - \sin\emptyset]$		
when $\emptyset = 30°$, $\frac{dA}{d\emptyset} = b^2[\cos^2 30° - \sin 30°(1+\sin 30°)]$	1M	1M For sub. $\emptyset = 30°$ in $\frac{dA}{d\emptyset}$
$= 0$	1A	
$\frac{d^2A}{d\emptyset^2} = b^2[-4\sin\emptyset\cos\emptyset - \cos\emptyset]$	1M	For finding 2nd derivative.
When $\emptyset = 30°$, $\frac{d^2A}{d\emptyset^2} < 0$,	1A	Do not award this mark if there is no 2nd derivative 2nd derivative is wrong.
$\therefore$ The area is a maximum.		
	10	

Alternatively:

SOLUTIONS	MARKS
$\frac{dA}{d\emptyset} = b^2[\cos^2\emptyset - \sin\emptyset(1+\sin\emptyset)]$	2A
$= 0$	1M
$\cos^2\emptyset - \sin\emptyset - \sin^2\emptyset = 0$	
$1 - \sin\emptyset - 2\sin^2\emptyset = 0$	
$2\sin^2\emptyset + \sin\emptyset - 1 = 0$	
$(2\sin\emptyset - 1)(\sin\emptyset + 1) = 0$	
$\sin\emptyset = \frac{1}{2}$ or -1 (rejected)	
$\emptyset = 30°$	1A
$\angle OQR = 30°$	
$\angle PQR = 30° + 30° = 60°$	
$\therefore \Delta PQR$ is equilateral	1A
$\frac{d^2A}{d\emptyset^2} = b^2[-2\cos\emptyset\sin\emptyset - \cos\emptyset - 2\sin\emptyset\cos\emptyset]$	1M
$= b^2[-4\cos\emptyset\sin\emptyset - \cos\emptyset]$	
$\left.\frac{d^2A}{d\emptyset^2}\right\|_{\emptyset = 30°} < 0$	1A
The area is a max. when Δ is equilateral.	

SOLUTIONS	MARKS	REMARKS
10.(a) $z^2 = (\cos\theta + i\sin\theta)^2$		Alternatively:
$= \cos 2\theta + i\sin 2\theta$ or $(\cos^2\theta - \sin^2\theta) + i2\sin\theta\cos\theta$	1A	$\frac{1}{z} = \bar{z}$ 1
$\bar{z} = \cos\theta - i\sin\theta$	1A	$z^2 - 2\bar{z} + \frac{1}{z} = z^2 - \bar{z}$ 1
$\frac{1}{z} = \frac{1}{\cos\theta + i\sin\theta} = \cos\theta - i\sin\theta$	1A	$z^2 - 2\bar{z} + \frac{1}{z}$ is real
$z^2 - 2\bar{z} + \frac{1}{z} = \cos 2\theta + i\sin 2\theta - \cos\theta + i\sin\theta$		$\therefore z^2 - \bar{z} = \overline{z^2 - \bar{z}}$ $= (\bar{z})^2 - z$ 1M+1
This is real,		
$\therefore \sin 2\theta + \sin\theta = 0$	1M+1A	$z^2 - (\bar{z})^2 = \bar{z} - z$ $(z - \bar{z})(z + \bar{z}) = \bar{z} - z$ $z + \bar{z} = -1$ 1
or $2\sin\theta\cos\theta + \sin\theta = 0$		$2\text{Re}(z) = -1$
$\sin\theta \neq 0$,		
$\therefore \cos\theta = -\frac{1}{2}$	1A	$\text{Re}(z) = -\frac{1}{2}$ 1
$\theta = 2n\pi \pm \frac{2\pi}{3}$		$\text{Im}(z) = \pm\sqrt{1 - \frac{1}{4}}$ $= \pm\frac{\sqrt{3}}{2}$
$z = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$ or $\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}$	2A+1A	$z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i$ 2A+1
$[z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ or $-\frac{1}{2} - \frac{\sqrt{3}}{2}i]$		
$[z = \text{cis}120°$ or $\text{cis}(-120°)]$ (Accept cis240°)	9	
(b) Take $z_1 = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$, $z_2 = \cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}$		Alternative Solution:
(i) $z_1{}^2 = (\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})^2$		$z_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$
$= \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$		$z_1{}^2 = (-\frac{1}{2} + \frac{\sqrt{3}}{2}i)(-\frac{1}{2} + \frac{\sqrt{3}}{2}i)$
$= \cos(2\pi - \frac{4\pi}{3}) - i\sin(2\pi - \frac{4\pi}{3})$		$= \frac{1}{4} - \frac{3}{4} - \frac{1}{2}(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2})i$
$= z_2$	1	$= -\frac{1}{2} - \frac{\sqrt{3}}{2}i$
$z_2{}^2 = (\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3})^2$		$= z_2$ 1
$= \cos\frac{4\pi}{3} - i\sin\frac{4\pi}{3}$		Similarly for $z_2{}^2 = z_1$ 1
$= \cos(2\pi - \frac{4\pi}{3}) + i\sin(2\pi - \frac{4\pi}{3})$		
$= z_1$	1	
(ii) $z_1{}^3 = (\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})^3$		$z_1{}^3 = z_1 z_1{}^2$
$= \cos 2\pi + i\sin 2\pi$		$= z_1 z_2$
$= 1$	1A	$= 1$ 1A
$z_2{}^3 = (\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3})^3$		$z_2{}^3 = z_2 z_2{}^2$
$= \cos 2\pi - i\sin 2\pi$		$= z_2 z_1$
$= 1$	1A	$= 1$ 1A

SOLUTIONS	MARKS	REMARKS
10.(b)(iii) $z_1^{3n} + z_2^{3n} = (z_1^3)^n + (z_2^3)^n$		
$= 1^n + 1^n$		
$= 2$	1A	
$z_1^{3n+1} + z_2^{3n+1} = z_1^{3n} \cdot z_1 + z_2^{3n} \cdot z_2$		
$= z_1 + z_2$		
$= -1$	1A	
$z_1^{3n+2} + z_2^{3n+2} = (z_1^{3n})z_1^2 + (z_2^{3n})z_2^2$		
$= z_1^2 + z_2^2$		
$= z_2 + z_1$		
$= -1$	1A	
Alternative Solution:		
$z_1^{3n} + z_2^{3n} = 2\cos 2n\pi$		
$= 2$	1A	
$z_1^{3n+1} + z_2^{3n+1} = 2\cos \frac{2(3n+1)\pi}{3}$		
$= 2\cos(2n\pi + \frac{2\pi}{3})$		
$= 2\cos \frac{2\pi}{3}$		
$= -1$	1A	
$z_1^{3n+2} + z_2^{3n+2} = 2\cos \frac{2(3n+2)\pi}{3}$		
$= 2\cos(2n\pi + \frac{4\pi}{3})$		
$= 2\cos \frac{4\pi}{3}$		
$= -1$	1A	
(iv) $z_1^{2k} + z_2^{2k} = (z_1^2)^k + (z_2^2)^k$	1A	
$= z_2^k + z_1^k$	2A	
$= \begin{cases} 2 & k \text{ is a multiple of } 3 \\ -1 & k \text{ is not a multiple of } 3 \end{cases}$	1	
	11	
Alternative Solution:		**Alt. Solution:**
$z_1^{2k} + z_2^{2k} = 2\cos \frac{4k\pi}{3}$	1A	$z_1^{2(3n)} + z_2^{2(3n)}$
$k = 3n$, $z_1^{2k} + z_2^{2k} = 2\cos \frac{4k\pi}{3} = 2\cos 4n\pi = 2$	1	$= \text{cis}\,4n\pi + \text{cis}(-4n\pi)$ [illegible]
$k = 3n+1$, $z_1^{2k} + z_2^{2k} = 2\cos(4n\pi + \frac{4\pi}{3}) = -1$	1	$= 2$ 1
$k = 3n+2$, $z_1^{2k} + z_2^{2k} = 2\cos(4n\pi + \frac{8\pi}{3}) = -1$	1	etc.

SOLUTIONS	MARKS
11.(a) $z^2 - 2z + k = 0$	
$(-2)^2 - 4k < 0$	1A
$k > 1$	1A
	2
(b) Let α and β be the roots of $z^2 - 2z + k = 0$	
$\alpha + \beta = 2$) $\alpha\beta = k$)	1A
$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha^2\beta - 3\alpha\beta^2$	
$= (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$	[illegible]
or $(\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$	
$= 8 - 6k$	1A
$\alpha^3\beta^3 = k^3$	1A
$\therefore$ Required equation is $z^2 + (6k - 8)z + k^3 = 0$	1A
$\Delta = (6k - 8)^2 - 4k^3$	1M
$= -4k^3 + 36k^2 - 96k + 64$ $= -4(k^3 - 9k^2 + 24k - 16)$ $= 4(1 - k)(4 - k)^2$	1
The equation has real roots,	
$4(1 - k)(4 - k)^2 \geq 0$	1A
but $k > 1$ 1	
$\therefore (k - 4)^2 \leq 0$ 1A	
$k = 4$ 1A	
Alt. Solution: $k = 4$ or $k \leq 1$	1A
but $k > 1$	1
$\therefore k = 4$	1A
	11
(c) $z = 1 \pm \sqrt{1 - k}$	1A
$z^2 = (2 - k) \pm 2\sqrt{1 - k}$	1A
$= (2 - k) \pm 2\sqrt{k - 1}\, i$	1A
$x = 2 - k$) $y = \pm 2\sqrt{k - 1}$)	1A
Eliminating k,	1M
$y = \pm 2\sqrt{2 - x - 1}$ $= \pm 2\sqrt{1 - x}$	1A
where $x \neq 1$	1A
i.e. $y^2 = 4(1 - x)$ where $x \neq 1$.	
	7

REMARKS

Alternative Solution:

$z = 1 \pm \sqrt{1 - k}$ 1

$z^2 = (2-k) \pm 2\sqrt{1 - k}$

$z^3 = (4-3k) \pm (4-k)\cdot\sqrt{1-k}$ 2

Required equation is

$\{z-[(4-3k)+(4-k)\sqrt{1-k}]\}$

$\times \{z-[(4-3k)-(4-k)\sqrt{1-k}]\} = 1$

$z^2 + (6k-8)z + k^3 = 0$ 1

Alternatively:

$\text{Arg}(z) = \pm \tan^{-1}(\sqrt{k - 1})$

z^3 is real

$\text{Arg}(z^3) = \pi$ 1

$3\text{Arg}(z) = \pi$ 1

$\tan^{-1}(\pm\sqrt{k - 1}) = \frac{\pi}{3}$ 1

$\pm\sqrt{k - 1} = \sqrt{3}$

$k - 1 = 3$

$k = 4$ 1A

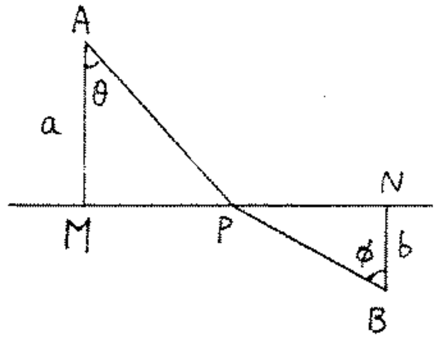
Alternatively:

$z = 1 \pm \sqrt{1 - k}$ 1

$z^2 = 2z - k$

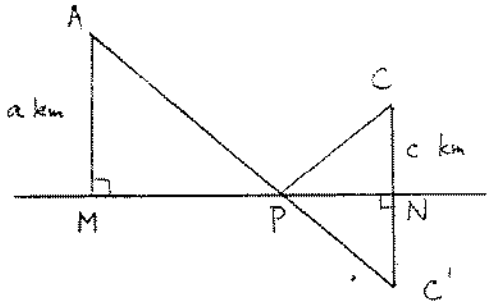
$= 2(1 \pm \sqrt{1 - k}) - k$ 1

$= (2-k) \pm \sqrt{k - 1}\, i$ 1A

SOLUTIONS	MARKS	REMARKS
12.(a) (i) $MN = MP + PN$	1A	
$= a\tan\theta + b\tan\phi$	1A	
$\frac{d(MN)}{d\theta} = a\sec^2\theta + b\sec^2\phi\,\frac{d\phi}{d\theta}$	1M+1A	or $\frac{d(MN)}{d\phi}$
$= 0$	1M	
$\therefore\ \frac{d\phi}{d\theta} = -\frac{a\sec^2\theta}{b\sec^2\phi}$		
(ii) $t = \frac{AP}{u} + \frac{BP}{v}$	1A	
$= \frac{a}{u}\sec\theta + \frac{b}{v}\sec\phi$		
$\frac{dt}{d\theta} = \frac{a}{u}\sec\theta\tan\theta + \frac{b}{v}\sec\phi\tan\phi\,\frac{d\phi}{d\theta}$	1M+2A	
$= 0$	1M	
From (a),		
$\frac{a}{u}\sec\theta\tan\theta + \frac{b}{v}\sec\phi\tan\phi\left(-\frac{a\sec^2\theta}{b\sec^2\phi}\right) = 0$	1M	
$\frac{a}{u}\frac{1}{\cos\theta}\cdot\frac{\sin\theta}{\cos\theta} + \frac{b}{v}\cdot\frac{\sin\phi}{\cos^2\phi}\left(-\frac{a}{b}\cdot\frac{\cos^2\phi}{\cos^2\theta}\right) = 0$		
$\frac{u}{v} = \frac{\sin\theta}{\sin\phi}$	1 12	
(b) (i) $t = \frac{AP}{u} + \frac{PN}{v}$		
$= \frac{a\sec\theta}{u} + \frac{h - a\tan\theta}{v}$	1A	
(ii) When t is a minimum,		
$\frac{dt}{d\theta} = 0$	1M	
$\frac{a\sec\theta\tan\theta}{u} - \frac{a\sec^2\theta}{v} = 0$		
$\frac{\tan\theta}{u} = \frac{\sec\theta}{v}$		
$\frac{u}{v} = \sin\theta$	1A	
$MP = a\tan\theta$		
$= \frac{a\frac{u}{v}}{\sqrt{1 - \frac{u^2}{v^2}}}$		
$= \frac{au}{\sqrt{v^2 - u^2}}$	2A 5	

SOLUTIONS | MARKS | REMARKS

12. (c)



Time required $= \dfrac{AP + CP}{u}$

$= \dfrac{AP + C'P}{u}$

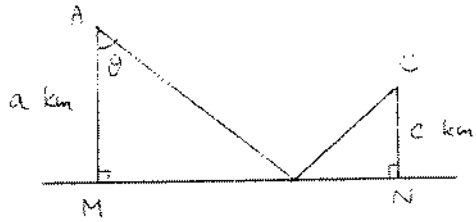
For minimum time, $(AP + C'P)$ is a minimum.

i.e. APC' is a straight line — 2

and MP : PN = a : c .... — 1A

Total: 3

Alternative Solution:



$$t = \frac{1}{u}\left[a\sec\theta + \sqrt{(h - a\tan\theta)^2 + c^2}\right]$$

$$\frac{dt}{d\theta} = \frac{1}{u}\left[a\sec\theta\tan\theta - \frac{2(h - a\tan\theta)\,a\sec^2\theta}{2\sqrt{(h - a\tan\theta)^2 + c^2}}\right] \text{........} \quad \text{1A}$$

$$= 0$$

$$\tan\theta\sqrt{(h - a\tan\theta)^2 + c^2} = (h - a\tan\theta)\sec\theta$$

$$\tan^2\theta(h - a\tan\theta)^2 + c^2\tan^2\theta = (h - a\tan\theta)^2\sec^2\theta$$

$$c^2\tan^2\theta = (h - a\tan\theta)^2$$

$$h - a\tan\theta = \pm c\tan\theta$$

$$(a \pm c)\tan\theta = h$$

$$\tan\theta = \frac{h}{a \pm c} \text{ ....} \quad \text{1A}$$

$MP = a\tan\theta = \dfrac{ah}{a \pm c}$

(Rejecting $\dfrac{ah}{a - c}$ $\because$ P lies between M and N)

$MP = \dfrac{ah}{a + c}$

$PN = \dfrac{ch}{a + c}$

MP : PN = a : c .... — 1A

RESTRICTED 內部文件

香港考試局

HONG KONG EXAMINATIONS AUTHORITY

一九八七年香港中學會考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1987

附加數學（卷二）

ADDITIONAL MATHEMATICS (Paper II)

評卷參考

MARKING SCHEME

請在學校任教之閱卷員特別留意

本評卷參考並非標準答案，故極不宜落於學生手中，以免引起誤會。

遇有學生求取此文件時，閱卷員應嚴予拒絕。閱卷員在任何情況下披露本評卷參考內容，均有違閱卷員守則及「一九七七年香港考試局法例」。

Special Note for Teacher Markers

It is highly undesirable that this marking scheme should fall into the hands of students. They are likely to regard it as a set of model answers, which it certainly is not.

Markers should therefore resist pleas from their students to have access to this document. Making it available would constitute misconduct on the part of the marker and is, moreover, in breach of the 1977 Hong Kong Examinations Authority Ordinace.

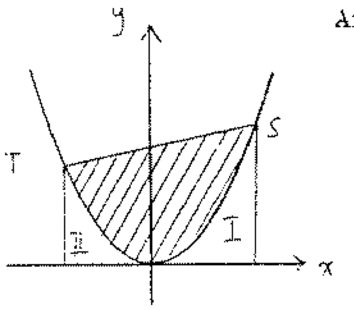
SOLUTIONS	MARKS	REMARKS
1. $(1 + x + x^2)^n$		
$= [1 + x(1 + x)]^n$	1M	
$= 1 + nx(1 + x) + \frac{n(n-1)}{2}(x^2)(1 + x)^2 + \ldots$		
Coeff. of $x^2 = n + \frac{n(n-1)}{2}$	2A	
$= 21$		
$n^2 + n - 42 = 0$		
$(n - 6)(n + 7) = 0$	1A	
$n = 6$ or -7 (rejected)	1A	
	5	
2. For $n = 1$, L.H.S. $= 1/4$		
R.H.S. $= 1/4 =$ L.H.S.	1	
Assume equality holds for some integer k .	1	
For $n = k + 1$,		
L.H.S. $= \frac{1}{(1)(4)} + \frac{1}{(4)(7)} + \ldots + \frac{1}{(3k-2)(3k+1)} + \frac{1}{(3k+1)(3k+4)}$		
$= \frac{k}{3k+1} + \frac{1}{(3k+1)(3k+4)}$	1	
$= \frac{3k^2 + 4k + 1}{(3k+1)(3k+4)}$		
$= \frac{k+1}{3k+4}$	1	
$=$ R.H.S.		
Therefore equality holds also for $n = k + 1$. mathematical induction, equality holds for all positive integers n .	1	Award this mark only if the candidate has scored the first four marks.
	5	
3. Let the slope of the required line be m .		
$\frac{m - 3}{1 + (m)(3)} = \pm\frac{1}{2}$	1A+1	1A for formula (excl. ±) 1 for ±
$2(m - 3) = 3m + 1$ or $2(m - 3) = -(3m + 1)$		
$m = -7$ or $m = 1$	1A+1A	
$\frac{y-2}{x-1} = -7$ $\frac{y-2}{x-1} = 1$		
$7x + y - 9 = 0$ $x - y + 1 = 0$	1A	For both equations
	5	

SOLUTIONS	MARKS	REMARKS
4. Put $x = \sin\theta$		
$dx = \cos\theta\, d\theta$	1A	
$x = 0,\ \theta = 0$)		
$x = \frac{1}{2},\ \theta = \frac{\pi}{6}$)	1A	
$\int_0^{\frac{1}{2}} \frac{2x^2}{\sqrt{1-x^2}}dx = \int_0^{\frac{\pi}{6}} \frac{2\sin^2\theta}{\sqrt{1-\sin^2\theta}}\cos\theta\, d\theta$		
$= \int_0^{\frac{\pi}{6}} 2\sin^2\theta\, d\theta$	1A	)
$= \int_0^{\frac{\pi}{6}} (1 - \cos 2\theta)\, d\theta$	1A	) for integrand)
$= [\theta - \frac{1}{2}\sin 2\theta]_0^{\frac{\pi}{6}}$	1A	
$= \frac{\pi}{6} - \frac{\sqrt{3}}{4}$ (0.0906)	1A	
	6	
5. $y = \int (3x^2 - 2)(x^3 - 2x + 1)^{\frac{1}{3}}\, dx$	1M	
put $u = x^3 - 2x + 1$		
$du = (3x^2 - 2)\, dx$	1A	
$y = \int u^{\frac{1}{3}}\, du$	1A	
$y = \frac{3}{4}u^{\frac{4}{3}} + c$		
$y = \frac{3}{4}(x^3 - 2x + 1)^{\frac{4}{3}} + c$	1A	
sub. $x = 0,\ y = 0$	1M	Do not award this mark if c is missing.
$c = -\frac{3}{4}$	1A	
$y = \frac{3}{4}(x^3 - 2x + 1)^{\frac{4}{3}} - \frac{3}{4}$	6	
6. $\sin 3\theta = \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$	1A	$(\cos\theta + i\sin\theta)^3$
$= 2\sin\theta\cos^2\theta + (1 - 2\sin^2\theta)\sin\theta$		$= \cos 3\theta + i\sin 3\theta$ 1A
$= 3\sin\theta - 4\sin^3\theta$	1A	⋮
Put $x = \sin\theta$	1M	$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ 1A
$8x^3 - 6x + 1 = 0$		
$8\sin^3\theta - 6\sin\theta + 1 = 0$		
$2(4\sin^3\theta - 3\sin\theta) + 1 = 0$		
$2\sin 3\theta = 1$		
$\sin 3\theta = \frac{1}{2}$	1A	
$3\theta = 180n^\circ + (-1)^n 30^\circ$		
$\theta = 60n^\circ + (-1)^n 10^\circ$		
$= 10^\circ, 50^\circ, 130^\circ, 170^\circ, 250^\circ, \ldots$		
$x = \sin 10^\circ, \sin 50^\circ, \sin 250^\circ$		
$= 0.17,\ 0.77,\ -0.94$ ————————	1A+1A	2 correct answers 1A 3 correct answers 2A
	6	

SOLUTIONS	MARKS	REMARKS
7. Tangents are of the form $y = 2x + k$	1A	Alternative Solution:
Sub. in $x^2 - y^2 = 3$	1M	Diff. $x^2 - y^2 = 3$ 1M
$x^2 - (2x + k)^2 = 3$		$2x - 2yy' = 0$ 1A
$-3x^2 - 4kx - k^2 = 3$		$y' = \frac{x}{y}$
$3x^2 + 4kx + k^2 + 3 = 0$	1A	$\frac{x}{y} = 2$ 1A
For tangents, $\Delta = 0$		$x = 2y$
$16k^2 - 4(3)(k^2 + 3) = 0$	1M	Sub. in $x^2 - y^2 = 3$ 1M
$k^2 = 9$		$3y^2 = 3$
		$y = \pm 1$
$k = \pm 3$	1A+1A	$x = \pm 2$
Equations of tangents $y = 2x + 3$ and $y = 2x - 3$		$y = 2x-3$ and $y = 2x+3$ 1A+1A
	6	

Alternative Solution:	
Eqt. of tangent: $x_1 x - y_1 y = 3$	1A
slope $= \frac{x_1}{y_1}$	1A
$\frac{x_1}{y_1} = 2$	1A
etc.	

SOLUTIONS	MARKS	REMARKS
8. (a) $du = \sec^2 x\,dx$,	1A	
$\int \tan^{n-2} x \sec^2 x\,dx = \int u^{n-2} du$	1A	
$= \frac{\tan^{n-1} x}{n-1} + c$	1A+1A	1A for c
	4	
(b) (i) $\int_0^{\frac{\pi}{4}} \tan^n x\,dx = \int_0^{\frac{\pi}{4}} \tan^{n-2} x \tan^2 x\,dx$		
$= \int_0^{\frac{\pi}{4}} \tan^{n-2} x (\sec^2 x - 1)\,dx$	1A	
$= \int_0^{\frac{\pi}{4}} \tan^{n-2} x \sec^2 x\,dx - \int_0^{\frac{\pi}{4}} \tan^{n-2} x\,dx$	1M	
$= \left[\frac{\tan^{n-1} x}{(n-1)} \right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \tan^{n-2} x\,dx$	1M	1M for using (a)
$= \frac{1}{n-1} - \int_0^{\frac{\pi}{4}} \tan^{n-2} x\,dx$	1	Alternative Solution:
(ii) $I_0 = \int_0^{\frac{\pi}{4}} dx = \frac{\pi}{4}$ or $I_2 = 1 - \frac{\pi}{4}$	1A	$\int_0^{\frac{\pi}{4}} \tan^6 x\,dx$
$I_6 = \int_0^{\frac{\pi}{4}} \tan^6 x\,dx = (\frac{1}{5} - I_4)$	2A	$= \int_0^{\frac{\pi}{4}} \tan^4 x (\sec^2 x - 1)dx$ 1A
$I_4 = (\frac{1}{3} - I_2)$	1A	⋮
$I_6 = [\frac{1}{5} - \frac{1}{3} + 1 - I_0]$		$= [\frac{1}{5}\tan^5 x - \frac{1}{3}\tan^3 x + (\tan x - x)]_0^{\frac{\pi}{4}}$ 1A+1A+1A
$= (\frac{13}{15} - \frac{\pi}{4})$ or 0.0813	1A	$= \frac{13}{15} - \frac{\pi}{4}$ 1A
	9	
(c) Putting $x = -v$	1A	Alternative Solution:
$dx = -dv$	1A	$\int_{-\frac{\pi}{4}}^{0} \tan^6 x\,dx$
$x = 0,\ v = 0$; $x = -\frac{\pi}{4},\ v = \frac{\pi}{4}$	1A	$= [\frac{1}{5}\tan^5 x - \frac{1}{3}\tan^3 x + (\tan x - x)]_{-\frac{\pi}{4}}^{0}$ 2A
$\int_{-\frac{\pi}{4}}^{0} \tan^6 x\,dx = \int_{\frac{\pi}{4}}^{0} \tan^6 (-v)(-dv)$		$= \frac{13}{15} - \frac{\pi}{4}$ 1A
$= \int_0^{\frac{\pi}{4}} \tan^6 v\,dv$	1	$= \int_0^{\frac{\pi}{4}} \tan^6 x\,dx$ 1
$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^6 x\,dx = \int_{-\frac{\pi}{4}}^{0} \tan^6 x\,dx + \int_0^{\frac{\pi}{4}} \tan^6 x\,dx$	1A	
$= 2\int_0^{\frac{\pi}{4}} \tan^6 x\,dx$	1A	
$= 2(\frac{13}{15} - \frac{\pi}{4})$ or 0.163	1A	
	7	

SOLUTIONS	MARKS	REMARKS
9. (a) Area of region I $= \int_0^s x^2\,dx$	1A	
$= [\frac{x^3}{3}]_0^s$		
$= \frac{s^3}{3}$	1A	
		
Area of (shaded region + I + II)		Alternative Solution:
$= \frac{1}{2}(s+t)(s^2+t^2)$	1A	ST : $\frac{y-s^2}{x-s} = \frac{s^2-t^2}{s-(-t)}$
		$y = (s-t)x + st$ 1A
Area of region II $= \frac{t^3}{3}$	1A	Shaded area
Shaded area $= \frac{1}{2}(s+t)(s^2+t^2) - \frac{1}{3}s^3 - \frac{1}{3}t^3$	1M+1A	$= \int_{-t}^{s} [(s-t)x+st-x^2]dx$ 1M+1A
$= \frac{1}{6}(s^3+3s^2t+3st^2+t^3)$ }		$= [\frac{(s-t)x^2}{2} + stx - \frac{x^3}{3}]_{-t}^{s}$ 1A
$= \frac{1}{6}(s+t)^3$ }	1	$= \frac{1}{6}(s^3+3s^2t+3st^2+t^3)$ } 1
	7	$= \frac{1}{6}(s+t)^3$ }
(b) (i) S, H, T are collinear.		Sub. (0, 1) in eqt. of ST 1M
$\frac{s^2-1}{s-0} = \frac{t^2-1}{-t-0}$	1M	$1 = st$
$-s^2t + t = st^2 - s$		$t = \frac{1}{s}$ 1
$s + t = st(t+s)$		
$st = 1$		
$t = \frac{1}{s}$	1	
(ii) Shaded area $A = \frac{1}{6}(s+\frac{1}{s})^3$	1A	
$\frac{dA}{ds} = \frac{1}{6}(3)(s+\frac{1}{s})^2(1-\frac{1}{s^2})$	1A	
$= 0$	1M	
$s = 1$ or -1 (rejected)		
$\therefore s = 1$	1A	
$s < 1, \frac{dA}{ds} < 0$)	1M	$\frac{d^2A}{ds^2} = \frac{1}{2}\,2(s+\frac{1}{s})(1-\frac{1}{s^2})^2 + \frac{1}{2}(s+\frac{1}{s})^2(\frac{2}{s^3})$
$s > 1, \frac{dA}{ds} > 0$)		
$\therefore s = 1$ corresponds to a minimum A .	7	When $s = 1$, $\frac{d^2A}{ds^2} > 0$ 1M

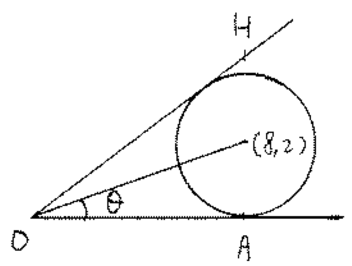
SOLUTIONS	MARKS	REMARKS
9. (c) For s = 1, ST is horizontal.		
Volume generated by region I $= \int_0^1 \pi y^2\,dx$	1M	For $\int_a^b \pi y^2 dx$
$= \pi\int_0^1 x^4\,dx$	1A	
$= \pi\left[\frac{x^5}{5}\right]_0^1$		
$= \frac{1}{5}\pi$	1A	
Volume of cylinder = $\pi(1)^2(2)$	1A	
Required volume = $\pi(1)^2(2) - \frac{1}{5}\pi - \frac{1}{5}\pi$	1M	
$= \frac{8\pi}{5}$ (or 5.03)	1A	
	6	

Alt. Solution		
Volume generated $= \int_a^b \pi(y_1{}^2 - y_2{}^2)\,dx$	1M+1M	1M for $\int_a^b \pi y^2 dx$
$= 2\int_0^1 \pi(1 - x^4)\,dx$	2A	
$= 2\pi\left[x - \frac{x^5}{5}\right]_0^1$	1A	
$= \frac{8\pi}{5}$ (or 5.03)	1A	

SOLUTIONS	MARKS	REMARKS
10.(a) PS = PN		
$\sqrt{(x-1)^2+y^2} = x+1$	1M+1A +1A	1A for L.S. 1A for R.S.
$(x-1)^2+y^2 = (x+1)^2$		
$y^2 = 4x$	1	
	4	
(b) (i) $y = 2t$		
$x = t^2$	1A	
(ii)(1) PN // x-axis and PR bisects ∠SPN.		**Alternative Solution:**
∴ ∠PRS = ∠RPS		PR intersects SN at M
SR = SP		
= PN		M is the mid-point of SN 3A
$= t^2+1$	2A	∴ M is the point (0, t) 2A
∴ OR = SR − SO		PR : $\frac{y-t}{x-0} = \frac{2t-t}{t^2-0}$ 1M
$= t^2$	1A	$x - ty + t^2 = 0$ 1
R is the point $(-t^2, 0)$	2A	
∴ the equation of PR is		
$y = \frac{2t-0}{t^2-(-t^2)}(x+t^2)$	1M	
i.e. $x - ty + t^2 = 0$	1	
Alternative Solution:		
PS : $\frac{y-0}{x-1} = \frac{2t}{t^2-1}$	1M	
$2tx + (1-t^2)y - 2t = 0$	1A	
PN : $y = 2t$	1A	
PR is the angle bisector. Its equation is		
$\frac{y-2t}{\sqrt{1^2+0^2}} = \frac{2tx+(1-t^2)y-2t}{\sqrt{(2t)^2+(1-t^2)^2}}$	2M+1A	
$y - 2t = \frac{2tx+(1-t^2)y-2t}{1+t^2}$)) ..... $x - ty + t^2 = 0$)	1	
(2) Sub. $x = ty - t^2$ in $y^2 = 4x$	1M	**Alternative Solution:**
$4(ty - t^2) = y^2$		Differentiating $y^2 = 4x$ 1M
$y^2 - 4ty + 4t^2 = 0$	1A	$y' = \frac{2}{y}$
$\Delta = (-4t)^2 - 4(4t^2)$ 1M ; $= 0$ 1A | $(y-2t)^2 = 0$	2A	slope of tangent at P = $\frac{1}{t}$ 1A
		Eqt. of tangent at P:
∴ it touches $y^2 = 4x$ at P.		$y - 2t = \frac{1}{t}(x - t^2)$ 1M
		$x - ty + t^2 = 0$ 1A
(3) R is the point $(-t^2, 0)$ ———————	1A	which is the eqt. of PR.
P is the point $(t^2, 2t)$		
Mid-point of PR is (0, t)	1A	
Equation of locus is x = 0.	2A	∴ PR touches $y^2 = 4x$ at P
	16	

SOLUTIONS	MARKS	REMARKS
11.(a) (i) $x^2 + y^2 - 16x - 4y + 64 = 0$		Centre = (8, 2))
Put $y = 0$,	1M	radius = 2)
$x^2 - 16x + 64 = 0$ $(x - 8)^2 = 0$ or $\Delta = (-16)^2 - 4(64) = 0$ $x = 8$	1A	Distance from centre to x-axis = radius 1
Therefore C_1 touches the x-axis at A		C_1 touches the x-axis at A
		Alternative Solution:
(ii) Let equation of OH be $y = mx$	1A	OH: $y = mx$ 1.
Sub. in equation of C_1 $x^2 + m^2x^2 - 16x - 4mx + 64 = 0$		C_1: centre = (8,2) radius = 2
$(1 + m^2)x^2 - 4(m + 4)x + 64 = 0$	1A	$\frac{8m - 2}{\sqrt{1 + m^2}} = \pm 2$ (± optional) 1M+1.
For tangents,		$(4m - 1)^2 = 1 + m^2$
$16(m + 4)^2 - (4)(64)(1 + m^2) = 0$	1M	$15m^2 - 8m = 0$ 1.
$m^2 + 8m + 16 - 16m^2 - 16 = 0$		$m = 0$ or $\frac{8}{15}$
$15m^2 - 8m = 0$	1A	OH: $y = \frac{8}{15}x$ 1.
$m = 0$ or $\frac{8}{15}$		
OH : $y = \frac{8}{15}x$	1A	

Alternative Solution:



Eqt. of OH: $y = mx$	1A	
$\tan\theta = \frac{2}{8} = \frac{1}{4}$	1A	
$m = \tan \angle AOH$		
$= \tan 2\theta$	1A	
$= \frac{2\tan\theta}{1 - \tan^2\theta}$	1M	
$= \frac{8}{15}$	1A	

SOLUTIONS	MARKS	REMARKS
		Alternative Solution:
(iii) Let coordinates of H be $(8, y_1)$	1A	By symmetry or $\angle HOB = \angle OBH$.
Sub. in equation of OH	1M	Slope of BH = $\tan(180° - \angle BOH)$
$y_1 = \frac{64}{15}$	1A	$= -\tan \angle BOH$
		$= -\frac{8}{15}$ 3
Equation of BH : $\frac{y - 0}{x - 16} = \frac{\frac{64}{15} - 0}{8 - 16}$	1M	BH: $\frac{y - 0}{x - 16} = -\frac{8}{15}$ 1
$\frac{y}{x - 16} = -\frac{8}{15}$		$8x + 15y - 128 = 0$ 1
$y = -\frac{8}{15}x + \frac{128}{15}$	1A	
$8x + 15y - 128 = 0$		
	12	

SOLUTIONS	MARKS	REMARKS
11.(b) (i) Sub. (8, 0) in equation of C_2	1M	Put y = 0 in eqt. of C_2
$64 - 128 + c = 0$		$x^2 - 16x + c = 0$
$c = 64$	1A	$\Delta = 16^2 - 4c = 0$ 1M
C_2 touches $4x + 3y = 0$		$c = 64$ 1A
Sub. in C_2		
$x^2 + \frac{16}{9}x^2 - 16x - \frac{8f}{3}x + 64 = 0$		
$25x^2 - (144 + 24f)x + (9)(64) = 0$		Alternative Solution:
For tangents,		OK is tangent.
$(144 + 24f)^2 - 4(25)(9)(64) = 0$	1M	Centre of C_2 = (8, −f) radius = f
$f = 4$ or -16	1A	$\therefore \frac{4(8) - 3(f)}{\sqrt{4^2 + 3^2}} = \pm f$ 1M
Rejecting f = −16,		$32 - 3f = \pm 5f$
$f = 4$	1A	$f = 4$ or -16 .... 1A
		Rejecting f = −16
		$f = 4$ 1A
		Alt. Solution:
(ii) $\frac{\Delta OBH}{\Delta OBK} = \frac{\frac{1}{2}(OB)(AH)}{\frac{1}{2}(OB)(AK)}$		K = (8, k)
$= \frac{AH}{AK}$		Sub. in 4x+3y = 0 1M
$= \frac{AH/OA}{AK/OA}$	1M	$k = -\frac{32}{3}$
$= \frac{8/15}{4/3}$		$\frac{\Delta OBH}{\Delta OBK} = \frac{AH}{AK}$
$= \frac{2}{5}$	2A	$= \frac{64/15}{32/3}$
	8	$= \frac{2}{5}$ 2A

Alternative Solution:

$\Delta OBH = \frac{1}{2}(16)(\frac{64}{15})$	1A
$\Delta OBK = \frac{1}{2}(16)(\frac{32}{3})$	1A
$\frac{\Delta OBH}{\Delta OBK} = \frac{2}{5}$	1A

SOLUTIONS	MARKS	REMARKS
12.(a)(i) $7\sin\theta - 24\cos\theta$		**Alternative Solutions:** $r\sin(\theta - A)$
$= \sqrt{7^2+24^2}\left(\frac{7}{\sqrt{7^2+24^2}}\sin\theta - \frac{24}{\sqrt{7^2+24^2}}\cos\theta\right)$	1A	$= r\sin\theta\cos A - r\cos\theta\sin A$ 1A
$= \sqrt{7^2+24^2}\sin(\theta - A)$	1A	$= 7\sin\theta - 24\cos\theta$
$r = \sqrt{7^2+24^2}$		$r\cos A = 7$)
$= 25$	1A	$r\sin A = 24$) 1A
$A = \tan^{-1}\frac{24}{7}$		$r = 25$ 1A
$\doteq 73.7°$ (73°42' or 1.29 rad.)	1A	$A = 73.7°$ 1A
(ii) $y = 2(7\sin\theta - 24\cos\theta) + 14$		
$= 2[25\sin(\theta - 73.7°)] + 14$	2M	**Alternative Solution:**
$-1 \leqslant \sin(\theta - 73.7°) \leqslant 1$	1M+1M	$y' = 50\cos(\theta - 1.29) = 0$ 1M
$-36 \leqslant y \leqslant 64$	1A+1A	$y'' = -50\sin(\theta - 1.29)$ 1M
When $y = 64$,		Max. $y = 64$ 1A
$\sin(\theta - 73.7°) = 1$		Min. $y = -36$ 1A
$\theta - 73.7° = 180n° + (-1)^n 90°$ or $360n° + 90°$	1A	
$\theta = 180n° + (-1)^n 90° + 73.7°$ or $360n° + 163.7°$	1A	
	12	
(b) $\cos\alpha\cos\beta = \frac{1}{6}$	1	
$\cos\alpha + \cos\beta = \frac{5}{6}$	1	
$\left(\cos\frac{\alpha+\beta}{2} + \cos\frac{\alpha-\beta}{2}\right)^2 = \left(2\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\right)^2$	2A	
$= \left(2\cos^2\frac{\alpha}{2}\right)\left(2\cos^2\frac{\beta}{2}\right)$	1A	
$= (1+\cos\alpha)(1+\cos\beta)$	1A	
$= 1 + \cos\alpha\cos\beta + \cos\alpha + \cos\beta$	1A	
$= 1 + \frac{1}{6} + \frac{5}{6}$	1M	
$= 2$	8	
$\cos\frac{\alpha+\beta}{2} + \cos\frac{\alpha-\beta}{2} = \sqrt{2}$		

Alternative Solution:

SOLUTIONS	MARKS
$\left(\cos\frac{\alpha+\beta}{2} + \cos\frac{\alpha-\beta}{2}\right)^2$	
$= \cos^2\frac{\alpha+\beta}{2} + \cos^2\frac{\alpha-\beta}{2} + 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$	1A
$= \frac{1}{2}[1+\cos(\alpha+\beta)] + \frac{1}{2}[1+\cos(\alpha-\beta)] + \cos\alpha + \cos\beta$	1A+1A
$= 1 + \cos\alpha + \cos\beta + \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$	
$= 1 + \cos\alpha + \cos\beta + \cos\alpha\cos\beta$	2A
$= 1 + \frac{1}{6} + \frac{5}{6}$	1M
$= 2$	
$\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = \sqrt{2}$	