香港考試局

HONG KONG EXAMINATIONS AUTHORITY

一九八六年香港中學會考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1986

附加數學（試卷一）
ADDITIONAL MATHEMATICS I

評卷參考
MARKING SCHEME

請在學校任教之閱卷員特別留意

本評卷參考並非標準答案，故極不宜落於學生手中，以免引起誤會。

遇有學生求取此文件時，閱卷員應嚴予拒絕。閱卷員在任何情況下披露本評卷參考內容，均有違閱卷員守則及「一九七七年香港考試局法例」。

Special Note for Teacher Markers

It is highly undesirable that this marking scheme should fall into the hands of students. They are likely to regard it as a set of model answers, which it certainly is not.

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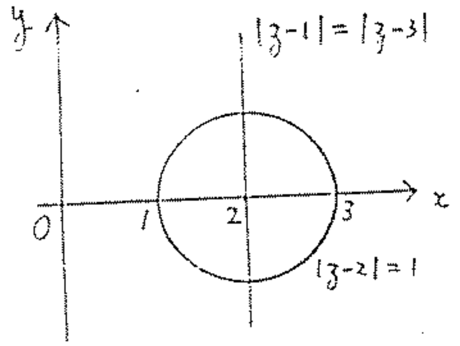
SOLUTIONS	MARKS	REMARKS
1. $\frac{d}{dx}(x^3) = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^3 - x^3}{\Delta x}$	1	
$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x}$	1A	For expanding $(x+\Delta x)^3$.
$= \lim_{\Delta x \to 0} [3x^2 + 3x(\Delta x) + (\Delta x)^2]$	1A	
$= 3x^2$	1A	
	4	
2. The discriminant $= (\log b)^2 - 4(\log a)(\log b)$	1A	Alt. Solution:
For equal roots, $(\log b)^2 - 4(\log a)(\log b) = 0$	1M	Differentiating, $2x\log a + \log b = 0$ 1M
The roots are non-zero, $\log b \neq 0$.	1	Solving with given eqt.
(Accept rejecting $\log b = 0$)		$(x+2)\log b = 0$ 1A; $x = -2$ 1A
$\therefore \log b = 4\log a$	1A	$-4\log a + \log b = 0$ 1A
$= \log a^4$		$b = a^4$ 1A
$b = a^4$	1A	
	5	
3. $f'(x) = 18 - 2kx$ $= 0$	1M	Alt. Solution: $4k + 18x - kx^2 = 45$ 1
$x = \frac{9}{k}$	1A	$kx^2 - 18x + 45 - 4k = 0$
Alt. Solution: f(x) is quadratic, its maximum occurs when $x = \frac{-18}{2(-k)}$	1A	For equal roots, $(-18)^2 - 4k(45-4k) = 0$ 1M+ 1A
$= \frac{9}{k}$	1A	$4k^2 - 45k + 81 = 0$ 1A
		$k = 9$ or $\frac{9}{4}$ 1A
Alt. Solution: $f(x) = -kx^2 + 18x + 4k$ $= -k[(x - \frac{9}{k})^2 - 4 - \frac{81}{k^2}]$	1M+1A	
$f(\frac{9}{k}) = 45$ or $4k + \frac{81}{k} = 45$	1M	
$4k^2 - 45k + 81 = 0$	1A	
$(k-9)(4k-9) = 0$		
$k = 9$ or $\frac{9}{4}$	1A	For both answers
	5	

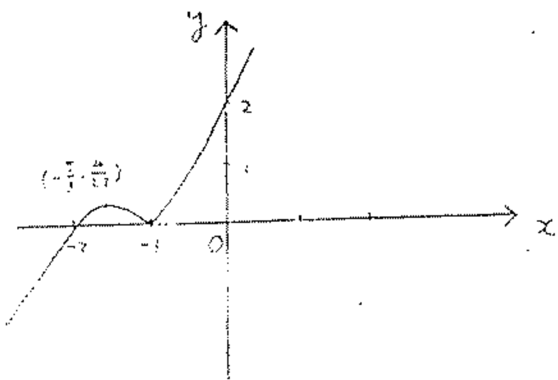
SOLUTIONS	MARKS	REMARKS
4. Differentiating with respect to x	1M	
$2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$	2A	
Substituting (2, 1),		
$4 + 2\frac{dy}{dx} + 1 + 2\frac{dy}{dx} = 0$		
$\therefore \frac{dy}{dx} = -\frac{5}{4}$	1A	
Equation of tangent: $\frac{y-1}{x-2} = -\frac{5}{4}$	1M	For point-slope form
$5x + 4y - 14 = 0$	1A	
	6	
5. $(\underline{i}+\underline{j})\cdot[(c+4)\underline{i} + (c-4)\underline{j}] = \lvert\underline{i}+\underline{j}\rvert\lvert(c+4)\underline{i} + (c-4)\underline{j}\rvert \cos\theta$	1M	Dot must be shown
$(c+4) + (c-4) = \sqrt{2}\,\sqrt{2}\,\sqrt{c^2+16}\,(-\frac{3}{5})$	1A+1A	1A for L.S. 1A for R.S.
$c = -\frac{3}{5}\sqrt{c^2+16}$		
$c^2 = 9$	1A	
$c = \pm 3$		
After checking,	1M	
$c = -3$	1A	
	[illegible]	
Alt. Solution:		
$\tan\alpha = 1$, $\tan\beta = \frac{c-4}{c+4}$	1A	
$\tan(\alpha-\beta) = \frac{1 - \frac{c-4}{c+4}}{1 + \frac{c-4}{c+4}}$	1M	
$\tan\theta = \pm\frac{4}{c}$		
$\cos\theta = -\frac{3}{5}$		
$\tan\theta = -\frac{4}{3}$	1A	
$\therefore \frac{4}{c} = \pm\frac{4}{3}$		
$c = \pm 3$	1A	
After checking,	1M	
$c = -3$	1A	

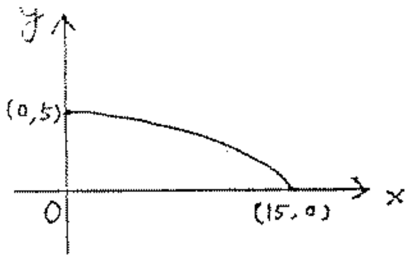
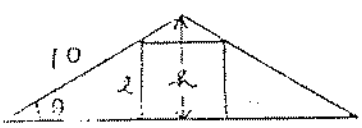
SOLUTIONS	MARKS	REMARKS
6. 		
Circle	1	
Radius & centre	1A	
Line ⊥ x-axis	1A	
Line passes through (2,0)	1A	
$2 + i$ and $2 - i$	1A+1A 6	Alt. Sol. for last part: $\|z - 2\| = 1$ $(x - 2)^2 + y^2 = 1$ $\|z - 1\| = \|z - 3\|$ $x = 2$ $y = \pm 1$ 2+i and 2-i 1A+1A
7. (a) $x > 0$, $x > \frac{3}{x} + 2$ $x^2 > 3 + 2x$	1A	
$x^2 - 2x - 3 > 0$ $(x - 3)(x + 1) > 0$ $x > 3$ or $x < -1$	1M	
but $x > 0$ $\therefore x > 3$	1A	
(b) $x < 0$, $x > \frac{3}{x} + 2$ $x^2 < 3 + 2x$	2A	
$x^2 - 2x - 3 < 0$ $(x - 3)(x + 1) < 0$ $3 > x > -1$	1M	
but $x < 0$ $\therefore -1 < x < 0$	1A 7	

SOLUTIONS	MARKS	REMARKS
3. (a) $\overrightarrow{OC} = \underline{a} + 2\underline{b}$	1A	If vector sign omitted, or division of vectors, pp-1.
$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$	1A	
$= \underline{a} + 2\underline{b} - \underline{b}$		
$= \underline{a} + \underline{b}$	1A	
$\overrightarrow{OQ} = \overrightarrow{OB} + \overrightarrow{BQ} = \overrightarrow{OB} + \frac{1}{3}\overrightarrow{BC}$		
$= \underline{b} + \frac{1}{3}(\underline{a} + \underline{b})$	1M	
$= \frac{1}{3}\underline{a} + \frac{4}{3}\underline{b}$	1A	
	5	
(b) $\overrightarrow{OR} = h\overrightarrow{OQ} + (1-h)\overrightarrow{OP}$	1	Alt. Solution:
		$\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR}$
$= h(\frac{1}{3}\underline{a} + \frac{4}{3}\underline{b}) + (1-h)\frac{1}{2}\underline{a}$	1M	$= \overrightarrow{OP} + h\overrightarrow{PQ}$
$= (\frac{1}{2} - \frac{h}{6})\underline{a} + \frac{4h}{3}\underline{b}$	1A	$= \overrightarrow{OP} + h(\overrightarrow{OQ} - \overrightarrow{OP})$ 1
		$= \frac{1}{2}\underline{a} + h[(\frac{1}{3}\underline{a} + \frac{4}{3}\underline{b}) - \frac{1}{2}\underline{a}]$ 1M
		$= (\frac{1}{2} - \frac{h}{6})\underline{a} + \frac{4h}{3}\underline{b}$ 1A
		$\overrightarrow{OC} = \underline{a} + 2\underline{b}$
$\overrightarrow{OR} = k\overrightarrow{OC}$		
$= k\underline{a} + 2k\underline{b}$	1A	OR // OC
		$\frac{4h/3}{2} = \frac{\frac{1}{2} - h/6}{1}$ 2M+1A
$\left.\begin{array}{l}\frac{1}{2} - \frac{h}{6} = k \\ \frac{4h}{3} = 2k\end{array}\right\}$	2M+1A	$h = 3/5$ 1A
Solving, $h = \frac{3}{5}$	1A	$\overrightarrow{OR} = 2/5\,\underline{a} + 4/5\,\underline{b}$
		$= 2/5(\underline{a} + 2\underline{b})$ 1A
$k = \frac{2}{5}$	1A	$= 2/5\,\overrightarrow{OC}$
	9	$\therefore k = 2/5$ 1A
(c) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$		
$= \frac{4}{3}\underline{b} - \frac{1}{6}\underline{a}$	1A	
$\overrightarrow{PT} = \overrightarrow{OT} - \overrightarrow{OP}$		
$= \lambda\underline{b} - \frac{1}{2}\underline{a}$	1A	
PQ // PT		Alt. Solution:
$\frac{\frac{4}{3}}{\lambda} = \frac{\frac{1}{6}}{\frac{1}{2}}$	2M+1A	Let $\overrightarrow{PT} = \mu\overrightarrow{PQ}$
		$= \frac{4}{3}\mu\underline{b} - \frac{\mu}{6}\underline{a}$
$\lambda = 4$	1A	$\left.\begin{array}{l}\frac{1}{2} = \frac{\mu}{6} \\ \lambda = \frac{4}{3}\mu\end{array}\right\}$ 2M+1A
	6	Solving $\mu = 3$, $\lambda = 4$ 1A

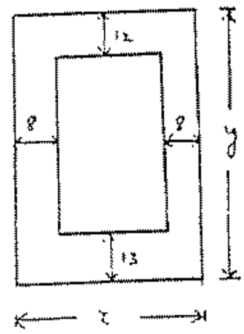
SOLUTIONS	MARKS	REMARKS
1.(a) $\cos x = \frac{1}{\sqrt{2}}$		If mixed units, pp-1.
$x = 2n\pi \pm \frac{\pi}{4}$ or $(n)(360°) \pm 45°$	2A	
(n is an integer)	1A	
	3	
(b)(i) $z = r(\cos\theta + i\sin\theta)$		
$z^m = r^m(\cos m\theta + i\sin m\theta)$	1A	
$\bar{z} = r[\cos(-\theta) + i\sin(-\theta)]$	1A	
$(\bar{z})^m = r^m[\cos(-m\theta) + i\sin(-m\theta)]$		
$= r^m(\cos m\theta - i\sin m\theta)$	1A	
$z^m + (\bar{z})^m = 2r^m\cos m\theta$	1	
(ii) $z = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$		
$= (\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$	1A	
$(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i)^m + (\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i)^m = \sqrt{2}$		
$2\cos\frac{m\pi}{4} = \sqrt{2}$	1M	
$\cos\frac{m\pi}{4} = \frac{1}{\sqrt{2}}$		
$\frac{m\pi}{4} = 2n\pi \pm \frac{\pi}{4}$	1M	
$m = 8n \pm 1$	1A	
$m = 1$ or $m = 8n\pm1$ where n is a positive integer.	1A	
	9	
(c) $(1 + i)^p - (1 - i)^p = 0$		Alt. Solution:
$(\frac{1+i}{1-i})^p = 1$	1A	$(\frac{1-i}{1+i})^p = 1$ 1A
$[\frac{(1+i)^2}{2}]^p = 1$	1A	$[\frac{(1-i)^2}{2}]^p = 1$ 1A
$i^p = 1$	1A	$(-i)^p = 1$ 1A
$p = 4n$, n is a +ve integer	1A	$p = 4n$,
(Accept p = 4, 8, 12, ...)		n is a +ve integer 1A

SOLUTIONS	MARKS	REMARKS
9.(c)(i)		
Alt. Solution:		
$(1+i)^p - (1-i)^p = 0$		
$1 + i = \sqrt{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$	1A	
$(1+i)^p - (1-i)^p$		
$= 2(\sqrt{2})^p\, i\sin\frac{p\pi}{4}$	1A	
$= 0$		
$\sin\frac{p\pi}{4} = 0$	1A	
$\frac{p\pi}{4} = n\pi$		
$p = 4n$, n is a +ve integer	1A	
(ii) $\frac{(1+i)^{4k+1}}{(1-i)^{4k-1}} = (\frac{1+i}{1-i})^{4k}(1+i)(1-i)$	1M+1A	
$= 1 - i^2$	1A	
$= 2$	1A	
	3	
Alt. Solution		
$\frac{(1+i)^{4k+1}}{(1-i)^{4k-1}} = \frac{(1+i)^{8k}}{2^{4k-1}}$	1M+1A	
$= \frac{(\sqrt{2})^{8k}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})^{8k}}{2^{4k-1}}$		
$= \frac{2^{4k}}{2^{4k-1}}(\cos 2k\pi + i\sin 2k\pi)$	1A	
$= 2$	1A	

SOLUTIONS	MARKS	REMARKS
10.(a) $f(x) = x^3 + hx^2 + kx + 2$		
$f'(x) = 3x^2 + 2hx + k$	1A	
$= 0$	1M	
For 2 distinct turning points,		
$(2h)^2 - 4(3)(k) > 0$	1M	
$h^2 > 3k$	1	
Put $y = 2$ in $y = x^3 + hx^2 + kx + 2$	1	
$x^3 + hx^2 + kx = 0$		
$x(x^2 + hx + k) = 0$	1A	
$x^2 + hx + k = 0$ has no real roots	1M	
$\therefore h^2 < 4k$	1	
	8	
(b)(i) Sub. $(-2, 0)$ in $f(x) = x^3 + hx^2 + kx + 2$	1	
$-8 + 4h - 2k + 2 = 0$		
$k = 2h - 3$	1A	
(ii) $4k > h^2 > 3k$		
$8h - 12 > h^2 > 6h - 9$	1M	
$h^2 - 8h + 12 < 0$ and $h^2 - 6h + 9 > 0$	1A+1A	1A For 2 ineq. 1A For 'and' (Accept omitting 'and'. Do not accept 'or'.)
$(h - 2)(h - 6) < 0$ and $(h - 3)^2 > 0$		
$6 > h > 2$ and $h \neq 3$	1A+1A	
h is an integer		
$\therefore h = 4$ or 5		
(iii) For $h = 4$,		
$f'(x) = 3x^2 + 8x + 5 = 0$		
$(3x + 5)(x + 1) = 0$		
$x = -\frac{5}{3}$ or -1	1A	
$f''(x) = 6x + 8$		
$x = -\frac{5}{3}$, $f''(x) < 0$, $\therefore (-\frac{5}{3}, \frac{4}{27})$ is a maximum point	1A	(1.67, 0.148)
$x = -1$, $f''(x) > 0$, $\therefore (-1, 0)$ is a minimum point	1A	
	1A	For shape
	1A	For 3 points out of the 4 points (-2, 0), (-5/3, 4/27) (-1, 0), (0, 2)
	12	

SOLUTIONS	MARKS	REMARKS
11.(a) $s = 20\cos\theta$	1A	
$\frac{ds}{dt} = \frac{ds}{d\theta}\cdot\frac{d\theta}{dt}$	1	
$\frac{ds}{d\theta} = -20\sin\theta$	1A	
$\frac{ds}{dt} = 10$		
$\therefore \frac{d\theta}{dt} = \frac{10}{-20\sin\theta}$		
$= \frac{-1}{2\sin\theta}$	1A	
When $s = 10$, $\theta = \frac{\pi}{3}$		
$\frac{d\theta}{dt} = -\frac{1}{\sqrt{3}}$ (s^{-1}) (or -0.577 s^{-1})	1A / 5	Unit optional
(b) $x = 15\cos\theta$	1A	
$y = 5\sin\theta$	1A	
$\frac{x^2}{15^2} + \frac{y^2}{5^2} = 1$	1A	
$(x, y \geq 0)$		
	1A	Shape
	1A / 5	Labelling the two end-points
(c) 		
$h = 10\sin\theta$	1A	
$\frac{h-l}{h} = \frac{l}{20\cos\theta}$	1M+1A	1M For similar Δ s.
$1 - \frac{l}{h} = \frac{l}{20\cos\theta}$		
$l(\frac{1}{h} + \frac{1}{20\cos\theta}) = 1$		Alt. Solution: Equating lengths 1M Correct equation 2A Final answer
$l = \frac{1}{(\frac{1}{10\sin\theta} + \frac{1}{20\cos\theta})}$		
$= \frac{20\sin\theta\cos\theta}{\sin\theta + 2\cos\theta}$	1	

SOLUTIONS	MARKS	REMARKS
11.(c) A = Area of square = ℓ^2		
$\frac{dA}{d\theta} = 2\ell\frac{d\ell}{d\theta}$	1M	
$= 2\ell\frac{(\sin\theta + 2\cos\theta)20(-\sin^2\theta+\cos^2\theta) - 20\sin\theta\cos\theta(\cos\theta-2\sin\theta)}{(\sin\theta + 2\cos\theta)^2}$		
$= 0$	1M	
$-\sin^3\theta-2\cos\theta\sin^2\theta+\sin\theta\cos^2\theta+2\cos^3\theta-\sin\theta\cos^2\theta+2\sin^2\theta\cos\theta = 0$		
Alt. Solution (1):		
$\frac{dA}{d\theta} = 2\ell\frac{d\ell}{d\theta}$	1M	
$= 2\ell\cdot 20\frac{d}{d\theta}\left[\frac{1}{\frac{1}{\cos\theta}+\frac{2}{\sin\theta}}\right]$		
$= 40\ell\frac{-1}{\left(\frac{1}{\cos\theta}+\frac{2}{\sin\theta}\right)^2}\left[\frac{\sin\theta}{\cos^2\theta}-\frac{2\cos\theta}{\sin^2\theta}\right]$		
$= 0$	1M	
Alt. Solution (2):		
$A = \ell^2$		
$= \frac{400\sin^2\theta\cos^2\theta}{(\sin\theta + 2\cos\theta)^2}$		
$\frac{dA}{d\theta} = 400\cdot\frac{(\sin\theta+2\cos\theta)^2(2\sin\theta\cos^3\theta-2\cos\theta\sin^3\theta)-\sin^2\theta\cos^2\theta\cdot 2(\sin\theta+2\cos\theta)(\cos\theta-2\sin\theta)}{(\sin\theta + 2\cos\theta)^4}$	1M	For quotient rule
$= 0$	1M	
$(\sin\theta + 2\cos\theta)(\cos^2\theta - \sin^2\theta) - \sin\theta\cos\theta(\cos\theta - 2\sin\theta) = 0$		
$-\sin^3\theta + 2\cos^3\theta = 0$	2A	
$\tan^3\theta = 2$	1A	
$\theta = 51.6°$	1A	51.561°
	10	

SOLUTIONS	MARKS	REMARKS
12. (a) $A = (x - 16)(y - 25)$	1A	
$= (x - 16)(\frac{3600}{x} - 25)$	1A / 2	
$= 4000 - 25x - \frac{16(3600)}{x}$		
Alt. Solution:		
$A = 3600 - 2(8y) - 12(x - 16) - 13(x - 16)$	1A	
$= 4000 - 25x - \frac{16(3600)}{x}$	1A	
(b) $\frac{dA}{dx} = -25 + \frac{16(3600)}{x^2}$	1A	
$= 0$	1M	
$x^2 = \frac{16(3600)}{25}$		
$x = \pm 48$		
Rejecting $x = -48$,		
$x = 48$	1A	
Maximum $A = 1600$		
Testing for maximum		
$\frac{d^2A}{dx^2} = -\frac{2(16)(3600)}{x^3}$	1A	
< 0		
Explaining why A is largest when $x = 48$	1M / 5	
c) (i) A decreases as x increases		
$\frac{dA}{dx} < 0$		
$-25 + \frac{16(3600)}{x^2} < 0$	1M	
$x > 48$ or $x < -48$ (rejected)		
$\therefore (144 >) x > 48$	1A	Accept $x \geqslant 48$
(ii) If $x \geqslant 50$,		
$\because A$ is decreasing	2	
$\therefore$ Largest value of A occurs when $x = 50$	1A	
Largest value of $A = 4000 - (25)(50) - \frac{16(3600)}{50}$		
$= 1598$	1A / 6	

SOLUTIONS	MARKS	REMARKS
12.(d) $\frac{4}{9} \leq \frac{x}{y} \leq \frac{9}{16}$		
$\frac{4}{9} \leq \frac{x}{\frac{3600}{x}} \leq \frac{9}{16}$	1A	
$\frac{4}{9} \leq \frac{x^2}{3600} \leq \frac{9}{16}$		
$1600 \leq x^2 \leq 2025$	1A	
$40 \leq x \leq 45$	1A	
For $x < 48$, $\frac{dA}{dx} > 0$		
A is increasing	2	
Largest value of A occurs when $x = 45$	1A	
Largest value of A $= 4000 - (25)(45) - \frac{16(3600)}{45}$		
$= 1595$	1A	
	7	

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附加數學（試卷二）

ADDITIONAL MATHEMATICS II

評卷參考

MARKING SCHEME

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SOLUTIONS	MARKS	REMARKS
1. When $n = 1$, L.S. $= \frac{1}{(1)(2)} = \frac{1}{2}$		
R.S. $= \frac{1}{1 + 1} = \frac{1}{2}$		
∴ the equality is true for $n = 1$.	1	
Assume that the equality holds for some positive integer k, then for $n = k + 1$,	1	
L.S. $= \frac{1}{(1)(2)} + \frac{1}{(2)(3)} + \dots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)}$		
$= \frac{k}{(k+1)} + \frac{1}{(k+1)(k+2)}$	1A	
$= \frac{k(k+2) + 1}{(k+1)(k+2)}$		
$= \frac{(k+1)^2}{(k+1)(k+2)}$		
$= \frac{k+1}{k+2}$	1A	
By mathematical induction, the equality is true for any positive integer n.	1 / 5	Awarded only if above correct
2. Coefficient of 3rd term $= {}_nC_2 \cdot 2^2$ or $\frac{n(n-1)}{2} \cdot 2^2$	1A	
$\frac{n(n-1)}{2} \cdot 4 = 40$	1M	
$n^2 - n - 20 = 0$	1A	
$(n - 5)(n + 4) = 0$		
$n = 5$	1A	
Coefficient of $x^4 = {}_5C_3 \cdot 2^3$		
$= 80$	1A / 5	
3. For equal roots, $(-4 \cos \theta)^2 - 4(3)(2)\sin \theta = 0$	2A	
$16 \cos^2\theta - 24 \sin \theta = 0$		
$2(1 - \sin^2\theta) - 3 \sin \theta = 0$		
$2\sin^2\theta + 3 \sin \theta - 2 = 0$	1A	
$(2 \sin \theta - 1)(\sin \theta + 2) = 0$		This may be omitted.
$\sin \theta = \frac{1}{2}$ or -2		
Rejecting $\sin \theta = -2$		
$\sin \theta = \frac{1}{2}$	1A	
θ is obtuse		
∴ $\theta = 150°$ $\left(\text{or } \frac{5\pi}{6}\right)$	1A / 5	

SOLUTIONS	MARKS	REMARKS
4. $\sin 2\theta + \sin 4\theta = \cos\theta$		For answers with mixed units, pp-1
$2\sin 3\theta\cos\theta = \cos\theta$	1A	
$\cos\theta = 0$ or $\sin 3\theta = \frac{1}{2}$	1A+1A	
$\theta = (2n+1)\frac{\pi}{2}$ or $3\theta = n\pi + (-1)^n\frac{\pi}{6}$	1A+1A	
[or $\theta = 2n\pi \pm \frac{\pi}{2}$]		
$\theta = \frac{n\pi}{3} + (-1)^n\frac{\pi}{18}$	1A	
(n is an integer)	6	
5.(a) $\frac{t+2}{s+1} = \frac{6-(-2)}{3-(-1)}$	1A	Alt. Solution: Equation of AB : $\frac{y+2}{x+1} = \frac{6-(-2)}{3-(-1)}$ 1A; $y = 2x$
$t = 2s$	1A	Sub. P(s, t), $t = 2s$ 1A
Alt. Solution: Area of $\triangle ABP = 2(2s - t)$ 1A; $= 0$; $t = 2s$ 1A		
Alt. Solution: Let $\frac{AP}{PB} = r$; $s = \frac{3-r}{1+r}$, $t = \frac{6-2r}{1+r}$ 1A; $t = 2s$ 1A		
(b) Area of $\triangle APC = \frac{1}{2}\begin{vmatrix} 3 & 6 \\ s & 2s \\ 5 & -3 \\ 3 & 6 \end{vmatrix}$		
$= \frac{1}{2}(-13s + 39)$	1A	
$\frac{1}{2}(-13s + 39) = \pm\frac{13}{2}$	1M	Accept no '±'
$s = 2$ or 4	1A+1A	
	6	
Alt. Solution:		
Height of $\triangle APC$ = distance of C from AB		
$= \frac{10+3}{\sqrt{5}}$		
$= \frac{13}{\sqrt{5}}$		
$AP = \sqrt{(s-3)^2 + (2s-6)^2}$		
$= \sqrt{5}\,\lvert s-3\rvert$,		
Area of $\triangle APC = \frac{1}{2}\frac{13}{\sqrt{5}}\cdot\sqrt{5}\,\lvert s-3\rvert$	1A	Accept (s-3) or (3-s)
$\frac{1}{2}\frac{13}{\sqrt{5}}\cdot\sqrt{5}\,(s-3) = \pm\frac{13}{2}$	1M	
$s = 2$ or 4	1A+1A	

SOLUTIONS	MARKS	REMARKS
6. AB : $\frac{y-2}{x-3} = m$	1A	
$y = mx + (2 - 3m)$		
Sub. in $y = (x-2)^2$		
$mx + (2 - 3m) = (x-2)^2$	1M	
$x^2 - (m+4)x + (3m+2) = 0$	1A	Alt. Solution:
$x_1 + x_2 = m + 4$		$x_1, x_2 = \frac{(m+4) \pm \sqrt{D}}{2}$
C is the mid-point, $\frac{m+4}{2} = 3$	1M+1A	$x_1 + x_2 = m + 4$
$m = 2$	1A	$\frac{m+4}{2} = 3$ 1M+1
		$m = 2$ 1
	6	
7 $\frac{dy}{d\theta} = \tan^2\theta\sec^2\theta - \sec^2\theta$	1A	
$= \tan^2\theta(1 + \tan^2\theta) - (1 + \tan^2\theta)$		
$= \tan^4\theta - 1$	1A	
$\tan^4\theta = \frac{dy}{d\theta} + 1$		
Integrating both sides		
$\int \tan^4\theta\, d\theta = \int (\frac{dy}{d\theta} + 1)\, d\theta$ or $\int \frac{dy}{d\theta} d\theta = \int (\tan^4\theta - 1) d\theta$	1A	
$= \int \frac{dy}{d\theta} d\theta + \int d\theta$		
$= y + \theta + C$	1M	For $\int \frac{dy}{d\theta} d\theta = y$
$= \frac{\tan^3\theta}{3} - \tan\theta + \theta + C$	2A	-1 if C omitted.
	6	
Alt. Solution:		
$\int \tan^4\theta d\theta$		
$= \int \tan^2\theta(\sec^2\theta - 1)d\theta$	1A	
$= \int \tan^2\theta\sec^2\theta d\theta - \int \tan^2\theta d\theta$		
$= \int \tan^2\theta d(\tan\theta) - \int (\sec^2\theta - 1)d\theta$	1M	For putting $u = \tan\theta$
$= \frac{\tan^3\theta}{3} - \tan\theta + \theta + c$	2A	-1 if c omitted.

SOLUTIONS	MARKS	REMARKS
8.(a) Putting $a - x = t$,	1A	
$dx = -dt$	1A	
When $x = 0$, $t = a$; When $x = a$, $t = 0$	1A	
$\int_0^a f(x)\,dx$		
$= \int_a^0 -f(a-t)\,dt$		
$= \int_0^a f(a-t)\,dt$	1	
$= \int_0^a f(a-x)\,dx$		
	4	
(b)(i) $\int_0^\pi \cos^{2n+1} x\,dx$		
$= \int_0^\pi \cos^{2n+1}(\pi - x)\,dx$	1A	
$= \int_0^\pi [-\cos x]^{2n+1}\,dx$	1A	
$= -\int_0^\pi \cos^{2n+1} x\,dx$	1A	
$\therefore\ 2\int_0^\pi \cos^{2n+1} x\,dx = 0$		
$\int_0^\pi \cos^{2n+1} x\,dx = 0$	1A	
(ii) $\int_0^\pi x\sin^2 x\,dx$		
$= \int_0^\pi (\pi - x)\sin^2(\pi - x)\,dx$	1A	
$= \int_0^\pi (\pi - x)\sin^2 x\,dx$		
$= \int_0^\pi \pi\sin^2 x\,dx - \int_0^\pi x\sin^2 x\,dx$	1M	
$\int_0^\pi x\sin^2 x\,dx = \frac{\pi}{2}\int_0^\pi \sin^2 x\,dx$	1A	
$= \frac{\pi}{2}\int_0^\pi \frac{1-\cos 2x}{2}\,dx$	1M	For $\sin^2 x = \frac{1-\cos 2x}{2}$
$= \frac{\pi}{4}\left[x - \frac{1}{2}\sin 2x\right]_0^\pi$	1A	
$= \frac{\pi^2}{4}$	1A	

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SOLUTIONS	MARKS	REMARKS
3.(b)(ii)		
Alt. Solution:		
$\int_0^{\pi} x \sin^2 x \, dx = \int_0^{\pi} x \frac{1-\cos 2x}{2} dx$	1M	
$= \frac{1}{2}\int_0^{\pi} x \, dx - \frac{1}{2}\int_0^{\pi} x \cos 2x \, dx$	1M	
$= \frac{1}{2}\left[\frac{x^2}{2}\right]_0^{\pi} - \frac{1}{2}\int_0^{\pi} x \cos 2x \, dx$		
$\int_0^{\pi} x \cos 2x \, dx = \int_0^{\pi} (\pi - x) \cos 2(\pi - x) \, dx$	1A	
$= \int_0^{\pi} (\pi - x) \cos 2x \, dx$		
$= \pi\int_0^{\pi} \cos 2x \, dx - \int_0^{\pi} x \cos 2x \, dx$		
$\therefore \int_0^{\pi} x \cos 2x \, dx = \frac{\pi}{2}\int_0^{\pi} \cos 2x \, dx$	1A	
$= \frac{\pi}{2}\left[\frac{1}{2}\sin 2x\right]_0^{\pi}$		
$= 0$	1A	
$\therefore \int_0^{\pi} x \sin^2 x \, dx = \frac{\pi^2}{4}$	1A	
(iii) $\int_0^{\frac{\pi}{2}} \frac{\sin x \, dx}{\sin x + \cos x} = \int_0^{\frac{\pi}{2}} \frac{\sin(\frac{\pi}{2} - x) \, dx}{\sin(\frac{\pi}{2} - x) + \cos(\frac{\pi}{2} - x)}$	1A	
$= \int_0^{\frac{\pi}{2}} \frac{\cos x \, dx}{\cos x + \sin x}$	1A	
$\int_0^{\frac{\pi}{2}} \frac{\sin x \, dx}{\sin x + \cos x} = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sin x + \cos x} dx$	2A	
$= \frac{1}{2}\int_0^{\frac{\pi}{2}} dx$	1A	
$= \frac{\pi}{4}$	1A	
	16	

SOLUTIONS	MARKS	REMARKS
6.(a)(i) slope of $L_1 = \frac{1}{2}$		Alt. Solution:
slope of reqd. line $= \frac{3k+2}{2k-1}$	1A	slope of required line $= \frac{3k+2}{2k-1}$ 1A
		$= m$
$\dfrac{\frac{3k+2}{2k-1} - \frac{1}{2}}{1 + \frac{1}{2}\cdot\frac{3k+2}{2k-1}} = \pm\tan 45°$ (Accept no "±")	1M	$\frac{m - \frac{1}{2}}{1 + \frac{1}{2}m} = \pm\tan 45°$ 1M
$= \pm 1$		$m = 3$ or $-\frac{1}{3}$
$\frac{6k + 4 - 2k + 1}{4k - 2 + 3k + 2} = \pm 1$		$\frac{3k+2}{2k-1} = 3$ or $-\frac{1}{3}$
$4k + 5 = \pm(7k)$		$k = \frac{5}{3}$ or $-\frac{5}{11}$ 1A+1A
$k = \frac{5}{3}$ or $-\frac{5}{11}$	1A+1A	$3x - y - 4 = 0$)
Equations of lines: $3x - y - 4 = 0$) $x + 3y - 18 = 0$)	1A	$x + 3y - 18 = 0$) 1A
Alt. Solution:		
The family of lines pass through (3, 5).	2A	
Let slope of required line be m.		
$\frac{m - \frac{1}{2}}{1 + \frac{1}{2}m} = \pm\tan 45°$	1M	
$m = 3$ or $-\frac{1}{3}$		
Equations of lines: $\frac{y - 5}{x - 3} = 3$ or $-\frac{1}{3}$	1A	
$3x - y - 4 = 0$) $x + 3y - 18 = 0$)	1A	
(ii) $\frac{3k+2}{2k-1} = \frac{1}{2}$	1M	
$6k + 4 = 2k - 1$		
$k = -\frac{5}{4}$		
L : $x - 2y + 7 = 0$	1A	
L_2 is of the form $x - 2y + c = 0$	1M	
Take (−7, 0) on L		
Distance from (−7, 0) to $L_1 = \left\lvert \frac{-7+4}{\sqrt{1^2 + 2^2}} \right\rvert$	1M	Accept expression with no absolute sign.
Distance from (−7, 0) to $L_2 = \left\lvert \frac{-7+c}{\sqrt{1^2 + 2^2}} \right\rvert$		
$-7 + c = \pm 3$	1M	Accept no '±'
$c = 10$ or 4 (rejected)		
L_2 : $x - 2y + 10 = 0$	1A	
	11	

SOLUTIONS	MARKS	REMARKS
7. (b) x - intercept $= \dfrac{11 - k}{3k + 2}$	1A	
y - intercept $= \dfrac{k - 11}{2k - 1}$	1A	
Area $S = -\dfrac{1}{2}\;\dfrac{(k - 11)^2}{(3k + 2)(2k - 1)}$	1M	For area $= \frac{1}{2}$(x-intercept)(y-intercept)
$\dfrac{dS}{dk} = \dfrac{(3k + 2)(2k - 1)(-2)(k - 11) + (k - 11)^2(12k + 1)}{4(3k + 2)^2(2k - 1)^2}$		
$= \dfrac{(k - 11)(-133k - 7)}{4(3k + 2)^2(2k - 1)^2}$		
$= 0$	1M	
$k = 11$ or $-\dfrac{1}{19}$	1A	
x-intercept and y-intercept are positive, reject $k = 11$		
$k = -\dfrac{1}{19}$ (or -0.0526)	1A	
Testing for minimum	1M	
	7	
(c) $3x - 2y + 1 = 0$	2A	

SOLUTIONS	MARKS	REMARKS
10.(a)(i) $C_1 - C_2$	1M	
$6x + 6y - 18 = 0$		
$x + y - 3 = 0$	1A	
(ii) $x^2 + y^2 - 4x + 2y + 1 + k(x + y - 3) = 0$	1M+1A	
$x^2 + y^2 + (k - 4)x + (k + 2)y + (1 - 3k) = 0$		
$r^2 = [\frac{1}{2}(k - 4)]^2 + [\frac{1}{2}(k + 2)]^2 + 3k - 1$	1M+1A	
$= \frac{1}{2}k^2 + 2k + 4$		
Area $S = \pi(\frac{1}{2}k^2 + 2k + 4)$		
$\frac{dS}{dk} = \pi(k + 2)$ or $\frac{d(r^2)}{dk} = (k + 2)$		
$= 0$	1M	
$k = -2$	1A	
$x^2 + y^2 - 6x + 7 = 0$	1A	
	9	
Alt. Solution (1):		
$x^2 + y^2 - 10x - 4y + 19 + k(x + y - 3) = 0$	1M+1A	
$x^2 + y^2 + (k - 10)x + (k - 4)y + (19 - 3k) = 0$		
$r^2 = [\frac{1}{2}(k - 10)]^2 + [\frac{1}{2}(k - 4)]^2 + 3k - 19$	1M+1A	
$= \frac{1}{2}k^2 - 4k + 10$		
Area $S = \pi(\frac{1}{2}k^2 - 4k + 10)$		
$\frac{dS}{dk} = \pi(k - 4)$		
$= 0$	1M	
$k = 4$	1A	
$x^2 + y^2 - 6x + 7 = 0$	1A	
Alt. Solution (2):		
$x^2 + y^2 - 4x + 2y + 1 + k(x^2+y^2-10x-4y+19) = 0$	1M+1A	
$(1+k)x^2 + (1+k)y^2 + (-4-10k)x + (2-4k)y + 19k + 1 = 0$		
$r^2 = (\frac{2 + 5k}{1+k})^2 + (\frac{2k - 1}{1+k})^2 - \frac{19k + 1}{1 + k}$	1M+1A	
$= \frac{2(5k^2 - 2k + 2)}{(1 + k)^2}$		
$\frac{d(r^2)}{dk} = \frac{2(1 + k)(12k - 6)}{(1 + k)^4}$		
$= 0$	1M	
$k = 1/2$	1A	
$\frac{3}{2}x^2 + \frac{3}{2}y^2 - 9x + \frac{21}{2} = 0$		
$x^2 + y^2 - 6x + 7 = 0$	1A	

SOLUTIONS	MARKS	REMARKS
10.(a)(ii) Alt. Solution (3):		
Solving equation of AB with equation of C_1 or C_2	1M	
Points of intersection: (2, 1) and (4, −1)	1A+1A	
For circle of minimum area, (2, 1) and (4, −1) are ends of a diameter.	2M	
$(x-2)(x-4)+(y-1)(y+1)=0$	1A	or centre: (3, 0), radius = $\sqrt{2}$ } 1A
$x^2+y^2-6x+7=0$	1A	
10.(b) Centre of C_3: (2, −1)	1A	
distance from (2, −1) to AB		
$= \left\lvert \frac{2-1-3}{\sqrt{1^2+1^2}} \right\rvert$ (Accept no absolute sign)	1M	
$=\sqrt{2}$	1A	
C_3: $(x-2)^2+(y+1)^2=2$ .... or $x^2+y^2-4x+2y+3=0$	1A	
	4	
Alt. Solution:		
Centre of C_3: (2, −1)	1A	
C_3: $(x-2)^2+(y+1)^2=R^2$		
Sub. $x+y-3=0$ in equation of C_3		
$2x^2-12x+(20-R^2)=0$	1A	
$(-12)^2-4(2)(20-R^2)=0$	1M	
$R^2=2$		
$(x-2)^2+(y+1)^2=2$	1A	
(c) Centre of C_1 = (2, −1), centre of C_2 = (5, 2)		
$\frac{\sqrt{(x-2)^2+(y+1)^2}}{\sqrt{(x-5)^2+(y-2)^2}}=\frac{1}{k}$	1M+1A	
$(k^2-1)x^2+(k^2-1)y^2+(10-4k^2)x+(4+2k^2)y+(5k^2-29)=0$	1	
(i) When k = 2,		
$3x^2+3y^2-6x+12y-9=0$		
$x^2+y^2-2x+4y-3=0$	1A	
a circle (with centre at (1, −2) and radius $2\sqrt{2}$)	1A	
(ii) The locus represents a straight line,		
$k^2-1=0$	1M	
$k=1$	1A	
	7	

SOLUTIONS	MARKS	REMARKS
11.(a)(i) Putting $x = 2\sin\theta$, $dx = 2\cos\theta\, d\theta$	1A	
When $x = 1$, $\theta = \frac{\pi}{6}$; When $x = 2$, $\theta = \frac{\pi}{2}$	1A	
$\int_1^2 \sqrt{4 - x^2}\, dx$		
$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 4\cos^2\theta\, d\theta$	1A	For integrand
$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 4 \cdot \frac{1}{2}(1 + \cos 2\theta)\, d\theta$	1M	For $\cos^2\theta = \frac{1 + \cos 2\theta}{2}$
$= 2[\theta + \frac{1}{2}\sin 2\theta]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$	1A	
$= 2[\frac{\pi}{3} - \frac{\sqrt{3}}{4}]$ or 1.23	1A	(1.228)
(ii) $3 + 2x - x^2$		
$= 2^2 - (x - 1)^2$	1A	
$\int_0^1 \sqrt{3 + 2x - x^2}\, dx$		
$= \int_0^1 \sqrt{4 - (x - 1)^2}\, dx$		
Putting $x - 1 = 2\sin\theta$, $dx = 2\cos\theta\, d\theta$	1A	
When $x = 0$, $\theta = -\frac{\pi}{6}$; When $x = 1$, $\theta = 0$	1A	
$= \int_{-\frac{\pi}{6}}^{0} 4\cos^2\theta\, d\theta$	1A	
$= 2[\theta + \frac{1}{2}\sin 2\theta]_{-\frac{\pi}{6}}^{0}$		
$= \frac{\pi}{3} + \frac{\sqrt{3}}{2}$ or 1.91	1A	(1.913)
	11	

SOLUTIONS	MARKS	REMARKS
11.(b)(i) $y = -\sqrt{4-(x-1)^2} + \sqrt{3}$	1A	Alt. Solution (2):
(ii) Required area = $A_1 + A_2$	1M	

$A_1 = \int_0^1 [\sqrt{3x} - (-\sqrt{4-(x-1)^2} + \sqrt{3})]\,dx$ — 1M+1A

$= [\frac{\sqrt{3}\cdot 2x^{3/2}}{3}]_0^1 + (\frac{\pi}{3} + \frac{\sqrt{3}}{2}) - \sqrt{3}$

$= (\frac{\sqrt{3}}{6} + \frac{\pi}{3})$ or 1.336 .... (Accept 1.34) — 1A

$A_2 = \int_1^2 [\sqrt{4-x^2} - (-\sqrt{4-(x-1)^2} + \sqrt{3})]\,dx$ — 1M+1A

$= 2[\frac{\pi}{3} - \frac{\sqrt{3}}{4}] + \int_1^2 \sqrt{4-(x-1)^2}\,dx - \sqrt{3}$

$= \frac{2\pi}{3} - \frac{3\sqrt{3}}{2} + \int_0^1 \sqrt{4-(x-1)^2}\,dx$

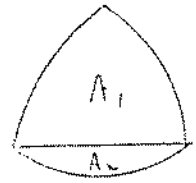
$= \frac{2\pi}{3} - \frac{3\sqrt{3}}{2} + (\frac{\pi}{3} + \frac{\sqrt{3}}{2})$

$= (\pi - \sqrt{3})$ or 1.410 .... — 1A — Accept 1.41

Area $= (\frac{4\pi}{3} - \frac{5\sqrt{3}}{6})$ or 2.75 .... — 1A

9

Alt. Solution (2):



Area = $A_1 + A_2$ — 1M

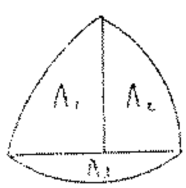
$A_1 = \int_0^{\sqrt{3}} (\sqrt{4-y^2} - \frac{y^2}{3})\,dy$ — 1M+1A

$= \frac{2\pi}{3} + \frac{\sqrt{3}}{6}$ 1A

(or 2.383)

A_2 same as A_3 in Alt. solution (1).

Alt. Solution (1)



Area $= A_1 + A_2 + A_3$ — 1M

$A_1 = \int_0^1 \sqrt{3x}\,dx$ — 1A

$= \frac{2\sqrt{3}}{3}$ or 1.155 — 1A — Accept 1.15

$A_2 = \int_1^2 \sqrt{4-x^2}\,dx$ — 1A

$= (\frac{2\pi}{3} - \frac{\sqrt{3}}{2})$ or 1.228 — Accept 1.23

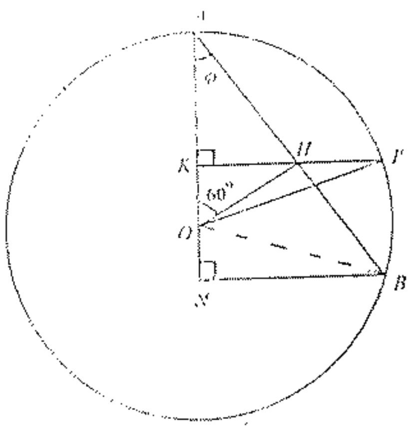
$A_3 = \left|\int_0^2 (-\sqrt{4-(x-1)^2} + \sqrt{3})\,dx\right|$

or $2\left|\int_0^1 (-\sqrt{4-(x-1)^2} + \sqrt{3})\,dx\right|$ .... — 1A — Accept no absolute sign

$= \left|-2(\frac{\pi}{3} + \frac{\sqrt{3}}{2}) + 2\sqrt{3}\right|$

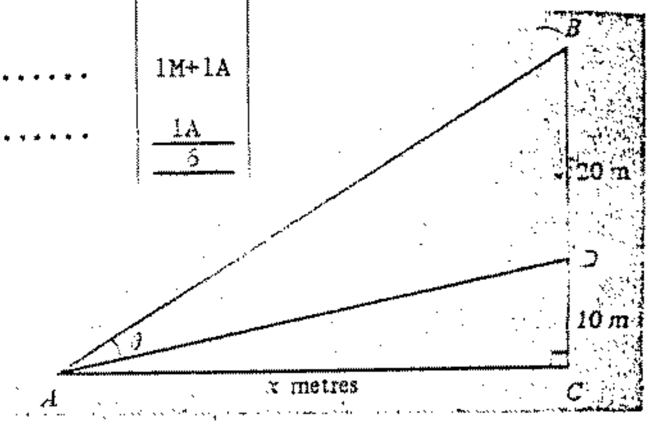
$= (\frac{2}{3}\pi - \sqrt{3})$ or 0.3623 .... — 1A — Accept 0.362

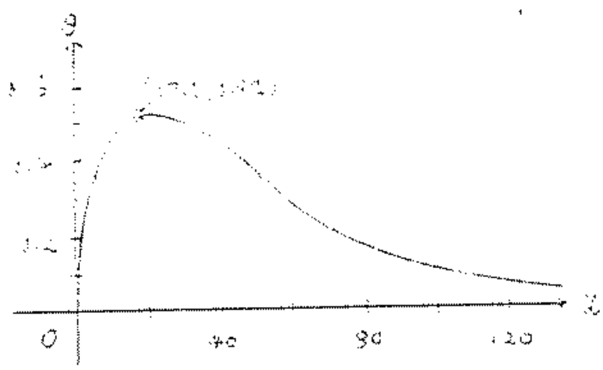
$A = (\frac{4\pi}{3} - \frac{5\sqrt{3}}{6})$ or 2.75 .... — 1A

SOLUTIONS	MARKS	REMARKS
12.(a)(i) $\sin 108^\circ = \sin(3 \times 36^\circ)$		$\sin 108^\circ$
$= 3\sin 36^\circ - 4\sin^3 36^\circ$	1A	$= \sin 72^\circ$ $= 2\sin 36^\circ \cos 36^\circ$ 1A
$\sin 72^\circ = 2\sin 36^\circ \cos 36^\circ$	1A	$= 2\sin 36^\circ \sqrt{1-\sin^2 36^\circ}$ 1A
$3\sin 36^\circ - 4\sin^3 36^\circ = 2\sin 36^\circ \cos 36^\circ$		
$3 - 4\sin^2 36^\circ = 2\cos 36^\circ$		
$3 - 4(1 - \cos^2 36^\circ) = 2\cos 36^\circ$		
$4\cos^2 36^\circ - 2\cos 36^\circ - 1 = 0$	1A	
$\cos 36^\circ = \frac{1 \pm \sqrt{5}}{4}$	1A	
$\cos 36^\circ > 0$		
$\therefore \cos 36^\circ = \frac{1+\sqrt{5}}{4}$	1	
(ii) $\cos 72^\circ = 2\cos^2 36^\circ - 1$	1A	
$= 2\left(\frac{1+\sqrt{5}}{4}\right)^2 - 1$		
$= \frac{\sqrt{5}-1}{4}$	1A	
	7	
(b)(i) 		
In $\triangle AOH$,		
$\frac{OH}{\sin \emptyset} = \frac{1}{\sin(180^\circ - 60^\circ - \emptyset)}$	1M	
$OH = \frac{\sin \emptyset}{\sin(60^\circ + \emptyset)}$ or $\frac{\sin \emptyset}{\sin(120^\circ - \emptyset)}$	1A	
$= \frac{\sin \emptyset}{\sin 60^\circ \cos \emptyset + \cos 60^\circ \sin \emptyset}$		
or $\frac{\sin \emptyset}{\sin 120^\circ \cos \emptyset - \cos 120^\circ \sin \emptyset}$		
$= \frac{\tan \emptyset}{\frac{\sqrt{3}}{2} + \frac{1}{2}\tan \emptyset}$	2A	
$= \frac{2\tan \emptyset}{\sqrt{3} + \tan \emptyset}$		

SOLUTIONS	MARKS	REMARKS
12.(b)(i) $\cos\angle POK$		
$= \dfrac{OK}{OP}$	1A	
$= OK$		
$= OH\cos 60^\circ$	1M	
$= \dfrac{2\tan\emptyset}{\sqrt{3}+\tan\emptyset}\cdot\dfrac{1}{2}$		
$= \dfrac{\tan\emptyset}{\sqrt{3}+\tan\emptyset}$	1A	
(ii)(1) $ON = \dfrac{1}{4}$		
$BN = \sqrt{OB^2 - ON^2}$	1	
$= \dfrac{\sqrt{15}}{4}$	1A	
$\tan\emptyset = \dfrac{BN}{AN}$		
$= \dfrac{\frac{\sqrt{15}}{4}}{\frac{5}{4}}$		
$= \dfrac{\sqrt{15}}{5}$	1A	
(2) $\cos\angle POK = \dfrac{\frac{\sqrt{15}}{5}}{\sqrt{3}+\frac{\sqrt{15}}{5}}$	1M	For substitution
$= \dfrac{(\sqrt{5})(\sqrt{3})}{5\sqrt{3}+(\sqrt{5})(\sqrt{3})}$		
$= \dfrac{\sqrt{5}}{5+\sqrt{5}}$		
$= \dfrac{1}{1+\sqrt{5}}$		
$= \dfrac{\sqrt{5}-1}{4}$	1A	
Compared with (a)(ii)		
$\angle POK = 72^\circ$	1A	Do not award this mark if a candidate had not completed (a)(ii).
	13	

SOLUTIONS	MARKS	REMARKS
11. (c) If $x = 50$, $\frac{d\theta}{dx} = \frac{20(300 - 50^2)}{30^4 + 1000(50)^2 + 90\ 000}$	1M	
$= \frac{-44\ 000}{8\ 840\ 000}$		
$= -0.0050$ (correct to 4 d.p.)	1A	Follow through for −0.005
$1° = 0.0175$ radians		
Since $\Delta x \doteq \Delta\theta \frac{1}{\frac{d\theta}{dx}}$ (or $\Delta\theta \doteq \frac{d\theta}{dx}\Delta x$), ...	1M	
at $x = 50$,		
$\Delta x \doteq \frac{-0.0175}{-0.005}$	1M+1A	
$= 3.5$ (correct to the nearest $\frac{1}{10}$ m)	1A	
	6	



SOLUTIONS	MARKS	REMARKS
(d) At $x = 0$, $\theta = 0$.	1A	May be indicated in diag.
At $x = \sqrt{300}$,		
$\tan\theta = 0.577$		
$\theta = 0.524$ (or 30°)	1A	
As $x \to \infty$, $\theta \to 0$	1A	
	2	1 shape, 1 tail
	5	