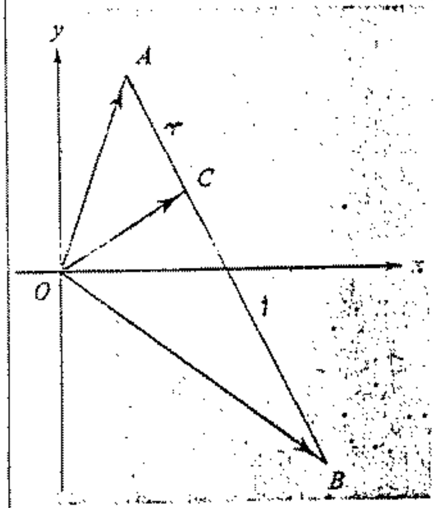
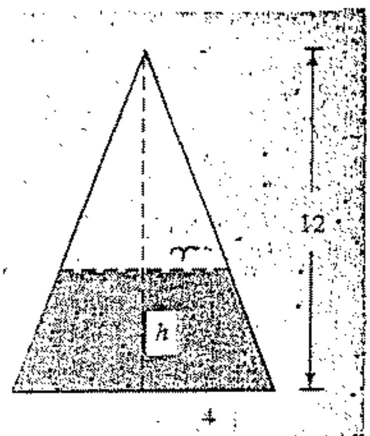
1985 PAPER I

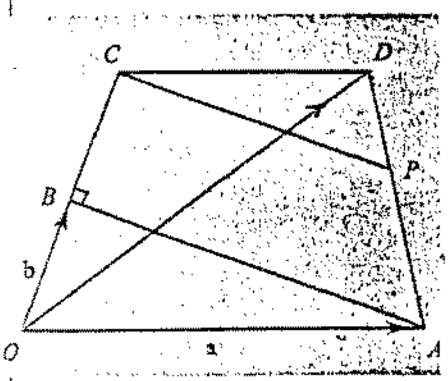
SOLUTIONS	MARKS	REMARKS
$f'(x) = \sqrt{1-x^2} + x \cdot \frac{1}{2\sqrt{1-x^2}}(-2x)$	1+1+1A	1 for product rule, 1 for chain rule
$= \frac{(1-2x^2)}{\sqrt{1-x^2}}$		
$\therefore f'(\frac{1}{2}) = \frac{1-\frac{2}{4}}{\sqrt{1-\frac{1}{4}}}$	1M	
$= \frac{1}{\sqrt{3}}$ (0.577)	1A	
	5	
$1 - i = \sqrt{2}(\cos -\frac{\pi}{4} + i \sin -\frac{\pi}{4})$	1A+1A	1 for mod., 1 for argument
(or $\sqrt{2}\text{cis}\,\frac{7\pi}{4}$, $\sqrt{2}\text{cis}\,315°$, etc.)		
$(1-i)^{\frac{1}{3}} = \sqrt[6]{2}(\cos \frac{-\frac{\pi}{4}+2k\pi}{3} + i \sin \frac{-\frac{\pi}{4}+2k\pi}{3})$	1M+1M	1M for general form 1M for De Moivre's Theorem
$= \sqrt[6]{2}(\cos \frac{(8k-1)\pi}{12} + i \sin \frac{(8k-1)\pi}{12})$,		
$k = 0, 1, 2$	2A	
$= \sqrt[6]{2}(\cos -\frac{\pi}{12} + i \sin -\frac{\pi}{12})$ or		
$\sqrt[6]{2}(\cos \frac{7\pi}{12} + i \sin \frac{7\pi}{12})$ or		
$\sqrt[6]{2}(\cos -\frac{3\pi}{4} + i \sin -\frac{3\pi}{4})$		
	6	
[Note other variants in arguments, e.g. $\theta = 105°, 225°, 345°$; $\theta = \frac{7\pi}{12}, \frac{5\pi}{4}, \frac{23\pi}{12}$]		
$x^2 - ax - 4 \leq 0$		
$\Leftrightarrow (x - \frac{a+\sqrt{a^2+16}}{2})(x - \frac{a-\sqrt{a^2+16}}{2}) \leq 0$		
$\therefore \frac{a-\sqrt{a^2+16}}{2} \leq x \leq \frac{a+\sqrt{a^2+16}}{2}$	1M+1A	for $\alpha \leq x \leq \beta$
$\frac{a+\sqrt{a^2+16}}{2} = 4$	1M	**Alt. Solution :**
$\Rightarrow \sqrt{a^2+16} = 8 - a$		Let $x^2 - ax - 4 = (x-\alpha)(x-\beta) \leq 0$, where $\alpha \leq \beta$
$\Rightarrow a^2 + 16 = 64 - 16a + a^2$		$\alpha \leq x \leq \beta$ ——— 1M Since $\alpha\beta = -4$ ——— 1A Sub. $\beta = 4$ ——— 1M $\alpha = -1$ ——— 1A
$\Rightarrow a = 3$	1A	
$\therefore$ the least possible value of x is $\frac{3-\sqrt{9+16}}{2} = -1$	1M+1A	
	6	

SOLUTIONS	MARKS	REMARKS
(a) $\overrightarrow{OC} = \frac{1}{1+r}(\overrightarrow{OA} + r\overrightarrow{OB})$	1A	
$= \frac{1}{1+r}[(\vec{i} + 3\vec{j}) + r(4\vec{i} - 3\vec{j})]$		
$= \frac{1}{1+r}[(1 + 4r)\vec{i} + (3 - 3r)\vec{j}]$	1A	
(b) $\overrightarrow{AB} = (4\vec{i} - 3\vec{j}) - (\vec{i} + 3\vec{j})$		
$= 3\vec{i} - 6\vec{j}$	1A	
$OC \perp AB \Rightarrow \overrightarrow{AB} \cdot \overrightarrow{OC} = 0$	1M	
$\Rightarrow \frac{1}{1+r}[(1 + 4r)3 - (3 - 3r)6] = 0$		
$\Rightarrow \frac{1}{1+r}(30r - 15) = 0$		
$\Rightarrow r = \frac{1}{2}$	1A	
$\therefore \overrightarrow{OC} = \frac{2}{3}[(1 + 2)\vec{i} + (3 - \frac{3}{2})\vec{j}] = 2\vec{i} + \vec{j}$		
i.e. $C = (2, 1)$	1A 6	
Let the radius of the water surface be r centimetres.		
By similar triangles	1M	Attempt to use similar triangles
$\frac{r}{12 - h} = \frac{4}{12}$		
$r = \frac{1}{3}(12 - h)$	1A	
Volume of water $V = \frac{1}{3}(\pi)(4^2)(12) - \frac{1}{3}\pi r^2(12 - h)$	1M	
$= \frac{\pi}{3}(192 - \frac{(12 - h)^3}{9})$	1A	
$= \frac{\pi}{27}(432h - 36h^2 + h^3)$		
$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$	1	
$= \frac{\pi}{9}(12 - h)^2 \cdot \frac{dh}{dt}$ - - - - - - - - - - - -	1A	
$\frac{\pi}{9}(12 - h)^2 \cdot \frac{dh}{dt} = \pi$	1M	
$\therefore$ at $h = 6$,		
$\frac{dh}{dt} = \frac{9}{(12 - 6)^2}$		
$= \frac{1}{4}$		
$\therefore$ the water level is rising at $\frac{1}{4}$ cm/s	1A 8	Accept $\frac{dh}{dt} = \frac{1}{4}$ cm/s

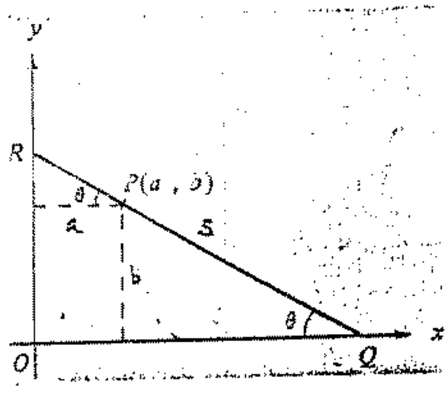
SOLUTIONS	MARKS	REMARKS
6. $\log_{10}\lvert x^2 + 2px\rvert = 0$ iff $\lvert x^2 + 2px\rvert = 1$	2A	'iff' optional
iff $x^2 + 2px = 1$ or $x^2 + 2px = -1$	1A+1A	-1A for 'and', accept ','
(i) Let $x^2 + 2px - 1 = 0$		
Discriminant $= 4p^2 + 4$		
> 0 for all real p	1A	
$\therefore$ the given equation has no double root.	1A	
(ii) Let $x^2 + 2px + 1 = 0$		
Discriminant $= 4p^2 - 4 = 0$	1A	
iff $p = \pm 1$	1A	
	8	
The given equation has a double root if $p = \pm 1$		

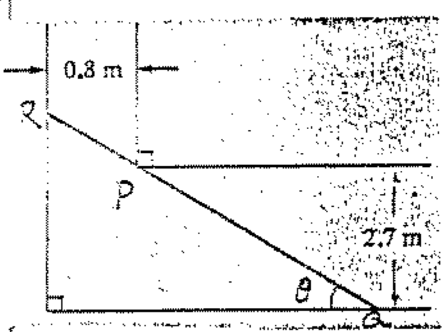
SOLUTIONS	MARKS	REMARKS
7. (a) (i) $ax^2 + bx + c$		
$= a(x^2 + \frac{b}{a}x + \frac{c}{a})$	1A	
$= a[(x + \frac{b}{2a})^2 - \frac{b^2 - 4ac}{4a^2}]$	1M+1A	1 M completing square
$= a(x + \frac{b - \sqrt{b^2-4ac}}{2a})(x + \frac{b + \sqrt{b^2-4ac}}{2a})$	1A	
(ii) The roots of the given equation are		
$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$	1A	
Since a, b are real, if $b^2 - 4ac < 0$, the roots are imaginary.	1A	Must mention a, b real.
(iii) If $a = 3i$, $b = -2$, $c = 5i$,		
$b^2 - 4ac = 4 - 4 \times 3 \times 5i^2$		
$= 64$	1A	
> 0		
But the roots $= \frac{2 \pm \sqrt{64}}{6i}$	1A+1A	
$= \frac{5}{3i}$ or $\frac{-1}{i}$ (or $\frac{-5i}{3}$, i),		
which are imaginary.	9	
(b) The discriminant $= 4\lambda^2 - 4(2\lambda^2 - 2\lambda\mu + \mu^2)$	1A	
$= -4(\lambda^2 - 2\lambda\mu + \mu^2)$		
$= -4(\lambda - \mu)^2$	1A	
Since the roots are real,		
$-4(\lambda - \mu)^2 \geqslant 0$	1M	
$\therefore \lambda = \mu$ (Since λ and μ are real)	1A	
	4	
(c) Since (1) and (2) have imaginary roots		
$a^2 < 4b$ and $c^2 < 4d$	1A	
The discriminant of (3) $= (a + c)^2 - 3(b + d)$	1A	
$< (a + c)^2 - 2(a^2 + c^2)$	1M+1A	1 M using $a^2 < 4b$ or $c^2 < 4d$
$= -(a - c)^2$	1A	
$\leqslant 0$	1A	
$\therefore$ the discriminant < 0. As the coefficients of (3) are real, it has imaginary roots.	1M	Must mention coeff. real
	7	

SOLUTIONS	MARKS	REMARKS
8. (a) (i) $\vec{OD} = \vec{OC} + \vec{CD}$		
$= 2\vec{b} + k\vec{a}$	1A	
$\vec{DA} = \vec{OA} - \vec{OD}$		
$= \vec{a} - (2\vec{b} + k\vec{a})$	1M	Sub. in correct expression
$= (1 - k)\vec{a} - 2\vec{b}$	1A	
(ii) $\vec{BA} = \vec{a} - \vec{b}$	1A	
$\vec{CP} = \vec{CD} + \vec{DP}$		
$= k\vec{a} + \lambda[(1 - k)\vec{a} - 2\vec{b}]$	1M	Same as above
$= (k + \lambda(1 - k))\vec{a} - 2\lambda\vec{b}$	1A	
Since CP // BA, $\dfrac{k + \lambda(1 - k)}{1} = \dfrac{-2\lambda}{-1}$	2M }	**Alt. Solution** : $t\,\vec{BA} = \vec{CP}$ .... 1M $t(\vec{a}-\vec{b}) = (k+\lambda(1-k))\vec{a} - 2\lambda\vec{b}$ $k + \lambda(1-k) = t$ $-2\lambda = -t$ 1M
$k = \lambda(1+k)$		
$\lambda = \dfrac{k}{1 + k}$	1A 9	
(b) (i) $\vec{a}\cdot\vec{b} = \|\vec{a}\|\|\vec{b}\|\cos AOB$	1A	
$= OB \times OA\cos AOB$ $= OB^2$	1A	Should not be omitted
(ii) $\vec{OD}\cdot\vec{DA} = (2\vec{b} + k\vec{a})\cdot((1 - k)\vec{a} - 2\vec{b})$		
$= k(1-k)\vec{a}\cdot\vec{a} - 4\vec{b}\cdot\vec{b} + [2(1-k) - 2k]\vec{a}\cdot\vec{b}$	1A	
$= k(1-k)\vec{a}\cdot\vec{a} - 4\vec{b}\cdot\vec{b} + (2-4k)OB^2$	1M	
$= 16k(1-k)OB^2 - 4OB^2 + (2-4k)OB^2$	1M+1M	
$= (-16k^2 + 12k - 2)OB^2$	1A	
If OD ⊥ AD, $-16k^2 + 12k - 2 = 0$	1M	
$(4k - 1)(2k - 1) = 0$		
$k = \frac{1}{4}$ or $\frac{1}{2}$	1A	
$\lambda = \dfrac{k}{1 + k}$		
$= \frac{1}{5}$ or $\frac{1}{3}$	1A+1A 11	



SOLUTIONS	MARKS	REMARKS
9. (a) (i) $RP = a\sec\theta \quad (=\frac{a}{\cos\theta})$	1A	
$PQ = b\operatorname{cosec}\theta \quad (=\frac{b}{\sin\theta})$	1A	
$\therefore s = RP + PQ$		
$= a\sec\theta + b\operatorname{cosec}\theta \quad (0 < \theta < \frac{\pi}{2})$	1A	
$(=\frac{a}{\cos\theta}+\frac{b}{\sin\theta}$ or $\sqrt{(\frac{a\tan\theta+b}{\tan\theta})^2+(a\tan\theta+b)^2}$		
(ii) $\frac{ds}{d\theta} = a\sec\theta\tan\theta - b\operatorname{cosec}\theta\cot\theta$	1A+1A	
$\frac{ds}{d\theta} = 0 \Rightarrow a\sec\theta\tan\theta - b\operatorname{cosec}\theta\cot\theta = 0$	1M	
$\Rightarrow \frac{a\tan\theta}{\cos\theta} = \frac{b}{\sin\theta\tan\theta}$		
$\Rightarrow \tan^3\theta = \frac{b}{a}$		
$\Rightarrow \tan\theta = \sqrt[3]{\frac{b}{a}}$	1A	
$\frac{d^2s}{d\theta^2} = a(\sec\theta\tan^2\theta+\sec^3\theta)-b(-\operatorname{cosec}\theta\cot^2\theta-\operatorname{cosec}^3\theta)$	2A	Alt. Solution :
$= a\sec\theta(\tan^2\theta+\sec^2\theta)+b\operatorname{cosec}\theta(\cot^2\theta+\operatorname{cosec}^2\theta)$		$\frac{ds}{d\theta} = \frac{a\sin\theta}{\cos^2\theta} - \frac{b\cos\theta}{\sin^2\theta}$
If $\tan\theta = \sqrt[3]{\frac{b}{a}}$, $0° < \theta < 90°$, $\sec\theta, \operatorname{cosec}\theta > 0$,		$= \frac{a\sin^3\theta - b\cos^3\theta}{\sin^2\theta\cos^2\theta}$
$\therefore \frac{d^2s}{d\theta^2} > 0$ .... (Knowledge of Test)	1M	$= \frac{\cos^3\theta(a\tan^3\theta - b)}{\sin^2\theta\cos^2\theta}$ 2A
$\therefore$ s will be least when $\tan\theta = \sqrt[3]{\frac{b}{a}}$.	1A	If $\theta < \tan^{-1}\sqrt[3]{\frac{b}{a}}$ slightly, $\frac{ds}{d\theta} < 0$.
	11	If $\theta > \tan^{-1}\sqrt[3]{\frac{b}{a}}$ slightly, $\frac{ds}{d\theta} > 0$ .... 1M (Knowledge of test)
		$\therefore$ s is least when $\tan\theta = \sqrt[3]{\frac{b}{a}}$ 1A



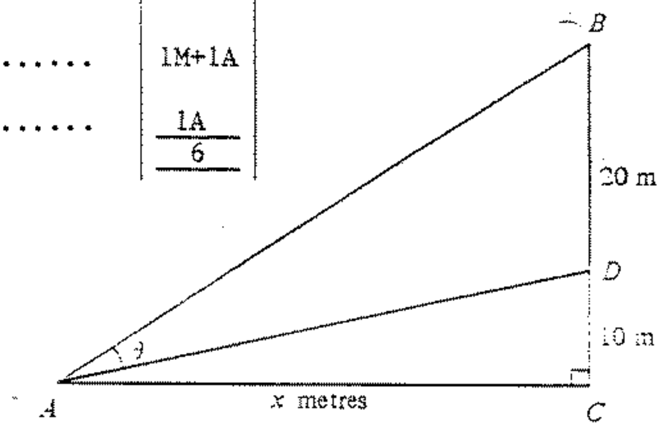
SOLUTIONS	MARKS	REMARKS
9. (b) (i) When being moved horizontally, the longest pipe will just touch the outside walls of both corridors while it is negotiating the corner P. The length of the pipe must not be longer than the shortest distance between Q and R. From (a), this occurs when		
$\tan\theta = \sqrt[3]{\frac{2.7}{0.8}}$	1M+1A	1M for attempting to use (a)
$= \frac{3}{2}$ $(\theta = 56.3°)$	1A	
$\therefore$ the length of the longest pipe that can be carried round the corner horizontally is		
$0.8 \sec\theta + 2.7 \operatorname{cosec}\theta$ $(\theta = 56.3°)$	1M+1M	1M for sub. a, b, 1M for sub. θ .
$= 0.8 \times \frac{\sqrt{13}}{2} + 2.7 \times \frac{\sqrt{13}}{3}$		
$= 4.69$ m (4.687)	1A	
(ii) If the height of the ceiling is 3 m, the length of the longest pipe that can be carried round the corner is		
$\sqrt{3^2 + 4.687^2}$	2M	
$= 5.57$ m	1A	
	9	

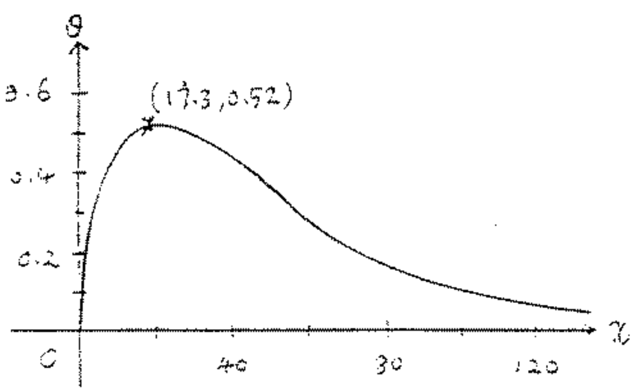
SOLUTIONS	MARKS	REMARKS
10. (a) $\frac{1}{2}(w + \overline{w}) = \frac{1}{2}[(p + qi) + (p - qi)]$		
$= p$	1A	
$\frac{1}{2i}(w - \overline{w}) = \frac{1}{2i}[(p + qi) - (p - qi)]$		
$= q$	1A	
$p = \frac{1}{2}(w + \overline{w})$		
$= \frac{1}{2}[\frac{z-1}{z+1} + (\overline{\frac{z-1}{z+1}})]$	1M	
$= \frac{(z-1)(\overline{z}+1) + (\overline{z}-1)(z+1)}{2(z+1)(\overline{z}+1)}$	1A	
$= \frac{z\overline{z} - \overline{z} + z - 1 + \overline{z}z - z + \overline{z} - 1}{2(z\overline{z} + z + \overline{z} + 1)}$		Show working
$= \frac{z\overline{z} - 1}{z\overline{z} + z + \overline{z} + 1}$	1A	
$q = \frac{1}{2i}(w - \overline{w})$		
$= \frac{1}{2i}[\frac{z-1}{z+1} - (\overline{\frac{z-1}{z+1}})]$	1M	
$= \frac{1}{2i}\ \frac{(z-1)(\overline{z}+1) - (\overline{z}-1)(z+1)}{(z+1)(\overline{z}+1)}$		
$= \frac{1}{2i}\ \frac{z\overline{z} + z - \overline{z} - 1 - \overline{z}z + z - \overline{z} + 1}{z\overline{z} + z + \overline{z} + 1}$		Show working
$= \frac{i(\overline{z} - z)}{z\overline{z} + z + \overline{z} + 1}$	1A 7	3+2 marks for p, q
(b) (i) w is real $\Leftrightarrow$ q = 0.	1	Optional
$\therefore z - \overline{z} = 0$	1A	
The locus of z is the real axis, excluding z = -1.	1A+1A	
(ii) w is purely imaginary $\Leftrightarrow$ p = 0, q ≠ 0	1	Optional
$\therefore\ z\overline{z} - 1 = 0$ i.e. $x^2 + y^2 = 1$	1A	
The locus of z is the circle, centre O, radius 1, excluding the points z = ±1 .	1A+1A	
(iii) $\|w\|^2 = w\overline{w}$		
$= \frac{z-1}{z+1} \times (\overline{\frac{z-1}{z+1}})$		
$= \frac{(z-1)(\overline{z}-1)}{(z+1)(\overline{z}+1)}$	1A	
$\|w\| = 1$		
$\Rightarrow 1 = \frac{z\overline{z} - z - \overline{z} + 1}{z\overline{z} + z + \overline{z} + 1}$	1M	
$z\overline{z} + z + \overline{z} + 1 = z\overline{z} - z - \overline{z} + 1$		
$\therefore z + \overline{z} = 0$	1A	
The locus of z is the imaginary axis.	2A 13	

SOLUTIONS	MARKS	REMARKS
10. (a) Alt. Solution :		
Let $z = x + iy$		
$w = \frac{(x-1)+iy}{(x+1)+iy}$	1M	
$= \frac{x^2+y^2-1}{(x+1)^2+y^2} + \frac{2yi}{(x+1)^2+y^2}$	1A	
$\overline{w} = \frac{x^2+y^2-1}{(x+1)^2+y^2} - \frac{2yi}{(x+1)^2+y^2}$		
$\therefore \frac{1}{2}(w+\overline{w}) = \frac{x^2+y^2-1}{(x+1)^2+y^2}$		
$= p$	1A	
$\frac{1}{2i}(w-\overline{w}) = \frac{1}{2i}\left(\frac{4yi}{(x+1)^2+y^2}\right)$		
$= q$	1A	
$\frac{z\overline{z}-1}{z\overline{z}+z+\overline{z}+1} = \frac{x^2+y^2-1}{x^2+y^2+(x+iy)+(x-iy)+1}$	1A	
$= \frac{x^2+y^2-1}{x^2+y^2+2x+1}$		
$= p$	1A	
$\frac{i(\overline{z}-z)}{z\overline{z}+z+\overline{z}+1} = \frac{i(-2iy)}{x^2+y^2+2x+1}$		
$= q$	1A	
	7	
(b) (i) w is real $\Leftrightarrow q = 0$	1	optional
$\therefore y = 0$	1A	
The locus of z is the real axis, excluding $z = -1$	1A+1A	
(ii) w is purely imaginary $\Leftrightarrow p = 0,\ q \neq 0$	1	optional
$\therefore x^2 + y^2 = 1$	1A	
The locus of z is the circle, centre 0, radius 1, excluding $z = \pm 1$.	1A+1A	
(iii) $\lvert w\rvert^2 = w\overline{w}$		
$= \frac{1}{[(x+1)^2+y^2]^2}[(x^2+y^2-1)^2+4y^2]$	1A	
$\lvert w\rvert = 1 \Leftrightarrow [(x+1)^2+y^2]^2 = (x^2+y^2-1)^2+4y^2$	1M	
$[(x+1)^2+y^2+x^2+y^2-1][2x+2] = 4y^2$		
$x[(x+1)^2+y^2] = 0$		
$\therefore x = 0$ (as $z = x+iy \neq -1$)	1A	
The locus is the imaginary axis.	2A	
	13	

SOLUTIONS	MARKS	REMARKS
11. (a) $\tan\theta = \tan(BAC - DAC)$	1	
$= \dfrac{\tan BAC - \tan DAC}{1 + \tan BAC \tan DAC}$	1M	
$= \dfrac{\frac{30}{x} - \frac{10}{x}}{1 + \frac{30}{x}\cdot\frac{10}{x}}$		Show working
$= \dfrac{20x}{x^2 + 300}$	1A / 3	
(b) Differentiating both sides w.r.t. x,		
$\dfrac{d}{dx}(\tan\theta) = \dfrac{d}{dx}\left(\dfrac{20x}{x^2 + 300}\right)$,		
$\sec^2\theta \dfrac{d\theta}{dx} = \dfrac{20(x^2 + 300) - 20x(2x)}{(x^2 + 300)^2}$	1A+1A	
But $\sec^2\theta = 1 + \tan^2\theta$		
$= \dfrac{(x^2 + 300)^2 + (20x)^2}{(x^2 + 300)^2}$	1A	
$\therefore \dfrac{d\theta}{dx} = \dfrac{(x^2 + 300)^2}{(x^2 + 300)^2 + (20x)^2} \cdot \dfrac{20(x^2 + 300) - 20x(2x)}{(x^2 + 300)^2}$		
$= \dfrac{20(300 - x^2)}{x^4 + 1000x^2 + 90\ 000}$	1A	
$\dfrac{d\theta}{dx} = 0 \Leftrightarrow x = \sqrt{300}$ ($\doteqdot 17.3$) (-ve root rejected)	1A	Accept $x = \pm\sqrt{300}$
When $x < \sqrt{300}$ slightly, $\dfrac{d\theta}{dx} > 0$.		
When $x > \sqrt{300}$ slightly, $\dfrac{d\theta}{dx} < 0$.		
$\therefore$ θ is maximum when $x = \sqrt{300}$	1A / 6	

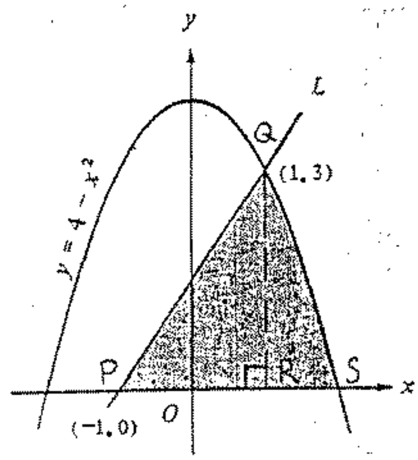
SOLUTIONS	MARKS	REMARKS
11. (c) If $x = 50$, $\frac{d\theta}{dx} = \frac{20(300 - 50^2)}{50^4 + 1000(50)^2 + 90\ 000}$	1M	
$= \frac{-44\ 000}{8\ 840\ 000}$		
$= -0.0050$ (correct to 4 d.p.)	1A	Follow through for -0.005
$1° = 0.0175$ radians		
Since $\Delta x \doteqdot \Delta\theta \frac{1}{\frac{d\theta}{dx}}$ (or $\Delta\theta \doteqdot \frac{d\theta}{dx}\Delta x$), ...	1M	
at $x = 50$,		
$\Delta x \doteqdot \frac{-0.0175}{-0.005}$	1M+1A	
$= 3.5$ (correct to the nearest $\frac{1}{10}$ m)	1A	
	6	



SOLUTIONS	MARKS	REMARKS
(d) At $x = 0$, $\theta = 0$.	1A	May be indicated in diag
At $x = \sqrt{300}$,		
$\tan\theta = 0.577$		
$\theta = 0.524$ (or 30°)	1A	
As $x \to \infty$, $\theta \to 0$	1A	
	2	1 shape, 1 tail
	5	

1985. PAPER II

SOLUTIONS	MARKS	REMARKS
1. General term $= C_r^n (ax)^{n-r} \frac{1}{x^{2r}}$.		If other terms given, disregard wrong but irrelevant terms.
The 4th term of the expansion		
$= C_3^n (ax)^{n-3} \frac{1}{x^6}$		
$= C_3^n a^{n-3} x^{n-9}$	2A	
If this term is independent of x, $n - 9 = 0$		
$n = 9$	1A	
$C_3^9 a^6 = \frac{21}{2}$	1M	
$a^6 = \frac{21}{2} \cdot \frac{3 \cdot 2}{9 \cdot 8 \cdot 7}$		
$= \frac{1}{8}$		
$a = \frac{1}{\sqrt{2}}$ (as $a > 0$) (or $\frac{\sqrt{2}}{2}$ or 0.707)	1A / 5	
2. For $n = 1$, L.S. $= \frac{1 \times (1 + 2)}{(1 + 1)^2} = \frac{3}{4}$		
R.S. $= \frac{1 + 2}{2(1 + 1)} =$ L.S.	1	
Assume that the equality holds for some positive integer k, ...	1	
then for $n = k + 1$,		
L.S. $= T_1 \times T_2 \times \ldots \times T_{k+1}$		
$= (T_1 \times T_2 \times \ldots \times T_k) \times T_{k+1}$		
$= \frac{k + 2}{2(k + 1)} \times \frac{(k + 1)(k + 3)}{(k + 2)^2}$	1A	
$= \frac{k + 3}{2(k + 2)}$	1A	
$=$ R.S.		
$\therefore$ the equality also holds for $n = k + 1$.		
By mathematical induction, the equality holds for all positive integers n.	1 / 5	Awarded only if above correct.

SOLUTIONS	MARKS	REMARKS
3. Let $u = 25 - x^2$, $du = -2x\,dx$.	1A	
When $x = 3$, $u = 16$; $x = 4$, $u = 9$	1A	
$\int_3^4 \frac{x}{\sqrt{25 - x^2}}\,dx = \int_{16}^{9} -\frac{1}{2\sqrt{u}}\,du$	1M+1A	1M for limits, 1A for $-\frac{1}{2\sqrt{u}}\,du$
$= \frac{1}{2}\left[2u^{\frac{1}{2}}\right]_9^{16}$		
$= 1$	1A	
	5	
Alt. Solution :		
Let $u = 25 - x^2$, $du = -2x\,dx$.	1A	
$\int \frac{x}{\sqrt{25 - x^2}}\,dx = -\frac{1}{2}\int \frac{1}{\sqrt{u}}\,du$	1A	
$= -\sqrt{u} + c$	1A	
$= -\sqrt{25 - x^2} + c$	1M	
$\therefore \int_3^4 \frac{x}{\sqrt{25 - x^2}}\,dx = \left[-\sqrt{25 - x^2}\right]_3^4$		
$= 1$	1A	
4. Area of the shaded part = area of PQR + area of RQS	1M	
Area of PQR $= \frac{1}{2} \times 3 \times 2 = 3$	1A	
Area of RQS $= \int_1^2 (4 - x^2)\,dx$	1A	
$= \left[4x - \frac{x^3}{3}\right]_1^2$		
$= (8 - \frac{8}{3}) - (4 - \frac{1}{3}) = 1\frac{2}{3}$ (or 1.67)	1A	
$\therefore$ total area $= 3 + 1\frac{2}{3} = 4\frac{2}{3}$ (or 4.67)	1A	
	5	
Alt. Solution :		
Equation of L : $y - 3 = \frac{3}{2}(x - 1)$		
$x = \frac{2}{3}y - 1$	1A	
Area $= \int_0^3 \left[\sqrt{4 - y} - (\frac{2}{3}y - 1)\right]\,dy$	1A+1A+1M	1A for limits 1A for integrand 1M for ' − '
$= \left[-\frac{2}{3}(4 - y)^{\frac{3}{2}} - \frac{1}{3}y^2 + y\right]_0^3$		
$= 4\frac{2}{3}$	1A	

SOLUTIONS	MARKS	REMARKS
5. The equation of the family of circles passing through A and B is		
$x^2 + y^2 - 2y + k(x - y) = 0$	1A	
$[\text{or } x - y + k(x^2 + y^2 - 2y) = 0 \,,\ (k \neq 0)]$		
The equation can be written as		
$x^2 + y^2 + kx - (2 + k)y = 0$		
Radius of the circle $= \sqrt{(\frac{k}{2})^2 + (\frac{2+k}{2})^2}$	1M	
$\sqrt{(\frac{k}{2})^2 + (\frac{2+k}{2})^2} = \sqrt{5}$	1M	
$k^2 + 2k - 8 = 0$	1A	
$\therefore k = 2$ or -4		If k is wrong, no marks below
The two circles are $x^2 + y^2 + 2x - 4y = 0$	1A	
and $x^2 + y^2 - 4x + 2y = 0$	1A	
	6	
(or $(x+1)^2 + (y-2)^2 = 5$		
$(x-1)^2 + (y+1) = 5$)		
6. Let the equation of the line through (-1, 0) be		Alt. Solution:
$y = m(x + 1)$	1A	Eqn. of the tangent at (x_1, y_1) is
Substituting in the equation of the parabola	1M	$y_1 y = 2(x_1 + x)$ 1A
$m^2(x + 1)^2 = 4x$		If the tangent passes through (-1, 0),
$m^2x^2 + (2m^2 - 4)x + m^2 = 0$	1A	$0 = 2(x_1 - 1)$ 1M
$(2m^2 - 4)^2 - 4m^4 = 0$	1M	$x_1 = 1$ 1A
For the line to be a tangent,		Putting $x=1$ in $y^2=4x$ 1M
$m^2 = 1$		$y_1 = \pm 2$
$m = \pm 1$		Equations of tangents are
$\therefore$ the equations of the tangents are		$y = \pm(x + 1)$.......1A+1A
$y = \pm(x + 1)$	1A+1A	i.e. $x - y + 1 = 0$
i.e. $x - y + 1 = 0$		and $x + y + 1 = 0$
$x + y + 1 = 0$		
	6	

SOLUTIONS	MARKS	REMARKS
7. (a) Since $A + B + C = \pi$		
$\sin C = \sin(\pi - (A + B))$	1A	
$= \sin(A + B)$		
$= \sin A \cos B + \cos A \sin B$	1A	
Since A, B, C are acute		
$\sin A = \frac{5}{13} \Rightarrow \cos A = \frac{12}{13}$; $\sin B = \frac{3}{5} \Rightarrow \cos B = \frac{4}{5}$	1A	
$\therefore \sin C = \frac{5}{13}\cdot\frac{4}{5} + \frac{12}{13}\cdot\frac{3}{5}$		
$= \frac{56}{65}$	1A	
(b) The 3 sides a, b, c satisfy		
$a : b : c = \sin A : \sin B : \sin C$	1M	for sine rule
$= \frac{5}{13} : \frac{3}{5} : \frac{56}{65}$		
$= 25 : 39 : 56$		
If the perimeter is 12 cm,		
the longest side $c = \frac{56}{120} \times 12 = 5.6$ cm	2A / 7	
8. (a) $\int_0^{\frac{\pi}{2}} \sin^3 t \cos^4 t \, dt = \int_0^{\frac{\pi}{2}} \sin t \cos^4 t (1 - \cos^2 t) \, dt$	1M	For $\sin^3 t = \sin t(1-\cos^2 t)$
$= \int_0^{\frac{\pi}{2}} \sin t \cos^4 t \, dt - \int_0^{\frac{\pi}{2}} \sin t \cos^6 t \, dt$		
$= \left[-\frac{1}{5}\cos^5 t + \frac{1}{7}\cos^7 t \right]_0^{\frac{\pi}{2}}$	1A+1A	
$= \frac{2}{35}$	1A / 4	
(b) Putting $t = \frac{\pi}{2} - u$, $dt = -du$		
When $t = 0$, $u = \frac{\pi}{2}$;		
$t = \frac{\pi}{2}$, $u = 0$	1A	
$\int_0^{\frac{\pi}{2}} \cos^3 t \sin^4 t \, dt = -\int_{\frac{\pi}{2}}^{0} \cos^3(\frac{\pi}{2} - u) \sin^4(\frac{\pi}{2} - u) \, du$	1A	
$= \int_0^{\frac{\pi}{2}} \sin^3 u \cos^4 u \, du$	1A	
$= \int_0^{\frac{\pi}{2}} \sin^3 t \cos^4 t \, dt$	1A / 4	

SOLUTIONS	MARKS	REMARKS
8. (c) Putting $t = -u$, $dt = -du$	1A	
When $t = -\frac{\pi}{2}$, $u = \frac{\pi}{2}$;		
$t = 0$, $u = 0$.	1A	
$\int_{-\frac{\pi}{2}}^{0} \cos^3 t \sin^4 t \, dt = -\int_{\frac{\pi}{2}}^{0} \cos^3(-u) \sin^4(-u) du$	1A	
$= \int_{0}^{\frac{\pi}{2}} \cos^3 u \sin^4 u \, du$		
$= \int_{0}^{\frac{\pi}{2}} \cos^3 t \sin^4 t \, dt$	1A	
$\int_{-\frac{\pi}{2}}^{0} \sin^3 t \cos^4 t \, dt = -\int_{\frac{\pi}{2}}^{0} \sin^3(-u) \cos^4(-u) \, du$	1A	
$= \int_{\frac{\pi}{2}}^{0} \sin^3 u \cos^4 u \, du$		
$= -\int_{0}^{\frac{\pi}{2}} \sin^3 t \cos^4 t \, dt$	1A 6	
(d) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{8} \sin^3 2t \, (\sin t + \cos t) \, dt$		
$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 t \cos^3 t \, (\sin t + \cos t) \, dt$	1M	for $\sin 2t = 2\sin t \cos t$
$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^3 t \sin^4 t \, dt + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 t \cos^4 t \, dt$	1A	
$= [\int_{-\frac{\pi}{2}}^{0} \cos^3 t \sin^4 t \, dt + \int_{0}^{\frac{\pi}{2}} \cos^3 t \sin^4 t \, dt]$		
$+ [\int_{-\frac{\pi}{2}}^{0} \sin^3 t \cos^4 t \, dt + \int_{0}^{\frac{\pi}{2}} \sin^3 t \cos^4 t \, dt]$	1M	
$= 2\int_{0}^{\frac{\pi}{2}} \cos^3 t \sin^4 t \, dt$		
$+ [-\int_{0}^{\frac{\pi}{2}} \sin^3 t \cos^4 t \, dt + \int_{0}^{\frac{\pi}{2}} \sin^3 t \cos^4 t \, dt]$		
$= 2\int_{0}^{\frac{\pi}{2}} \sin^3 t \cos^4 t \, dt$	2A	
$= 2 \times \frac{2}{35} = \frac{4}{35}$ (or 0.114)	1A 6	

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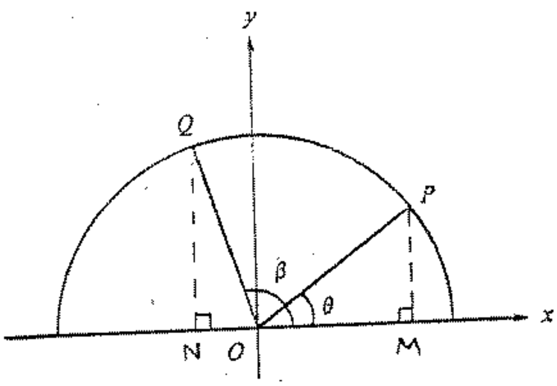
SOLUTIONS	MARKS	REMARKS
9.		
		
(a) The radius of the semi-circle is 1		
$\therefore$ $P = (\cos\theta\ ,\ \sin\theta)$		
$Q = (\cos\beta\ ,\ \sin\beta)$		
Volume generated by rotating PONM about the x-axis.		
$= \int_{\cos\beta}^{\cos\theta} \pi(1 - x^2)\,dx$	1M+1M + 1A	1M for vol. 1M for limits, accept -cos 1A for integrand
$= \pi\left[x - \frac{x^3}{3}\right]_{\cos\beta}^{\cos\theta}$		
$= \pi\left[(\cos\theta - \cos\beta) - \frac{1}{3}(\cos^3\theta - \cos^3\beta)\right]$	1A	
Volume of the two cones generated by rotating POM and QON are $\left\lvert\frac{1}{3}\pi\sin^2\theta\cos\theta\right\rvert$, $\left\lvert\frac{1}{3}\pi\sin^2\beta\cos\beta\right\rvert$	1A+1A	Accept vol. without absolute value signs
Volume V of the solid		
$= \pi\left[(\cos\theta - \cos\beta) - \frac{1}{3}(\cos^3\theta - \cos^3\beta)\right]$ $- \frac{1}{3}\pi\sin^2\theta\cos\theta + \frac{1}{3}\pi\sin^2\beta\cos\beta$	1M+2A	
$= \frac{\pi}{3}\left[3(\cos\theta-\cos\beta)-\cos^3\theta+\cos^3\beta-\sin^2\theta\cos\theta+\sin^2\beta\cos\beta\right]$		
$= \frac{\pi}{3}\left[3(\cos\theta-\cos\beta)-\cos\theta(\cos^2\theta+\sin^2\theta)+\cos\beta(\cos^2\beta+\sin^2\beta)\right]$		
$= \frac{2\pi}{3}(\cos\theta - \cos\beta)$	1A 10	

SOLUTIONS	MARKS	REMARKS

9. (b) If $\beta = 2\theta$, $V = \frac{2}{3}\pi(\cos\theta - \cos 2\theta)$, ... 1A

$\frac{dV}{d\theta} = \frac{2}{3}\pi(-\sin\theta + 2\sin 2\theta)$ 1M

Putting $\frac{dV}{d\theta} = 0$, $-\sin\theta + 2\sin 2\theta = 0$ ....

$4\sin\theta\cos\theta - \sin\theta = 0$

$\sin\theta\,(4\cos\theta - 1) = 0$

$\therefore$ $\sin\theta = 0$ or $\cos\theta = \frac{1}{4}$ ($\theta = 0$ or 1.318) 1A

Obviously the volume is minimum if $\sin\theta = 0$.

$\frac{d^2V}{d\theta^2} = \frac{2\pi}{3}(-\cos\theta + 4\cos 2\theta)$

$\frac{d^2V}{d\theta^2} < 0$ if $\cos\theta = \frac{1}{4}$ 1A

V is maximum at $\cos\theta = \frac{1}{4}$

Its max. value is

$\frac{2\pi}{3}(\frac{1}{4} - 2(\frac{1}{4})^2 + 1) = \frac{3}{4}\pi$ (or 2.36) (cu. units) 1A

5

Alt. Solution:

If $\beta = 2\theta$,

$V = \frac{2}{3}\pi(\cos\theta - \cos 2\theta)$

$= \frac{2}{3}\pi(1 + \cos\theta - 2\cos^2\theta)$ ...1A

$= \frac{4}{3}\pi(\frac{9}{16} - (\frac{1}{4} - \cos\theta)^2)$ 1M+1A

$\therefore$ V is a max. when

$\cos\theta = \frac{1}{4}$ & the max. value

is $\frac{3}{4}\pi$ (cu. units) 2A

(c) If $\beta - \theta = \frac{\pi}{3}$, $V = \frac{2\pi}{3}(\cos\theta - \cos(\frac{\pi}{3} + \theta))$

$\frac{dV}{d\theta} = \frac{2\pi}{3}(-\sin\theta + \sin(\frac{\pi}{3} + \theta))$ 1A

$= \frac{2\pi}{3}(-\sin\theta + \sin\frac{\pi}{3}\cos\theta + \cos\frac{\pi}{3}\sin\theta)$

$= \frac{2\pi}{3}(-\frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta)$

Putting $\frac{dV}{d\theta} = 0$, $\tan\theta = \sqrt{3}$ 1M

$\theta = \frac{\pi}{3}$ 1A

$\frac{d^2V}{d\theta^2} = \frac{2\pi}{3}(-\cos\theta + \cos(\frac{\pi}{3} + \theta)) < 0$ if $\theta = \frac{\pi}{3}$ 1A

$\therefore$ V is max. at $\theta = \frac{\pi}{3}$ and its value is

$\frac{2\pi}{3}(\frac{1}{2} + \frac{1}{2}) = \frac{2\pi}{3}$ (or 2.09) (cu. units) 1A

5

Alt. Solution:

If $\beta - \theta = \frac{\pi}{3}$,

$V = \frac{2\pi}{3}(\cos\theta - \cos(\frac{\pi}{3} + \theta))$

$= \frac{2}{3}\pi\,[2\sin\frac{1}{2}(\frac{\pi}{3} + 2\theta)\sin\frac{\pi}{6}]$ ..1A

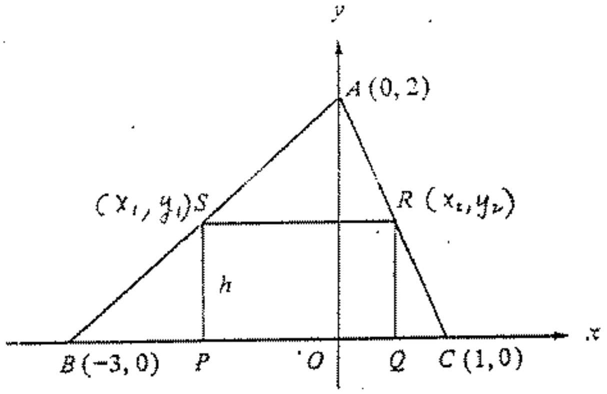
$= \frac{2\pi}{3}\sin(\frac{\pi}{6} + \theta)$ 1A

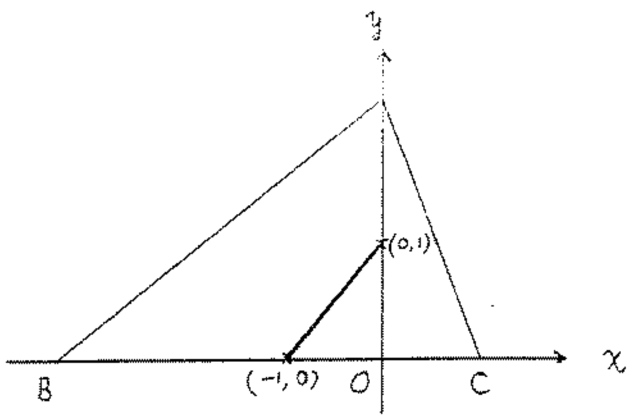
$\therefore$ V is a max. if $\theta = \frac{\pi}{3}$ 1M

[1M for $\sin(\ ) \leq 1$]

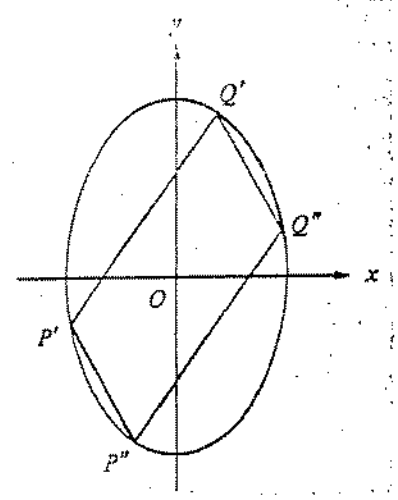
and the max. value is

$\frac{2}{3}\pi$ (cu. units) 2A

SOLUTIONS	MARKS	REMARKS
10.		
		
(a) Let $S = (x_1, y_1)$, $R = (x_2, y_2)$		**Alt. Solution :**
y_1 $(= y_2) = h$	1A	y_1 $(= y_2) = h$ 1A
By similar triangles		Equation of AB is
$\frac{-3 - x_1}{-3} = \frac{h}{2}$	1A	$y = \frac{2}{3}x + 2$ 1A
$\therefore x_1 = \frac{3h}{2} - 3$	1A	Substituting $y = h$
$\frac{1 - x_2}{1} = \frac{h}{2}$	1A	$x_1 = \frac{3}{2}(h - 2)$ 1A
$\therefore x_2 = 1 - \frac{h}{2}$	1A	Equation of AC is $y = 2 - 2x$ 1A
		Substituting $y = h$
	5	$x_2 = 1 - \frac{h}{2}$ 1A
(b) If PQRS is a square $x_2 - x_1 = h$	1M	
$4 - 2h = h$		
$h = \frac{4}{3}$	1A	
$\therefore A_1 = h^2$ $(= \frac{16}{9})$	1A	
Area of rectangle $= h(4 - 2h)$	1A	
$= -2(h^2 - 2h + 1) + 2$		
$= 2 - 2(h - 1)^2$	1M	or $\frac{dA}{dh} = 0$
$\therefore A_2 = 2$	1A	
$A_3 = \frac{1}{2} \times 2 \times (1 - (-3)) = 4$	1A	
$\therefore A_1 : A_2 : A_3 = \frac{16}{9} : 2 : 4$ (or 8 : 9 : 18)	1A	
	8	

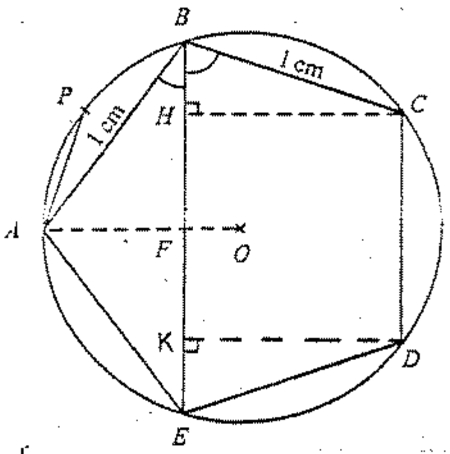
SOLUTIONS	MARKS	REMARKS
10. (c) The coordinates of the centre M(x, y) of PQRS are given by		
$x = \frac{x_1 + x_3}{2}$		
$= \frac{1}{2}(h - 2)$	1A	
$y = \frac{h}{2}$	1A	
Eliminating h, ...	1M	Attempt to eliminate
$x - y = \frac{1}{2}(h - 2) - \frac{h}{2}$		
$= -1$	1A	
Since $0 \leqslant h \leqslant 2$ (or $0 < h < 2$), the locus of M is the part of the straight line $x - y = -1$ lying between (-1, 0) and (0, 1) (end-points included/excluded)		
	3A	Line segment with end-point on axes ....2 End points correct ......1 (only awarded if equation correct)
Locus of M	7	
11. (a) (i) $PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$		
$= \sqrt{(x_1 - x_2)^2 + [(2x_1 + c) - (2x_2 + c)]^2}$	1M+1A	1M for sub. y
$= \sqrt{5}\,\lvert x_1 - x_2 \rvert$	1A	
(ii) Putting $y = 2x + c$, $x^2 + \frac{(2x + c)^2}{16} = 1$	1M	
$16x^2 + (4x^2 + 4cx + c^2) = 16$		
$20x^2 + 4cx + (c^2 - 16) = 0$ ....(*) ...	1A	
Since (x_1, y_1) (x_2, y_2) satisfy the equations		
$y = 2x + c$ and $x^2 + \frac{y^2}{16} = 1$,		
x_1, x_2 are the roots of (*)		

SOLUTIONS	MARKS	REMARKS
11. (a) (iii) If $PQ = 2\sqrt{2}$, since x_1, x_2 are roots of (*),		
$\sqrt{5}\,\lvert x_1 - x_2\rvert = \sqrt{5}\sqrt{(x_1 + x_2)^2 - 4x_1x_2}$	1A	
$= \sqrt{5}\sqrt{\left(\frac{-4c}{20}\right)^2 - \frac{4(c^2 - 16)}{20}}$	1M+1M	Sub. $x_1 + x_2 = \frac{-4c}{20}$, $x_1x_2 = \frac{c^2 - 16}{20}$
$= \sqrt{\frac{80 - 4c^2}{5}}$	1A	
$= 2\sqrt{2}$	1M	
$\Longrightarrow$ $80 - 4c^2 = 40$ $c^2 = 10$		
$c = \pm\sqrt{10}$	1A 11	
(b) Let the equations of P'Q' and P"Q" be $y = 2x + \sqrt{10}$ and $y = 2x - \sqrt{10}$ respectively.		
(i) $(0, \sqrt{10})$ is a point on P'Q'.	1A	
Distance between P'Q' and P"Q" is $\frac{2 \times 0 - \sqrt{10} - \sqrt{10}}{\pm\sqrt{2^2 + 1^2}}$	1M	
$= 2\sqrt{2}$	1A	
Area of parallelogram $= 2\sqrt{2} \times 2\sqrt{2}$	1M	
$= 8$ (sq. units)	1A	
(ii) If $P' = (x_1, y_1)$, $Q' = (x_2, y_2)$ by symmetry $P'' = (-x_2, -y_2)$	1M	
$\therefore$ $P'P'' = \sqrt{(x_1 + x_2)^2 + (y_1 + y_2)^2}$		
$= \sqrt{(x_1 + x_2)^2 + 4(x_1 + x_2 + c)^2}$	1M	
$= \sqrt{\left(\frac{-c}{5}\right)^2 + 4\left(\frac{4c}{5}\right)^2}$		
$= \sqrt{\frac{65}{25}c^2}$	1A	
$= \sqrt{\frac{130}{5}}$		
$= \sqrt{26}$	1A 9	



SOLUTIONS	MARKS	REMARKS
Alt. Solution:		
11. (a)		
(iii) $x = \dfrac{-4c \pm \sqrt{16c^2 - 80(c^2 - 16)}}{40} = \dfrac{-c \pm \sqrt{80 - 4c^2}}{10}$	1A	
$y = 2x + c = \dfrac{-c \pm \sqrt{80 - 4c^2}}{5} + c$		
$= \dfrac{4c \pm \sqrt{80 - 4c^2}}{5}$	1A	
Let $P = \left(\dfrac{-c - \sqrt{80 - 4c^2}}{10}, \dfrac{4c - \sqrt{80 - 4c^2}}{5}\right)$		
$Q = \left(\dfrac{-c + \sqrt{80 - 4c^2}}{10}, \dfrac{4c + \sqrt{80 - 4c^2}}{5}\right)$		
$PQ^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$	1M	
$= \left(\dfrac{\sqrt{80 - 4c^2}}{5}\right)^2 + \left(\dfrac{2\sqrt{80 - 4c^2}}{5}\right)^2$		
$= \dfrac{80 - 4c^2}{5}$	1A	
$PQ = 2\sqrt{2} \Rightarrow \dfrac{80 - 4c^2}{5} = (2\sqrt{2})^2$	1M	
i.e. $c = \pm\sqrt{10}$	1A	
(b) (i) $\sqrt{80 - 4c^2} = \sqrt{40} = 2\sqrt{10}$	1A	
$P' = \left(\dfrac{-\sqrt{10} - 2\sqrt{10}}{10}, \dfrac{4\sqrt{10} - 2\sqrt{10}}{5}\right)$		
$= \left(\dfrac{-3\sqrt{10}}{10}, \dfrac{2\sqrt{10}}{5}\right)$	1A	
$Q' = \left(\dfrac{-\sqrt{10} + 2\sqrt{10}}{10}, \dfrac{4\sqrt{10} + 2\sqrt{10}}{5}\right)$		
$= \left(\dfrac{\sqrt{10}}{10}, \dfrac{6\sqrt{10}}{5}\right)$	1A	
$P'' = \left(\dfrac{\sqrt{10} - 2\sqrt{10}}{10}, \dfrac{-4\sqrt{10} - 2\sqrt{10}}{5}\right)$		
$= \left(\dfrac{-\sqrt{10}}{10}, \dfrac{-6\sqrt{10}}{5}\right)$	1A	
Area of parallelogram P'Q'Q"P" = 2 Δ P'Q'P"	1M	
$= \left\| \dfrac{-3\sqrt{10}}{10}\left(\dfrac{6\sqrt{10}}{5} - \dfrac{-6\sqrt{10}}{5}\right) + \dfrac{\sqrt{10}}{10}\left(\dfrac{-6\sqrt{10}}{5} - \dfrac{2\sqrt{10}}{5}\right) - \dfrac{\sqrt{10}}{10}\left(\dfrac{2\sqrt{10}}{5} - \dfrac{6\sqrt{10}}{5}\right) \right\|$		
$= \left\| -\dfrac{36}{5} - \dfrac{8}{5} + \dfrac{4}{5} \right\|$		
$= 8$	2A	
(ii) $(P'P'')^2 = \left(\dfrac{2\sqrt{10}}{10}\right)^2 + \left(\dfrac{8\sqrt{10}}{5}\right)^2$	1M	
$= \dfrac{2}{5} + \dfrac{128}{5}$		
$= 26$		
$\therefore$ $P'P'' = \sqrt{26}$	1A	

SOLUTIONS	MARKS	REMARKS
12. (a) $\angle ABC = \frac{2 \times 5 - 4}{5} \times 90°$		
$= 108°$	1A	
$\angle ABE = \frac{(180 - 108)°}{2}$		
$= 36°$	1A	
$\angle CBE = 108° - 36° = 72°$	1A	
$BE = BH + HK + KE$	1M	
$= \cos 72° + 1 + \cos 72°$		
$= 2 \cos 72° + 1$	1A	
Also, $BE = 2BF = 2 \cos 36°$	1A	see Alt. Solution
$\therefore\ 2 \cos 72° + 1 = 2 \cos 36°$		
i.e. $\cos 36° - \cos 72° = \frac{1}{2}$	1A	
$\cos 36° - (2 \cos^2 36° - 1) = \frac{1}{2}$	1A	
$4 \cos^2 36° - 2 \cos 36° - 1 = 0$		
$\cos 36° = \frac{2 + \sqrt{4 + 16}}{8}$ (-ve root rejected as $\cos 36° > 0$)		
$= \frac{1 + \sqrt{5}}{4}$	1A	
	9	
(b) $\frac{\frac{1}{2}AB}{OA} = \cos 54°$	1A	Alt. Solution:
$= \sin 36°$	1A	$OA^2 + OB^2 - AB^2 = 2OA \cdot OB \cos AOB$ 1A
$\therefore\ OA = \frac{1}{2 \sin 36°}$		$2OA^2 - 1 = 2OA^2 \cos 72°$
$= \frac{1}{2\sqrt{1 - \cos^2 36°}}$	1A	$OA^2 = \frac{1}{2(1 - \cos 72°)}$ ..1A
$= \frac{1}{2\sqrt{1 - \frac{(1 + \sqrt{5})^2}{16}}}$	1M	$= \frac{1}{2(1 - \cos 36° + \frac{1}{2})}$ ...1M
$= \frac{2}{\sqrt{16 - (1 + 5 + 2\sqrt{5})}}$		$= \frac{1}{3 - \frac{1 + \sqrt{5}}{2}}$
$= \frac{2}{\sqrt{10 - 2\sqrt{5}}}$ cm	1A	$= \frac{2}{5 - \sqrt{5}}$ 1A
		$\therefore\ OA = \sqrt{\frac{2}{5 - \sqrt{5}}}$
		$= \frac{2}{\sqrt{10 - 2\sqrt{5}}}$ ...1A
	5	



SOLUTIONS	MARKS	REMARKS
12. (c) Each angle of a regular decagon		
$= \frac{2 \times 10 - 4}{10} \times 90° = 144°$	1A	or $\angle AOP = 36°$
$\therefore \angle PAO = 72°$	1A	
$\frac{\frac{1}{2}AP}{AO} = \cos 72°$	1A	
$AP = 2\cos 72° \times AO$		
$= 2(\cos 36° - \frac{1}{2}) \times AO$	1A	
$= 2(\frac{\sqrt{5}-1}{4})\frac{2}{\sqrt{10-2\sqrt{5}}}$	1M	see Alt. Solution
$= \frac{(\sqrt{5}-1)(\sqrt{5}+1)}{\sqrt{10-2\sqrt{5}}(\sqrt{5}+1)}$		
$= \frac{4}{\sqrt{(10-2\sqrt{5})(6+2\sqrt{5})}}$		
$= \frac{4}{\sqrt{40+8\sqrt{5}}}$		
$= \frac{2}{\sqrt{10+2\sqrt{5}}}$ cm	1A	
	6	

Alt. Solution

12. (a) In $\triangle ABE$,		
$BE = \sqrt{1+1-2\cos 108°}$		
$= \sqrt{2+2\cos 72°}$	1A	
In $\triangle BCE$, $BE = EC$,		
$1^2 = BE^2 + BE^2 - 2BE^2 \cos 36°$		
$BE = \frac{1}{\sqrt{2-2\cos 36°}}$	1A	
$2 + 2\cos 72° = \frac{1}{2-2\cos 36°}$		
$\cos 36° - \cos 72° = \frac{3}{4} - \cos 72° \cos 36°$		
$= \frac{3}{4} - \frac{\cos 36° \cos 72° \cdot \sin 36°}{\sin 36°}$	1	
$= \frac{3}{4} - \frac{1}{2}\frac{\sin 72° \cos 72°}{\sin 36°}$		
$= \frac{3}{4} - \frac{1}{4}\frac{\sin 144°}{\sin 36°}$		
$= \frac{1}{2}$	1A	

SOLUTIONS	MARKS	REMARKS
Alt. Solution (1)		
12. (c) $\angle PAB = 72° - 54°$		
$= 18°$ …………	1A	
$\cos 18° = \frac{\frac{1}{2}}{AP}$		
$AP = \frac{1}{2\cos 18°}$		
$= \frac{1}{2\sqrt{\frac{1+\cos 36°}{2}}}$ …………	1A	
$= \frac{1}{2\sqrt{\frac{1+\frac{1+\sqrt{5}}{4}}{2}}}$ …………	1M	
$= \frac{\sqrt{2}}{\sqrt{5+\sqrt{5}}}$		
$= \frac{2}{\sqrt{10+2\sqrt{5}}}$ …………	1A	
Alt. Solution (2)		
In $\triangle PAO$, $AP = AO$,		
$AP^2 = AO^2 + AO^2 - 2(AO)^2 \cos 36°$ …………	2A	
$= 2\left(\frac{2}{\sqrt{10-2\sqrt{5}}}\right)^2 (1 - \cos 36°)$		
$= \frac{8}{10-2\sqrt{5}}\left(1 - \frac{1+\sqrt{5}}{4}\right)$ …………	1M	
$= \frac{2(3-\sqrt{5})(3+\sqrt{5})}{(10-2\sqrt{5})(3+\sqrt{5})}$		
$= \frac{4}{10+2\sqrt{5}}$		
$\Rightarrow AP = \frac{2}{\sqrt{10+2\sqrt{5}}}$ …………	1A	

RESTRICTED 内部文件