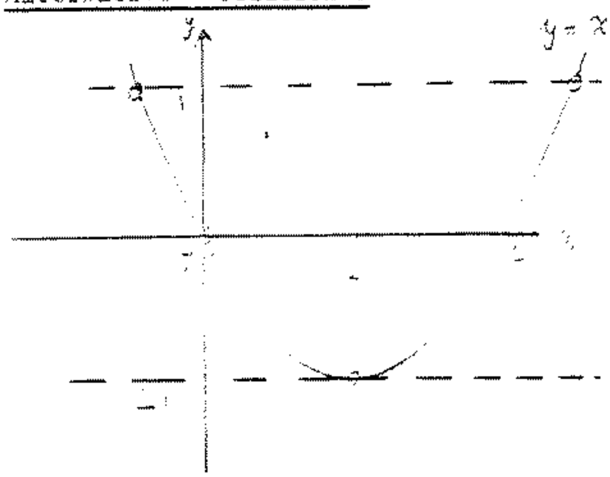
1983 PAPER I

Solution	Marks	Remarks
1. $x^2 - 4x + 2 - \lambda(2x - 1) = 0$		
$\Rightarrow x^2 - (4 + 2\lambda)x + (2 + \lambda) = 0$	1A	
For the equation to have no real roots,		
$(4 + 2\lambda)^2 - 4(2 + \lambda) < 0$	1M	
$4\lambda^2 + 12\lambda + 8 < 0$		
$4(\lambda + 1)(\lambda + 2) < 0$	1A	
$-2 < \lambda < -1$	1M+1A	
	5	
2. a, b, c in A.P. $\Rightarrow$ $b = \frac{1}{2}(a+c)$	1A	
x, y, z in G.P. $\Rightarrow$ $y = \sqrt{xz}$	1A	
$(b - c)\log x + (c - a)\log y + (a - b)\log z$		
$= [\frac{1}{2}(a+c) - c]\log x + (c-a)\log\sqrt{xz} + [a - \frac{1}{2}(a+c)]\log z$	1M	elimination of y and b, etc
$= (\frac{a-c}{2})\log x + (c-a)\frac{1}{2}(\log x + \log z) + (\frac{a-c}{2})\log z$	1M+1M+1A	1M for log MN = log M + log N; 1M for log √N = ½ log N
$= 0$		
	6	
Alternatively		
Let $b = a + d$, $c = a + 2d$	1A	
$y = xr$, $z = xr^2$	1A	
$(b - c)\log x + (c - a)\log y + (a - b)\log z$		
$= -d\log x + 2d\log xr - d\log xr^2$	1M	
$= -d\log x + 2d(\log x + \log r) - d(\log x + 2\log r)$	1M+1M+1A	
$= 0$		
	6	

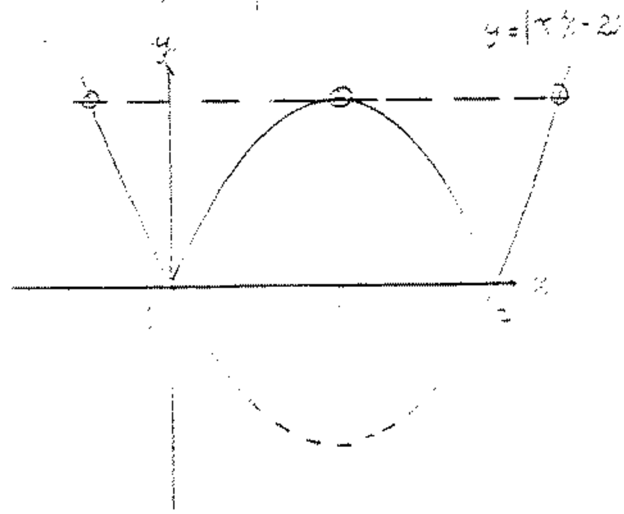
Solution	Marks	Remarks
3. $AB = AC = 1 - x$		
$\therefore\ AD = \sqrt{(1-x)^2 - x^2}$		
$= \sqrt{1 - 2x}$	1A	
Volume formed $= 2 \times \frac{1}{3}\pi AD^2 \times BD$	1M	
$= \frac{2}{3}\pi(1-2x)x$	1A	
$V = \frac{2}{3}\pi(x - 2x^2)$		
$\frac{dV}{dx} = \frac{2}{3}\pi(1-4x)$	1A	
$\frac{dV}{dx} = 0$	1M	
$\Rightarrow x = \frac{1}{4}$		
$\frac{d^2V}{dx^2} = \frac{2}{3}\pi(-4) < 0$		
$\therefore$ V is maximum at $x = \frac{1}{4}$	1A	
	6	
4. $(1+ax)^4(1-4x)^3 = (1 + 4ax + 6a^2x^2 + \ldots)\times(1 - 12x + 48x^2 - \ldots)$	1+1+1A	1 for "+ ..."
$= 1 + (4a-12)x + (6a^2-48a+48)x^2 + \ldots$	2A	-1 for 1 wrong term
As the coefficient of x is zero, $4a - 12 = 0$, $a = 3$	1A	
$\therefore$ coefficient of x^2 is $54 - 144 + 48 = -42$	1A	
	7	
5. $\lvert x(x-2)\rvert < 1$		
$\Leftrightarrow -1 < x(x-2) < 1$	1A	
$\Leftrightarrow x^2 - 2x + 1 > 0$ and $x^2 - 2x - 1 < 0$	1+1+1A	
$\Leftrightarrow (x-1)^2 > 0$ and $[x - (1+\sqrt{2})][x - (1-\sqrt{2})] < 0$	1A	
$\Leftrightarrow x \neq 1$ and $1 - \sqrt{2} < x < 1 + \sqrt{2}$	1+1+1A	OR $1-\sqrt{2}<x<1$ or $1<x<1+\sqrt{2}$
	8	

Solution	Marks	Remarks
5. <u>Alternative Solution 1</u>		
Case (i) $x(x-2) \geq 0$ and $x(x-2) < 1$	1A	
$\Rightarrow x(x-2) \geq 0$ and $x^2 - 2x - 1 < 0$	1A	
$\Rightarrow (x \geq 2$ or $x \leq 0)$ and $(1-\sqrt{2}) < x < (1+\sqrt{2})$		
$\Rightarrow (1-\sqrt{2} < x \leq 0)$ or $(2 \leq x < 1+\sqrt{2})$	1+1A	
Case (ii) $x(x-2) < 0$ and $-x(x-2) < 1$	1A	
$\Rightarrow 2 > x > 0$ and $x^2 - 2x + 1 > 0$	1A	
$\Rightarrow 2 > x > 0$ and $x \neq 1$	1A	
Solution of $\lvert x(x-2)\rvert < 0$ is $x \neq 1$ and $(1-\sqrt{2}) < x < 1+\sqrt{2}$	1A	
	8	

<u>Alternative Solution 2</u>



OR



2 Marks for curve.
2 Marks for necessary line(s) or points.
4 Marks for answers.

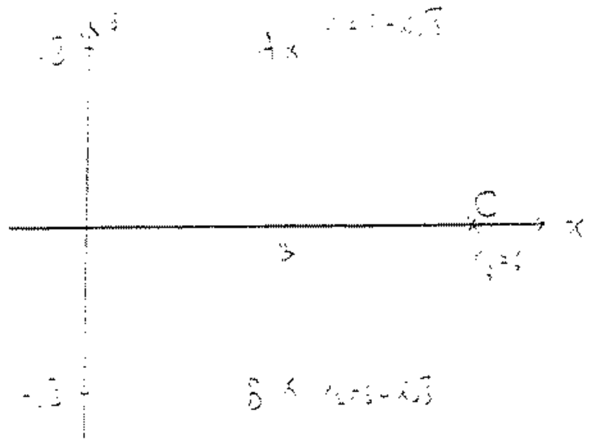
$x \neq 1$ and $-0.4 < x < 2.4$
1 1 2 (deduct one mark if there is equality sign).

Solution	Marks	Remarks
5. $\lvert z - (3 + i)\rvert = \lvert z - (5 + 5i)\rvert$		
$\Rightarrow \lvert (x - 3) + (y - 1)i\rvert = \lvert (x - 5) + (y - 5)i\rvert$	1A	
$\Rightarrow (x - 3)^2 + (y - 1)^2 = (x - 5)^2 + (y - 5)^2$	1M	
$4x + 8y - 40 = 0$	1A	
i.e. $x + 2y - 10 = 0$		
As the locus of z is a line of slope $-\frac{1}{2}$, the required z with the smallest modulus corresponds to the foot of the perpendicular from the origin to this line.	1M	-1 if omitted but continued as below
Equation of perpendicular is $y = 2x$	1A	
Solving this with the locus of z,		
$x + 4x - 10 = 0$ $x = 2$ $y = 4$	 1A 1A	
$\therefore z = 2 + 4i$	7	
Alternatively		
$\lvert z\rvert = \sqrt{x^2 + y^2}$	1A	
$= \sqrt{(10 - 2y)^2 + y^2}$	1M	
$= \sqrt{5y^2 - 40y + 100}$		
$\frac{d\lvert z\rvert}{dy} = \frac{10y - 40}{2\sqrt{5y^2 - 40y + 100}}$	1M	
$\frac{d\lvert z\rvert}{dy} = 0$ when $y = 4$ and $\frac{d\lvert z\rvert}{dy}$ changes sign at $y = 4$.	1M	
$\lvert z\rvert$ is minimum at $x = 2,\ y = 4$.	1A	

Solution	Marks	Remarks
(a) $(\cos\theta + i\sin\theta)^4 = \cos^4\theta + 4i\cos^3\theta\sin\theta +$	1A	
$6i^2\cos^2\theta\sin^2\theta + 4i^3\cos\theta\sin^3\theta +$		
$i^4\sin^4\theta$		
$= (\cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta) +$		
$(4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta)\,i$	1+1A	
But $(\cos\theta + i\sin\theta)^4 = \cos4\theta + i\sin4\theta$	1A	
Comparing real and imaginary parts, we have	1M	
(i) $\cos4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$		
(ii) $\sin4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$		
	5	
(b) $\tan4\theta = \dfrac{\sin4\theta}{\cos4\theta}$	1M	
$= \dfrac{4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta}{\cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta}$	1M	
$= \dfrac{4\dfrac{\sin\theta}{\cos\theta} - 4\dfrac{\sin^3\theta}{\cos^3\theta}}{1 - 6\dfrac{\sin^2\theta}{\cos^2\theta} + \dfrac{\sin^4\theta}{\cos^4\theta}}$		some working must be shown.
$= \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$	1A	
	3	
<u>Alternatively</u>		
$\tan4\theta = \dfrac{2\tan2\theta}{1 - \tan^2 2\theta}$	1A	
$= \dfrac{\dfrac{4\tan\theta}{1 - \tan^2\theta}}{1 - \dfrac{4\tan^2\theta}{1 - 2\tan^2\theta + \tan^4\theta}}$	1A	
$= \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$	1A	
	3	

Solution	Marks	Remarks
7. (c) Putting $x = \tan\theta$ in (*)		
$x^4 + 4x^3 - 6x^2 - 4x + 1 = 0$		
$\tan^4\theta + 4\tan^3\theta - 6\tan^2\theta - 4\tan\theta + 1 = 0$	1A	
$4\tan\theta - 4\tan^3\theta = 1 - 6\tan^2\theta + \tan^4\theta$	1A	
$\therefore \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta} = 1$	1A	
By (b) $\tan 4\theta = 1$	1A	
$4\theta = \dfrac{4n+1}{4}\pi$	1A	
$\theta = \dfrac{(4n+1)\pi}{16}$, $n = 0, \pm 1, \pm 2, \ldots$	1+1A	
$\therefore x = \tan\theta$	1M	
$= \tan\dfrac{(4n+1)\pi}{16}$	1A	
$x_1 = \tan\dfrac{\pi}{16}$ ([illegible]π)		
$x_2 = \tan\dfrac{5\pi}{16}$ (0.476π)		
$x_3 = \tan\dfrac{9\pi}{16}$ (-1.60π)		
$x_4 = \tan\dfrac{13\pi}{16}$ (-0.213π)	2A	-1 for each wrong answer
As these are all distinct, they are the four roots of (*)	12	

Solution	Marks	Remarks
3. (a) $f(x) = x^3 + ax^2 + bx - 72$		
$f'(x) = 3x^2 + 2ax + b$	1A	
$\because x = 4$ is a double root of $f'(x) = 0$		
$\frac{-2a}{3} = 8$	1M	Or $4a^2 - 12b = 0$,
$\frac{b}{3} = 16$	1M	$8a + b = -48$.
$\therefore a = -12$	1A	
$b = 48$	1A	
	5	
(b) $x^3 - 12x^2 + 48x - 72 = (x + p)^3 + q$		OR
$= x^3 + 3px^2 + 3p^2x + p^3 + q$	1A	$x^3-12x^2+48x-72$
$\Longleftrightarrow \begin{cases} 3p = -12 \\ 3p^2 = 48 \\ p^3 + q = -72 \end{cases}$	1M	$= x^3 - 3\times4x^2 + 3\times4^2x - 4^3$
The system is consistent (or rejecting $p = 4$)	1M	$+(4^3-72)$ 1M+1A
$p = -4,\ q = -8$	1A+1A	$= (x-4)^3 - 8$ 1A
$\therefore f(x) = (x - 4)^3 - 8$		$p = -4$ 1A
		$q = -8$ 1A
$x^3 - 12x^2 + 48x - 72 = 0$		
$\Rightarrow (x - 4)^3 - 8 = 0$	1M	
$\Rightarrow (x - 4 - 2)[(x - 4)^2 + 2(x - 4) + 4] = 0$		
$\Rightarrow (x - 6)(x^2 - 6x + 12) = 0$	1A	
$\therefore x = 6$ or $3 \pm \sqrt{3}i$	1+1A	-1 for each wrong ans.
	9	
4. Let $x_1 = 3 + \sqrt{3}i$, $x_2 = 3 - \sqrt{3}i$	1A	



Solution	Marks	Remarks
(diagram)	2A	All 3 points correct.

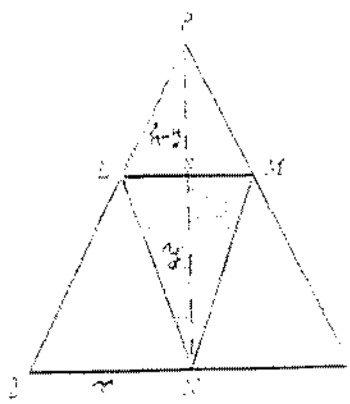
The three roots of $z^3 = 8$ form an equilateral triangle with 0 as the centre and $2\text{cis}120°$, $2\text{cis}240°$ (or $2\text{cis}-120°$), $2\text{cis}0°$ as vertices.

Putting $z = x - 4$, the three roots x_1, x_2, x_3 of $f(x) = 0$ form an equilateral (1A) triangle with $4 + 0i$ as the centre.

Solution	Marks	Remarks
triangle with $4 + 0i$ as the centre.	1A	OR
$\therefore \arg\left(\frac{x_2 - 4}{x_1 - 4}\right) = 120°$ (or $-240°$)	1A	(1A) $\arg\left(\frac{x_2-4}{x_1-4}\right) = \arg\left(\frac{-1+\sqrt{3}i}{-1-\sqrt{3}i}\right)$
	5	(1A) $= \arg\frac{1}{2}(-1+\sqrt{3}i)$
		(1A) $= 120°$ (or $-240°$)

Solution	Marks	Remarks
9. (a) For $n = 1$, L.S. $= 1\cdot 2$		
R.S. $= \frac{1}{3}(1)(2)(3)$		
$=$ L.S.	1A	
Assume that for some $k \geq 1$, $1\cdot 2 + 2\cdot 3 + \ldots + k(k+1) = \frac{1}{3}k(k+1)(k+2)$	1M	
For $n = k+1$, L.S. $= 1\cdot 2 + 2\cdot 3 + \ldots + k(k+1) + (k+1)(k+2)$	1A	
$= \frac{1}{3}k(k+1)(k+2) + (k+1)(k+2)$	1M	
$= \frac{1}{3}(k+1)(k+2) \times [k+3]$	1A	
$= \frac{1}{3}(k+1)[(k+1)+1][(k+1)+2]$		
$=$ R.S.		
$\therefore$ by induction, $1\cdot 2 + 2\cdot 3 + \ldots + n(n+1) = \frac{1}{3}n(n+1)(n+2)$ for all $n \geq 1$.	1M	
	6	
(b) (i) The number of balls in the r-th layer		
$= 1 + 2 + \ldots + r$	1A	
$= \frac{1}{2}r(r+1)$	1A	
(ii) The total number of balls in a heap of n layers		
$= \sum_{r=1}^{n} \frac{1}{2}r(1+r)$	1M+1M	[illegible]
$= \frac{1}{2}\sum_{r=1}^{n} r(1+r)$		
$= \frac{1}{2}\{\frac{1}{3}n(n+1)(n+2)\}$		
$= \frac{1}{6}n(n+1)(n+2)$	1A	
(iii) The time required to deliver and fire all balls in the r-th layer		
$= \frac{1}{2}r(r+1) \times \frac{2}{r}$ minutes	1M	
$= (r+1)$ minutes	1A	
The total time required $= \sum_{r=1}^{10}(r+1)$	1M	
$= \sum_{r=1}^{10} r + 10$	1A	
$= \frac{1}{2}(10)(11) + 10$	1A	
$= 65$ minutes	1A	
	14	

Solution	Marks	Remarks
10. (a) Since LM // QR, $\triangle PQR \sim \triangle PLM$	1M	
Let the heights of $\triangle PQR$ and $\triangle LMN$ be h and y, respectively.		
$\dfrac{h-y}{x} = \dfrac{h}{2r}$	1A	
$h - y = \dfrac{h}{2r}x$		
$y = h - \dfrac{h}{2r}x$	1A	
Area of $\triangle LMN = A = \dfrac{1}{2}xy$		
$= \dfrac{1}{2}(h - \dfrac{h}{2r}x)x$	1M	
$\dfrac{dA}{dx} = \dfrac{1}{2}(h - \dfrac{h}{r}x)$	1A	
$\dfrac{dA}{dx} = 0$ if $x = r$	1M+1A	
$\dfrac{d^2A}{dx^2} = -\dfrac{h}{2r} < 0$		
$\therefore$ A is maximum at $x = r$.	1A	
	8	
(b) Volume of cone $= V = \dfrac{1}{3}\pi(\dfrac{x}{2})^2 y$	...	
$= \dfrac{1}{3}\pi\dfrac{x^2}{4}(h - \dfrac{h}{2r}x)$	1M	
$= \dfrac{\pi}{12}(hx^2 - \dfrac{h}{2r}x^3)$		
$\dfrac{dV}{dx} = \dfrac{\pi h}{12}(2x - \dfrac{3}{2r}x^2)$	1A	
$\dfrac{dV}{dx} = 0$ if $x = 0$ or $\dfrac{4}{3}r$	1A	
$\dfrac{d^2V}{dx^2} = \dfrac{\pi h}{12}(2 - \dfrac{3}{r}x)$	1M	
< 0 at $x = \dfrac{4}{3}r$		
$\therefore$ V is maximum at $x = \dfrac{4}{3}r$	1A	
	6	

Solution	Marks	Remarks
10. (c) Volume of cone generated by revolving $\triangle LMN$ in (a)		
about $PN = \frac{1}{3}\pi\left(\frac{r}{2}\right)^2 y$	1M	
where $y = h - \frac{h}{2r}r$		
$= \frac{h}{2}$	1A	
Volume of cone in (b) $= \frac{1}{3}\pi\left(\frac{2}{3}r\right)^2 y$,	1A	
where $y = h - \frac{h}{2r}\left(\frac{4}{3}r\right)$		
$= \frac{h}{3}$	1A	
$\therefore$ ratio of 2 volumes $= \dfrac{\frac{1}{3}\pi\left(\frac{r}{2}\right)^2\frac{h}{2}}{\frac{1}{3}\pi\left(\frac{2}{3}r\right)^2\frac{h}{3}}$	1M	
$= \frac{27}{32}$	1A	
	6	

Solution	Marks	Remarks
11. (a) (i) $x^2 + y^2 - 2xy\cos 120^\circ = 7$	2A	
$x^2 + y^2 + xy = 7$ ………………… (*)	1A	
$x = 2$, $y^2 + 2y - 3 = 0$	1M	
$(y - 1)(y + 3) = 0$	1A+1A	[illegible]
$y = 1$ (-ve value rejected)		
(ii) Diff. (*) w.r.t. x,		
$2x + 2y\frac{dy}{dx} + x\frac{dy}{dx} + y = 0$	1M+1A	
$\frac{dy}{dx} = -\frac{2x+y}{x+2y}$	1A	
$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$	1M	
$= -\left(\frac{2x+y}{x+2y}\right)\left(\frac{dx}{dt}\right)$		
	10	
(b) At $x = 2$, $\frac{dx}{dt} = -\frac{1}{2}$	1M	accept $\frac{1}{2}$
$\frac{dy}{dt} = -\left(\frac{2x+y}{x+2y}\right)\left(\frac{dx}{dt}\right)$		
$= -\left(\frac{4+1}{2+2}\right)\left(-\frac{1}{2}\right)$	1M	
$= \frac{5}{8}$	1A	
The speed of B is $\frac{5}{8}$ m/s.	1A	
	4	
(c) Area of $\triangle ABO = \frac{1}{2}xy\sin 120^\circ$		
$= \frac{\sqrt{3}}{4}xy$	1A	
Area of $\triangle ABO$ is also equal to $\frac{1}{2}p\sqrt{7}$	1A	
$\therefore \frac{\sqrt{3}}{4}xy = \frac{1}{2}p\sqrt{7}$	1M	
$p = \frac{xy}{2}\sqrt{\frac{3}{7}}$		
$\frac{dp}{dt} = \frac{1}{2}\sqrt{\frac{3}{7}}\left[y\frac{dx}{dt} + x\frac{dy}{dt}\right]$	1A	
When $x = 2$, $y = 1$, $\frac{dx}{dt} = -\frac{1}{2}$, $\frac{dy}{dt} = \frac{5}{8}$		
$\therefore \frac{dp}{dt} = \frac{1}{2}\sqrt{\frac{3}{7}}\left(-\frac{1}{2} + 2\left(\frac{5}{8}\right)\right)$	1M	
$= \frac{3}{8}\sqrt{\frac{3}{7}}$	1A	
	6	

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HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1983

Additional Mathematics II

MARKING SCHEME

Solution	Marks	Remarks
1. Area of $\Delta PQR = \pm\frac{1}{2}[11k + 7 + 21 - 11 - 1 - 3k]$		
$= \pm\frac{1}{2}(8k + 16)$	1+1A	or $\left\lvert\frac{1}{2}(8k + 16)\right\rvert$
If this is 20 units		
$\pm(4k + 8) = 20$	1M	
$k = 3$ or -7	1+1A	
	5	
2. Let $u = x^2$ $du = 2xdx$	1A	
$\int x\sin^2(x^2)dx = \frac{1}{2}\int \sin^2 u du$	1A	
$= \frac{1}{2}\int\frac{1 - \cos 2u}{2}du$	1M	for $\sin^2 u = \frac{1 - \cos 2u}{2}$
$= \frac{1}{4}u - \frac{1}{8}\sin 2u + c$	1A	-1 if omit either "c"
$= \frac{x^2}{4} - \frac{1}{8}\sin 2x^2 + c$	1A	
	5	
3. Put $u = 1 + 3x^2$, $du = 6xdx$	1A	
$x = 0 \Rightarrow u = 1$; $x = 1 \Rightarrow u = 4$	1A	
$\therefore \int_0^1 x^3\sqrt{1 + 3x^2}dx = \int_1^4 \frac{u - 1}{3}\frac{\sqrt{u}}{6}du$	1A	for integrand
$= \frac{1}{18}\int_1^4 (u^{\frac{3}{2}} - u^{\frac{1}{2}})du$		
$= \frac{1}{18}\left[\frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}}\right]_1^4$	1A	
$= \frac{38}{135}$ (0.4296)	1A	Any figure roundable to 0.43.
	5	
4. (a) Equation of L is $y - (-3) = \frac{5 - -3}{5 - 1}(x - 1)$	1M	
or $y = 2x - 5$	1A	
(b) Area between curves $= \int_a^b (y_1 - y_2)dx$	1M	For other methods: Area between curves $= \int_a^b (y_1 - y_2)dx$
Area req'd $= \int_0^1 [(x^2 - 4x) - (2x - 5)]dx + \int_1^5 [(2x - 5) - (x^2 - 4x)]dx$	1M	Required area $= A_1 + A_2 + \ldots + A_n$
$= \int_0^1 (x^2 - 6x + 5)dx + \int_1^5 (-x^2 + 6x - 5)dx$		Answer.
$= \left[\frac{x^3}{3} - 3x^2 + 5x\right]_0^1 + \left[-\frac{x^3}{3} + 3x^2 - 5x\right]_1^5$		
$= \left[\frac{1}{3} - 3 + 5\right] - \frac{125}{3} + 75 - 25 + \frac{1}{3} - 3 + 5 = 13$	2A	
	6	

Solution	Marks	Remarks
5. Let $y = \frac{3}{2}x + c$ be a tangent. (or $3x - 2y + c = 0$)	1A	
Substituting in equation of ellipse	1M	
$4x^2 + (\frac{3}{2}x + c)^2 = 16$		
$(4 + \frac{9}{4})x^2 + 3cx + (c^2 - 16) = 0$	1A	
For tangency, $9c^2 - 4(4 + \frac{9}{4})(c^2 - 16) = 0$	1M	
$16c^2 = 25 \times 16$		
$c = \pm 5$	1+1A	
equations of tangents are $y = \frac{3}{2}x \pm 5$		
	6	
5. <u>Alternatively</u>		
$8x + 2yy' = 0$		
Slope of tangent is $y' = -\frac{4x}{y}$	1A	
But slope of line $= \frac{3}{2}$		
$\therefore -\frac{4x}{y} = \frac{3}{2}$	1M	
or $y = -\frac{8}{3}x$		
Substituting in equation of ellipse	1M	
$4x^2 + (-\frac{8}{3}x)^2 = 16$		
$100x^2 = 144$		
$x = \pm\frac{6}{5}$		
$y = \mp\frac{16}{5}$	1A	For either x or y.
$\therefore$ equation of tangents required are		
$y \pm \frac{16}{5} = \frac{3}{2}(x \mp \frac{6}{5})$		
i.e. $3x - 2y - 10 = 0$	1A	
and $3x - 2y - 10 = 0$	1A	
	6	

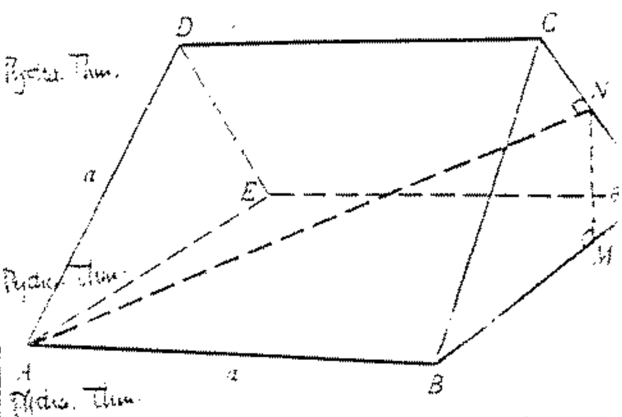
Solution	Marks	Remarks
6. (a) The family of circles passing through the points of intersection of C_1 and C_2 is		Note
$x^2 + y^2 - 3x + 2y - 2 + k(x^2 + y^2 + x + 3y - 10) = 0$	1M	$kC_1 + C_2 = 0$
or $(1+k)x^2 + (1+k)y^2 + (k-3)x + (3k+2)y - (10k+2) = 0$		$\Rightarrow k = -\frac{1}{2}$
Substituting $P(1, 2)$ in the equation	1M	
$(1+k) + (1+k)4 + (k-3) + (3k+2)2 - (10k+2) = 0$		
$2k + 4 = 0$		
$k = -2$	1A	
equation of C is $x^2 + y^2 + 5x + 4y - 18 = 0$	1A	
	4	
Alternatively		
$C_2 - C_1$: $4x + y - 8 = 0$.		
Substituting $y = 8 - 4x$ in C_1,	1M	
$x^2 + (8 - 4x)^2 - 3x + 2(8 - 4x) - 2 = 0$		
$17x^2 - 75x + 78 = 0$		
$x = \frac{75 \pm \sqrt{321}}{34}$ $\begin{pmatrix} 2.7328, \\ 1.6789 \end{pmatrix}$		
$y = \frac{-14 \mp 2\sqrt{321}}{17}$ $\begin{pmatrix} -2.9313, \\ 1.2843 \end{pmatrix}$		
Let C : $x^2 + y^2 + ax + by + c = 0$		
Substituting the three points in C and solving,	1M	
$a = 5,\ b = 4,\ c = -18$.	2A	
	4	
(b) Equation of tangent at P is		
$1x + 2y + \frac{5}{2}(x + 1) + 2(y + 2) - 18 = 0$	1A	
or $7x + 8y - 23 = 0$	1A	
	2	
Alternatively		
$2x + 2yy' + 5 + 4y' = 0$		
$y' = -\frac{2x + 5}{2y + 4}$		
At $P(1, 2)$, slope $= -\frac{7}{8}$	1A	
$y - 2 = -\frac{7}{8}(x - 1)$		
$7x + 8y - 23 = 0$	1A	
	2	

Solution	Marks	Remarks
7. $\sin(n+m)\theta \sin(n-m)\theta$		
$= [\sin n\theta \cos m\theta + \cos n\theta \sin m\theta] \times [\sin n\theta \cos m\theta - \cos n\theta \sin m\theta]$	1A	
$= \sin^2 n\theta \cos^2 m\theta - \cos^2 n\theta \sin^2 m\theta$		
$= \sin^2 n\theta(1 - \sin^2 m\theta) - (1 - \sin^2 n\theta)\sin^2 m\theta$		
$= \sin^2 n\theta - \sin^2 n\theta \sin^2 m\theta - \sin^2 m\theta + \sin^2 n\theta \sin^2 m\theta$		
$= \sin^2 n\theta - \sin^2 m\theta$	1A	
$\sin^2 3\theta - \sin^2 2\theta - \sin\theta = 0$		
$\Rightarrow \sin(3+2)\theta \sin(3-2)\theta - \sin\theta = 0$	1M	
$\Rightarrow \sin 5\theta \sin\theta - \sin\theta = 0$	1A	
$\Rightarrow \sin\theta(\sin 5\theta - 1) = 0$		
$\Rightarrow \sin\theta = 0$ or $\sin 5\theta = 1$	1A	
$\Rightarrow \theta = 0$ or π or $5\theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$		
$\because 0 \leq \theta \leq \pi$		
$\theta = 0, \frac{\pi}{10}, \frac{\pi}{2}, \frac{9\pi}{10}$ or π	3A	−1 for each missing or wrong answer
(0°, 18°, 90°, 162°, 180°)		
	7	

<u>Alternatively</u>

Solution	Marks	Remarks
(i) $\sin^2 n\theta - \sin^2 m\theta = \frac{1}{2}(1 - \cos 2n\theta) - \frac{1}{2}(1 - \cos 2m\theta)$	1A	
$= \frac{1}{2}(\cos 2m\theta - \cos 2n\theta)$		
$= \sin(n+m)\theta \sin(n-m)\theta$	1A	
(ii) $\sin^2 n\theta - \sin^2 m\theta$		
$= (\sin n\theta - \sin m\theta)(\sin n\theta + \sin m\theta)$		
$= \left[2\cos\frac{n+m}{2}\theta \sin\frac{n-m}{2}\theta\right]\left[2\sin\frac{n+m}{2}\theta \cos\frac{n-m}{2}\theta\right]$	1A	
$= 4\sin\frac{n-m}{2}\theta \sin\frac{n+m}{2}\theta \cos\frac{n-m}{2}\theta \cos\frac{n+m}{2}\theta$		
$= \sin(n-m)\theta \sin(n+m)\theta$	1A	

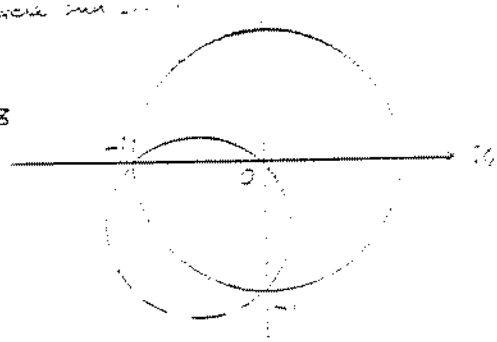
Solution	Marks	Remarks
8. (a) $BM = 2a\cos\theta - x\cos\theta$	2A+1A	
$AM = \sqrt{AB^2 + BM^2}$	1M	Pyth. Thm.
$= \sqrt{a^2 + (2a - x)^2 \cos^2\theta}$	1A	
	4	
(b) $AF^2 = AB^2 + BF^2$	2M	Pyth. Thm.
$= a^2 + 4a^2\cos^2\theta$	1A	
$AN = \sqrt{AF^2 - NF^2}$	1M	Pyth. Thm.
$= \sqrt{a^2 + 4a^2\cos^2\theta - x^2}$	1A	
	5	



Solution	Marks	Remarks
(c) $NM = x\sin\theta$	1A	
Consider $\triangle$ AMN,		
$AN^2 = NM^2 + AM^2$	1M	Pyth. Thm.
$a^2 + 4a^2\cos^2\theta - x^2 = x^2\sin^2\theta + a^2 + (2a - x)^2\cos^2\theta$		
$2x^2 - 4ax\cos^2\theta = 0$	2A ~~1A~~	any equivalent step
$x = 2a\cos^2\theta$ (∵ $x \neq 0$)	1A	(∵ $x \neq 0$) optional
	5	
(d) If $x = \frac{a}{2}$, by (c) $\cos^2\theta = \frac{1}{4}$	1A	
$\theta = \frac{\pi}{3}$ 60°	1A	Or
Let β be the inclination.		$\sin\beta = \frac{NM}{AN}$
$\tan\beta = \frac{NM}{AM}$	1A	$= \frac{x\sin\theta}{\sqrt{a^2+4a^2\cos^2\theta-x^2}}$
$= \frac{x\sin\theta}{\sqrt{a^2 + (2a - x)^2\cos^2\theta}}$ ← 可以錯，但必須已交過	1M	
$= \frac{\frac{a}{2} \times \frac{\sqrt{3}}{2}}{\sqrt{a^2 + \frac{9}{4}a^2\frac{1}{4}}}$		$= \frac{\sqrt{3}}{2\sqrt{7}}$ ($= 0.327$)
$= \frac{\sqrt{3}}{5}$ ($= 0.346$)	1A	Any fig. rounded to 0.346
$\beta = 19.1°$		
$\beta \doteq 19°$ (correct to the nearest degree)	1A	
分兩步，唔可以合	6	

Solution	Marks	Remarks
9. (a) $y^2 = 4x$ $2yy' = 4$		
Slope of tangent $= \frac{2}{y}$	1A	
Slope of $L_1 = 1$,	1A	
of $L_2 = -2$	1A	
Equation of L_1 is $x - y - 3 = 0$	1A	
of L_2 is $2x + y - 12 = 0$	1A	
Solving the above, the coordinates of N are $x = 5$, $y = 2$.	1+1A	
Slope of ON $= \frac{2}{5}$	1A	
	8	
(b) Coordinates of P are $x = \frac{4+k}{1+k}$, $y = \frac{4-2k}{1+k}$	1+1A	
Slope of OP $= \frac{4 - 2k}{4 + k}$	1A	
$\tan \angle PON = \pm \left\| \frac{\frac{4-2k}{4+k} - \frac{2}{5}}{1 + \frac{4-2k}{4+k} \times \frac{2}{5}} \right\|$	2M	for $\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$
$= \pm \left\| \frac{12 - 12k}{28 + k} \right\|$ according as $\angle PON$ is acute or obtuse	1+1A	
	7	
(c) (i) If $\left\| \frac{12 - 12k}{28 + k} \right\| = 1$ $\pm\frac{12-12k}{28+k} = 1$ 試圖	1M	
$k = -\frac{16}{13}$ or $\frac{40}{11}$	1A	
By inspection, $k = -\frac{16}{13}$ corresponds to the case $\angle PON = 135°$. ∴ if $\angle PON = 45°$, $k = \frac{40}{11}$	1A	
(ii) When PON is a straight line $\frac{12 - 12k}{28 + k} = 0$	1M	
$k = 1$	1A	← reduce to $\pm\frac{12-12k}{28+k}$
	5	

Solution	Marks	Remarks
10. (a) Let the line be		
$y + 1 = m(x + 1)$	1M	
$y = mx + (m - 1)$	1A	
Substituting in the circle,		
$x^2 + [mx + (m - 1)]^2 = 1$	1M	
$(1 + m^2)x^2 + 2m(m - 1)x + (m - 1)^2 - 1 = 0$	1A	
If $A = (x_1, y_1)$, $B = (x_2, y_2)$		
$x_1 + x_2 = -\dfrac{2m(m - 1)}{1 + m^2}$	1M	also for $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
∴ the coordinates of P are		
$x = \dfrac{x_1 + x_2}{2}$	1M	
$= -\dfrac{m(m - 1)}{1 + m^2}$ .... (i)	1A	
$y = mx + (m - 1)$		
$= -\dfrac{m^2(m - 1)}{1 + m^2} + (m - 1)$	1M	
$= \dfrac{m - 1}{1 + m^2}$ .... (ii)	1A	
	9	
(b) (i) ÷ (ii) $\dfrac{x}{y} = -m$		
Substituting in (ii) $y = \dfrac{-\frac{x}{y} - 1}{1 + \frac{x^2}{y^2}}$	2M	Attempt to eliminate m between x, y.
$x^2 + y^2 + x + y = 0$	3A	
which is a circle.	1A	
	6	
(c) Or Sketch by joining mid-points. 2A Proof for circle. 3A	3	for circle passing through (0,0), (-1,0)(0,-1)
	1	for labelling.
	1	for indicating correct part of circle as locus
	5	



Solution	Marks	Remarks

11. (a) $\frac{\sin 3\theta}{\sin\theta} = \frac{\sin 2\theta\cos\theta + \cos 2\theta\sin\theta}{\sin\theta}$ — 1A

$= \frac{2\sin\theta\cos^2\theta + \cos 2\theta\sin\theta}{\sin\theta}$

$= 2\cos 2\theta + 1$ — 1A

Or

$\frac{\sin 3\theta}{\sin\theta} = \frac{3\sin\theta - 4\sin^3\theta}{\sin\theta}$ — 1A

$= 3 - 4\sin^2\theta$

$= 3 - 4\left(\frac{1 - \cos 2\theta}{2}\right)$

$= 2\cos 2\theta + 1$ — 1A

Putting $\theta = \frac{\pi}{4} + \phi$,

L.S. $= \frac{\sin 3\theta}{\sin\theta}$

$= \frac{\sin\left(\frac{3\pi}{4} + 3\phi\right)}{\sin\left(\frac{\pi}{4} + \phi\right)}$

$= \frac{\sin\frac{3\pi}{4}\cos 3\phi + \cos\frac{3\pi}{4}\sin 3\phi}{\sin\frac{\pi}{4}\cos\phi + \cos\frac{\pi}{4}\sin\phi}$ — 1A, 1A

$= \frac{\cos 3\phi - \sin 3\phi}{\cos\phi + \sin\phi}$ — 1A

R.S. $= 2\cos\left(\frac{\pi}{2} + 2\phi\right) + 1$

$= 1 - 2\sin 2\phi$ — 1A

$\therefore \frac{\cos 3\phi - \sin 3\phi}{\cos\phi + \sin\phi} = 1 - 2\sin 2\phi$ — 7

(b) Putting $\phi = \frac{\pi}{2} - u$, $d\phi = -du$ — 1A

when $\phi = 0$, $u = \frac{\pi}{2}$ — 1A

$\phi = \frac{\pi}{2}$, $u = 0$

$\int_0^{\frac{\pi}{2}} \frac{\cos 3\phi}{\cos\phi + \sin\phi}\,d\phi = -\int_{\frac{\pi}{2}}^{0} \frac{\cos\left(\frac{3\pi}{2} - 3u\right)}{\cos\left(\frac{\pi}{2} - u\right) + \sin\left(\frac{\pi}{2} - u\right)}\,du$

$= -\int_{\frac{\pi}{2}}^{0} \frac{-\sin 3u}{\sin u + \cos u}\,du$

$= \int_{\frac{\pi}{2}}^{0} \frac{\sin 3u}{\cos u + \sin u}\,du$ — 2A

	Solution	Marks	Remarks
11. (b)	$\therefore \int_0^{\frac{\pi}{2}} \frac{\cos 3\theta - \sin 3\theta}{\cos\theta + \sin\theta}\,d\theta = \int_0^{\frac{\pi}{2}} \frac{\cos 3\theta}{\cos\theta + \sin\theta}\,d\theta -$		
	$\int_0^{\frac{\pi}{2}} \frac{\sin 3\theta}{\cos\theta + \sin\theta}\,d\theta$	2M	must check steps
	$= 2\int_0^{\frac{\pi}{2}} \frac{\cos 3\theta}{\cos\theta + \sin\theta}\,d\theta$	2M	
	$\int_0^{\frac{\pi}{2}} \frac{\cos 3\theta}{\cos\theta + \sin\theta}\,d\theta = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{\cos 3\theta - \sin 3\theta}{\cos\theta + \sin\theta}\,d\theta$		
		8	
(c)	$\int_0^{\frac{\pi}{2}} \frac{\cos 3\theta}{\cos\theta + \sin\theta}\,d\theta = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{\cos 3\theta - \sin 3\theta}{\cos\theta + \sin\theta}\,d\theta$	1A	
	$= \frac{1}{2}\int_0^{\frac{\pi}{2}} (1 - 2\sin 2\theta)\,d\theta$	2A	
	$= \frac{1}{2}[\theta + \cos 2\theta]_0^{\frac{\pi}{2}}$	1A	
	$= \frac{1}{2}(\frac{\pi}{2} - 1 - 1)$		
	$= \frac{\pi}{4} - 1 \quad (\doteq -0.215)$	1A	Any figure roundable -0.215.
		5	

Solution	Marks	Remarks
12. (a) Putting $y = x + ks$, $dy = dx$	1A	
when $x = 0$, $y = ks$; $x = s$, $y = (k+1)s$	1A	
$\int_0^s f(x+ks)dx = \int_{ks}^{(k+1)s} f(y)dy$	2A	
$= \int_{ks}^{(k+1)s} f(x)dx$	1A	
	5	
$\int_0^s [f(x) + f(x+s) + \ldots + f(x+(n-1)s)]dx$		
$= \int_0^s f(x)dx + \int_0^s f(x+s)dx + \ldots + \int_0^s f(x+(n-1)s)dx$	1A	
$= \int_0^s f(x)dx + \int_s^{2s} f(x)dx + \ldots + \int_{(n-1)s}^{ns} f(x)dx$	0+1A+2A	
$= \int_0^{ns} f(x)dx$	1A	
	5	
(b) Putting $x = \sin\theta$, $dx = \cos\theta d\theta$	1A	
when $x = 0$, $\theta = 0$; $x = \frac{1}{2}$, $\theta = \frac{\pi}{6}$	1A	
$\int_0^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^2}} = \int_0^{\frac{\pi}{6}} \frac{\cos\theta d\theta}{\cos\theta}$	1A	any form in θ only.
$= [\theta]_0^{\frac{\pi}{6}}$	1A	
$= \frac{\pi}{6}$ (0.524)	1A	Any figure roundable 0.524.
Putting $f(x) = \frac{1}{\sqrt{1-x^2}}$, $s = \frac{1}{2n}$, by (a)	1+1A	may be omitted.
$\int_0^{\frac{1}{2n}} \left[\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-\left(x+\frac{1}{2n}\right)^2}} + \ldots + \frac{1}{\sqrt{1-\left(x+\frac{n-1}{2n}\right)^2}}\right] dx$		
$= \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} dx$	2A (~~2A~~)	
$= \frac{\pi}{6}$	1A	
	10	