香港考試局

HONG KONG EXAMINATIONS AUTHORITY

一九八三年香港中學會考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1983

附加數學 試卷一 / ADDITIONAL MATHEMATICS PAPER I

二小時完卷 / Two hours

上午八時三十分至十時三十分 / 8.30 a.m.—10.30 a.m.

本試卷必須用英文作答 / This paper must be answered in English

Answer ALL questions in Section A and any THREE questions from Section B.

All working must be clearly shown.

SECTION A (40 marks)
Answer ALL questions in this section.

1. Determine the range of values of λ for which the equation

$$x^2 + 4x + 2 + \lambda(2x + 1) = 0$$

has no real roots.

(5 marks)

2. Given that a, b, c are in arithmetic progression and the positive numbers x, y, z are in geometric progression, prove that

$$(b - c)\log x + (c - a)\log y + (a - b)\log z = 0 .$$

(6 marks)

3. Figure 1 shows an isosceles triangle ABC with $BC = 2x$ and $AB = AC$. The perimeter of the triangle is 2 metres. The triangle is revolved about BC so as to form a solid consisting of two cones with a common base of radius AD. Express the volume of this solid in terms of x. Hence find the value of x for which this volume is a maximum.

(6 marks)

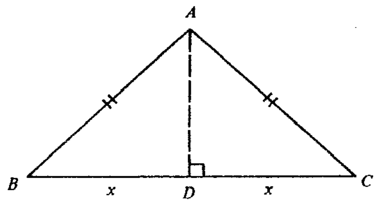

Figure 1

4. Expand $(1 + ax)^4(1 - 4x)^3$ in ascending powers of x up to and including the term containing x^2.

Given that the coefficient of x is zero, evaluate the coefficient of x^2.

(7 marks)

5. Solve the inequality $|x(x - 2)| < 1$. (8 marks)

6. The complex number z satisfies the condition

$$|z - (3 + i)| = |z - (5 + 5i)| .$$

If $z = x + iy$, where x and y are real, find and simplify the relation between x and y.

Find also the values of x and y for which $|z|$ is a minimum.

(8 marks)

SECTION B (60 marks)
Answer any THREE questions from this section.
Each question carries 20 marks.

7. (a) Using De Moivre's theorem, or otherwise, show that

(i) $\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$

(ii) $\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$.

(5 marks)

(b) Using (a), or otherwise, show that

$$\tan 4\theta = \frac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta} .$$

(3 marks)

(c) By putting $x = \tan\theta$ and using the result of (b), show that the equation

$x^4 + 4x^3 - 6x^2 - 4x + 1 = 0$ .... (*)

can be transformed to

$\tan 4\theta = 1$ . ... (**)

Find the general solution of equation (**) in terms of π.

Hence deduce the four roots of (*) leaving your answers in terms of π.

(12 marks)

8. Let $f(x) = x^3 + ax^2 + bx - 72$.

(a) Given that $x = 4$ is a double root of $\frac{df}{dx} = 0$, find the values of a and b. (5 marks)

(b) Show that $f(x)$ can be expressed in the form $(x + p)^3 + q$, and find p and q.
Hence find the three roots of $f(x) = 0$.

(9 marks)

(c) Represent the three roots x_1, x_2, x_3 of $f(x) = 0$ on an Argand diagram by the points A, B, C, respectively, x_1 and x_2 being complex conjugates and $0 < \arg(x_1) < \pi$.

By considering triangle ABC, or otherwise, determine $\arg\left(\frac{x_2 - 4}{x_1 - 4}\right)$.

(6 marks)

9. (a) Prove, by mathematical induction, that for all positive integers n,

$$1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots + n(n + 1) = \frac{1}{3}n(n + 1)(n + 2) .$$

(6 marks)

(b) On a battle field, cannon-balls are stacked as shown in Figure 2. For a stack with n layers, the balls in the bottom layer are arranged as shown in Figure 3 with n balls on each side. For the second bottom layer, the arrangement is similar but each side consists of $(n - 1)$ balls; for the third bottom layer, each side has $(n - 2)$ balls, and so on. The top layer consists of only one ball.

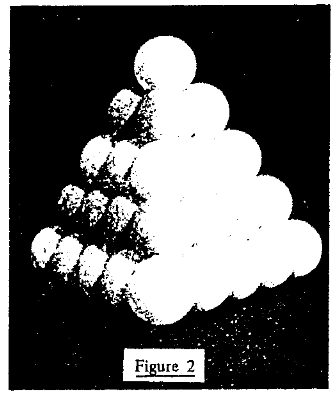

Figure 2

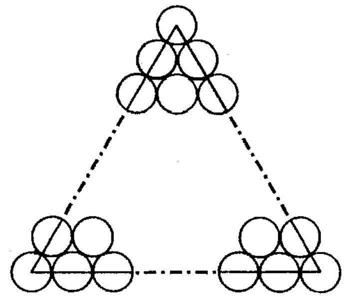

Figure 3

(i) Find the number of balls in the r-th layer counting from the top.

(ii) Using the result of (a), or otherwise, find the total number of cannon-balls in a stack consisting of n layers.

(iii) If the time required to deliver and fire a cannon-ball taken from the r-th layer is $\frac{2}{r}$ minutes, find the time required to deliver and fire all the cannon-balls in the r-th layer.

Hence find the total time needed to use up all the cannon-balls in a stack of 10 layers.

(14 marks)

10. In Figure 4, PQR is an isosceles triangle with base $QR = 2r$. N is the mid-point of QR. L and M are variable points on PQ and PR, respectively, such that $LM /\!/ QR$. Let $LM = x$.

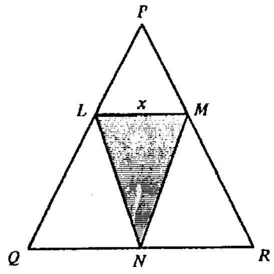

Figure 4

(a) Find x such that the area of ΔLMN is a maximum.

(8 marks)

(b) If the figure is revolved about PN, find x so that the volume of the cone generated by ΔLMN is a maximum.

(6 marks)

(c) Show that the volume of the cone generated by revolving the ΔLMN specified in (a) about PN is only $\frac{27}{32}$ of the volume generated in (b).

(6 marks)

11. Figure 5 shows a rail POQ with $\angle POQ = 120°$. A rod AB of length $\sqrt{7}$ m is free to slide on the rail with its end A on OP and end B on OQ. Let $OA = x$ metres and $OB = y$ metres.

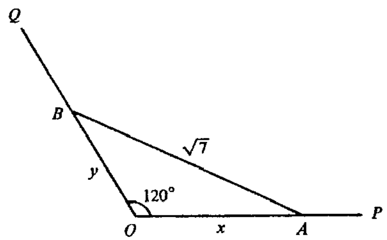

Figure 5

(a) (i) Find a relation between x and y and hence find the value of y when $x = 2$.

(ii) Find $\frac{dy}{dx}$.

Given that x and y are functions of time t (in seconds), show that

$$\frac{dy}{dt} = -\left(\frac{2x + y}{x + 2y}\right)\frac{dx}{dt}.$$

(10 marks)

(b) The end A is pushed towards O with a uniform speed of $\frac{1}{2}$ m/s. When A is at a distance of 2 metres from O, find the speed of the end B.

(4 marks)

(c) Suppose the perpendicular distance from O to the rod is p metres. Show that

$$p = \frac{xy}{2}\sqrt{\frac{3}{7}}.$$

Hence find $\frac{dp}{dt}$ when $x = 2$.

(6 marks)

END OF PAPER

香 港 考 試 局

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一 九 八 三 年 香 港 中 學 會 考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1983

附加數學 試卷二

ADDITIONAL MATHEMATICS PAPER II

二小時完卷

上午十一時十五分至下午一時十五分

本試卷必須用英文作答

Two hours

11.15 a.m.—1.15 p.m.

This paper must be answered in English

Answer ALL questions in Section A and any THREE questions from Section B.

All working must be clearly shown.

SECTION A (40 marks)
Answer ALL questions in this section.

1. A triangle has vertices $P(k, -1)$, $Q(7, 11)$ and $R(1, 3)$. Given that the area of the triangle is 20 units, find the two values of k.

(5 marks)

2. Use the substitution $u = x^2$ to find the indefinite integral

$$\int x \sin^2(x^2)\,dx.$$

(5 marks)

3. Use the substitution $u = 1 + 3x^2$ to evaluate

$$\int_0^1 x^3\sqrt{1 + 3x^2}\,dx.$$

(5 marks)

4. Figure 1 shows the curve $y = x^2 - 4x$. A straight line L intersects the curve at the points $P(1, -3)$ and $Q(5, 5)$.

Find (a) the equation of L,

and (b) the area of the shaded region.

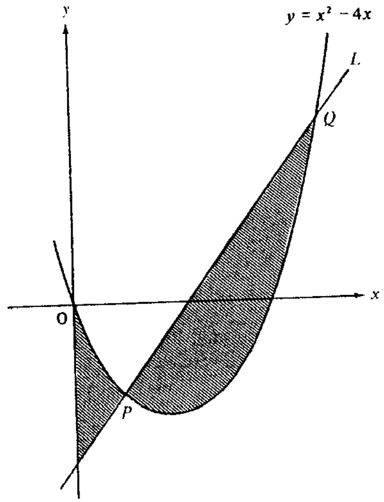

Figure 1

(6 marks)

5. Find the equations of the two lines which are both parallel to the line $3x - 2y = 0$ and tangent to the ellipse

$$4x^2 + y^2 = 16.$$

(6 marks)

6. A circle C passes through the point $P(1, 2)$ and the points of intersection of the circles

$$C_1 : x^2 + y^2 - 3x + 2y - 2 = 0$$

and $$C_2 : x^2 + y^2 + x + 3y - 10 = 0.$$

Find the equations of (a) the circle C,

and (b) the tangent to C at P.

(6 marks)

7. Show that $\sin^2 n\theta - \sin^2 m\theta = \sin(n + m)\theta \sin(n - m)\theta$.

Hence, or otherwise, solve the equation

$$\sin^2 3\theta - \sin^2 2\theta - \sin\theta = 0$$

for $0 \leqslant \theta \leqslant \pi$.

(7 marks)

SECTION B (60 marks)

Answer any THREE questions from this section.
Each question carries 20 marks.

8. Figure 2 shows a tent consisting of two inclined square planes $ABCD$ and $EFCD$ standing on the horizontal ground $ABFE$. The length of each side of the inclined planes is a. N is a point on CF such that $AN \perp CF$. Let $NF = x (\neq 0)$, $\angle CFB = \theta$ and M be a point on BF such that $NM \perp BF$.

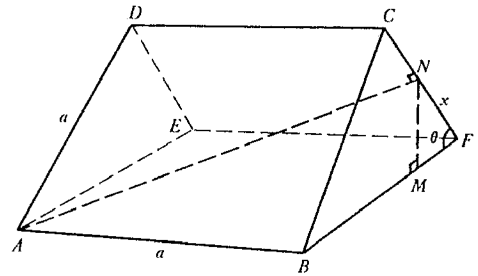

Figure 2

(a) By considering ΔABM, express AM in terms of a, x and θ.
(4 marks)

(b) By considering ΔANF, express AN in terms of a, x and θ.
(5 marks)

(c) Using the results of (a) and (b), or otherwise, show that $x = 2a\cos^2\theta$.
(5 marks)

(d) Given that $x = \frac{a}{2}$, find (correct to the nearest degree) the inclination of AN to the horizontal.
(6 marks)

9. $A(1, -2)$ and $B(4, 4)$ are two points on the parabola $y^2 = 4x$. P is a point on the line AB such that $AP : PB = 1 : k$. A line L_1 through A is perpendicular to the tangent at A. Another line L_2 through B is perpendicular to the tangent at B. L_1 and L_2 intersect at N. Let O be the origin.

(a) Find the coordinates of the point N and the slope of ON.
(8 marks)

(b) (i) Express the slope of OP in terms of k.

(ii) Express $\tan \angle PON$ in terms of k when

(1) $\angle PON$ is acute,

(2) $\angle PON$ is obtuse.

(7 marks)

(c) Find the value of k in each of the following cases :

(i) when $\angle PON = 45°$;

(ii) when OPN is a straight line.

(5 marks)

10. A straight line through the point $R(-1, -1)$ has a variable slope m. It intersects the circle

$$x^2 + y^2 = 1$$

at A and B. Let P be the mid-point of AB.

(a) Find the coordinates of P in terms of m. (9 marks)

(b) The locus of P is a part of a curve C. Find the equation of C and name it. (6 marks)

(c) Sketch the locus of P. (5 marks)

11. (a) Show that $\dfrac{\sin 3\theta}{\sin\theta} = 2\cos 2\theta + 1$.

By putting $\theta = \dfrac{\pi}{4} + \phi$ in the above identity, show that

$$\frac{\cos 3\phi - \sin 3\phi}{\cos\phi + \sin\phi} = 1 - 2\sin 2\phi .$$

(7 marks)

(b) Using the substitution $\phi = \dfrac{\pi}{2} - u$, show that

$$\int_0^{\frac{\pi}{2}} \frac{\cos 3\phi}{\cos\phi + \sin\phi}\,d\phi = \int_{\frac{\pi}{2}}^{0} \frac{\sin 3u}{\cos u + \sin u}\,du .$$

Hence, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} \frac{\cos 3\phi}{\cos\phi + \sin\phi}\,d\phi = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{\cos 3\phi - \sin 3\phi}{\cos\phi + \sin\phi}\,d\phi .$$

(8 marks)

(c) Using the results in (a) and (b), evaluate

$$\int_0^{\frac{\pi}{2}} \frac{\cos 3\phi}{\cos\phi + \sin\phi}\,d\phi .$$

(5 marks)

12. Let $f(x)$ be a function of x and let k and s be constants.

(a) By using the substitution $y = x + ks$, show that

$$\int_0^{s} f(x + ks)\,dx = \int_{ks}^{(k+1)s} f(x)\,dx .$$

Hence show that, for any positive integer n,

$$\int_0^{s} [f(x) + f(x+s) + \ldots + f(x + (n-1)s)]\,dx = \int_0^{ns} f(x)\,dx .$$

(10 marks)

(b) Evaluate $\displaystyle\int_0^{\frac{1}{2}} \frac{dx}{\sqrt{1 - x^2}}$ by using the substitution $x = \sin\theta$.

Using this result together with (a), evaluate

$$\int_0^{\frac{1}{2n}} \left(\frac{1}{\sqrt{1 - x^2}} + \frac{1}{\sqrt{1 - (x + \frac{1}{2n})^2}} + \frac{1}{\sqrt{1 - (x + \frac{2}{2n})^2}} + \ldots + \frac{1}{\sqrt{1 - (x + \frac{n-1}{2n})^2}} \right) dx .$$

(10 marks)

END OF PAPER

HONG KONG EXAMINATIONS AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1984

附加數學　試卷一

ADDITIONAL MATHEMATICS PAPER I

8.30 am–10.30 am (2 hours)

This paper must be answered in English

Answer ALL questions in Section A and any THREE questions from Section B.

All working must be clearly shown.

NUMERICAL ANSWERS

1983

Additional Mathematics I

1. $-2 < \lambda < -1$

3. $x = \frac{1}{4}$

4. -42

5. $x \neq 1$ and $1 - \sqrt{2} < x < 1 + \sqrt{2}$

6. $x + 2y - 10 = 0$

   $x = 2$, $y = 4$

7. (c) $\theta = \frac{(4n+1)\pi}{16}$, $n = 0, \pm 1, \pm 2, \ldots$

   $\tan\frac{\pi}{16}$, $\tan\frac{5\pi}{16}$, $\tan\frac{9\pi}{16}$, $\tan\frac{13\pi}{16}$

8. (a) $a = -12$, $b = 48$

   (b) $p = -4$, $q = -8$

   $x = 6$ or $3 \pm i\sqrt{3}$

   (c) $\arg\left(\frac{x_2 - 4}{x_1 - 4}\right) = 120°$

9. (b) (i) $\frac{r}{2}(1 + r)$

   (ii) $\frac{1}{6}n(n+1)(n+2)$

   (iii) $(r + 1)$ minutes

   65 minutes

10. (a) $x = r$

    (b) $x = \frac{4}{3}r$

11. (a) (i) $x^2 + y^2 + xy = 7$

    When $x = 2$, $y = 1$.

    (ii) $\frac{dy}{dx} = -\frac{2x + y}{x + 2y}$

    (b) $\frac{5}{8}$ m/s

    (c) $\frac{3}{8}\sqrt{\frac{3}{7}}$

1983

Additional Mathematics II

1. $k = 3$ or -7

2. $\frac{x^2}{4} - \frac{1}{8}\sin 2x^2 + c$

3. $\frac{58}{135}$

4. (a) $y = 2x - 5$

   (b) 13

5. $y = \frac{3}{2}x \pm 5$

6. (a) $x^2 + y^2 + 5x + 4y - 18 = 0$

   (b) $7x + 8y - 23 = 0$

7. $\theta = 0$, $\frac{\pi}{10}$, $\frac{\pi}{2}$, $\frac{9\pi}{10}$ or π

8. (a) $AM = \sqrt{a^2 + (2a - x)^2 \cos^2\theta}$

   (b) $AN = \sqrt{a^2 + 4a^2\cos^2\theta - x^2}$

   (d) 19°

9. (a) $N = (5 , 2)$

   Slope of $ON = \frac{2}{5}$

   (b) (i) Slope of $OP = \frac{4 - 2k}{4 + k}$

   (ii) $\tan\angle PON = \pm\left|\frac{12 - 12k}{28 + k}\right|$

   according as $\angle PON$ is acute or obtuse

   (c) (i) $k = \frac{40}{11}$

   (ii) $k = 1$

10. (a) $P = \left(\frac{m - m^2}{1 + m^2} , \frac{m - 1}{1 + m^2}\right)$

    (b) The circle $x^2 + y^2 + x + y = 0$

11. (c) $\frac{\pi}{4} - 1$

12. (b) $\frac{\pi}{6}$

    $\frac{\pi}{6}$