香港考試局

HONG KONG EXAMINATIONS AUTHORITY

一九八二年香港中學會考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1982

Additional Mathematics I

MARKING SCHEME

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Solutions	Marks	Remarks
$2^x = 10^{x+1}$		
$x \log 2 = (x+1)\log 10$	1M+1A	1M for taking log.
$x(\log 2 - 1) = 1$ or $(0.3010-1)x = 1$	1A	
$x = \dfrac{1}{\log 2 - 1}$		
$= -1.431$	1A	or any figure roundable to -1.431
$= -1.43$ (correct to 3 sig fig.)	1A	
	5	
$\dfrac{\log\sqrt[3]{4} + \log\sqrt[3]{25} - \log\sqrt[3]{9}}{\log 8 + \log 5 - \log 12}$		
$= \dfrac{\frac{1}{3}(\log 4 + \log 25 - \log 9)}{\log 8 + \log 5 - \log 12}$	1M	$\log a^p = p \log a$
$= \dfrac{1}{3}\dfrac{\log\frac{4\times 25}{9}}{\log\frac{8\times 5}{12}}$	1M + 1M	$\log a + \log b = \log ab$ $\log a - \log b = \log\frac{a}{b}$
$= \dfrac{1}{3}\dfrac{\log\frac{2^2\times 5^2}{3^2}}{\log\frac{2\times 5}{3}}$		
$= \dfrac{2}{3}$	2A	
	5	

Alternatively,

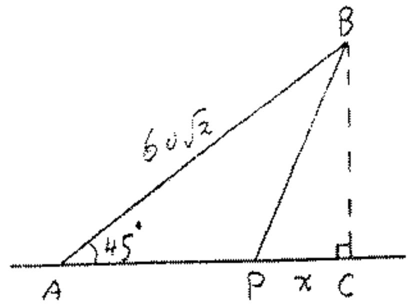
Solutions	Marks	Remarks
$\dfrac{\log\sqrt[3]{4} + \log\sqrt[3]{25} - \log\sqrt[3]{9}}{\log 8 + \log 5 - \log 12} = \dfrac{\log 2^{\frac{2}{3}} + \log 5^{\frac{2}{3}} - \log 3^{\frac{2}{3}}}{\log 2^3 + \log 5 - \log 2^2\times 3}$	1M	Expressing in powers of 2, 3, 5
$= \dfrac{\frac{2}{3}[\log 2 + \log 5 - \log 3]}{3\log 2 + \log 5 - 2\log 2 - \log 3}$	1M + 1M	$\log a^p = p \log a$ $\log ab = \log a + \log b$
$= \dfrac{2}{3}$	2A	
	5	

RESTRICTED 内部文件

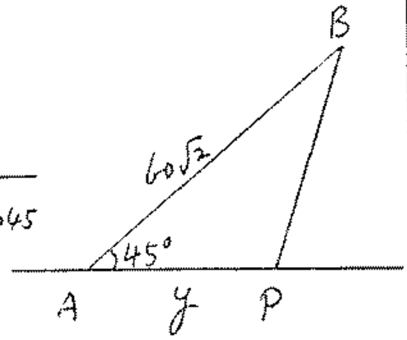
Solution	Marks	Remarks
(3) $\dfrac{d}{dt}\left(\dfrac{\sin kt}{3+2\cos kt}\right) = \dfrac{(3+2\cos kt)k\cos kt - \sin kt(-2k\sin kt)}{(3+2\cos kt)^2}$	1M + 1M + 1A	Chain Rule Quotient Rule
$= \dfrac{3k\cos kt + 2k(\cos^2 kt + \sin^2 kt)}{(3+2\cos kt)^2}$		—
$= \dfrac{3k\cos kt + 2k}{(3+2\cos kt)^2}$		
When $t = \frac{3\pi}{2k}$, $\dfrac{d\theta}{dt} = \dfrac{3k\cos\frac{3\pi}{2} + 2k}{(3+2\cos\frac{3\pi}{2})^2}$	1M	Substitution
$= \dfrac{2k}{9}$	2A	
	6	
(4) $-1-i = \sqrt{2}\left[\cos(-\frac{3}{4}\pi) + i\sin(-\frac{3}{4}\pi)\right]$ $\left(\text{or } \sqrt{2}\operatorname{cis}(-\frac{3\pi}{4})\right)$	1A	or $\frac{5}{4}\pi$, $225°$, $-135°$
$1-i = \sqrt{2}\left[\cos(-\frac{1}{4}\pi) + i\sin(-\frac{1}{4}\pi)\right]$	1A	or $\frac{7}{4}\pi$, $315°$, $-45°$
$\dfrac{-1-i}{(1-i)^5} = \dfrac{\sqrt{2}\left[\cos(-\frac{3}{4}\pi) + i\sin(-\frac{3}{4}\pi)\right]}{\left[\sqrt{2}\left(\cos(-\frac{1}{4}\pi) + i\sin(-\frac{1}{4}\pi)\right)\right]^5}$		
$= \dfrac{\sqrt{2}\left[\cos(-\frac{3}{4}\pi) + i\sin(-\frac{3}{4}\pi)\right]}{(\sqrt{2})^5\left[\cos(-\frac{5}{4}\pi) + i\sin(-\frac{5}{4}\pi)\right]}$	1M	De Moivre's Thm
$= \frac{1}{4}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$	1M +1A	$\dfrac{\operatorname{cis}\theta}{\operatorname{cis}\phi} = \operatorname{cis}(\theta-\phi)$
$\left(\text{or } = \dfrac{-1-i}{4(-1+i)} = \dfrac{(-1-i)^2}{4(-1+i)(-1-i)}\right)$	(1M)	For multiplying $-1-i$ in denominator
$= \frac{1}{4}i$	1~~2~~A	
	6	
(5) $y = x^3 - 9x^2 + 30x + 4$		
Slope of tangent $= \dfrac{dy}{dx} = 3x^2 - 18x + 30$	1M+1A	1M for attempt to diff.
A tangent // x-axis iff $3x^2 - 18x + 30 = 0$ for some x.		
Since discriminant $= 18^2 - 4\times3\times30$ $= -36 < 0$	2M+1A	or $3x^2 - 18x + 30 = 3[(x-3)^2 + 1] \neq 0$ 2M+1A
$3x^2 - 18x + 30 = 0$ has no real roots. Hence tangent are never // x-axis	1M	Association of "slope = 0" with "tangent // x-axis".
	6	

Solutions	Marks	Remarks
(6) $\frac{d}{dx}[x+x^2+\dots+x^{n-1}+x^n]$		
$=1+2x+\dots+(n-1)x^{n-2}+nx^{n-1}$	1A	
$\frac{d}{dx}\left[\frac{x(x^n-1)}{x-1}\right]=\frac{(x-1)[(n+1)x^n-1]-(x^{n+1}-x)}{(x-1)^2}$	1M +1A	Quotient Rule
$=\frac{nx^{n+1}-(n+1)x^n+1}{(x-1)^2}$		
$\therefore 1+2x+\dots+(n-1)x^{n-2}+nx^{n-1}=\frac{nx^{n+1}-(n+1)x^n+1}{(x-2)^2}$	1A	
Putting $n=10, x=2$,		
$1+2(2)+3(2)^2+\dots+9(2)^8+10(2)^9$		
$=\frac{10\times2^{11}-11\times2^{10}+1}{(2-1)^2}$	1M	
$=9\times2^{10}+1$	1A	no mark for direct calculation
$=9217$ (9208 from tables)	6	
(7) $\lvert x-2\rvert\le1 \iff -1\le x-2\le1$	1M	e.g. 1A for ① $1<x<3$ ② $1\le x\le a$ ③ $6\le x\le3$
$\iff 1\le x\le3$	2A	
$1\le x\le3 \Rightarrow 1\le x^2\le9$	1M	Graphical method or Checking end-points + sketching graph to support
$-5\le x^2-6\le3$	1M	
$\therefore \lvert x^2-6\rvert\le5$	1A	
the max. value of x^2-6 is 5	6	

Solutions	Marks	Remarks
8 (a) At B, $\frac{ds}{dt} = 0$	2M	
$t = 60 (\text{sec})$	1A	
The boat stops after 60 sec		
$S = \int_0^{60} \frac{ds}{dt}\, dt$	1A+1M +1M	1A 0 1M 60 1M $\int \frac{ds}{dt} dt$
$= \int_0^{60} \sqrt{2}\left(2 - \frac{t}{30}\right) dt$		Alt. method $s = \int \frac{ds}{dt} dt$ 1M $= 2\sqrt{2}t - \frac{\sqrt{2}t^2}{60} + C$ 1A
$= \left[2\sqrt{2}t - \frac{\sqrt{2}t^2}{60}\right]_0^{60}$	1A	$t = 0$ 1M $c = 0$ $t = 60$ 1M
$= 60\sqrt{2}$ m	1A	$S = 60\sqrt{2}$ 1A
$\therefore AB = 60\sqrt{2}$ m	8	
(b) $BC = AB \cos 45°$		
$= 60$	1A	
$AC = BC = 60$	1A	
Let $PC = x$, $AP = 60 - x$		
$PB = \sqrt{PC^2 + BC^2}$		
$= \sqrt{x^2 + 3600}$	1A	
Time required $t = \frac{60 - x}{5} + \frac{\sqrt{x^2+3600}}{3}$	1M+1M	
$\frac{dt}{dx} = -\frac{1}{5} + \frac{2x}{6\sqrt{x^2+3600}}$	2A	
$\frac{dt}{dx} = 0 \Rightarrow \frac{1}{5} = \frac{x}{3\sqrt{x^2+3600}}$	1M	
$\Rightarrow 25x^2 = 9x^2 + 9 \times 3600$	1A	
$x = 45$ ($x = -45$ rejected)	1A	Accept giving 45 also or $x = \pm 45$



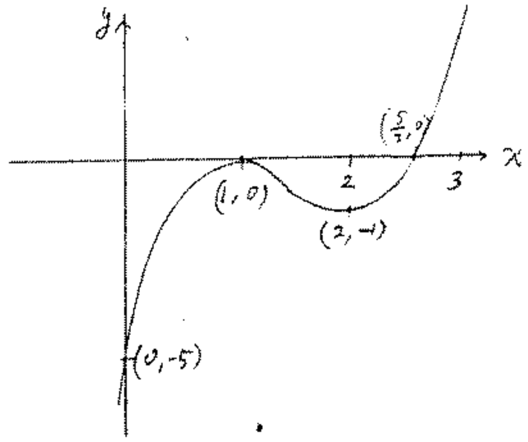
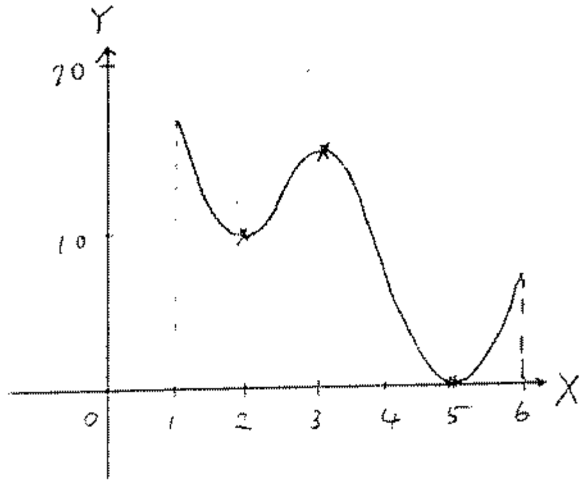
Solutions	Marks	Remarks
8 (cont'd) On checking, it is found that t is a min. at $x=45$		Optional
$t = \frac{60-45}{5} + \frac{\sqrt{45^2+3600}}{3}$	1M	
$= 28$ sec.	1A	
	12	
Alt		
(b) Let $AP = y$		
$PB = \sqrt{(60\sqrt{2})^2 + y^2 - 120\sqrt{2}\,y\cos 45}$	1A	
$= \sqrt{y^2 - 120y + 7200}$	2A ~~1A~~	
$t = \frac{y}{5} + \frac{\sqrt{y^2-120y+7200}}{3}$	1M+1M	
$\frac{dt}{dy} = \frac{1}{5} + \frac{1}{6}\frac{2y-120}{\sqrt{y^2-120y+7200}}$	2A	
$\frac{dt}{dy} = 0 \Rightarrow \frac{3\sqrt{y^2-120y+7200} + 5y - 300}{15} = 0$	1M	
$\Rightarrow 9(y^2-120y+7200) = (300-5y)^2$		
$= 90000 - 3000y + 25y^2$		
$y^2 - 120y + 1575 = 0$	1A ~~1A~~	
$(y-15)(y-105) = 0$		
$y = 15$ ($y = 105$ rejected)	1A	Accept "$y=15$" or "$y=15$ or 105"
On checking, t is min at $y=15$		
$t = \frac{15}{5} + \frac{\sqrt{15^2 - 120\times 15 + 7200}}{3}$	1M	
$= 28$ (sec)	1A	If a cand. write: $t = 28$ or 46, no mark.
	12	



G.P. 405 (2/77)

Solutions	Marks	Remarks
(9) (a) $f(x) \equiv x^3-(p+1)x^2+(p-q)x+q$		
$f(1) = 1-(p+1)+(p-q)+q$ $= 0$	1M	
$\therefore$ $(x-1)$ is a factor of $f(x)$	1A	
$f(x) \equiv (x-1)(x^2-px-q)$	1A	
$x=1$ is a solution of $f(x)=0$		
Let $\sin A = 1$, $A = 90°$	1+1A	
$\therefore \triangle ABC$ is right-angled		
$\sin B$ and $\sin C$ are the roots of $x^2-px-q=0$	1M	
Since ABC is a $\triangle$, $\sin B, \sin C \neq 0$, $\therefore$ $q \neq 0$	1A	
	7	
(b) $R_1 = f(0)$ $= q$	1A	
$R_2 = f(p)$ $= p^3-(p+1)p^2+(p-q)p+q$ $= q-pq$	1A	
$\frac{2q}{p} = R_1-R_2$ $= q-(q-pq)$ $= pq$		
$q(p^2-2)=0$	1A	
Since $q \neq 0$, $p = \pm\sqrt{2}$	1+1A	
$\sin B+\sin C = p$	1M	
$= \sqrt{2}$ (-ve root rejected)	1M	rejecting -ve root and subst.

Solutions	Marks	Remarks
(9) (Cont'd) As $B + C = 90°$		
$C = 90 - B$		
$\sin B + \cos B = \sqrt{2}$	1M+1A	
$\frac{1}{\sqrt{2}}\sin B + \frac{1}{\sqrt{2}}\cos B = 1$		$\sin^2 B + \cos^2 B + 2\sin B\cos B = 2$
$\sin(45° + B) = 1$	1M	$\sin 2B = 1$
$45° + B = 90°$		$2B = 90°$
$B = 45°$, $C = 45°$	1A	$B = 45°$, $C = 45°$
$\therefore \triangle ABC$ is isosceles		
$q = -\sin B \sin C$	1A	May be awarded if given at end of (a)
$= -\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = -\frac{1}{2}$	1A	
	13	
(10) (a) $f(x) = 2x^3 - 9x^2 + 12x - 5$		
$f'(x) = 6x^2 - 18x + 12$	1A	
$= 6(x^2 - 3x + 2)$		
$= 6(x-1)(x-2)$		
$f'(x) = 0 \Leftrightarrow x = 1$ or 2	1M+1A	
$f''(x) = 12x - 18$	1A	
At $x = 1$, $f''(x) = -6 < 0$		
$\therefore (1, 0)$ is a max pt	1A	
At $x = 2$, $f''(x) = 6 > 0$		
$\therefore (2, -1)$ is a min. pt.	1A	

Solutions	Marks	Remarks
(10) (Cont'ed) $f(x) = 2x^3 - 9x^2 + 12x - 5$		
$= (x-1)(x-1)(2x-5)$		
$(1,0)$, $(\frac{5}{2},0)$ and $(0,-5)$ are the points where the curve & the axes meet	0+1+1A	
	1+1	1 mark for shape 1 mark for marking the four points
	10	
(b) (i) Since $\frac{dY}{dX} = 0$ at $X = 2, 3, 5$ Y has stationary values there	1	Accept max/min
At $X=2$, $\frac{d^2Y}{dX^2} > 0$, $(2,10)$ is a min. pt.	1+1A	1A for sign of $\frac{d^2Y}{dX^2}$ 1A for correct conclusion
At $X=3$, $\frac{d^2Y}{dX^2} < 0$, $(3, \cancel{14}\ 15)$ is a max. pt.	1+1A	
At $X=5$, $\frac{d^2Y}{dX^2} > 0$, $(5,0)$ is a min. pt.	1+1A	
	2+1	2 for shape 1 for $1 < X < 6$
	10	

Solutions	Marks	Remarks
(11) (a) Let $z = a+bi$		
$\bar{z} = a-bi$	1A	
(i) $\lvert z\rvert^2 = a^2+b^2$	1A	
$z\bar{z} = (a+bi)(a-bi)$		
$= a^2+b^2$	1A	
$= \lvert z\rvert^2$		
(ii) $-\frac{i}{2}(z-\bar{z}) = -\frac{i}{2}\left[(a+bi)-(a-bi)\right]$		
$= -\frac{i}{2}(2bi)$		
$= b$	1A	
$= \mathrm{Im}(z)$	1A	
	5	
(b)(i) $\lvert p-q\rvert = \lvert p+q\rvert \Rightarrow \lvert p-q\rvert^2 = \lvert p+q\rvert^2$	1M	
$\Rightarrow (p-q)(\overline{p-q}) = (p+q)(\overline{p+q})$	1M	
$\Rightarrow (p-q)(\bar{p}-\bar{q}) = (p+q)(\bar{p}+\bar{q})$	1M	
$\Rightarrow p\bar{p}+q\bar{q}-p\bar{q}-\bar{p}q = p\bar{p}+q\bar{q}+p\bar{q}+\bar{p}q$	1A	
$\therefore\ 2(p\bar{q}+\bar{p}q)=0$		
$p\bar{q}+\bar{p}q=0$	1A	
$\mathrm{Im}\left(\frac{ip}{q}\right) = -\frac{i}{2}\left[\frac{ip}{q}-\left(\overline{\frac{ip}{q}}\right)\right]$	1M	*
$= -\frac{i}{2}\left[\frac{ip}{q}-\frac{\bar{i}\,\bar{p}}{\bar{q}}\right]$	1M	
$= -\frac{i}{2}\left[\frac{ip\bar{q}-\bar{i}\bar{p}q}{q\bar{q}}\right]$		
$= -\frac{i^2}{2}\left[\frac{p\bar{q}+\bar{p}q}{q\bar{q}}\right]$	1A	
$= 0$	1M	

Solutions	Marks
(11) (Cont'd) (b)(ii) $\operatorname{Im}\left(\frac{ip}{q}\right)=0 \Rightarrow \frac{ip}{q}$ is real	2(i)
$\therefore \frac{p}{q}$ is purely imaginary	1M
$\arg\left(\frac{p}{q}\right)=\pm\frac{\pi}{2}$ (Accept $\frac{\pi}{2}$)	1A
$\arg(p)-\arg(q)=\pm\frac{\pi}{2}$	1M
$OP \perp OQ$	1A
	15

Alt

Solutions	Marks
(b)(i) Let $p=a+bi$, $q=c+di$	
$\lvert p-q\rvert=\sqrt{(a-c)^2+(b-d)^2}$	1A
$=\sqrt{a^2+b^2+c^2+d^2-2(ac+bd)}$	1A
$\lvert p-q\rvert=\sqrt{(a+c)^2+(b+d)^2}$	1A
$=\sqrt{a^2+b^2+c^2+d^2+2(ac+bd)}$	1A
$\lvert p-q\rvert=\lvert p+q\rvert$	
$\Rightarrow \sqrt{a^2+b^2+c^2+d^2-2(ac+bd)}=\sqrt{a^2+b^2+c^2+d^2+2(ac+bd)}$	1M
$\therefore ac+bd=0$	1A
$p\bar{q}+\bar{p}q=(a+bi)(c-di)+(a-bi)(c+di)$	1A
$=ac+bd+(bc-ad)i+ac+bd+(ad-bc)i$	
$=2(ac+bd)=0$	1A
$\operatorname{Im}\left(\frac{ip}{q}\right)=\operatorname{Im}\left[\frac{i(a+bi)}{c+di}\right]=\operatorname{Im}\left[\frac{(-b+ai)(c-di)}{c^2+d^2}\right]$	1A
$=\operatorname{Im}\left[\frac{-(ad+bc)+(ac+bd)i}{c^2+d^2}\right]$	1A
$=\frac{ac+bd}{c^2+d^2}=0$	1A

Remarks

Alt(i) $\lvert\vec{p}\rvert^2=a^2+b^2$ (1A)

$\lvert\vec{q}\rvert^2=c^2+d^2$ (1A)

$\lvert\vec{p}-\vec{q}\rvert^2=(a-c)^2+(b-d)^2$ (1A)

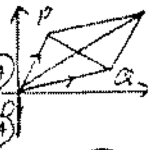
$=a^2+b^2+c^2+d^2-2(ac+bd)$

$=a^2+b^2+c^2+d^2$ (1A)

$\therefore \lvert\vec{p}\rvert^2+\lvert\vec{q}\rvert^2=\lvert\vec{p}-\vec{q}\rvert^2$

$OP \perp OQ$ (2M)

Alt(ii)



$OR=\lvert\vec{p}+\vec{q}\rvert$ (1A)

$PQ=\lvert\vec{p}-\vec{q}\rvert$ (1A)

$\lvert\vec{p}+\vec{q}\rvert=\lvert\vec{p}-\vec{q}\rvert$ (1M)

$\therefore OPRQ$ is a rectangle (2M)

$OP \perp OQ$ (1A)

$\tan(\arg p)=\frac{b}{a}$ ①A

$\tan(\arg q)=\frac{d}{c}$ ①A

$\tan(\arg p-\arg q)=\frac{\frac{b}{a}-\frac{d}{c}}{1+\frac{b}{a}\cdot\frac{d}{c}}=\infty$ ①

$\arg p-\arg q=\frac{\pi}{2}$ ①

$\therefore OP \perp OQ$ ①A

$\frac{p}{q}=\frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$

$\tan\left(\arg\frac{p}{q}\right)=\frac{bc-ad}{ac+bd}$ ①

$=\infty$ ①

$\therefore \arg\frac{p}{q}=\frac{\pi}{2}$ ①A

$\arg p-\arg q=\frac{\pi}{2}$ ①M

$\therefore OP \perp OQ$ ①A

$m_1=\frac{b}{a}$ (1A)

$m_2=\frac{d}{c}$ (1A)

$m_1m_2=\frac{b}{a}\cdot\frac{d}{c}$ (1M)

$=-\frac{ac}{ac}$ (1M)

$=-1$ (1A)

$\therefore OP \perp OQ$ (1A)

G.F. 405 (2/77)

Solutions	Marks	Remarks
(12) (a)(i) $t^2-(b+1)t+(b-1)=0 \quad \ldots\ldots(*)$		
Discriminant $=(b+1)^2-4(b-1)$	1A	
$=b^2-2b+5$		
$=(b-1)^2+4$	1A	
Since discriminant >0, λ_1, λ_2 are real and distinct.	1	
(ii) $(1-\lambda_1)(1-\lambda_2)=1-(\lambda_1+\lambda_2)+\lambda_1\lambda_2$		
$=1-(b+1)+(b-1)$	1M	
$=-1$	1A	
<0		
$\therefore$ either $1-\lambda_1>0$ and $1-\lambda_2<0$	1A	
or $1-\lambda_1<0$ and $1-\lambda_2>0$	1A	
i.e. either $\lambda_1<1<\lambda_2$		
or $\lambda_2<1<\lambda_1$		
	7	
(b) If λ is a root of $(*)$,		
$\lambda^2-(b+1)\lambda+(b-1)=0$	1M	
$b(1-\lambda)=1+\lambda-\lambda^2$		
$b=\dfrac{1+\lambda-\lambda^2}{1-\lambda}$	1A	
$(1-\lambda)\left[(x^2+2x+b)-\lambda(x^2+1)\right]$		
$=(1-\lambda)\left[\left(x^2+2x+\dfrac{1+\lambda-\lambda^2}{1-\lambda}\right)-\lambda(x^2+1)\right]$	1M	
$=(1-\lambda)\left[\dfrac{(1-\lambda)(x^2+2x)+1+\lambda-\lambda^2-\lambda(1-\lambda)(x^2+1)}{1-\lambda}\right]$		
$=(1-2\lambda+\lambda^2)x^2+2(1-\lambda)x+1$	1A	
$=\left[(1-\lambda)x+1\right]^2$	1A	
	5	

Solutions	Marks	Remarks
(12) (c) $(1-\lambda)[(x^2+2x+b)-\lambda(x^2+1)] = [(1-\lambda)x+1]^2$		
≥ 0 (for all real x)	1M	
$\therefore (1-\lambda)\left[\dfrac{x^2+2x+b}{x^2+1}-\lambda\right] \geq 0 \quad (x^2+1>0)$	1M	
Since λ_1, λ_2 are roots of (*)		
$(1-\lambda_1)\left[\dfrac{x^2+2x+b}{x^2+1}-\lambda_1\right] \geq 0$	1M	
and $(1-\lambda_2)\left[\dfrac{x^2+2x+b}{x^2+1}-\lambda_2\right] \geq 0$ for all real x	1M	
If $\lambda_1 < \lambda_2$, by (a) $\lambda_1 < 1 < \lambda_2$	1M	
$1-\lambda_1 > 0 \Rightarrow \dfrac{x^2+2x+b}{x^2+1} - \lambda_1 \geq 0$	1M	
$\dfrac{x^2+2x+b}{x^2+1} \geq \lambda_1$		
and $1-\lambda_2 < 0 \Rightarrow \dfrac{x^2+2x+b}{x^2+1} - \lambda_2 \leq 0$	1M	
$\dfrac{x^2+2x+b}{x^2+1} \leq \lambda_2 \quad \forall x \in R$		
$\therefore \lambda_1 \leq \dfrac{x^2+2x+b}{x^2+1} \leq \lambda_2$	1	
	8	

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Additional Mathematics II

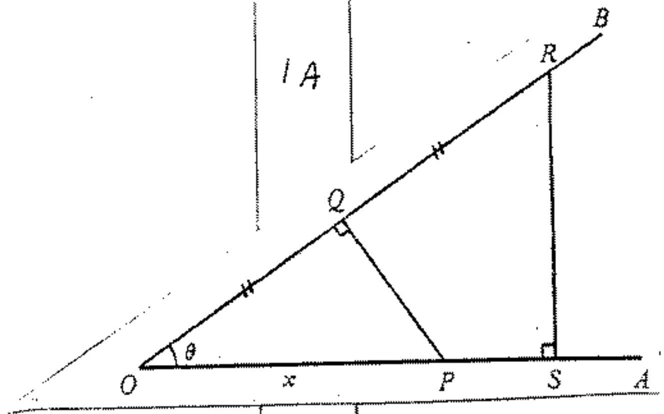
MARKING SCHEME

Solutions	Marks	Remarks
(1) Let $u=\sqrt{x+9}$		
$u^2 = x+9$		
$2u\,du = dx$	1A	
$\int \frac{x}{\sqrt{x+9}}dx = \int \frac{(u^2-9)}{u} 2u\,du$	1A	no mark for ... dx but may proceed to mark remaining part
$= 2\int(u^2-9)du$		
$= \frac{2}{3}u^3 - 18u + C$ (1A, 1A)	1+1A	−1 if omit C throughout
$= \frac{2}{3}(x+9)^{\frac{3}{2}} - 18(x+9)^{\frac{1}{2}} + C$	1A	
	5	
(2) The line through A and B is given by		Alternatively
$y+1 = \frac{-1-1}{3+1}(x-3)$		Let (a,b) divides AB in the ratio
i.e. $x+2y-1=0$ ($2x+4y-2=0$ etc.)	1A	$a=\frac{3r-1}{1+r}$ (1A)
Solving this with $x-y-1=0$	1M	$b=\frac{-r+1}{1+r}$ (1A)
In case of a graph, this is awarded for drawing 2 lines (may be ...)		
$y=0$		Sub. in given line
$x=1$		
$\therefore$ the two lines meet at $C=(1,0)$ (r=1 ... yield)	1A	$\frac{3r-1}{1+r} - \frac{-r+1}{1+r} - 1 =$ (2)
If C divides AB in the ratio $1:r$,		$r=1$ (1)
$1 = \frac{3r-1}{1+r}$ $\left(\text{or } 0=\frac{1-r}{1+r}\right)$	1M	Alt $\frac{AC}{CB} = \frac{\sqrt{(3-1)^2+(-1-0)^2}}{\sqrt{(-1-1)^2+(1-0)^2}}$ (1M)
$r = 1$	1A	$=\frac{\sqrt{5}}{\sqrt{5}}$
$\therefore$ C divides AB in the ratio $1:1$	5	$=1$ (1A)
Alt		
Graphical method acceptable.		

Solutions	Marks	Remarks
(3) $\cos 2\theta - \sqrt{3}\cos\theta + 1 = 0$		
$2\cos^2\theta - \sqrt{3}\cos\theta = 0$	1M	Attempt to express $\cos 2\theta$ in terms of $\cos^2\theta$.
$\cos\theta\,(2\cos - \sqrt{3}) = 0$		
$\cos\theta = 0$ or $\frac{\sqrt{3}}{2}$	1+1A	
For $0 \le \theta \le \frac{\pi}{2}$, $\theta = \frac{\pi}{2}$ or $\frac{\pi}{6}$ (90° or 30°)		
The general soln. is $\theta = 2n\pi \pm \frac{\pi}{2}$	1M + 1A	for $2n\pi \pm \alpha$
or $2n\pi \pm \frac{\pi}{6}$,	1A	
where $n = 0, \pm 1, \pm 2, \ldots$	6	−1 if mixing degree with radian.
Note other variations of answers e.g. $(2n+1)\pi \pm \frac{\pi}{2}$, $(2n+1)\frac{\pi}{2}$, etc.		
(4) Volume $= \pi \int_0^{2\pi} x^2\,dy$	1A	[illegible] wrong limits
$= \pi \int_0^{2\pi} (4 + 4\sin y + \sin^2 y)\,dy$		
$= \pi \int_0^{2\pi} \left(4 + 4\sin y + \frac{1-\cos 2y}{2}\right)dy$	1M	Attempt to express $\sin^2 y$ in terms of $\cos 2y$
$= \pi \left[\frac{9}{2}y - 4\cos y - \frac{\sin 2y}{4} \right]_0^{2\pi}$	1+1+1A	provided limits correct
$= 9\pi^2$	1A	−2 if omit π but otherwise correct
must have the above part.	6	

Solutions	Marks	Remarks
(5) $\int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\cos x+\sin x}dx + \int_0^{\frac{\pi}{2}} \frac{\cos^3 x}{\cos x+\sin x}dx$		
$= \int_0^{\frac{\pi}{2}} \frac{\sin^3 x+\cos^3 x}{\cos x+\sin x}dx$	1A	
$= \int_0^{\frac{\pi}{2}} (\sin^2 x-\sin x\cos x+\cos^2 x)dx$	1A	
$= \int_0^{\frac{\pi}{2}} (1-\frac{\sin 2x}{2})dx$	1A	
$= \left[x+\frac{\cos 2x}{4}\right]_0^{\frac{\pi}{2}}$	1A	
$= \frac{\pi}{2}-\frac{1}{2}$	1A	
$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\cos x+\sin x}dx = \frac{1}{2}\left(\frac{\pi}{2}-\frac{1}{2}\right)$	1M	
	6	
(6) Let $Q=(x,y)$		
$x=\frac{2x_1+9}{5}$	1A	
$y=\frac{2y_1}{5}$	1A	
$\therefore x_1=\frac{5x-9}{2}$, $y_1=\frac{5y}{2}$	1M+1A	1M for attempt to change subject
Since (x_1,y_1) lies on the circle		
$\left(\frac{5x-9}{2}\right)^2+\left(\frac{5y}{2}\right)^2=4$	1M	
$5x^2+5y^2-18x+13=0$	1A	$5a^2-5b^2-18x+13=$ etc. acceptable
or $\left(x-\frac{9}{5}\right)^2+y^2=\frac{16}{25}$		

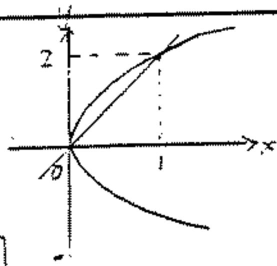
Solutions	Marks	Remarks
7 (a) $\Delta_1 =$ area of $\triangle OPQ$		
$= \frac{1}{2} PQ \times OQ$		
$= \frac{1}{2} x^2 \sin\theta \cos\theta$	1A	
$\Delta_2 =$ area of $\triangle ORS$		
$= \frac{1}{2} RS \times OS$		
$= \frac{1}{2} (2x\cos\theta\sin\theta)(2x\cos^2\theta)$		
$= 2x^2 \sin\theta \cos^3\theta$	1A	
(b) $\frac{d\Delta_1}{dx} = x\cos\theta\sin\theta$	1A	-1 if write $\frac{d\Delta_1}{dt}$
$\frac{d\Delta_2}{dx} = 4x\sin\theta\cos^3\theta$	1A	-1 if write $\frac{d\Delta_2}{dt}$
$\frac{d\Delta_1}{dx} = \frac{d\Delta_2}{dx}$	1M	
$\Rightarrow x\cos\theta\sin\theta = 4x\sin\theta\cos^3\theta$		
$\Rightarrow \cos^2\theta = \frac{1}{4}$		
$\Rightarrow \cos\theta = \frac{1}{2}$		
$\theta = 60^\circ$ $\left(\frac{\pi}{3}\right)$	1A	
	6	



Solutions	Marks	Remarks
8 (a) Distance from M to L $= \left\lvert \dfrac{5x+12y-32}{\sqrt{25+144}} \right\rvert$	1M	
$= \dfrac{65}{13} = 5$ (r)	1A	
Equation of C is $(x-5)^2+(y-6)^2=25$	1M+1A	for r^2
or $x^2+y^2-10x-12y+36=0$ ....(*)	4	
(b) Since distance from M to y-axis $=5$,	2A	
C also touches the y-axis	2	
(c) Let $y=mx$ be a tangent to C through O.	1A	Alt: $y=mx$ (1A)
Solving with (*), $x^2+m^2x^2-10x-12mx+36=0$	1M+1A	Dist. from M to $y=m$[x]
$(1+m^2)x^2-(10+12m)x+36=0$		$= \left\lvert \dfrac{5m-6}{\sqrt{m^2+1}} \right\rvert$ (1M)
For tangency, $(10+12m)^2-4(1+m^2)36=0$	1M	$=5$ (1A); $\dfrac{(5m-6)^2}{m^2+1}=25$ (1M)
$240m-44=0$		$25m^2-60m+36=25m^2+2$[5]
$m=\dfrac{11}{60}$	1A	$m=\dfrac{11}{60}$ (1[A])
∴ the other tangent is $y=\dfrac{11}{60}x$	1A	$y=\dfrac{11}{60}x$ (1[A])
or $11x-60y=0$	6	
(d) Slope of PQ $=\dfrac{6-2}{5-2}=\dfrac{4}{3}$	1A	Alt: $P=(2,2)$, $M=(5,6)$
Eqn of PQ is $y-2=\dfrac{4}{3}(x-2)$		∴ $Q=(8,10)$
$4x-3y-2=0$ $\left\{ \begin{array}{l} x^2+y^2+ax+by+c=0 \\ c=0 \end{array} \right.$	1A	Let circle be $x^2+y^2+ax+by=0$
Eqn of family of circle is		Sub. P, Q.
$x^2+y^2-10x-12y+36+k(4x-3y-2)=0$	2M+1A	$4+4+2a+2b=0$; $64+100+8a+10b=$[0]
[or $4x-3y-2+k(x^2+y^2-10x-12y+36)=0$]		$b=-66$ (1A)
Putting $(x,y)=(0,0)$, $36-2k=0$	1M	$a=62$ (1A)
$k=18$ (or $\frac{1}{18}$)	1A	∴ $x^2+y^2+62x-66y$[=0]
∴ req'd eqn is $x^2+y^2+62x-66y=0$	1A	
	8	

Solutions	Marks	Remarks
9 (a) $y^2 = 4x$		Alt
$2yy' = 4$		Eqn. of tangent
$y' = \frac{2}{y}$.	1A	is $yy_1 = 2(x + x_1)$ 1M
Eqn of PR is $y - 2s = \frac{2}{2s}(x - s^2)$	1M	At $(s^2, 2s)$,
		$2sy = 2(x + s^2)$
i.e. $y = \frac{1}{s}x + s$ (or $x - sy + s^2 = 0$)	1A	or $x - sy + s^2 = 0$ 1A
Similarly, eqn of QR is		
$y = \frac{1}{t}x + t$ (or $x - ty + t^2 = 0$)	1A	
Solving these two eqns.		
$\frac{1}{s}x + s = \frac{1}{t}x + t$		
$x = st$	1A	
$y = s + t$	1A	
$\therefore R = (st, s+t)$	6	
(b) If $\frac{1}{s} + \frac{1}{t} = 2$	1M+1A	
$\frac{s+t}{st} = 2$		
$\frac{y}{x} = 2$	2A	
$\therefore R$ must lie on the line $y = 2x$	4	
(c) Solving $\begin{cases} y^2 = 4x \\ y = 2x \end{cases}$		
$y = 0$ or 2		
($x = 0$ or 1)		
$(x, y) = (0, 0)$ or $(1, 2)$	1+1A	

Solutions	Marks	Remarks
9 (cont'd) Area enclosed $= \int_0^2 \left(\frac{y}{2} - \frac{y^2}{4}\right) dy$	1M	for ~
$= \left[\frac{y^2}{4} - \frac{y^3}{12}\right]_0^2$ must be correct	1A+1A	Alt $\int_0^1 (2\sqrt{x} - 2x)\,dx$
$= \frac{1}{3}$	1A	$= \left[\frac{4}{3}x^{\frac{3}{2}} - x^2\right]_0^1$ must be correct limits
Alt $A = \int_0^2 \frac{y}{2}dy - \int_0^2 \frac{y^2}{4}dy$ (1M) $= \left[\frac{y^2}{4}\right]_0^2 - \left[\frac{y^3}{12}\right]_0^2$ (1+1A) $= \frac{1}{3}$ (1A)		$= \frac{1}{3}$
	6	
(d) Vol. generated $= \pi \int_0 [4x - (2x)^2]\,dx$	1M	for $V_1 \sim V_2$
$= \pi \left[2x^2 - \frac{4}{3}x^3\right]_0^1$ must be correct	1A+1A	-2 if omit π but otherwise correct
$= \frac{2}{3}\pi$	1A	
Alt $V = \pi\int_0^1 4x\,dx - \pi\int_0^1 (2x)^2 dx$ (1M) $= \pi[2x^2]_0^1 - \pi\left[\frac{4}{3}x^3\right]_0^1$ (1+1A) $= \frac{2}{3}\pi$ (1A)		
	4	
10 (a) Solving $\begin{cases} 3x - 2y - 8 = 0 \\ x - y - 2 = 0 \end{cases}$	1M	Alt $3x - 2y - 8 + k(x - y - 2)$
$x = 4,\ y = 2$ $\therefore P = (4, 2)$	1+1A	Slope $= \frac{3+k}{2+k}$
Eqn. of L_1 is $y - 2 = \frac{1}{2}(x - 4)$	1M	$\frac{3+k}{2+k} = \frac{1}{2} \Rightarrow k = -4$ either one (1M) $\frac{3+k}{2+k} = 2 \Rightarrow k = -1$
$x - 2y = 0$	1A	L_1: $x - 2y = 0$
Eqn. of L_2 is $y - 2 = 2(x - 4)$		L_2: $2x - y - 6 = 0$
$2x - y - 6 = 0$	1A	
	6	
(b) Eqn. of L is $y = m(x - 2)$	1A	
Solving with L_1, $2m(x - 2) = x$		
$x = \frac{4m}{2m-1},\ y = \frac{2m}{2m-1}$	1+1A	

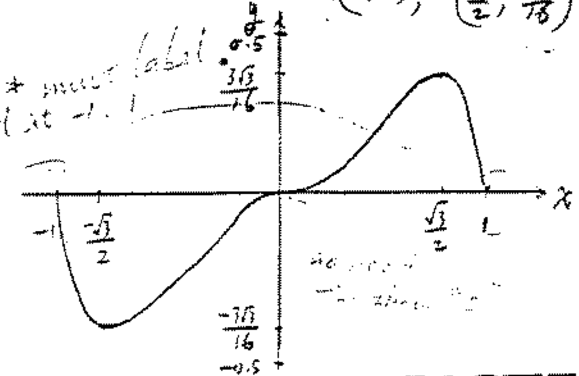


G.F. 405 (2/77)

Solutions	Marks	Remarks
(13) (Cont'd) $\therefore A = \left(\frac{4m}{2m-1}, \frac{2m}{2m-1}\right)$		
Solving L with L_2,		
$m(x-2) = 2x-6$		
$x = \frac{2m-6}{m-2}$	1A	
$y = m\left[\frac{2m-6}{m-2} - 2\right]$		
$= \frac{-2m}{m-2}$	1A	
$\therefore B = \left(\frac{2m-6}{m-2}, \frac{-2m}{m-2}\right)$		
Area of $\Delta PAB = \frac{1}{2}\begin{vmatrix} 4 & 2 \\ \frac{4m}{2m-1} & \frac{2m}{2m-1} \\ \frac{2m-6}{m-2} & \frac{-2m}{m-2} \\ 4 & 2 \end{vmatrix}$	1M + 2A	−1 if omit $\frac{1}{2}$
$= \frac{1}{2}\left[\frac{8m}{2m-1} - \frac{8m^2}{(2m-1)(m-2)} + \frac{2(2m-6)}{m-2} - \frac{2m(2m-6)}{(2m-1)(m-2)} - \frac{8m}{2m-1} + \frac{8m}{m-2}\right]$		
$= \frac{6(m^2-2m+1)}{(m-2)(2m-1)}$		
$= \frac{6(m-1)^2}{(m-2)(2m-1)}$		
$\frac{d\Delta}{dm} = 6\,\frac{(m-2)(2m-1)(2m-2) - (m-1)^2(4m-5)}{(m-2)^2(2m-1)^2}$	1M	attempt to diff.
$= \frac{6(1-m^2)}{(m-2)^2(2m-1)^2}$	1A	
$\frac{d\Delta}{dm} = 0 \Rightarrow m = \pm 1$	1A	
If $m=1$, L is the line PQ, rejected. Take $m=-1$	1A	
Checking that $m=-1$, Δ is a min.	1M.	Attempt to check
Eqn. of L is $y = -(x-2)$ $x+y-2=0$	1A	
	14	

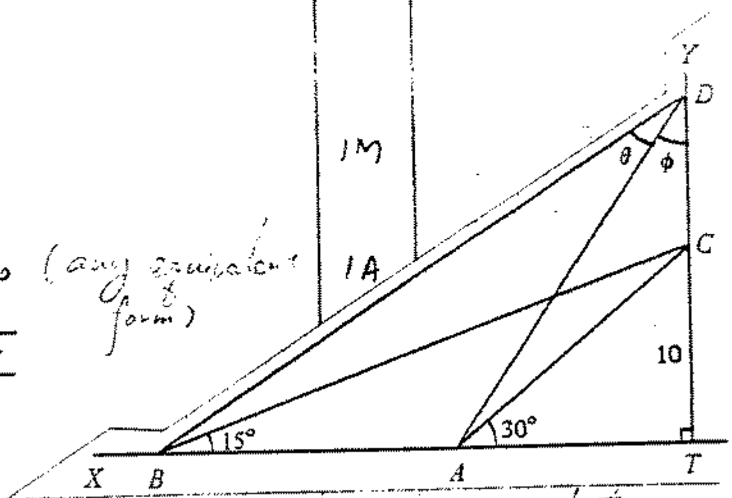
G.F. 405 (2/77)

Solution	Marks	Remarks
(11) (a) Putting $u = \cos\theta$, $du = -\sin\theta\, d\theta$	1A	
When $\theta = 0$, $u = 1$; $\theta = \frac{\pi}{2}$, $u = 0$.	1A	
$\int_0^{\frac{\pi}{2}} \sin^3\theta\cos^2\theta\, d\theta = -\int_1^0 (1-u^2)u^2\, du$	1M + 1A	for sub. of limits; for integrand
$= \int_0^1 (u^2 - u^4)\, du$		Alt $I = -\int_0^{\frac{\pi}{2}} \sin^2\theta\cos^2\theta\, d\cos\theta$ (1A) + (1
$= \left[\frac{u^3}{3} - \frac{u^5}{5}\right]_0^1$	1A	$= -\int_0^{\frac{\pi}{2}} (1-\cos^2\theta)\cos^2\theta\, d\cos\theta$ (1
$= \frac{2}{15}$ be correct	1A	$= -\int_0^{\frac{\pi}{2}} (\cos^2\theta - \cos^4\theta)\, d\cos\theta$ (1
	6	$= -\left[\frac{\cos^3\theta}{3} - \frac{\cos^5\theta}{5}\right]_0^{\frac{\pi}{2}}$
(b) (i) The range is $-1 \le x \le 1$	1A	$= \frac{2}{15}$ (1A)
(ii) Putting $y = 0$, $x = 0$ or ± 1		
$\therefore$ C meets the x-axis at $(0,0)$, $(1,0)$, $(-1,0)$	1A	
(iii) $y = x^3\sqrt{1-x^2}$		
$\frac{dy}{dx} = 3x^2(1-x^2)^{\frac{1}{2}} + \frac{1}{2}\cdot\frac{x^3(-2x)}{(1-x^2)^{\frac{1}{2}}}$		
$= \frac{x^2(3-4x^2)}{(1-x^2)^{\frac{1}{2}}}$		
$\frac{dy}{dx} = 0 \Rightarrow x = 0$ or $\pm\frac{\sqrt{3}}{2}$ (± 0.8660); $y = 0$ or $\pm\frac{3\sqrt{3}}{16}$ (± 0.3248)	1M	
$\therefore$ the 3 pts. are $(0,0)$, $\left(\frac{\sqrt{3}}{2}, \frac{3\sqrt{3}}{16}\right)$, $\left(-\frac{\sqrt{3}}{2}, -\frac{3\sqrt{3}}{16}\right)$	1+1+1A	$\begin{cases} x = \dots \\ y = \dots \end{cases}$
	6	
(c) * must label [illegible]	1	general shape
	1*	range
	1	tangent at $(0,0)$
	3	



G.P. 405 (2/77)

Solutions	Marks	Remarks
(11) (cont'd)		
(d) Putting $x = \sin\theta$, $dx = \cos\theta\, d\theta$	1A	
when $x = 0$, $\theta = 0$; $x = 1$, $\theta = \frac{\pi}{2}$.	1A	
Area bounded $= 2\int_0^1 x^3\sqrt{1-x^2}\,dx$	1M	for $2 \times$ area $\int_0^1 y\,dx$ or sum of areas $\left\lvert\int_{-1}^{0} y\,dx\right\rvert + \left\lvert\int_0^1 y\,dx\right\rvert$
$= 2\int_0^{\frac{\pi}{2}} \sin^3\theta\cos^2\theta\, d\theta$	1A	
$= \frac{4}{15}$	1A	
	5	
(12) (a) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$	1A	
$\frac{1}{\sqrt{3}} = \tan 30^\circ$ $= \dfrac{2\tan 15^\circ}{1-\tan^2 15^\circ}$	1M	
$\therefore \tan^2 15^\circ + 2\sqrt{3}\tan 15^\circ - 1 = 0$ (any equivalent form)	1A	
$\tan 15^\circ = \dfrac{-2\sqrt{3} \pm \sqrt{12+4}}{2}$		
$= -\sqrt{3} \pm 2$		
$= 2-\sqrt{3}$ (-ve root rejected)	1A	no need to simplify
	4	
(b) (i) $BT = \dfrac{10}{\tan 15^\circ} = \boxed{\dfrac{10}{2-\sqrt{3}}}$ $(= \boxed{37.32})$	1A	**Alt**
$AT = \dfrac{10}{\tan 30^\circ} = \boxed{10\sqrt{3}}$ $(= \boxed{17.32})$	1A	$AC = \dfrac{10}{\sin 30^\circ} = 20$ (1A)
$\therefore AB = \dfrac{10}{2-\sqrt{3}} - 10\sqrt{3}$		$\angle ACB = (30^\circ - 15^\circ) = 15^\circ$ (1
$= 20$	1A	$\therefore AB = AC = 20$ (1



Solutions	Marks	Remarks
12 (cont'd) (b)(ii) $\tan\phi = \frac{10\sqrt{3}}{h}$ $\left(\frac{17.3}{h}\right)$	1A	
$\tan(\theta+\phi) = \frac{10}{(2-\sqrt{3})h}$	1A	
$\tan(\theta+\phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$	1A	
$\frac{10}{(2-\sqrt{3})h} = \frac{\tan\theta + \frac{10\sqrt{3}}{h}}{1 - \tan\theta \times \frac{10\sqrt{3}}{h}}$	1M+ 1A	
$= \frac{h\tan\theta + 10\sqrt{3}}{h - 10\sqrt{3}\tan\theta}$		
$(2-\sqrt{3})h^2\tan\theta + 10\sqrt{3}(2-\sqrt{3})h = 10h - 100\sqrt{3}\tan\theta$		
$[(2-\sqrt{3})h^2 + 100\sqrt{3}]\tan\theta = 10h - (20\sqrt{3}-30)h$		
$\tan\theta = \frac{(40-20\sqrt{3})h}{(2-\sqrt{3})h^2 + 100\sqrt{3}}$		
$= \frac{20h}{h^2 + 100(3+2\sqrt{3})}$		
(iii) If AB subtends equal angles at D & C,		Alt
since $\angle ACB = \angle ABC = 15^\circ$		If $\angle ADB = \angle ACB$, ABDC are concyclic
$\theta = \angle ADB = 15^\circ$	1A	$\angle\phi = \angle ABC = 15^\circ$ (1A)
$\therefore \tan\theta = \frac{20h}{h^2 + 100(3+2\sqrt{3})}$		
$= 2-\sqrt{3}$	1M	$h = \frac{AT}{\tan 15^\circ}$ (1M)
$(2-\sqrt{3})h^2 - 20h + 100\sqrt{3} = 0$ (any equivalent form)	1A	$= \frac{10\sqrt{3}}{2-\sqrt{3}}$ (1A)
$h = \frac{20 \pm \sqrt{400 - 400\sqrt{3}(2-\sqrt{3})}}{2(2-\sqrt{3})}$		$= 10(3+2\sqrt{3})$
$= \frac{10 \pm 10\sqrt{4-2\sqrt{3}}}{2-\sqrt{3}}$		
$= \frac{10 \pm 10(\sqrt{3}-1)}{2-\sqrt{3}}$ $(64.64$ or $10)$		
$= \frac{10\sqrt{3}}{2-\sqrt{3}}$ $(64.64$, $h=10$ rejected$)$	1A	
$= 10(3+2\sqrt{3})$		

Solutions	Marks	Remarks
(12) (Cont'd)		
θ is greatest when $\tan\theta$ is greatest.		
$\dfrac{d(\tan\theta)}{dh} = \dfrac{20(h^2+100(3+2\sqrt{3})) - 40h^2}{[h^2+100(3+2\sqrt{3})]^2}$	1M	Attempt to diff
$= \dfrac{2000(3+2\sqrt{3}) - 20h^2}{[h^2+100(3+2\sqrt{3})]^2}$		
$\dfrac{d(\tan\theta)}{dh} = 0 \Rightarrow h = 10\sqrt{3+2\sqrt{3}}$ $\doteqdot \boxed{25.42}$ (−ve root rejected)	2A	
If $h \lessgtr 10\sqrt{3+2\sqrt{3}}$ slightly, $\dfrac{d(\tan\theta)}{dh} \gtrless 0$	1M	Attempt to check
$\therefore$ when θ is max., $h = 10\sqrt{3+2\sqrt{3}}$		
	16	

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