香港考試局
HONG KONG EXAMINATIONS AUTHORITY

一九八一年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1981

Additional Mathematics I

MARKING SCHEME
評卷參考

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Solution	Marks	Remarks
1. $(1+2x)^4(1-x)^7$		
$=[1+4(2x)+6(2x)^2+\cdots][1+7(-x)+21(-x)^2+\cdots]$	2+1A	2A for first given factor
$=1+x-11x^2+\cdots$	1A	
Coefficient of x^2 is -11.	1A	
	5	
Alternatively,		
$(1+2x)^4(1-x)^7$		
$=[1+4(2x)+6(2x)^2+\cdots][1+7(-x)+21(-x)^2+\cdots]$	2+1A	
Coeff. of $x^2 = 1\times 21+8\times(-7)+24\times 1$	1A	
$=-11$	1A	
	5	
Alternatively,		
Coeff of x^2		
$={}_4C_2(2)^2\times 1+{}_4C_1\times 2\times {}_7C_1\times(-1)+1\times {}_7C_2$	1+1+1A	
$=24\times 1+8\times(-7)+1\times 21$	1A	
$=-11$	1A	
	5	
2. $\log_{78}52=\dfrac{\log_3 52}{\log_3 78}$	1M	or any ~~suitable~~ base
$=\dfrac{\log_3 4+\log_3 13}{\log_3 2+\log_3 3+\log_3 13}$	1M	for $\log(ab)=\log a+\log b$
$=\dfrac{2\log_3 2+\log_3 13}{\log_3 2+\log_3 3+\log_3 13}$	1M	for $\log(a^p)=p\log a$
$=\dfrac{2a+b}{1+a+b}$	2A	有闲 award
	5	
Alternatively,		
$\left.\begin{array}{l}\log_3 2=a\Rightarrow 2=3^a\\ \log_3 13=b\Rightarrow 13=3^b\end{array}\right\}$	1A	
Let $\log_{78}52=x$	1M	
$78^x=52$		
$(2\times 3\times 13)^x=2\times 2\times 13$		
$2^{x-2}\times 3^x\times 13^{x-1}=1$		Put $2=3^a$, $13=3^b$
$(3^a)^{x-2}\times 3^x\times(3^b)^{x-1}=3^0$	1M	
$ax-2a+x+bx-b=0$	1M	Equate indices
$x=\dfrac{2a+b}{1+a+b}$	1A	
	5	

G.P. 405 (2/77)

Solution	Marks	Remarks
3. Put $u = \frac{x-1}{x+1}$		
$\frac{dy}{du} = \sec^2 u$	1A	
$\frac{du}{dx} = \frac{(x+1)-(x-1)}{(x+1)^2}$		
$= \frac{2}{(x+1)^2}$	1A	
$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ Chain Rule	2M	
$= \frac{2}{(x+1)^2} \cdot \sec^2\left(\frac{x-1}{x+1}\right)$	1A	
	5	
4. $x = \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2}$		
$= \cos\theta \pm i\sin\theta$	1A	De Moivre's Theorem
$x^n = \cos n\theta \pm i \sin n\theta$	1M+1A	
$\therefore$ Reqd eqt. is		Alternatively,
$(x - \cos n\theta - i\sin n\theta)(x - \cos n\theta + i\sin n\theta) = 0$	1M	Sum of roots $= 2\cos n\theta$
		Prod. of roots $= 1$
$x^2 - 2\cos n\theta \cdot x + 1 = 0$ deduct 1 mark	2A	1M + 1A
	6	$x^2 - 2\cos n\theta \cdot x + 1 = 0$ 1A
5. $f(x) = (x + \frac{a}{2})^2 + (b - \frac{a^2}{4})$	1+1A	Alternatively,
[illegible] $\geq b - \frac{a^2}{4}$ (realize $(x+\frac{a}{2})^2 \geq 0$)	1M	$\frac{df}{dx} = 2x + a$ 1A
$= f(-\frac{a}{2})$	1A	$\frac{df}{dx} = 0$
		$\Rightarrow x = -\frac{a}{2}$ 1A
Alternatively,		
$f(x) - f(-\frac{a}{2})$ 相減	1M	$\frac{d^2f}{dx^2} = 2 > 0$
$= (x^2 + ax + b) - (\frac{a^2}{4} - \frac{a^2}{2} + b)$		$\therefore f$ is min. at $-\frac{a}{2}$ 1A
$= x^2 + ax + \frac{a^2}{4}$		Since f is quadratic,
$= (x + \frac{a}{2})^2$	1A	$\therefore f(x) \geq f(-\frac{a}{2})$. 1M
≥ 0 ~~for all x~~	1A	
$\therefore f(x) \geq f(-\frac{a}{2})$ for all x	1A	explain this is absolute minimum
The min. value of $x^2 - \sqrt{13}x + 5$ is		
$(\frac{\sqrt{13}}{2})^2 - \sqrt{13} \cdot \frac{\sqrt{13}}{2} + 5$ or $5 - \frac{1}{4}(\sqrt{13})^2$	1M	
$= \frac{7}{4}$	1A	
	6	

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Solution $\log_a x^y = y\log_a x$	Marks	Remarks

6. $a^x = (ab)^z$, $b^y = (ab)^z$

$x\log a = z\log(ab)$, $y\log b = z\log(ab)$ — 1+1M

$x = \dfrac{\log(ab)}{\log a}\cdot z$ $\quad y = \dfrac{\log(ab)}{\log b}\cdot z$ — 1+1A

$\dfrac{xy}{x+y} = \dfrac{\dfrac{\log(ab)}{\log a}\cdot z\cdot\dfrac{\log ab}{\log b}\cdot z}{\dfrac{\log ab}{\log a}\cdot z + \dfrac{\log ab}{\log b}\cdot z}$ — 1M

$= \dfrac{\log(ab)\cdot z}{\log a + \log b}$

$= z$ — 1A

6

Remarks: Alternatively,

$x\log a = y\log b$ 1M

$x\log a = z(\log a + \log b)$ 1M

$(x-z)\log a = z\log b$ 1M + 1A

$\therefore \dfrac{x}{x-z} = \dfrac{y}{z}$ 1M

$z = \dfrac{xy}{x+y}$ 1A

eliminate $\log a$, $\log b$

Alternatively,

$a^x = b^y \Rightarrow b = a^{\frac{x}{y}}$ — 1A

$(ab)^z = (a\cdot a^{\frac{x}{y}})^z$ 代 $b = a^{\frac{x}{y}}$ — 1M

$= a^{\frac{yz+xz}{y}}$ — 1A

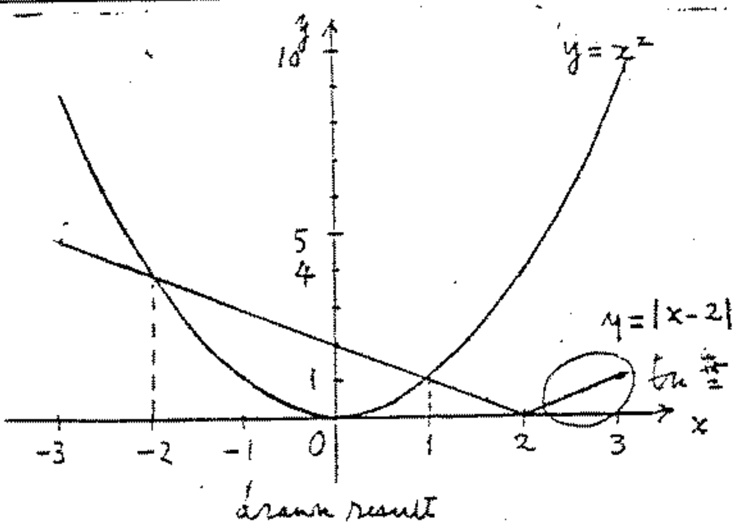
$\therefore a^x = a^{\frac{yz+xz}{y}}$ equate — 1M

$x = \dfrac{yz+xz}{y}$ equate index — 1M

$z = \dfrac{xy}{x+y}$ — 1A

6

7.



drawn result

x	−3	−2	−1	0	1	2	3
x^2	9	4	1	0	1	4	9
$\lvert x-2\rvert$	5	4	3	2	1	0	1

1A For graph of $y = x^2$

2A For graph of $y = |x-2|$

如全線則為 2A 及 1+1+1 A

From the graphs, $|x-2| < x^2$ — 1M

iff $\underline{x < -2}$ $\underline{\text{or}}$ $\underline{x > 1}$ — 1+1+1A

(1 1 1)

If equality sign included in the answer, deduct 1 mark.

7

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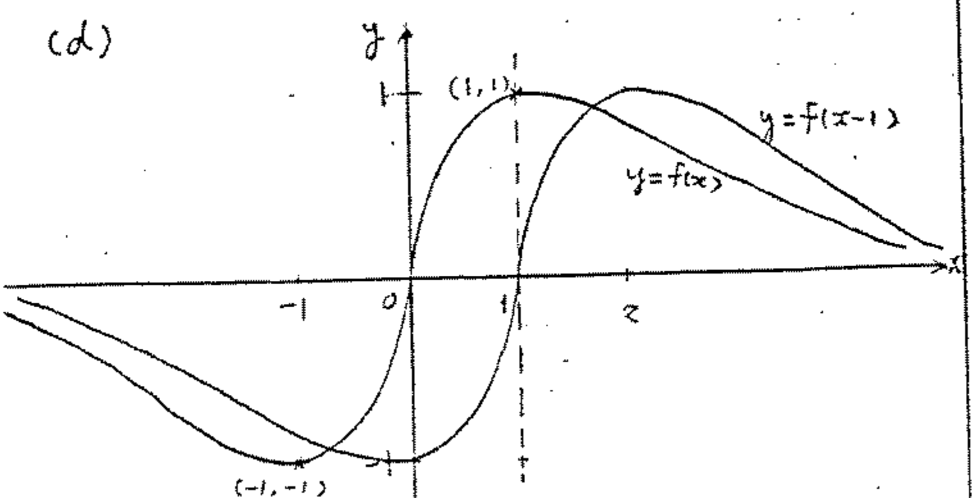
Solution	Marks	Remarks
8. (a) $f(-x) = \dfrac{2(-x)}{(-x)^2+1}$ 代入$(-x)$ →	1M	il must
$= \dfrac{-2x}{x^2+1}$		
$= -f(x)$ for all x	1A	
	2	
(b) $\dfrac{dy}{dx} = \dfrac{2(x^2+1)-2x \cdot 2x}{(x^2+1)^2}$	1M	← For Quotient Rule
$= \dfrac{2-2x^2}{(x^2+1)^2}$	1A	
$\dfrac{d^2y}{dx^2} = \dfrac{-4x(x^2+1)^2-(2-2x^2)\cdot 2(x^2+1)\cdot 2x}{(x^2+1)^4}$	1M	← For Quotient Rule
$= \dfrac{4x(x^2+1)[-(x^2+1)-(2-2x^2)]}{(x^2+1)^4}$		
$= \dfrac{4x(x^2-3)}{(x^2+1)^3}$	2A	Accept $\dfrac{d^2y}{dx^2} = \dfrac{4(x^5-2x^3-3x)}{(x^2+1)^4}$
	5	
(c) Put $\dfrac{dy}{dx}=0$ Put $\dfrac{dy}{dx}=0$	1M	
$x = \pm 1$	1+1A	
At $x=1$, $\dfrac{d^2y}{dx^2}<0$, ∴ f is max.	1M	← Test max. or min. from $\dfrac{d^2y}{dx^2}$
Awarded independent of the acy of $\dfrac{d^2y}{dx^2}$ → ∴ Max. point $=(1,1)$ 擡部有分	1A	
At $x=-1$, $\dfrac{d^2y}{dx^2}>0$, ∴ f is min.	1M	← and realise that max. or min. may be determined from y''
→ ∴ Min. point $=(-1,-1)$	1A	
	7	
(d) 	1A	For (0,0)
	1A	For the 2 turning pts
	1A	For 2 tails not cutting x-axis
	1M	shifting horizontally
	1M	shifting 1 unit towards the right
	1A	for graph of $y=f(x-1)$
	6	

Solutions	Marks	Remarks
(9) (a) $V = \frac{4}{3}\pi r^3 + \pi r^2 h$	1+1A	
$\therefore h = \frac{V}{\pi r^2} - \frac{4}{3}r$	1A	
	3	
(b) (i) Surface area of cylinder $= 2\pi r h$	1A	
$= 2\pi r\left(\frac{V}{\pi r^2} - \frac{4}{3}r\right)$	1M	Sub. h
Surface area of ends $= 4\pi r^2$	1A	
$\therefore C = 2\pi r\left(\frac{V}{\pi r^2} - \frac{4}{3}r\right)k + 4\pi r^2 \times 2k$	1+1M	$\leftarrow C = \underset{\text{cylinder}}{S_C}\, k + \underset{\text{ends}}{S_E}\, 2k$
$= \frac{2kV}{r} + \frac{16}{3}k\pi r^2$	1A	
(ii) $\frac{dC}{dr} = -\frac{2kV}{r^2} + \frac{32}{3}k\pi r$	1A	
用 $-\frac{1}{\pi}\frac{dc}{dr}$		
$\frac{dC}{dr} = 0 \Rightarrow (32k\pi r^3 - 6kV) = 0$	1M	Put $\frac{dc}{dr} = 0$
$r = \sqrt[3]{\frac{3V}{16\pi}}$	1A	Alternatively, Checking sign of $\frac{dc}{dr}$ 2M
$\frac{d^2C}{dr^2} = \frac{4kV}{r^3} + \frac{32}{3}k\pi$ $\qquad$ $\frac{d^2c}{dr^2}$ 用 $\rightarrow$	1M+1A	Correct working 2A
If $r = \sqrt[3]{\frac{3V}{16\pi}}$, $\frac{d^2C}{dr^2} > 0$ $\qquad$ Test sign of $\frac{d^2c}{dr^2}$ $\rightarrow$	1M+1A	
$\therefore$ C is min. awarded only if the above is correct		
(iii) $\frac{r}{h} = \frac{r}{\frac{V}{\pi r^2} - \frac{4}{3}r}$ $\qquad$ 代 h from (a)	1M	
$= \frac{3\pi r^3}{3V - 4\pi r^3}$		
$= \frac{3\pi \cdot \frac{3V}{16\pi}}{3V - 4\pi \cdot \frac{3V}{16\pi}}$	1M	
$= \frac{1}{4}$	2A	
$\therefore r : h = 1 : 4$	17	

Solutions	Marks	Remarks
(10) (a) $\frac{df}{dx} = 3x^2 + 2ax + b$ 識 $\frac{df}{dx}$	1M+1A	
f has stationary values at α, β		
$\Rightarrow \alpha, \beta$ are roots of $3x^2+2ax+b=0$	2M	← may be omitted if the followings are correct
$\therefore \alpha+\beta = -\frac{2a}{3}$	1A	
$\alpha\beta = \frac{b}{3}$	1A	
	6	
(b) Discriminant of $f'(x)=0$ is	1M	Alternatively,
$(2a)^2 - 4(3b)$		$(\alpha-\beta)^2 > 0$ 1M
$= 4a^2 - 12b$ 搵 D		$\alpha^2+\beta^2 > 2\alpha\beta$
Since $\alpha \neq \beta$ (α, β real)		$(\alpha+\beta)^2 > 4\alpha\beta$ 1A
$\therefore 4a^2-12b > 0$ 証 $D > 0$	1M	$\therefore \frac{4a^2}{9} > 4 \cdot \frac{b}{3}$
i.e. $a^2 > 3b$	1A	i.e. $a^2 > 3b$ 1A
	3	$a = -\frac{3}{2}(\alpha+\beta)$, $b = 3\alpha\beta$ 1M
(c) $\frac{f(\alpha)-f(\beta)}{\alpha-\beta} = \frac{(\alpha^3+a\alpha^2+b\alpha+c)-(\beta^3+a\beta^2+b\beta+c)}{\alpha-\beta}$	1M 代 α, β	$a^2-3b = \frac{9}{4}(\alpha+\beta)^2 - 9\alpha\beta$
$= \frac{(\alpha^3-\beta^3)+a(\alpha^2-\beta^2)+b(\alpha-\beta)}{\alpha-\beta}$		$= \frac{9}{4}(\alpha-\beta)^2$ 1A
		> 0 1A
$= \alpha^2+\alpha\beta+\beta^2+a(\alpha+\beta)+b$	2A	
$= (\alpha+\beta)^2-\alpha\beta+a(\alpha+\beta)+b$	1M	For $\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta$
以 $\alpha+\beta$, $\alpha\beta$ → $= \frac{4a^2}{9} - \frac{b}{3} - \frac{2a^2}{3} + b$	1M+1A	
$= \frac{2}{9}(3b-a^2)$	1A	
	7	
(d) Since $a^2 > 3b$, $\frac{2}{9}(3b-a^2) < 0$	1M	
$\therefore \frac{f(\alpha)-f(\beta)}{\alpha-\beta} < 0$	1M	
$\therefore f(\alpha)-f(\beta) < 0$ if $\alpha-\beta > 0$	2A	
i.e. $f(\alpha) < f(\beta)$ if $\alpha > \beta$		
	4	

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Solutions	Marks	Remarks
(11) (a) Let $P(n)$: $1^2+2^2+\dots+n^2=\frac{1}{6}n(n+1)(2n+1)$		
$P(1)$ is true since $R.S.=\frac{1}{6}\times1\times2\times3=1=L.S.$	1A	
Assume $P(n)$ is true for $n=k$,	1M	Assume $n=k$ is true
i.e. $1^2+2^2+\dots+k^2=\frac{1}{6}k(k+1)(2k+1)$		
$1^2+2^2+\dots+k^2+(k+1)^2=\frac{1}{6}k(k+1)(2k+1)+(k+1)^2$	1A	Add $(k+1)^2$ to both sides
$=\frac{(k+1)}{6}\left[(2k^2+k)+6(k+1)\right]$		
$=\frac{(k+1)}{6}(k+2)(2k+3)$	1A	
$=\frac{1}{6}(k+1)((k+1)+1)(2(k+1)+1)$	1A	
$\therefore$ $P(n)$ is also true for $n=k+1$		
By M.I. $P(n)$ is true $\forall n\in N$	1M	Only awarded if above correct
	6	
(b) (i) Length of each brick $=\frac{x}{n}$	1A	
$\therefore$ vol. of each brick $=\left(\frac{x}{n}\right)^3$		
No. of bricks in the r-th layer $=r^2$	1A	
vol. of the r-th layer $=r^2\times\left(\frac{x}{n}\right)^3$	1A	
There are altogether n layers	1A	$V=V_1+V_2+\dots V_n$
$\therefore$ vol. of the solid $=\left(\frac{x}{n}\right)^3(1^2+2^2+\dots+n^2)$	1M+1A	
(ii) Height of pyramid $=n\times\frac{x}{n}=x$	1A	
$\therefore$ vol. of pyramid $=\frac{1}{3}x^2\cdot x=\frac{x^3}{3}$	1A	
Vol. of solid − vol. of pyramid		
$=\left(\frac{x}{n}\right)^3(1^2+2^2+\dots+n^2)-\frac{x^3}{3}$	1M	Volume 相減
$=\left(\frac{x}{n}\right)^3\cdot\frac{1}{6}n(n+1)(2n+1)-\frac{x^3}{3}$	1M	代 (x) result 入
$=\frac{x^3}{3}\left[\frac{1}{2n^2}(n+1)(2n+1)-1\right]$		
$=\frac{x^3}{3}\left(\frac{3n+1}{2n^2}\right)$ $\quad\left(\text{or } \frac{x^3}{6}\left(\frac{3}{n}+\frac{1}{n^2}\right)\right)$	2A	
>0	1A	
$\therefore$ vol. of solid is always greater than vol. of pyramid. If n is very large, the difference is close to zero	1A	quite independent of above

Solutions	Marks	Remarks
2) (a) $\omega^n + \omega^{-n} = \left[\cos\frac{2nk\pi}{5} + i\sin\frac{2nk\pi}{5}\right] + \left[\cos\frac{-2nk\pi}{5} + i\sin\frac{-2nk\pi}{5}\right]$	1+1M	De Moivre's Thm
$= 2\cos\frac{2nk\pi}{5}$	1A	
	3	
(b) $\omega^5 = \cos 5\times\frac{2k\pi}{5} + i\sin 5\times\frac{2k\pi}{5}$	1A	
$= \cos 2k\pi + i\sin 2k\pi$		
$= 1$	1A	Alt
		$\omega^5 - 1 = 0$ (1A)
$\therefore 1+\omega+\omega^2+\omega^3+\omega^4 = \frac{1-\omega^5}{1-\omega}$	3A	$(\omega-1)(1+\omega+\omega^2+\omega^3+\omega^4)=0$ (2A)
$= 0$ $\boxed{\cancel{(\because \omega \neq 1)}}$ X	1A	$\because \omega \neq 1$, $1+\omega+\omega^2+\omega^3+\omega^4=0$ (1A)
	6	
Alt		
$1+\omega+\omega^2+\omega^3+\omega^4 = \omega^5+\omega+\omega^2+\omega^3+\omega^4$	1A	代無分
$= \omega(\omega^4+1+\omega+\omega^2+\omega^3)$	1A	
$\therefore (1-\omega)(1+\omega+\omega^2+\omega^3+\omega^4)=0$	1A	
$1+\omega+\omega^2+\omega^3+\omega^4=0$ $\cancel{(\because \omega \neq 1)}$	1A	
		Alt
(c) $(\omega+\omega^{-1})^2 + (\omega^2+\omega^{-2})^2 = \omega^2+2+\omega^{-2}+\omega^4+2+\omega^{-4}$	1A	$(\omega+\omega^{-1})^2 + (\omega^2+\omega^{-2})^2$
$\omega^{-2} + 1 = \omega^{-2}\times\omega^5$ $= \omega^2+2+\omega^3+\omega^4+2+\omega$	2M* +1A	$= \omega^2+2+\frac{1}{\omega^2}+\omega^4+2+\frac{1}{\omega^4}$ (1A)
$= 3+(1+\omega+\omega^2+\omega^3+\omega^4)$	1A	$= \frac{\omega^6+2\omega^4+\omega^2+\omega^8+2\omega^4+1}{\omega^4}$
$= 3$	1A	$= \frac{\omega+2\omega^4+\omega^2+\omega^3+2\omega^4+1}{\omega^4}$ (2M+1A)
* 2M for $\omega^{-n} = \omega^{-n+5}$	6	$= \frac{(1+\omega+\omega^2+\omega^3+\omega^4)+3\omega^4}{\omega^4}$ (1A)
		$= 3$ (1A)
		2M for $\omega^n = \omega^{n-5}$
(d) From (a) $\cos\frac{2k\pi}{5} = \frac{\omega+\omega^{-1}}{2}$, $\cos\frac{4k\pi}{5} = \frac{\omega^2+\omega^{-2}}{2}$	1+1A	$\omega^n = \omega^{n-5}$
$\therefore \left(\cos\frac{2k\pi}{5}\right)^2 + \left(\cos\frac{4k\pi}{5}\right)^2 = \left(\frac{\omega+\omega^{-1}}{2}\right)^2 + \left(\frac{\omega^2+\omega^{-2}}{2}\right)^2$	1M	Sub.
可以 omit $= \frac{1}{4}\left[(\omega+\omega^{-1})^2 + (\omega^2+\omega^{-2})^2\right]$	1A	
$= \frac{3}{4}$ from (c)	1A	
	5	

81 I END

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Additional Mathematics II

MARKING SCHEME

評卷參考

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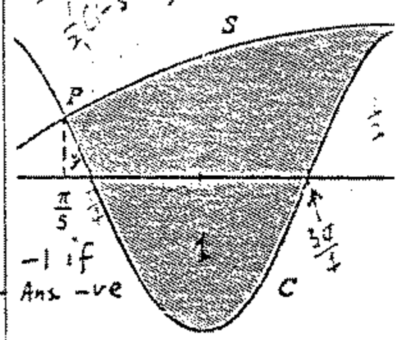
G.P. 195 (2/77)

Solutions	Marks	Remarks
(1) $\int (1+\cos\theta)^2 d\theta = \int (1+2\cos\theta+\cos^2\theta)\,d\theta$	1A	
$= \int \left(1+2\cos\theta+\frac{\cos 2\theta+1}{2}\right)d\theta$	1A	
$= \frac{3}{2}\theta + 2\sin\theta + \frac{\sin 2\theta}{4} + C$	1+1+1A	−1 if omit C
	5	
(2) Area of region $= \int_{\frac{\pi}{5}}^{\pi} \left(\sin\frac{x}{2} - \cos 2x\right)dx$	1M	
$= \left[-2\cos\frac{x}{2} - \frac{\sin 2x}{2}\right]_{\frac{\pi}{5}}^{\pi}$	1+1A	
$= 2\cos\frac{\pi}{10} + \frac{1}{2}\sin\frac{2\pi}{5}$	1+1A	
$= \frac{5}{2}\cos\frac{\pi}{10} \quad (\doteqdot 2.378)$		−1 if Ans −ve
	5	
Alt		
$\int_{\frac{\pi}{5}}^{\pi} \sin\frac{x}{2}\,dx = -2\cos\frac{x}{2}\Big\vert_{\frac{\pi}{5}}^{\pi}$	1A	
$= 2\cos\frac{\pi}{10}$	1A	
$\int_{\frac{\pi}{5}}^{\pi} \cos 2x\,dx = \frac{\sin 2x}{2}\Big\vert_{\frac{\pi}{5}}^{\pi}$	1A	
$= -\frac{1}{2}\sin\frac{2\pi}{5} \quad [-0.476$	1A	
Area $= 2\cos\frac{\pi}{10} + \frac{1}{2}\sin\frac{2\pi}{5}$	1M	−1 for −ve answer
(3) Putting $u^2 = 9-x$, $2u\,du = -dx$	1A	
When $x=0$, $u=3$; $x=9$, $u=0$.	1A	
$\int_0^9 \frac{x}{\sqrt{9-x}}dx = \int_3^0 \frac{-2u(9-u^2)}{u}du$	1M+1A	1-M for change of limi[ts] 1A for integrand
$= \int_0^3 2(9-u^2)\,du$		
$= \left[18u - \frac{2}{3}u^3\right]_0^3$	1A	
$= 36$	1A	
	6	

Solutions	Marks	Remarks
(4) $12\cos 3x - 5\sin 3x = r\cos(3x+\theta)$		
$= r(\cos 3x\cos\theta - \sin 3x\sin\theta)$		
Putting		
$r\cos\theta = 12$		
$r\sin\theta = 5$		
$r = \sqrt{12^2+5^2} = 13$	1A	
$\cos\theta = \frac{12}{13}$ $(= 0.9231)$		
$\therefore \theta = \cos^{-1}\frac{12}{13}$		
$= 22.62^\circ$	1A	Accept 22.6°
$= 22^\circ 37'$		Accept $\pm 1'$
$12\cos 3x - 5\sin 3x = 13$		
$13\cos(3x+22.62^\circ) = 13$	1M	for sub.
$\cos(3x+22.62^\circ) = 1$		
$3x+22.62^\circ = 360^\circ\times n$, $n = 0, \pm1, \pm2, \ldots$	1A	$n = 0, \ldots$ optional
$\therefore x = 120^\circ\times n - 7.54^\circ$ $(7^\circ 32')$	1A	
$= 120^\circ\times n - 8^\circ$, $n = 0, \pm1, \pm2, \ldots$	1A	$n = 0, \ldots$ optional
(corr. to the nearest degree)	6	
(5) $\sin\theta + \cos\theta = \frac{h}{2}$	1A	
$\sin\theta\cos\theta = \frac{1}{2}$	1A	Alt
$h^2 = 4(\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta)$	1M	$\sin\theta\cos\theta = \frac{1}{2} \Rightarrow \sin 2\theta =$
$= 4(1+1)$	1M+1A for sub.	$\therefore \theta = 45^\circ$, as $0^\circ < \theta < 90^\circ$
$h = 2\sqrt{2}$, since $0^\circ < \theta < 90^\circ$	1A	$\sin\theta = \cos\theta = \frac{1}{\sqrt{2}}$
	6	$\therefore h = 2(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}})$
		$= 2\sqrt{2}$

(5) Alt

$2\sin^2\theta - h\sin\theta + 1 = 0 \quad \ldots (i)$

$2\cos^2\theta - h\cos\theta + 1 = 0 \quad \ldots (ii)$ } 1M +1A | 1M for sub.

(i) + (ii) $\quad 2 - h(\sin\theta + \cos\theta) + 2 = 0$

$$h = \frac{4}{\sin\theta + \cos\theta}$$

(i) − (ii) $\quad 2(\sin^2\theta - \cos^2\theta) - h(\sin\theta - \cos\theta) = 0$

$$(\sin\theta - \cos\theta)\left[2(\sin\theta + \cos\theta) - h\right] = 0$$ 1M

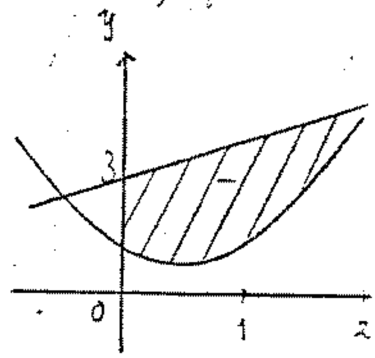
$\therefore \quad \sin\theta - \cos\theta = 0 \quad$ or $\quad \sin\theta + \cos\theta = \frac{h}{2}$ 1+1A

In both cases $\sin\theta = \cos\theta = \frac{1}{\sqrt{2}}$

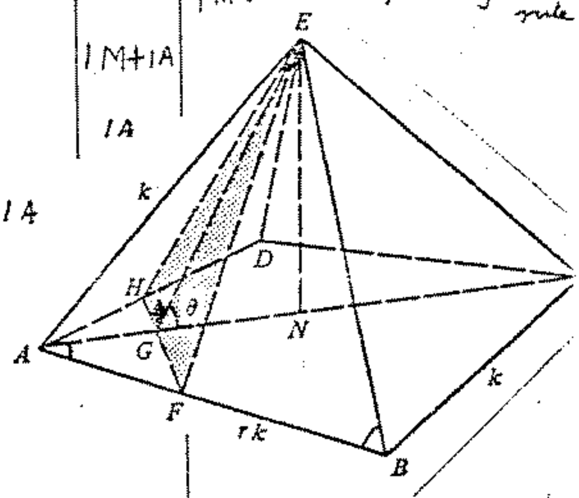
$\therefore h = 2\sqrt{2}$ 1A

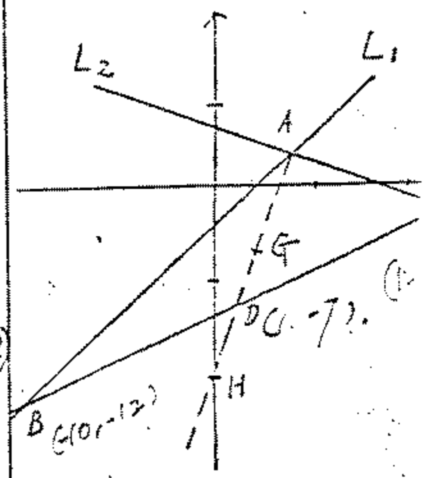
Solutions	Marks	Remarks
3) (a) $C_2 - C_1$, $6x-3y-3=0$	1M	
$y=2x-1$		
Sub. in C_1, $x^2+(2x-1)^2+7(2x-1)+11=0$	1M	
$5x^2+10x+5=0$		
$\therefore x=-1$, $y=-3$	1A	
(b) Centre of $C_2=(-3,-2)$ [of $C_1=(0,-\frac{7}{2})$]	1A	(b) Alt I: Tangent to C_1 at P: $-1x+-3y+\frac{7}{2}(y+-3)+11=0$ or $2x-y-1=0$
Slope of line joining P and centre $=-\frac{1}{2}$		
Slope of tangent $=2$	2A	
$\therefore$ common tangent at P is $y+3=2(x+1)$		
or $2x-y-1=0$	1A	(b) Alt III: $\left.\frac{dy}{dx}\right\|_{x=-1}=2$ (2A)
	6	Tangent is $2x-y-1=0$ (1A)
(b) Alt II: when C_1 and C_2 meet, the common chord is $6x-3y-3=0$		
or $2x-y-1=0$	2A	
Since they touch each other ext., the above equation is that of the common tangent at P	1A	
4) Let $P=(x,y)$		
$x=\frac{s+t}{2}$	1A	
$y=\frac{3}{2}(s-t)$	1A	
$s=x+\frac{y}{3}$, $t=x-\frac{y}{3}$ $\left(\text{or } \begin{array}{l} s+t=2x \\ s-t=\frac{2}{3}y \end{array}\right)$	1A	
$ST=\sqrt{(s-t)^2+9(s+t)^2}=2$	1A	
$\Rightarrow \left(\frac{2y}{3}\right)^2+9(2x)^2=4$	1M	
$\therefore \frac{y^2}{9}+9x^2=1$	1A	
i.e. $81x^2+y^2-9=0$		
	6	

Solutions	Marks	Remarks
8 (a) Putting $y = \sin x$, $dy = \cos x\, dx$	1A	
when $x = 0$, $y = 0$; $x = \frac{\pi}{2}$, $y = 1$.	1A	
$\int_0^{\frac{\pi}{2}} \cos^3 x \sin^2 x\, dx = \int_0^1 (1 - y^2) y^2\, dy$	1M+1A	
$= \left[\frac{y^3}{3} - \frac{y^5}{5}\right]_0^1$	1A	
$= \frac{2}{15}$ $(\doteqdot 0.1333)$	1A	
	6	
(b) (i) $\frac{1}{x^2+3} - \frac{1}{(x+1)^2} = \frac{(x+1)^2 - (x^2+3)}{(x^2+3)(x+1)^2}$		
$= \frac{2(x-1)}{(x^2+3)(x+1)^2}$, $(x \neq -1.)$	2A	
(ii) Putting $x = \sqrt{3}\tan\theta$, $dx = \sqrt{3}\sec^2\theta\, d\theta$	1A	
when $x = 0$, $\theta = 0$; $x = 3$, $\theta = \frac{\pi}{3}$.	1A	
$\therefore \int_0^3 \frac{dx}{x^2+3} = \int_0^{\frac{\pi}{3}} \frac{\sqrt{3}\sec^2\theta}{3(\tan^2\theta + 1)}\, d\theta$	1M+1A	
$= \frac{\sqrt{3}}{3} \int_0^{\frac{\pi}{3}} d\theta$	1A	
$= \frac{\pi\sqrt{3}}{9}$	1A	
(iii) $\int_0^3 \frac{2(x-1)}{(x^2+3)(x+1)^2}\, dx = \int_0^3 \left[\frac{1}{x^2+3} - \frac{1}{(x+1)^2}\right] dx$	2M	Using result in (b)(i
$= \int_0^3 \frac{dx}{x^2+3} - \int_0^3 \frac{dx}{(x+1)^2}$	1A	
$= \frac{\pi\sqrt{3}}{9} \mp \left[\frac{1}{x+1}\right]_0^3$	0+2A	
$= \frac{\pi\sqrt{3}}{9} - \frac{3}{4}$ $(\doteqdot -0.1454)$	1A	
	14	

Solutions	Marks	Remarks
9 (a) $y = \int k(x-\frac{1}{4})dx$		
$= k(\frac{x^2}{2} - \frac{x}{4}) + C$	1A+1A	
Sub. $(x, y) = (-1, 4), (0, 1)$,	1M	
$\begin{cases} 4 = k(\frac{1}{2}+\frac{1}{4}) + C \\ 1 = C \end{cases}$	1A	
$\therefore k = 4$	1A	
Equation of curve is $y = 2x^2 - x + 1$	1A	
	6	
(b) Solving $y = 2x^2 - x + 1$ with $y = 2x+3$	1M	
$2x^2 - 3x - 2 = 0$		
$\therefore x = 2$ or $-\frac{1}{2}$	1A	
Area of region $= \int_0^2 [(2x+3) - (2x^2 - x + 1)]dx$	1M+1M	Alt: Area $= \int_a^b y\,dx$
$= \int_0^2 [-2x^2 + 3x + 2]dx$	2A	$\int_0^2 (2x+3)dx = x^2 + 3x \Big\vert_0^2 = 10$
$= \left[-\frac{2}{3}x^3 + \frac{3}{2}x^2 + 2x\right]_0^2$		$\int_0^2 (2x^2 - x + 1)dx = \frac{2}{3}x^3 - \frac{x^2}{2} + x = \frac{16}{3}$
$= \frac{14}{3}$ $(\doteqdot 4.667)$	1A	Area req'ed $= 10 - \frac{16}{3} = \frac{14}{3}$ (1M
	7	
(c) Volume $= \pi\int_0^2 (2x+3)^2 dx - \pi\int_0^2 (2x^2 - x + 1)^2 dx$	1M +1M	for $V = \pi\int_a^b y^2 dx$; for $V_1 - V_2$
$= \pi\int_0^2 (4x^2 + 12x + 9)dx - \pi\int_0^2 [4x^4 - 4x^3 + 5x^2 - 2x + 1]dx$		Alt: $V_1 = \pi\int_0^2 (2x+3)^2 dx$ (1
$= \pi\left[-\frac{4}{5}x^5 + x^4 - \frac{x^3}{3} + 7x^2 + 8x\right]_0^2$ -1 for 1 wrong max. 3	3A	$= \pi\int_0^2 (4x^2 + 12x + 9)dx$
$= \pi\left[-\frac{4}{5}\times 32 + 16 - \frac{8}{3} + 7\times 4 + 8\times 2\right]$		$= \pi\left[\frac{4}{3}x^3 + 6x^2 + 9x\right]_0^2$ (
$= 31\frac{11}{15}\pi$ $(\doteqdot 31.73\pi \doteqdot 99.69)$	2A	$= 52\frac{2}{3}\pi$ (1
	7	$V_2 = \pi\int_0^2 (2x^2 - x + 1)^2 dx$
If omit π, award at most 5 marks		$= \pi\left[\frac{4}{5}x^5 - x^4 + \frac{5}{3}x^3 - x^2 + x\right]$
		$= 20\frac{14}{15}\pi$ (1
		$V = V_1 - V_2 = 31\frac{11}{15}\pi$ (1M+1A)

Solutions	Marks	Remarks
10(a) In ΔBFE,		1M for idea of using cos rule
$FE^2 = BE^2 + BF^2 - 2BE\cdot BF\cos FBE$	1M+1A	
$= k^2 + r^2k^2 - 2rk^2\cos 60°$	1A	
$= k^2(1-r+r^2)$	1A	
In ΔAFG, $FG \perp AC$	1A	
$FG^2 = (AF\sin FAG)^2$	1A	
$= [(1-r)k\sin 45°]^2$	1A	
$= \frac{k^2}{2}(1-2r+r^2)$	1A	
$\frac{k^2}{2}(1-r)^2$		
	8	
(b) In ΔEFG,		
$EG = \sqrt{FE^2 - FG^2}$	1M	
$= \sqrt{k^2(1-r+r^2) - \frac{k^2}{2}(1-2r+r^2)}$	1A	
$= k\left(\frac{1+r^2}{2}\right)^{\frac{1}{2}}$	1A	
$EN = AE\cos 45°$	1M+1A	1A for 45°
$= \frac{k}{\sqrt{2}}$	1A	
$\therefore \sin\theta = \frac{EN}{EG}$	1M	
$= \frac{1}{\sqrt{1+r^2}}$	1A	
	8	
(c) Since $0 \le r \le 1$, $\frac{1}{\sqrt{2}} \le \sin\theta \le 1$	1+1A	Accept strict inequalities
$\therefore 45° \le \theta \le 90°$	1+1A	
	4	



Solutions	Marks	Remarks
1)(a) Solving L_1 and L_2, $3y-6=0$	1M	
$y=2$, $x=4$	1A	
$\therefore A=(4,2)$.		
Let $D=(x_1,y_1)$, then $AG:GD=2:1$ (or $AD:DG=-3:1$)	1A (optional)	
$\therefore \begin{cases} t=\dfrac{2x_1+4}{2+1} \\ t-6=\dfrac{2y_1+2}{2+1}\end{cases}$		
$x_1=\dfrac{3t-4}{2}$	1A	
$y_1=\dfrac{3t-20}{2}$	1A	
	5	
(b) Slope of $AH=\dfrac{2-{-10}}{4}=3$	1A	**Alt** Eqn of AH is $y+10=\dfrac{2+10}{4}x$ or $3x-y-10=0$ (1A)
Slope of $AG=\dfrac{2-(t-6)}{4-t}=3$	1M	Sub. $G(t,t-6)$ (1M)
$\therefore t=2$	1A	$t=2$ (1A)
$D=(1,-7)$	1A	
$AD=\sqrt{3^2+9^2}$ $\sqrt{90}$		
$=3\sqrt{10}$ $(\doteqdot 9.487)$	1A	
$\tan\angle BAD=\dfrac{3-1}{1+3\times1}$	1M	
$=\dfrac{1}{2}$	1A	
	7	
(c) $\sin BAD=\dfrac{1}{\sqrt{1^2+2^2}}=\dfrac{1}{\sqrt5}$ 0.447	1A	
Area of $\triangle ABD=\dfrac12 AB\times AD\sin BAD$	1M	
$=\dfrac12\times14\sqrt2\times3\sqrt{10}\times\dfrac{1}{\sqrt5}$		
$=42$	1A	
$\therefore$ area of $\triangle ABC=84$ units	1A	
	4	

Solutions	Marks	Remarks
11 (d) Let $P=(x,y)$.		
Area of $\triangle APD = \pm\frac{1}{2}\left[(2x-4y)+(-28-2)+(y+7x)\right]$	1+1A	for + and −
$= 42$	~~2A~~	signs optional
$\therefore$ Locus of P is $9x-3y-30=\pm 84$		
i.e. $3x-y+18=0$	1A	
$3x-y-38=0$	1A	
	4	
Alt		
(d) Let $P=(x,y)$, $h=$ height of $\triangle ADC$ with AD as base.		
$\frac{1}{2}\times h\times 3\sqrt{10}=42$		
$h=\frac{28}{\sqrt{10}}$	1A	
$\frac{3x-y-10}{\pm\sqrt{10}}=\frac{28}{\sqrt{10}}$	1A	
Locus of P is $3x-y+18=0$	1A	
$3x-y-38=0$	1A	

G.F. 405 (2/77)

Solutions	Marks	Remarks
12) $L: y = mx+2$		$x_1+x_2 = -\frac{4m}{m^2+1}$
(a) $C: x^2+y^2=1$		$x_1x_2 = \frac{3}{m^2+1}$
Sub. L in C,		$y_1+y_2 = \frac{4}{m^2+1}$
$x^2+(mx+2)^2=1$	1M	$y_1y_2 = \frac{4-m^2}{m^2+1}$
$(m^2+1)x^2+4mx+3=0$	1A	Alt
$x = \frac{-4m \pm \sqrt{16m^2-12(m^2+1)}}{2(m^2+1)}$		$(x_1-x_2)^2 = (x_1+x_2)^2-4x_1x_2$
$= \frac{-2m \pm \sqrt{m^2-3}}{m^2+1}$	1A	$= \frac{16m^2}{(m^2+1)^2} - \frac{12}{(m^2+1)}$ $= \frac{4(m^2-3)}{(m^2+1)^2}$ (1A)
$y = mx+2$ $= \frac{2 \pm m\sqrt{m^2-3}}{m^2+1}$	1A	$(y_1-y_2)^2 = m^2(x_1-x_2)^2$ $= \frac{4m^2(m^2-3)}{(m^2+1)^2}$ (1A)
Let $A = \left(\frac{-2m+\sqrt{m^2-3}}{m^2+1}, \frac{2+m\sqrt{m^2-3}}{m^2+1}\right)$		
$B = \left(\frac{-2m-\sqrt{m^2-3}}{m^2+1}, \frac{2-m\sqrt{m^2-3}}{m^2+1}\right)$		
$AB = \sqrt{\left(\frac{2\sqrt{m^2-3}}{m^2+1}\right)^2 + \left(\frac{2m\sqrt{m^2-3}}{m^2+1}\right)^2}$	1M	Distance formula
$= \sqrt{\frac{4(m^4-2m^2-3)}{m^4+1}}$		
$= 2\sqrt{\frac{m^2-3}{m^2+1}}$	1A	
	6	

Solutions	Marks	Remarks
12 (b)(i) L meets C at two distinct points		In (i), (ii), (iii), 2M for 1st correct idea about length of AB.
iff $2\sqrt{\frac{m^2-3}{m^2+1}} > 0$	2M	or $\Delta = m^2-3$
i.e. $m < -\sqrt{3}$ or $m > \sqrt{3}$	1A	
(ii) L is a tangent to C iff $AB = 0$		
i.e. $m = \pm\sqrt{3}$	1A	
(iii) L does not meet C iff $AB < 0$ imaginary		
i.e. $-\sqrt{3} < m < \sqrt{3}$	1A	
	5	
(c) Since $m = \pm\sqrt{3}$, sub. in A or B of (a)		In this case, A, B are identical
$P = \left(\frac{-2m+\sqrt{m^2-3}}{m^2+1}, \frac{2+m\sqrt{m^2-3}}{m^2+1}\right)$	~~1A~~	
$= \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	1+1A	
$Q = \left(\frac{-2m-\sqrt{m^2-3}}{m^2+1}, \frac{2-m\sqrt{m^2-3}}{m^2+1}\right)$ vice versa	~~1A~~	
$= \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	1+1A	
$\therefore$ eqn. of PQ is $y = \frac{1}{2}$ (or $2y-1=0$)	1A	
	5	
(d) Eqn. req'd is $x^2+y^2-1+k(2y-1)=0$.	2M+2A	
	4	