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香港考試局

HONG KONG EXAMINATIONS AUTHORITY

一九八〇年香港中學會考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1980

SUBJECT

Additional Mathematics I

MARKING SCHEME

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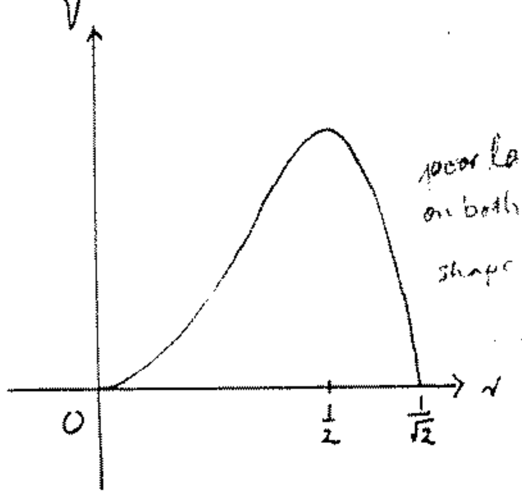
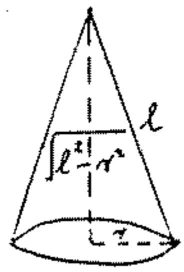
Solution	Marks	Notes
1. $2x^2 + x + 5 = k(x+1)^2$		
$(2-k)x^2 + (1-2k)x + (5-k) = 0$	1A	→ for RHS = 0
It has no real roots if $(1-2k)^2 - 4(2-k)(5-k) < 0$	2M+1A	→ for setting $\Delta < 0$
i.e. $k < \frac{39}{24}$ (or $\frac{13}{8}$) accept	1A	→ deduct if "$\leq$" (1A)
	5	
2. $\lim\limits_{\Delta x \to 0} \frac{1}{\Delta x}\left[\frac{2x+3+2\Delta x}{x+4+\Delta x} - \frac{2x+3}{x+4}\right]$		−1 if omit "$\lim\limits_{\Delta x \to 0}$" at any line
$\lim\limits_{\Delta x \to 0} \frac{1}{\Delta x}\left[\frac{(2x+3+2\Delta x)(x+4) - (2x+3)(x+4+\Delta x)}{(x+4+\Delta x)(x+4)}\right]$	1M ~~2M~~	→ common deno. Alternatively, $\lim\limits_{\Delta x \to 0}[\quad] = \frac{d}{dx}\left[\frac{2x+3}{x+4}\right]$ (2A)
$= \lim\limits_{\Delta x \to 0} \frac{1}{\Delta x}\left[\frac{2\Delta x(x+4) - \Delta x(2x+3)}{(x+4+\Delta x)(x+4)}\right]$ (1A) $5\Delta x$	1A	$= \frac{2(x+4)-(2x+3)}{(x+4)^2}$
$= \lim\limits_{\Delta x \to 0} \frac{5\Delta x}{\Delta x(x+4+\Delta x)(x+4)}$ 1A for cancelling Δx	1A	$= \frac{5}{(x+4)^2}$ (2A)
$= \frac{5}{(x+4)^2}$	2A	
	5	
3. Slope of $L_3 = \frac{-5}{3}$		Alternatively,
∴ slope of altitude $= \frac{3}{5}$ (1A)	1A	Solving L_1, L_2 1M
System of lines thro' intersection of L_1 and L_2		$x+2y+1-(12x+2y+6)=0$
is $6x+y+3+k(x+2y+1)=0$ combining (1A)	1M	$x = -\frac{5}{11}$ 1A
or $(6+k)x + (1+2k)y + (3+k) = 0$	equal slopes	$y = -\frac{3}{11}$ 1A
It is the altitude iff $-\frac{6+k}{1+2k} = \frac{3}{5}$	1M+1A	Slope of altitude $= \frac{3}{5}$ 1A
∴ $k = -3$ (1A)	1A	Altitude is for pt.-slope form
Eqn. of altitude is $(6-3)x + (1-6)y + (3-3) = 0$		$y + \frac{3}{11} = \frac{3}{5}(x + \frac{5}{11})$ 1M
$3x - 5y = 0$ (1A) accept if no simplification on line	1A	$3x - 5y = 0$ 1A
	6	

Solution	Marks	Notes
$\int \frac{2}{\cot\frac{x}{2}+\tan\frac{x}{2}}\,dx = \int \frac{2}{\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}+\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}\,dx$	~~1A~~	
$= \int 2\sin\frac{x}{2}\cos\frac{x}{2}\,dx$	2A	
$= \int \sin x\,dx$ or $\int 4\sin\frac{x}{2}\,d(\sin\frac{x}{2})$ 2A	2A	for C
$= -\cos x + C$ or $[2\sin^2\frac{x}{2} + C$ 1A+1M]	1A+1M	1M for constant
	6	
5. $\rightarrow \sin^2 A = 3 + 6\cos^2 A$		
$\sin^2 A = \frac{9}{25}$ (or $\cos^2 A = \frac{16}{25}$, etc)	2A	
$\therefore \sin A = \pm\frac{3}{5}$		
$\cos A = \pm\sqrt{1-\sin^2 A}$		
$= \pm\frac{4}{5}$		
If $90° < A < 180°$, $\sin A = \frac{3}{5}$ (1A)	1A	
$\cos A = -\frac{4}{5}$ (1A)	1A	
$\therefore \frac{\sin A}{1+2\cos A} = \frac{\frac{3}{5}}{1-\frac{8}{5}}$	1M	for sub.
$= -1$	1A	
	6	
6. (i) For $x \le 0$, accept if write $x < 0$	1M	
$x^2 - \lvert x\rvert - x < 0$		$x^2 - x < \lvert x\rvert$
$\Rightarrow x^2 + x - x < 0$		$x^4 - 2x^3 + x^2 < x^2$
$\Rightarrow x^2 < 0$, which is impossible	1A	$x^2(x-2)x < 0$
(ii) For $x > 0$,	1M	$(x-2)x < 0$
$x^2 - \lvert x\rvert - x < 0$		$0 < x < 2$
$\Rightarrow x^2 - 2x < 0$	1A	Incomplete
$\Rightarrow x(x-2) < 0$		award 3 marks
$\Rightarrow 0 < x < 2$	1+1A	or 0 mark if
(i) and (ii) $\Rightarrow 0 < x < 2$ (−1) if emphasis on set is ϕ	6	any mistake was made in between

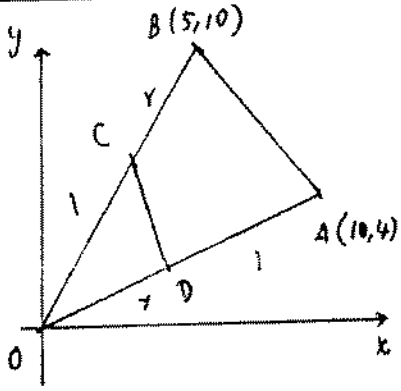
Solution	Marks	Notes
7. $\tan 7\theta + \cot 2\theta = 0$		
$\dfrac{\sin 7\theta}{\cos 7\theta} + \dfrac{\cos 2\theta}{\sin 2\theta} = 0$		
$\dfrac{\sin 2\theta \sin 7\theta + \cos 2\theta \cos 7\theta}{\cos 7\theta \sin 2\theta} = 0$	1A	
$\dfrac{\cos 5\theta}{\cos 7\theta \sin 2\theta} = 0$	1A	for $\cos 5\theta$
If $\sin 2\theta, \cos 7\theta \neq 0$, i.e. $\theta \neq n\pi, \frac{\pi}{2}+n\pi, \frac{4n+1}{14}\pi, \frac{4n+3}{14}\pi$, $n = 0, \pm 1, \pm 2, \ldots$		
then $\cos 5\theta = 0$ (1A)	1A	general solution for cosine
$5\theta = 2n\pi \pm \frac{\pi}{2}$ (or $\frac{\pi}{2}+n\pi$)	1M+1A	
$\therefore \theta = \frac{4n\pm 1}{10}\pi$ (1A), $n = 0, \pm 1, \pm 2, \ldots$ or $\frac{2n+1}{10}\pi$ accept if n is not defined.	1A	
	6	
Alternatively,		
$\tan 7\theta + \cot 2\theta = 0$		
$\tan 7\theta + \tan(\frac{\pi}{2} - 2\theta) = 0$	1A	to $\tan(\frac{\pi}{2}-2\theta)$
$\tan 7\theta = \tan(2\theta - \frac{\pi}{2})$ [or $\tan(2\theta+\frac{\pi}{2})$]	2A	"transfer" + sign
$7\theta = 2\theta - \frac{\pi}{2} + n\pi$, $n = 0, \pm 1, \pm 2, \ldots$ (or $2\theta+\frac{\pi}{2}+n\pi$)	1M+1A	general formula
$5\theta = n\pi - \frac{\pi}{2}$ [or $\frac{\pi}{2}+n\pi$]		accept if in terms of "degree" or mixture
$\therefore \theta = \frac{1}{10}(2n-1)\pi$, $n = 0, \pm 1, \pm 2, \ldots$ [or $\frac{1}{10}(2n+1)\pi$]	1A	(-1) if no degree "°" written.

Solution	Marks	Notes
8. (a) Area of curved surface		
$= \pi r l$ (1A)	1A	→ can skip
$\therefore \pi \overset{1M}{=} \pi r^2 + \pi r l$ (1A)	1M+1A	
$l = \frac{1-r^2}{r}$ (1A)	1A	
Height of cone $= \sqrt{l^2-r^2}$ (1A)	1A	
$\therefore$ volume $= (\frac{1}{3})(\pi r^2)\sqrt{l^2-r^2}$	1M	For formula of vol. $\frac{1}{3}$(base)(ht.)
$V^2 = \frac{1}{9}\pi^2 r^4 (l^2-r^2)$		
$= \frac{1}{9}\pi^2 r^2 (1-2r^2)$ (2A)	2A	
	8	
(b) $\frac{d(V^2)}{dr} = \frac{\pi^2}{9}(2r-8r^3)$ (1M) (1A)	1M+1A	
$\frac{d(V^2)}{dr} = 0$ (1M)	1M	
$\therefore r = 0, \pm\frac{1}{2}$	~~2A~~ 1A ← must have $r=\frac{1}{2}$ (or more)	Accept $r = 0, \frac{1}{2}$; or $r = \frac{1}{2}$.
Test for max. (1M) → either mention or do it.	1M	
$\therefore V^2$ is max when $r = \frac{1}{2}$	1A	
Max. value of $V = \frac{1}{3}\pi r\sqrt{1-2r^2} = \frac{\sqrt{2}}{12}\pi$ (1M (formula)) (1A) omit/ignore unit	1M+1A	can leave unit.
	~~9~~ 8	
(c) 	1	for label on x, y axes
poor labelling on both x & y axes (−1)	1	for $\frac{1}{2}$ max
shape ... not ... (−1)	1	for 0 origin
3 marks not awarded if wrong ...	1	for $\frac{1}{\sqrt{2}}$ on x-axis
		−1 if outside range $(0 \le r \le \frac{1}{\sqrt{2}})$
	4	

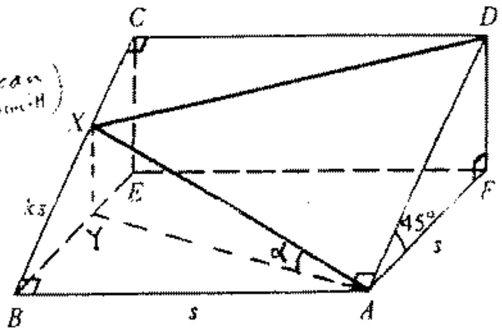


Solution	Marks	Notes
9. (a) $\tan 3\theta = \tan(2\theta + \theta)$		
$= \dfrac{\tan 2\theta + \tan\theta}{1 - \tan 2\theta \tan\theta}$ must have this line	2A	
$= \dfrac{\frac{2\tan\theta}{1-\tan^2\theta} + \tan\theta}{1 - \frac{2\tan\theta}{1-\tan^2\theta}\cdot\tan\theta}$ " " "	1A	
$= \dfrac{\tan^3\theta - 3\tan\theta}{3\tan^2\theta - 1}$ " " "	1A	
	4	
(b) (i) Putting $x = 1$ (1M), $m + n + 2 = 0$ (1A) .... (1)	1M+1A	
Putting $x = 2$ (1M), $4m + n + 11 = 0$ (1A) .... (2)	1M+1A	
(1) and (2) $\Rightarrow$ $m = -3$, $n = 1$	1A	Awarded only if (1), (2) correct
	5	
(ii) $f(x) = 0$ iff $3x^3 - 3x^2 - 9x + 1 = 0$ for m, n subst.	1A	
Putting $x = \tan\theta$, subst. (1M)		
$3\tan^3\theta - 3\tan^2\theta - 9\tan\theta + 1 = 0$	1M	Alternatively,
$3(\tan^3\theta - 3\tan\theta) - (3\tan^2\theta - 1) = 0$		$\dfrac{\tan^3\theta - 3\tan\theta}{3\tan^2\theta - 1} = \dfrac{1}{3}$, 1+1A
$3\underbrace{\tan 3\theta}_{\text{subst (1A) on tan 3A}}(3\tan^2\theta - 1) - (3\tan^2\theta - 1) = 0$ by (a)	1A	
$(3\tan 3\theta - 1)(3\tan^2\theta - 1) = 0$ factorization	1A	$\therefore \tan 3\theta = \dfrac{1}{3}$ 1M
$3\tan 3\theta - 1 = 0$ (1M)	1M	
$[3\tan^2\theta - 1 \neq 0$ for the validity of identity$]$ accept if not mentioned this line		
$\tan 3\theta = \frac{1}{3}$		
$\Rightarrow 3\theta = 18.43°, 198.43°, 378.43°$ accept for gen. solution	(2A)	1 for 1, 1 for 2, 2 for 3 ~~-1 for one wrong~~
allow $\pm 0.01°$ (or $18°26'$, $198°26'$, $378°26'$)		
or allow ± 1 min $\theta = 6.14°, 66.14°, 126.14°$	~~2A~~	~~for 2 or more correct~~
(or $6°9'$, $66°9'$, $126°9'$)		
Solutions are $x = \tan\theta$ (1M)	1M	1 mark each (to 2 d.p.)
only here lost 3 marks $\rightarrow$ $= 0.108, 2.262, -1.369$	3A	~~for 2 or more correct~~
$= 0.11, 2.26, -1.37$ (corr. to 2 d.p.)	~~1A~~	
	11	

Solution	Marks	Notes
(a) $\lvert z-(3+4i)\rvert = 4$		
$\lvert (x-3)+(y-4)i\rvert = 4$	1A	mod.
$(x-3)^2+(y-4)^2=16$	2A	either 3rd or 4th line.
$x^2+y^2-6x-8y+9=0 \quad \ldots\ldots (i)$		
	3	
(b) $\dfrac{z-1}{z+1} = \dfrac{(x-1)+yi}{(x+1)+yi}$ (1A)	1A	
$= \dfrac{[(x-1)+yi][(x+1)-yi]}{(x+1)^2+y^2}$ (1M)	1M	kn. conjugate
$= \dfrac{1}{(x+1)^2+y^2}\left[(x^2+y^2-1)+2yi\right]$ (1A)	1A	separate Real + Imag.
If its amp. $=\dfrac{\pi}{2}$, $\quad x^2+y^2-1=0 \ldots\ldots (ii)$	1M+1A	equate real = 0. Real part = 0
	5	
(c) If $z_1 = x+yi$ satisfies (a) and (b),		
(ii) − (i) $\Rightarrow \quad x=\dfrac{5-4y}{3} \quad \left(\text{or } y=\dfrac{5-3x}{4}\right)$	2M+1A	for combining (i) + (ii) together (solving) x or y; for right x or right y.
Sub. in (ii), $\left(\dfrac{5-4y}{3}\right)^2+y^2-1=0 \quad \left(x^2+\left(\dfrac{5-3x}{4}\right)^2-1=0\right)$	1M	subst. x or y
$25y^2-40y+16=0 \quad (25x^2-30x+9=0)$	1A	find Quad. in x (or y).
$y=\dfrac{4}{5}$ 1A	1A	
$x=\dfrac{3}{5}$ 1A	1A	
$\therefore z_1 = \dfrac{3}{5}+\dfrac{4}{5}i$ 1A	1A	
If $z_1=(p+qi)^2$,		
$\dfrac{3}{5}+\dfrac{4}{5}i = (p+qi)^2$ 1M	1M	
$= (p^2-q^2)+2pqi$	1A	
$\therefore pq=\dfrac{2}{5}$	1M+1A	kn. (f.?)
	12	

Solution	Marks	Notes
(a) Let $C = (x_1, y_1)$, $D = (x_2, y_2)$.		
Then $x_1 = \frac{5}{1+r}$	1A	
$y_1 = \frac{10}{1+r}$	1A	
$x_2 = \frac{10r}{1+r}$	1A	
$y_2 = \frac{4r}{1+r}$	1A	
	4	
(b) Area of $\triangle ODC = \frac{1}{2}(x_2 y_1 - x_1 y_2)$	1M	→ for area formula.
$= \frac{1}{2}\left(\frac{10r}{1+r}\cdot\frac{10}{1+r} - \frac{5}{1+r}\cdot\frac{4r}{1+r}\right)$		
$= \frac{40r}{(1+r)^2}$ 2A	2A	
	3	
(c) Area of $\triangle OAB = \frac{1}{2}(10\times10 - 4\times5)$		for area formula.
$= 40$ (1A)	1M+1A	
Since $\triangle ODC = k\times\triangle OAB$, $\frac{40r}{(1+r)^2} = 40k$	1M+1A	→ equate 2 areas → correct equation
$kr^2 + (2k-1)r + k = 0$	1A	→ correct quadratic equ.
$r = \frac{(1-2k) \pm \sqrt{(2k-1)^2 - 4k^2}}{2k}$	1A	→ correct r.
$= \frac{(1-2k) \pm \sqrt{1-4k}}{2k}$ (Ans.)		
r is real ⇒ $1 - 4k \geq 0$	2M	For $D \geq 0$
$\therefore k \leq \frac{1}{4}$ 1A	1A	
	9	
(d) Area of $\triangle ODC$ is max if k is max.	2M	→ may use differentiation.
$\therefore$ max. area of $\triangle ODC = \frac{1}{4}\times 40$		
$= 10$ sq. units (accept omit units)	2A	
	4	

Solution	Marks	Notes
12. (a) $AX=\sqrt{k^2s^2+s^2}$	1M+1A	Pythagoras Thm. For correct AX
$=s\sqrt{1+k^2}$ (can omit)		
	2	
(b) $XY=ks\sin 45°$	1A	By diag for XY.
$=\frac{ks}{\sqrt{2}}$	1A	
$\therefore \sin\alpha=\frac{XY}{AX}$		
$=\frac{\frac{ks}{\sqrt{2}}}{s\sqrt{1+k^2}}$	1M	subst. XY, AX.
$=\frac{k}{\sqrt{2(1+k^2)}}$ 1A	1A	
	4	
(c) If $\alpha\le 30°$, then $\frac{k}{\sqrt{2(1+k^2)}}\le\sin 30°$	3M	−1 for giving "<" any time
$=\frac{1}{2}$ 1A	1A	If "=" given, award only 3 marks unless followed by explanation.
$\therefore \frac{k^2}{2(1+k^2)}\le\frac{1}{4}$		
can be omitted $k^2\le 1$. 1A	1A	
$(0<)\ k\le 1$ 1A	1A	
i.e. $XB\le s$.		
Similarly, if the inclination of XD to the horizontal is not to exceed 30°, $\boxed{XC\le s}$ 2A	2A.	
Under this condition, $XB=BC-XC$		
$=\sqrt{2}s-XC$ (1A)	1A	
$\geq\sqrt{2}s-s$ (2A)	2A	for $\geq$ and subst. of s.
Combining $XB\le s$, $XB\geq\sqrt{2}s-s$, we have	1M	[illegible]
$\sqrt{2}s-s\le ks\le s$		
$\sqrt{2}-1\le k\le 1$	2A	whole statements correct not on 1 side only
	14	

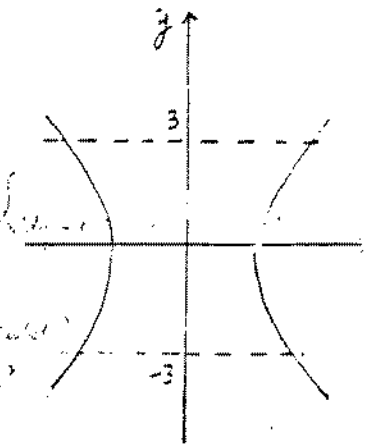


Solution	Marks	Notes
1. $(1+2x)^3(1+3x)^4 = (1+6x+12x^2+\dots)(1+12x+54x^2+\dots)$	2A+1A	2A for 1st correct expansion (3 terms only)
$= (1+18x+138x^2+\dots)$	2A	−1 if omit "+…"
		−1 if include x^3, etc.
	5	−1 for each wrong term
2. Let $u = x-1$		
$\int (x+2)\sqrt{x-1}\,dx = \int (u+3)\sqrt{u}\,du$	1A	Full marks given if no intermediate steps
$= \int (u^{\frac{3}{2}} + 3u^{\frac{1}{2}})\,du$	1M	
$= \frac{2}{5}u^{\frac{5}{2}} + 2u^{\frac{3}{2}} + C$	1+1A	−1 if omit C.
$= \frac{2}{5}(x-1)^{\frac{5}{2}} + 2(x-1)^{\frac{3}{2}} + C$	1A	
	5	
3. Diff. w.r.t. x,		
$4xy + 2x^2y' + 2x + 2yy' = 0$	3A	−1 for each wrong term
$y' = -\frac{2xy+x}{x^2+y}$	1A	
Putting $x=2$, $y=0$,	1M	
slope of tangent at $(2,0) = -\frac{1}{2}$	1A	
	6	
4. $\frac{dy}{dx} = -\sin(\sin x)\frac{d}{dx}\sin x$	1M	optional
$= -[\sin(\sin x)]\cos x$	1A	
$\frac{d^2y}{dx^2} = -\cos x\frac{d}{dx}[\sin(\sin x)] - [\sin(\sin x)]\frac{d}{dx}\cos x$	1A+1M	Optional.
$= -[\cos(\sin x)]\cos^2 x + [\sin(\sin x)]\sin x$	1+1A	
	6	

Solution	Marks	Notes
5. (a) $\log_4 x - \log_x 16 = 1$		
$\log_4 x - \dfrac{\log_4 16}{\log_4 x} = 1$	2A	Cand. not expected to write $\log_4 x \neq 0$
$(\log_4 x)^2 - \log_4 x - 2 = 0$	1A	
$(\log_4 x + 1)(\log_4 x - 2) = 0$		
$\log_4 x = -1$ or 2	1A	For any one correct
$\therefore\ x = \frac{1}{4}$ or 16	1+1A	
	6	
6. Eqn. of AB is $y-2 = \dfrac{2-\frac{1}{2}}{-4-2}(x+4)$	1A	
or $y = -\frac{1}{4}(x-\ldots)$		
Area required $= \int_{-4}^{2}\left[-\frac{1}{4}(x-4) - \ldots\right]dx$		
$= \int_{-4}^{2}\left(-\frac{1}{8}x^2 - \frac{x}{4} + \ldots\right)dx$		
$= \left[-\frac{x^3}{24} - \frac{x^2}{8} + x\right]$		
$= \left(-\frac{8}{24} - \frac{4}{8} + 2\right) - \ldots$		
$= \frac{9}{2}$		
Alternatively		
Area under AB	1A	
$= \left(\frac{1}{2}+2\right)(2--4)\times\frac{1}{2}$		
$= \frac{15}{2}$		
Area under curve		
$= \int_{-4}^{2} y\,dx = \int_{-4}^{2} \frac{x^2}{8}\,dx = \ldots$	1M+1A	
$\therefore$ area required $= \frac{15}{2} - \ldots$	1M	
$= \frac{9}{2}$	1A	
	6	

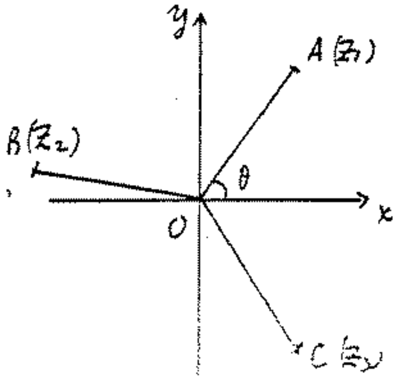
Solution	Marks	Notes
7. Let $P=(a,b)$, $M=(x,y)$.		
$x=\dfrac{a-1}{2}$	1A	
$y=\dfrac{b+2}{2}$	1A	
$\therefore\ a=2x+1$, $b=2y-2$	1M	
P lies on the circle		
$\Rightarrow (2x+1)^2+(2y-2)^2-2(2x+1)-4(2y-2)-5=0$	1M+1A	for wrong a, b
$2x^2+2y^2-8y+3=0$	1A	for $4x^2+4y^2-16y+6=0$ awc
	6	
8. (a) $\dfrac{dy}{dx}=\dfrac{2}{y}$		
Putting $x=1$, $y=2$, $\dfrac{dy}{dx}=1$	1M+1A	
Eqn. of tangent is $y-2=x-1$	1A	Optional
$x-y+1=0$	1A	-1 if not simplified
$y=x+1$		
	4	
<u>Alternatively</u>		
$y_1y=2(x+x_1)$	1A	
$2y=2(x+1)$	1M+1A	
$x-y+1=0$	1A	

Solution	Marks	Notes
8.(b) (i) Expanding C		
(I) $x^2+y^2-(2+k)x-(4-k)y+(5-k)=0$	2A	
$\left[x-(1+\frac{k}{2})\right]^2+\left[y-(2-\frac{k}{2})\right]^2=\frac{k^2}{2}$	1M+1A (R.S) 1M+1A (L.S.)	If r^2 given $\neq \frac{k^2}{2}$, awarded for checking $r=$
(1,2) satisfies C	1M	
∴ C represents a circle passing thro' (1,2).		**Alt. soln. for (b) (i)** Expanding (optional) or coeff x^2 = coeff y^2
(ii) Centre of C $=\left(1+\frac{k}{2},\ 2-\frac{k}{2}\right)$	1+1A	coeff. $xy=0$
(II) (iii) $2(x-1)+2(y-2)\frac{dy}{dx}+k\left(\frac{dy}{dx}-1\right)=0$	1M	$g^2+f^2-c=\frac{k^2}{2}$ 1M 1A
$\frac{dy}{dx}=\frac{k+2-2x}{k-4+2y}$	1A	(1,2) satisfies C
At (1,2), $\frac{dy}{dx}=1$	1A	
Equ. required is $y-x-1=0$	1A	
	11	
(c) (I) By (a) and (b), C represents a variable circle which shares a common tangent with the parabola $y^2=4x$ at (1,2)	1M	
If its centre lies on $x-y=3$, from (b)(ii)		
$\left(1+\frac{k}{2}\right)-\left(2-\frac{k}{2}\right)=3$	1M	(I) for sub. into $x-y=$
$k=4$	1A	
∴ the required equ. is $(x-1)^2+(y-2)^2+4(y-x-1)=0$	1M	
$x^2+y^2-6x+1=0$	1A	
	5	

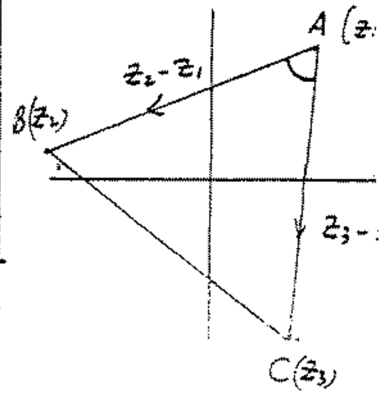
Solution	Marks	Notes
9. (a) Let $P=(x,y)$.		
Slope of $PA = \frac{y-3}{x-5}$	1A	
Slope of $PB = \frac{y+3}{x+5}$	1A	
$\frac{y-3}{x-5}\times\frac{y+3}{x+5} = k$	2M	
Locus of P is $kx^2-y^2=25k-9$	1A	
	5	
(b) (i) Circle, $k=-1$.	1A	
(ii) Ellipse but not circle, $k<0$, $k\neq -1$.	1+1A	
(iii) Hyperbola, $k>0$, $k\neq\frac{9}{25}$.	1+1A	
If $k=0$, two parallel lines	1A	
If $k=\frac{9}{25}$, two intersecting lines	2A	
	8	
(c) When $k=1$, the locus is the hyperbola $x^2-y^2=16$	1A	
Vol. req. $=\int_{-3}^{3}\pi x^2\,dy$	1M+1M+1A	
$=\int_{-3}^{3}\pi(16+y^2)\,dy$	1M	
$=\left[\pi\left(16y+\frac{y^3}{3}\right)\right]_{-3}^{3}$	1A	
$=114\pi$	1A	
	7	

Solution	Marks	Notes
10. $x^2-2x-1=0$		
$x=\frac{2\pm\sqrt{8}}{2}$	1A	
$\therefore\ \alpha=1+\sqrt{2},\quad \beta=1-\sqrt{2}$ $\alpha+\beta=2$, $\alpha\beta=-1$		
(a) $U_{n+2}=\frac{1}{2\sqrt{2}}(\alpha^{n+2}-\beta^{n+2})$		6 marks for 1st given part, 3 m for 2nd part.
$=\frac{1}{2\sqrt{2}}\left[\alpha^n(3+2\sqrt{2})-\beta^n(3-2\sqrt{2})\right]$	1M+1A	
$=\frac{1}{2\sqrt{2}}\left[3(\alpha^n-\beta^n)+2\sqrt{2}(\alpha^n+\beta^n)\right]$	1A	
$\therefore 2U_{n+1}+U_n=\frac{2}{2\sqrt{2}}(\alpha^{n+1}-\beta^{n+1})+\frac{1}{2\sqrt{2}}(\alpha^n-\beta^n)$	1A	
$=\frac{1}{2\sqrt{2}}\left[2\alpha^n(1+\sqrt{2})-2\beta^n(1-\sqrt{2})+\alpha^n-\beta^n\right]$	1A	
$=\frac{1}{2\sqrt{2}}\left[3(\alpha^n-\beta^n)+2\sqrt{2}(\alpha^n+\beta^n)\right]$	1A	
$=U_{n+2}$		
$V_{n+2}=\frac{1}{2\sqrt{2}}(\alpha^{n+2}+\beta^{n+2})$		
$=\frac{1}{2\sqrt{2}}\left[\alpha^n(3+2\sqrt{2})+\beta^n(3-2\sqrt{2})\right]$		
$=\frac{1}{2\sqrt{2}}\left[3(\alpha^n+\beta^n)+2\sqrt{2}(\alpha^n-\beta^n)\right]$	2A	
$\therefore 2V_{n+1}+V_n=\frac{2}{2\sqrt{2}}\left[\alpha^n(1+\sqrt{2})+\beta^n(1-\sqrt{2})\right]+\frac{1}{2\sqrt{2}}(\alpha^n+\beta^n)$		
$=\frac{1}{2\sqrt{2}}\left[3(\alpha^n+\beta^n)+2\sqrt{2}(\alpha^n-\beta^n)\right]$		
$=V_{n+2}$	1A	
	10	

Solution	Marks	Notes
10. (b) (i) $U_1 = \frac{1}{2\sqrt{2}}(\alpha - \beta) = 1$	2A	
$U_2 = \frac{1}{2\sqrt{2}}(\alpha^2 - \beta^2)$		
$= \frac{1}{2\sqrt{2}}\left[(3+2\sqrt{2}) - (3-2\sqrt{2})\right] = 2$	2A	
(ii) If U_n, U_{n+1} are integers,		+ [illegible] result in (a)
$U_{n+2} = 2U_{n+1} + U_n$ must also be an integer	1+1M	→ for integers.
(iii) Since U_1, U_2 are integers and from (b)(ii), if U_k, U_{k+1} are integers, U_{k+2} is also an integer, by induction,	Yes	
U_n is an integer for all n.	1+1	1 for answer
	8	1 for reasons.
(c) V_n is not an integer for some n		1 for answer
e.g. $V_1 = \frac{1}{\sqrt{2}}$	1+1	1 for reasons.
	2	
$V_1 = \frac{1}{2\sqrt{2}}(1+\sqrt{2} + (1-\sqrt{2}))$		
$= \frac{1}{\sqrt{2}}$		

Solution	Marks	Notes
11. $\omega^3-1=(\omega-1)(\omega^2+\omega+1)=0$	1A	
$\omega\neq 1 \Rightarrow \quad \omega=\frac{-1\pm\sqrt{3}i}{2}$	1A	Accept $\frac{-1+\sqrt{3}i}{2}$
$=\operatorname{cis}\frac{2\pi}{3}\quad (\because 0<\operatorname{amp}(\omega)<\pi)$	1A	
Let $z_1=3\operatorname{cis}\theta$,		
then $z_2=\omega z_1=3\operatorname{cis}(\theta+\frac{2}{3}\pi)$ 120° acceptable.	1A	
$z_3=\omega z_2=3\operatorname{cis}(\theta+\frac{4}{3}\pi)$	1A	
(a)  Show $\angle AOB = \angle BOC = \angle COA$ app. equal	2M	(2M)
(b) $\lvert z_2\rvert=\lvert z_3\rvert=3$	1A	
	8	
(c) Since $\lvert z_1\rvert=\lvert z_2\rvert=\lvert z_3\rvert$,		
A, B, C lie on a circle centred O	2M	4 marks for any correct method.
$\angle AOB=\angle AOC=\angle BOC=\frac{2}{3}\pi$		
$\therefore$ Δs AOB, AOC, BOC are congruent	2M	
$\therefore$ ΔABC is equilateral.	4	

Solution	Marks	Notes
11. (d) $z_1^2+z_2^2+z_3^2+z_1z_2+z_2z_3+z_3z_1$		
$=z_1^2+(\omega z_1)^2+(\omega^2 z_1)^2+z_1(\omega z_1)+(\omega z_1)(\omega^2 z_1)+(\omega^2 z_1)z_1$	2M	Write in polar form ... (z
$=z_1^2(1+\omega^2+\omega^4+\omega+\omega^3+\omega^2)$	1A	
$=2z_1^2(1+\omega+\omega^2)$	1A	
$=0$, (since $1+\omega+\omega^2=0$)		-1 if omit "= 0"
	4	
(e) $\text{Amp}\,\frac{z_3-z_1}{z_2-z_1}=\text{amp}(z_3-z_1)-\text{amp}(z_2-z_1)$	1A	
$=\angle BAC$	2A	
$=60^\circ$ $\omega+1$ ( ) $=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$, $\frac{1}{2}+\frac{\sqrt{3}}{2}$	1A	
	4	
<u>Alternatively</u>		
$\text{Amp}\,\frac{z_3-z_1}{z_2-z_1}=\text{amp}\,\frac{(\omega^2-1)z_1}{(\omega-1)z_1}$	1A	
$=\text{amp}(\omega+1)$ $(\because \omega\neq 1)$	1A	
$=\text{amp}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$	1A	
$=60^\circ$	1A	



Solution	Marks	Notes
12. (a) Let $u = a - x$, $x = a - u$, $dx = -du$, $f(x) = f(a-x)$		
$\int_0^a x f(x)\,dx = -\int_a^0 (a-u) f(a-u)\,du$ → must be written out	1A+1A	for limits
$= \int_0^a (a-u) f(a-u)\,du$ give 2M	1A	for $\int_0^a$
$= \int_0^a (a-u) f(u)\,du$	1M	Using $f(x) = f(a-x)$
$= a\int_0^a f(u)\,du - \int_0^a u f(u)\,du$	1A 1M	
$f(x) = f(a-x)$ $= f(u)$ $\quad = a\int_0^a f(x)\,dx - \int_0^a x f(x)\,dx$	1A	
$\therefore \int_0^a x f(x)\,dx = \frac{a}{2}\int_0^a f(x)\,dx$	1A	
	6	
(b) Putting $u = x - \frac{\pi}{2}$, by (a),		
$\int_{\frac{\pi}{2}}^{\pi} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}\,dx = \int_0^{\frac{\pi}{2}} \frac{\sin^4(u+\frac{\pi}{2})}{\sin^4(u+\frac{\pi}{2}) + \cos^4(u+\frac{\pi}{2})}\,du$	1M+1A	limits
$= \int_0^{\frac{\pi}{2}} \frac{\cos^4 u}{\cos^4 u + \sin^4 u}\,du$	1A	
$\therefore \int_0^{\pi} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}\,dx = \int_0^{\frac{\pi}{2}} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}\,dx + \int_{\frac{\pi}{2}}^{\pi} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}\,dx$	2A	
$= \int_0^{\frac{\pi}{2}} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}\,dx + \int_0^{\frac{\pi}{2}} \frac{\cos^4 x}{\sin^4 x + \cos^4 x}\,dx$	1A	
$= \int_0^{\frac{\pi}{2}} \frac{\sin^4 x + \cos^4 x}{\sin^4 x + \cos^4 x}\,dx$	2A	
$= \int_0^{\frac{\pi}{2}} dx = \frac{\pi}{2}$	1A	
	9	

Solution	Marks	Notes
12. (c) Let $f(x) = \dfrac{\sin^4 x}{\sin^4 x + \cos^4 x}$, then $f(x) = f(\pi - x)\ \forall x$.	2M	
$\displaystyle\int_0^{\pi} \frac{x \sin^4 x}{\sin^4 x + \cos^4 x}\,dx = \frac{\pi}{2}\int_0^{\pi} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}\,dx$, by (a)	2M	
$= \dfrac{\pi}{2} \cdot \dfrac{\pi}{2}$, by (b)	1A	
$= \dfrac{\pi^2}{4}$ $(= 2.467)$		
	5	