Application of Quadratic Equations

In this section we're going to go back and revisit some of the applications that we saw in the Linear Applications section and see some examples that will require us to solve a quadratic equation to get the answer.

Note that the solutions in these cases will almost always require the quadratic formula so expect to use it and don't get excited about it. Also, we are going to assume that you can do the quadratic formula work and so we won't be showing that work. We will give the results of the quadratic formula, we just won't be showing the work.

Also, as we will see, we will need to get decimal answer to these and so as a general rule here we will round all answers to 4 decimal places.

Example 1 We are going to fence in a rectangular field and we know that for some reason we want the field to have and enclosed area of 75 ft^2. We also know that we want the width of the field to be 3 feet longer than the length of the field. What are the dimensions of the field?

Solution

So, we'll let x be the length of the field and so we know that $x+3$ will be the width of the field. Now, we also know that area of a rectangle is length times width and so we know that,

$$x\left(x+3\right)=75$$

Now, this is a quadratic equation so let's first write it in standard form.

$$x^2+3x=75$$
$$x^2+3x-75=0$$

Using the quadratic formula gives,

$$x=\frac{-3\pm\sqrt{309}}{2}$$

Now, at this point, we've got to deal with the fact that there are two solutions here and we only want a single answer. So, let's convert to decimals and see what the solutions actually are.

$$x=\frac{-3+\sqrt{309}}{2}=7.2892 \qquad \text{and} \qquad x=\frac{-3-\sqrt{309}}{2}=-10.2892$$

So, we have one positive and one negative. From the stand point of needing the dimensions of a field the negative solution doesn't make any sense so we will ignore it.

Therefore, the length of the field is 7.2892 feet. The width is 3 feet longer than this and so is 10.2892 feet.

Notice that the width is almost the second solution to the quadratic equation. The only difference is the minus sign. Do NOT expect this to always happen. In this case this is more of a function of the problem. For a more complicated set up this will NOT happen.

Now, from a physical standpoint we can see that we should expect to NOT get complex solutions to these problems. Upon solving the quadratic equation we should get either two real distinct solutions or a double root. Also, as the previous example has shown, when we get two real distinct solutions we will be able to eliminate one of them for physical reasons.

Let's work another example or two.

Example 2 Two cars start out at the same point. One car starts out driving north at 25 mph. Two hours later the second car starts driving east at 20 mph. How long after the first car starts traveling does it take for the two cars to be 300 miles apart?

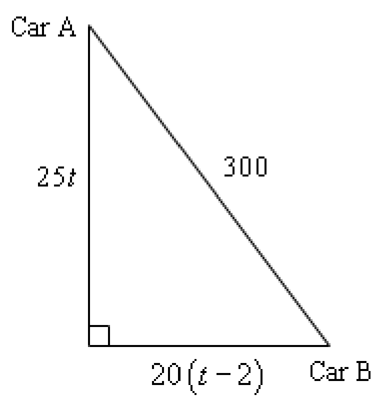
Solution

We'll start off by letting t be the amount of time that the first car, let's call it car A, travels. Since the second car, let's call that car B, starts out two hours later then we know that it will travel for $t-2$ hours.

Now, we know that the distance traveled by an object (or car since that's what we're dealing with here) is its speed times time traveled. So we have the following distances traveled for each car.

$$\text{distance of car A} : 25t$$
$$\text{distance of car B} : 20\left(t-2\right)$$

At this point a quick sketch of the situation is probably in order so we can see just what is going on. In the sketch we will assume that the two cars have traveled long enough so that they are 300 miles apart.



So, we have a right triangle here. That means that we can use the Pythagorean Theorem to say,

$$\left(25t\right)^2+\left(20\left(t-2\right)\right)^2=\left(300\right)^2$$

This is a quadratic equation, but it is going to need some fairly heavy simplification before we can solve it so let's do that.

$$\begin{aligned} 625t^2+\left(20t-40\right)^2&=90000\\ 625t^2+400t^2-1600t+1600&=90000\\ 1025t^2-1600t-88400&=0 \end{aligned}$$

Now, the coefficients here are quite large, but that is just something that will happen fairly often with these problems so don't worry about that. Using the quadratic formula (and simplifying that answer) gives,

$$t = \frac{1600 \pm \sqrt{365000000}}{2050} = \frac{1600 \pm 1000\sqrt{365}}{2050} = \frac{32 \pm 20\sqrt{365}}{41}$$

Again, we have two solutions and we're going to need to determine which one is the correct one, so let's convert them to decimals.

$$t = \frac{32 + 20\sqrt{365}}{41} = 10.09998 \qquad \text{and} \qquad t = \frac{32 - 20\sqrt{365}}{41} = -8.539011$$

As with the previous example the negative answer just doesn't make any sense. So, it looks like the car A traveled for 10.09998 hours when they were finally 300 miles apart.

Also, even though the problem didn't ask for it, the second car will have traveled for 8.09998 hours before they are 300 miles apart. Notice as well that this is NOT the second solution without the negative this time, unlike the first example.

Example 3 An office has two envelope stuffing machines. Working together they can stuff a batch of envelopes in 2 hours. Working separately it will take the second machine 1 hour longer than the first machine to stuff a batch of envelopes. How long would it take each machine do stuff a batch of envelopes by themselves?

Solution

Let t be the amount of time it takes the first machine (Machine A) to stuff a batch of envelopes by itself. That means that it will take the second machine (Machine B) $t+1$ hours to stuff a batch of envelopes by itself.

The word equation for this problem is then,

$$\left(\begin{array}{c}\text{Portion of job}\\ \text{done by Machine A}\end{array}\right) + \left(\begin{array}{c}\text{Portion of job}\\ \text{done by Machine B}\end{array}\right) = 1\text{ Job}$$

$$\left(\begin{array}{c}\text{Work Rate}\\ \text{of Machine A}\end{array}\right)\left(\begin{array}{c}\text{Time Spent}\\ \text{Working}\end{array}\right) + \left(\begin{array}{c}\text{Work Rate}\\ \text{of Machine B}\end{array}\right)\left(\begin{array}{c}\text{Time Spent}\\ \text{Working}\end{array}\right) = 1$$

We know the time spent working together (2 hours) so we need to work rates of each machine. Here are those computations.

$$1\text{ Job} = \left(\text{Work Rate of Machine A}\right) \times \left(t\right) \qquad \Rightarrow \qquad \text{Machine A} = \frac{1}{t}$$

$$1\text{ Job} = \left(\text{Work Rate of Machine B}\right) \times \left(t+1\right) \qquad \Rightarrow \qquad \text{Machine B} = \frac{1}{t+1}$$

Note that it's okay that the work rates contain t. In fact they will need to so we can solve for it! Plugging into the word equation gives,

$$\left(\frac{1}{t}\right)(2)+\left(\frac{1}{t+1}\right)(2)=1$$
$$\frac{2}{t}+\frac{2}{t+1}=1$$

So, to solve we'll first need to clear denominators and get the equation in standard form.

$$\begin{aligned}\left(\frac{2}{t}+\frac{2}{t+1}\right)(t)(t+1)&=(1)(t)(t+1)\\ 2(t+1)+2t&=t^2+t\\ 4t+2&=t^2+t\\ 0&=t^2-3t-2\end{aligned}$$

Using the quadratic formula gives,

$$t=\frac{3\pm\sqrt{17}}{2}$$

Converting to decimals gives,

$$t=\frac{3+\sqrt{17}}{2}=3.5616 \qquad \text{and} \qquad t=\frac{3-\sqrt{17}}{2}=-0.5616$$

Again, the negative doesn't make any sense and so Machine A will work for 3.5616 hours to stuff a batch of envelopes by itself. Machine B will need 4.5616 hours to stuff a batch of envelopes by itself. Again, unlike the first example, note that the time for Machine B was NOT the second solution from the quadratic without the minus sign.

Equations Reducible to Quadratic Form

In this section we are going to look at equations that are called **quadratic in form** or **reducible to quadratic in form**. What this means is that we will be looking at equations that if we look at them in the correct light we can make them look like quadratic equations. At that point we can use the techniques we developed for quadratic equations to help us with the solution of the actual equation.

It is usually best with these to show the process with an example so let's do that.

Example 1 Solve $x^4 - 7x^2 + 12 = 0$

Solution
Now, let' start off here by noticing that

$$x^4 = \left(x^2\right)^2$$

In other words, we can notice here that the variable portion of the first term (*i.e.* ignore the coefficient) is nothing more than the variable portion of the second term squared. Note as well that all we really needed to notice here is that the exponent on the first term was twice the exponent on the first term.

This, along with the fact that third term is a constant, means that this equation is reducible to quadratic in form. We will solve this by first defining,

$$u = x^2$$

Now, this means that

$$u^2 = \left(x^2\right)^2 = x^4$$

Therefore, we can write the equation in terms of u's instead of x's as follows,

$$x^4 - 7x^2 + 12 = 0 \qquad \Rightarrow \qquad u^2 - 7u + 12 = 0$$

The new equation (the one with the u's) is a quadratic equation and we can solve that. In fact this equation is factorable, so the solution is,

$$u^2 - 7u + 12 = \left(u-4\right)\left(u-3\right) = 0 \qquad \Rightarrow \qquad u = 3,\ u = 4$$

So, we get the two solutions shown above. These aren't the solutions that we're looking for. We want values of x, not values of u. That isn't really a problem once we recall that we've defined

$$u = x^2$$

To get values of x for the solution all we need to do is plug in u into this equation and solve that for x. Let's do that.

$$\begin{aligned} u = 3: &\qquad 3 = x^2 &\qquad \Rightarrow &\qquad x = \pm\sqrt{3} \\ u = 4: &\qquad 4 = x^2 &\qquad \Rightarrow &\qquad x = \pm\sqrt{4} = \pm 2 \end{aligned}$$

So, we have four solutions to the original equation, $x = \pm 2$ and $x = \pm\sqrt{3}$.

So, the basic process is to check that the equation is reducible to quadratic in form then make a quick substitution to turn it into a quadratic equation. We solve the new equation for *u*, the variable from the substitution, and then use these solutions and the substitution definition to get the solutions to the equation that we really want.

In most cases to make the check that it's reducible to quadratic in form all that we really need to do is to check that one of the exponents is twice the other. There is one exception to this that we'll see here once we get into a set of examples.

Also, once you get "good" at these you often don't really need to do the substitution either. We will do them to make sure that the work is clear. However, these problems can be done without the substitution in many cases.

Example 2 Solve each of the following equations.

(a) $x^{\frac{2}{3}} - 2x^{\frac{1}{3}} - 15 = 0$ [Solution]

(b) $y^{-6} - 9y^{-3} + 8 = 0$ [Solution]

(c) $z - 9\sqrt{z} + 14 = 0$ [Solution]

(d) $t^4 - 4 = 0$ [Solution]

Solution

(a) $x^{\frac{2}{3}} - 2x^{\frac{1}{3}} - 15 = 0$

Okay, in this case we can see that,

$$\frac{2}{3} = 2\left(\frac{1}{3}\right)$$

and so one of the exponents is twice the other so it looks like we've got an equation that is reducible to quadratic in form. The substitution will then be,

$$u = x^{\frac{1}{3}} \qquad\qquad u^2 = \left(x^{\frac{1}{3}}\right)^2 = x^{\frac{2}{3}}$$

Substituting this into the equation gives,

$$u^2 - 2u - 15 = 0$$

$$(u-5)(u+3) = 0 \qquad \Rightarrow \qquad u = -3,\ u = 5$$

Now that we've gotten the solutions for *u* we can find values of *x*.

$$u = -3: \qquad x^{\frac{1}{3}} = -3 \qquad \Rightarrow \qquad x = (-3)^3 = -27$$

$$u = 5: \qquad x^{\frac{1}{3}} = 5 \qquad \Rightarrow \qquad x = (5)^3 = 125$$

So, we have two solutions here $x = -27$ and $x = 125$.

[Return to Problems]

(b) $y^{-6} - 9y^{-3} + 8 = 0$

For this part notice that,

$$-6 = 2(-3)$$

and so we do have an equation that is reducible to quadratic form. The substitution is,

$$u = y^{-3} \qquad\qquad u^2 = \left(y^{-3}\right)^2 = y^{-6}$$

The equation becomes,

$$u^2 - 9u + 8 = 0$$
$$(u-8)(u-1) = 0 \qquad\qquad u = 1,\ u = 8$$

Now, going back to y's is going to take a little more work here, but shouldn't be too bad.

$$u = 1: \quad \Rightarrow \quad y^{-3} = \frac{1}{y^3} = 1 \quad \Rightarrow \quad y^3 = \frac{1}{1} = 1 \quad \Rightarrow \quad y = (1)^{\frac{1}{3}} = 1$$

$$u = 8: \quad \Rightarrow \quad y^{-3} = \frac{1}{y^3} = 8 \quad \Rightarrow \quad y^3 = \frac{1}{8} \quad \Rightarrow \quad y = \left(\frac{1}{8}\right)^{\frac{1}{3}} = \frac{1}{2}$$

The two solutions to this equation are $y = 1$ and $y = \frac{1}{2}$.

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(c) $z - 9\sqrt{z} + 14 = 0$

This one is a little trickier to see that it's quadratic in form, yet it is. To see this recall that the exponent on the square root is one-half, then we can notice that the exponent on the first term is twice the exponent on the second term. So, this equation is in fact reducible to quadratic in form.

Here is the substitution.

$$u = \sqrt{z} \qquad\qquad u^2 = \left(\sqrt{z}\right)^2 = z$$

The equation then becomes,

$$u^2 - 9u + 14 = 0$$
$$(u-7)(u-2) = 0 \qquad\qquad u = 2,\ u = 7$$

Now go back to z's.

$$u = 2: \quad \Rightarrow \quad \sqrt{z} = 2 \quad \Rightarrow \quad z = (2)^2 = 4$$
$$u = 7: \quad \Rightarrow \quad \sqrt{z} = 7 \quad \Rightarrow \quad z = (7)^2 = 49$$

The two solutions for this equation are $z = 4$ and $z = 49$

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(d) $t^4 - 4 = 0$

Now, this part is the exception to the rule that we've been using to identify equations that are reducible to quadratic in form. There is only one term with a t in it. However, notice that we can write the equation as,

$$\left(t^2\right)^2 - 4 = 0$$

So, if we use the substitution,

$$u = t^2 \qquad\qquad u^2 = \left(t^2\right)^2 = t^4$$

the equation becomes,

$$u^2 - 4 = 0$$

and so it is reducible to quadratic in form.

Now, we can solve this using the square root property. Doing that gives,

$$u = \pm\sqrt{4} = \pm 2$$

Now, going back to t's gives us,

$$\begin{aligned} u &= 2 & \Rightarrow \quad t^2 &= 2 & \Rightarrow \quad t &= \pm\sqrt{2} \\ u &= -2 & \Rightarrow \quad t^2 &= -2 & \Rightarrow \quad t &= \pm\sqrt{-2} = \pm\sqrt{2}\, i \end{aligned}$$

In this case we get four solutions and two of them are complex solutions. Getting complex solutions out of these are actually more common that this set of examples might suggest. The problem is that to get some of the complex solutions requires knowledge that we haven't (and won't) cover in this course. So, they don't show up all that often.

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All of the examples to this point gave quadratic equations that were factorable or in the case of the last part of the previous example was an equation that we could use the square root property on. That is need not always be the case however. It is more than possible that we would need the quadratic formula to do some of these. We should do an example of one of these just to make the point.

Example 3 Solve $2x^{10} - x^5 - 4 = 0$.

Solution

In this case we can reduce this to quadratic in form by using the substitution,

$$u = x^5 \qquad\qquad u^2 = x^{10}$$

Using this substitution the equation becomes,

$$2u^2 - u - 4 = 0$$

This doesn't factor and so we'll need to use the quadratic formula on it. From the quadratic formula the solutions are,

$$u = \frac{1 \pm \sqrt{33}}{4}$$

Now, in order to get back to x's we are going to need decimals values for these so,

$$u = \frac{1+\sqrt{33}}{4} = 1.68614 \qquad\qquad u = \frac{1-\sqrt{33}}{4} = -1.18614$$

Now, using the substitution to get back to x's gives the following,

$$u = 1.68614 \qquad x^5 = 1.68614 \qquad x = \left(1.68614\right)^{\frac{1}{5}} = 1.11014$$

$$u = -1.18614 \qquad x^5 = -1.18614 \qquad x = \left(-1.18614\right)^{\frac{1}{5}} = -1.03473$$

Of course we had to use a calculator to get the final answer for these. This is one of the reasons that you don't tend to see too many of these done in an Algebra class. The work and/or answers tend to be a little messy.

Equations with Radicals

The title of this section is maybe a little misleading. The title seems to imply that we're going to look at equations that involve any radicals. However, we are going to restrict ourselves to equations involving square roots. The techniques we are going to apply here can be used to solve equations with other radicals, however the work is usually significantly messier than when dealing with square roots. Therefore, we will work only with square roots in this section.

Before proceeding it should be mentioned as well that in some Algebra textbooks you will find this section in with the equations reducible to quadratic form material. The reason is that we will in fact end up solving a quadratic equation in most cases. However, the approach is significantly different and so we're going to separate the two topics into different section in this course.

It is usually best to see how these work with an example.

Example 1 Solve $x = \sqrt{x+6}$.

Solution
In this equation the basic problem is the square root. If that weren't there we could do the problem. The whole process that we're going to go through here is set up to eliminate the square root. However, as we will see, the steps that we're going to take can actually cause problems for us. So, let's see how this all works.

Let's notice that if we just square both sides we can make the square root go away. Let's do that and see what happens.

$$\begin{aligned} \left(x\right)^2 &= \left(\sqrt{x+6}\right)^2 \\ x^2 &= x+6 \\ x^2 - x - 6 &= 0 \\ \left(x-3\right)\left(x+2\right) &= 0 \qquad \Rightarrow \qquad x = 3,\; x = -2 \end{aligned}$$

Upon squaring both sides we see that we get a factorable quadratic equation that gives us two solutions $x = 3$ and $x = -2$.

Now, for no apparent reason, let's do something that we haven't actually done since the section on solving linear equations. Let's check our answers. Remember as well that we need to check the answers in the original equation! That is very important.

Let's first check $x = 3$

$$\begin{aligned} 3 &\overset{?}{=} \sqrt{3+6} \\ 3 &= \sqrt{9} \qquad \text{OK} \end{aligned}$$

So $x = 3$ is a solution. Now let's check $x = -2$.

$$-2 \stackrel{?}{=} \sqrt{-2+6}$$
$$-2 \neq \sqrt{4} = 2 \qquad \text{NOT OK}$$

We have a problem. Recall that square roots are ALWAYS positive and so $x = -2$ does not work in the original equation. One possibility here is that we made a mistake somewhere. We can go back and look however and we'll quickly see that we haven't made a mistake.

So, what is the deal? Remember that our first step in the solution process was to square both sides. Notice that if we plug $x = -2$ into the quadratic we solved it would in fact be a solution to that. When we squared both sides of the equation we actually changed the equation and in the process introduced a solution that is not a solution to the original equation.

With these problems it is vitally important that you check your solutions as this will often happen. When this does we only take the values that are actual solutions to the original equation.

So, the original equation had a single solution $x = 3$.

Now, as this example has shown us, we have to be very careful in solving these equations. When we solve the quadratic we will get two solutions and it is possible both of these, one of these, or none of these values to be solutions to the original equation. The only way to know is to check your solutions!

Let's work a couple more examples that are a little more difficult.

Example 2 Solve each of the following equations.

(a) $y + \sqrt{y-4} = 4$ [Solution]

(b) $1 = t + \sqrt{2t-3}$ [Solution]

(c) $\sqrt{5z+6} - 2 = z$ [Solution]

Solution

(a) $y + \sqrt{y-4} = 4$

In this case let's notice that if we just square both sides we're going to have problems.

$$\left(y + \sqrt{y-4}\right)^2 = \left(4\right)^2$$
$$y^2 + 2y\sqrt{y-4} + y - 4 = 16$$

Before discussing the problem we've got here let's make sure you can do the squaring that we did above since it will show up on occasion. All that we did here was use the formula

$$\left(a+b\right)^2 = a^2 + 2ab + b^2$$

with $a = y$ and $b = \sqrt{y-4}$. You will need to be able to do these because while this may not have worked here we will need to this kind of work in the next set of problems.

Now, just what is the problem with this? Well recall that the point behind squaring both sides in the first problem was to eliminate the square root. We haven't done that. There is still a square root in the problem and we've make the remainder of the problem messier as well.

So, what we're going to need to do here is make sure that we've got a square root all by itself on one side of the equation before squaring. Once that is done we can square both sides and the square root really will disappear.

Here is the correct way to do this problem.

$$\begin{aligned} \sqrt{y-4} &= 4-y & \text{now square both sides} \\ \left(\sqrt{y-4}\right)^2 &= (4-y)^2 \\ y-4 &= 16-8y+y^2 \\ 0 &= y^2-9y+20 \\ 0 &= (y-5)(y-4) \quad \Rightarrow \quad & y=4,\ y=5 \end{aligned}$$

As with the first example we will need to make sure and check both of these solutions. Again, make sure that you check in the original equation. Once we've square both sides we've changed the problem and so checking there won't do us any good. In fact checking there could well lead us into trouble.

First $y=4$.

$$\begin{aligned} 4+\sqrt{4-4} &\stackrel{?}{=} 4 \\ 4 &= 4 \qquad \text{OK} \end{aligned}$$

So, that is a solution. Now $y=5$.

$$\begin{aligned} 5+\sqrt{5-4} &\stackrel{?}{=} 4 \\ 5+\sqrt{1} &\stackrel{?}{=} 4 \\ 6 &\neq 4 \qquad \text{NOT OK} \end{aligned}$$

So, as with the first example we worked there is in fact a single solution to the original equation, $y=4$.

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(b) $1=t+\sqrt{2t-3}$

Okay, so we will again need to get the square root on one side by itself before squaring both sides.

$$\begin{aligned} 1-t &= \sqrt{2t-3} \\ (1-t)^2 &= \left(\sqrt{2t-3}\right)^2 \\ 1-2t+t^2 &= 2t-3 \\ t^2-4t+4 &= 0 \\ (t-2)^2 &= 0 \qquad \Rightarrow \qquad t=2 \end{aligned}$$

So, we have a double root this time. Let's check it to see if it really is a solution to the original equation.

$$1 \stackrel{?}{=} 2 + \sqrt{2(2) - 3}$$

$$1 \stackrel{?}{=} 2 + \sqrt{1}$$

$$1 \neq 3$$

So, $t = 2$ isn't a solution to the original equation. Since this was the only possible solution, this means that there are **no solutions** to the original equation. This doesn't happen too often, but it does happen so don't be surprised by it when it does.

[Return to Problems]

(c) $\sqrt{5z+6} - 2 = z$

This one will work the same as the previous two.

$$\sqrt{5z+6} = z + 2$$

$$\left(\sqrt{5z+6}\right)^2 = (z+2)^2$$

$$5z + 6 = z^2 + 4z + 4$$

$$0 = z^2 - z - 2$$

$$0 = (z-2)(z+1) \qquad \Rightarrow \qquad z = -1, \quad z = 2$$

Let's check these possible solutions start with $z = -1$.

$$\sqrt{5(-1)+6} - 2 \stackrel{?}{=} -1$$

$$\sqrt{1} - 2 \stackrel{?}{=} -1$$

$$-1 = -1 \qquad \text{OK}$$

So, that's was a solution. Now let's check $z = 2$.

$$\sqrt{5(2)+6} - 2 \stackrel{?}{=} 2$$

$$\sqrt{16} - 2 \stackrel{?}{=} 2$$

$$4 - 2 = 2 \qquad \text{OK}$$

This was also a solution.

So, in this case we've now seen an example where both possible solutions are in fact solutions to the original equation as well.

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So, as we've seen in the previous set of examples once we get our list of possible solutions anywhere from none to all of them can be solutions to the original equation. Always remember to check your answers!

Okay, let's work one more set of examples that have an added complexity to them. To this point all the equations that we've looked at have had a single square root in them. However, there can be more than one square root in these equations. The next set of examples is designed to show us how to deal with these kinds of problems.

Example 3 Solve each of the following equations.

(a) $\sqrt{2x-1}-\sqrt{x-4}=2$ [Solution]

(b) $\sqrt{t+7}+2=\sqrt{3-t}$ [Solution]

Solution

In both of these there are two square roots in the problem. We will work these in basically the same manner however. The first step is to get one of the square roots by itself on one side of the equation then square both sides. At this point the process is different so we'll see how to proceed from this point once we reach it in the first example.

(a) $\sqrt{2x-1}-\sqrt{x-4}=2$

So, the first thing to do is get one of the square roots by itself. It doesn't matter which one we get by itself. We'll end up the same solution(s) in the end.

$$\sqrt{2x-1}=2+\sqrt{x-4}$$
$$\left(\sqrt{2x-1}\right)^2=\left(2+\sqrt{x-4}\right)^2$$
$$2x-1=4+4\sqrt{x-4}+x-4$$
$$2x-1=4\sqrt{x-4}+x$$

Now, we still have a square root in the problem, but we have managed to eliminate one of them. Not only that, but what we've got left here is identical to the examples we worked in the first part of this section. Therefore, we will continue now work this problem as we did in the previous sets of examples.

$$\left(x-1\right)^2=\left(4\sqrt{x-4}\right)^2$$
$$x^2-2x+1=16\left(x-4\right)$$
$$x^2-2x+1=16x-64$$
$$x^2-18x+65=0$$
$$\left(x-13\right)\left(x-5\right)=0 \qquad \Rightarrow \qquad x=13,\ x=5$$

Now, let's check both possible solutions in the original equation. We'll start with $x=13$

$$\sqrt{2(13)-1}-\sqrt{13-4}\stackrel{?}{=}2$$
$$\sqrt{25}-\sqrt{9}\stackrel{?}{=}2$$
$$5-3=2 \qquad \text{OK}$$

So, the one is a solution. Now let's check $x=5$.

$$\sqrt{2(5)-1}-\sqrt{5-4}\stackrel{?}{=}2$$
$$\sqrt{9}-\sqrt{1}\stackrel{?}{=}2$$
$$3-1=2 \qquad \text{OK}$$

So, they are both solutions to the original equation.

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(b) $\sqrt{t+7}+2=\sqrt{3-t}$

In this case we've already got a square root on one side by itself so we can go straight to squaring both sides.

$$\left(\sqrt{t+7}+2\right)^2=\left(\sqrt{3-t}\right)^2$$
$$t+7+4\sqrt{t+7}+4=3-t$$
$$t+11+4\sqrt{t+7}=3-t$$

Next, get the remaining square root back on one side by itself and square both sides again.

$$\begin{aligned}4\sqrt{t+7}&=-8-2t\\ \left(4\sqrt{t+7}\right)^2&=\left(-8-2t\right)^2\\ 16\left(t+7\right)&=64+32t+4t^2\\ 16t+112&=64+32t+4t^2\\ 0&=4t^2+16t-48\\ 0&=4\left(t^2+4t-12\right)\\ 0&=4\left(t+6\right)\left(t-2\right) \qquad \Rightarrow \qquad t=-6,\quad t=2\end{aligned}$$

Now check both possible solutions starting with $t=2$.

$$\begin{aligned}\sqrt{2+7}+2&\stackrel{?}{=}\sqrt{3-2}\\ \sqrt{9}+2&\stackrel{?}{=}\sqrt{1}\\ 3+2&\neq 1 \qquad \text{NOT OK}\end{aligned}$$

So, that wasn't a solution. Now let's check $t=-6$.

$$\begin{aligned}\sqrt{-6+7}+2&\stackrel{?}{=}\sqrt{3-\left(-6\right)}\\ \sqrt{1}+2&\stackrel{?}{=}\sqrt{9}\\ 1+2&=3 \qquad \text{OK}\end{aligned}$$

It looks like in this case we've got a single solution, $t=-6$.

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So, when there is more than one square root in the problem we are again faced with the task of checking our possible solutions. It is possible that anywhere form none to all of the possible solutions will in fact be solutions and the only way to know for sure is to check them in the original equation.